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AN ANALYSIS OF THE FINITE ELEMENT METHOD APPLIED ON DYNAMIC MOTION AND MAXIMUM PAYLOAD PLANNING OF FLEXIBLE MANIPULATORS

Summary

This paper is concerned with the dynamic motion analysis and the planning of maximum payload path of flexible manipulators. The finite element method was employed for dynamic modelling of the system and the motion of the model was considered as a combination of the rigid displacement and the elastic deformation of each link. Each manipulator link was treated as a finite number of elements and total displacement was derived by means of the shape functions of flexible elements. The problem of maximum payload trajectory planning was formulated as an optimal control problem. An indirect optimal control solution was employed. This method converts an optimality problem to a two-point boundary value problem. The effect of the number of elements on the dynamic motion, optimal trajectory and maximum allowable dynamic payload of the system was studied. Finally, a number of simulations were performed to verify the applicability and capability of the method for the nonlinear dynamic modelling and the control of flexible manipulators.

Key words: *flexible manipulators, finite element method, nonlinear dynamic motion, optimal control, maximum payload*

1. Introduction

Flexible manipulators exhibit many potential advantages over their rigid counterparts. They have been developed in various scientific fields such as space exploration [1, 2] and surgical applications [3, 4]. Therefore, analyzing the dynamic behavior and control of flexible arms has been considered as an appealing task and it has been attracting much attention especially during the last two decades [5-7].

To enable an analysis of flexible robot manipulators, the robot model must capture the nonlinear flexible dynamics of the robot. The finite Element (FE) method has been used to solve complex structural engineering problems during the past years. One of the main advantages of the FE method over most of other approximate solution methods is that the FE method can handle irregular geometries routinely. Another significant advantage of the FE method, especially over the analytical solution techniques, is the ease with which nonlinear conditions can be handled in this approach. In the FE modelling of dynamical manipulators, elastic link deformations are analyzed by assuming a known rigid body motion and later superposing the elastic deformation with the rigid body motion. A comprehensive literature review on the dynamic analysis of flexible manipulators can be found in [8].

Several studies have focused on the trajectory optimization of flexible manipulators. The first formulation for obtaining the maximum payload of a manipulator in the point-to-point motion was presented by Wang and Ravani [9]. They used the iterative linear programming (ILP) method to solve the problem. The effects of payload on the vibration excitation of a 2R robot manipulator were studied by Parks and Pak [10]. In [11] a formulation on the dynamic analysis and maximum payload calculation of a flexible manipulator was presented based on the ILP approach. A FE procedure was employed in [12] to find the maximum payload of a flexible robotic system along the given trajectory, but this study employed only one mode shape element in the dynamic analysis of the system; also, the planning of the optimal path the system was not considered in this study.

In this paper, the non-linear dynamic modelling and the planning of the optimal path of flexible link manipulators are studied. The paper especially focuses on investigating the effects of the number of element per each link on the motion analysis and the maximum allowable payload capacity of the system. The finite element approach and the Bernoulli–Euler beam theory are employed for the dynamic modelling of elastic manipulators; each link is treated as an amount of equal length elements and the displacement of the robot is presented in a global coordinate system using element shape functions. Then, by implementing Lagrange's principle, nonlinear dynamic equations of the system are derived. After that, the trajectory optimization of the system is formulated as an optimal control problem. Accordingly, Pontraygin's minimum principle is employed to solve the problem. In this way, the optimal control problem is converted into a two-point boundary value problem which is solved numerically. Finally, a number of simulations are carried out and the results are discussed. Also, it is shown that robot motion characteristics converge by increasing the number of elements.

2. Mathematical dynamic modelling of flexible manipulators

2.1 Background

To derive the dynamic modelling of flexible manipulators using the FE method and the Bernoulli–Euler beam theory, each link of the manipulator is treated as an assembly of n_i elements of length l_{ji}; where i and ij are related to i^{th} link and j^{th} element of i^{th} link of the flexible manipulator, respectively. Let T_{ij} and V_{ij} be kinetic energy and potential energy of the element ij in the global coordinate system, respectively. The total energy of the system can be expressed as:

$$T(\vec{q},\dot{\vec{q}}) = \sum_{i=1}^{m}\sum_{j=1}^{n_i} T_{ij} \quad V(\vec{q}) = \sum_{i=1}^{m}\sum_{j=1}^{n_i} V_{ij} \tag{1}$$

where $\vec{q}$ and $\dot{\vec{q}}$ are the generalized coordinates and velocities of the system. By assuming the Lagrangian $L(\vec{q},\dot{\vec{q}}) = T(\vec{q},\dot{\vec{q}}) - V(\vec{q})$ and implementing Lagrange's principle, the dynamic equations of the system can be written in a compact form as:

$$M(\vec{q})\ddot{\vec{q}} + \vec{f}(\vec{q},\dot{\vec{q}}) = B(\vec{q})\vec{\tau} \tag{2}$$

In the above equation, M is the generalized inertia matrix, $\vec{f}$ is a vector of the nonlinear forces and $\vec{\tau}$ represents the torques applied to the joints. Details on dynamic modelling of flexible manipulators using the FE method can be found in [8, 13].

2.2 Boundary condition of flexible manipulator

In the present study, to model the flexible manipulators it is assumed that the first element of each link of the robot manipulator is placed with the same point to the associated joint. In other words, the joint of each link is constrained to have zero displacement (transitional) and angular displacement with respect to the coordinate system attached to the beginning of the link. Thus, the displacement and slope of the first node for each link due to the local coordinate system attached to the link is determined to be zero.

3. Trajectory optimization of the system

The problem of the trajectory optimization of flexible manipulators is dealt with by finding the optimum values of the system generalized coordinates and control inputs. In the proposed method, the optimal trajectory planning is suitably formulated as an optimal control problem. It is acknowledged that this approach attracts great interest of researchers, and has resulted in extensive theoretical development in a wide range of optimal path planning problems [14-16]. The set of dynamic equations of the system (Eq. 2) is rewritten in the state space form as $\dot{X} = F\big(X(t), U(t)\big)$. The purpose is to determine the optimal state vector X^* and the optimal control vector U^* where the following objective function is minimized [17]:

$$J(X,U) = \int_{t_o}^{t_f} L\big(X(t), U(t), t\big)\,dt \tag{3}$$

In the present study, it is presumed that the objective function is a function of the actuator velocities and torques of the system:

$$L\big(X(t), U(t)\big) = \frac{1}{2}\|X\|_W^2 + \frac{1}{2}\|U\|_R^2, \tag{4}$$

where $\|X\|_W^2$ is the generalized squared norm of the state vector *X* with respect to a state weighting matrix *W*, and $\|U\|_R^2$ is the generalized squared norm of the control vector *U* with respect to a control weighting matrix *R*.

By implementing the indirect solution of the optimal control problem, necessary conditions for the optimal trajectory planning for the system can be achieved. It must be mentioned that the indirect method of the optimal control problem does not require linearization of the dynamics equations and it is known as an appropriate method for the path planning of the system with a large number of degrees of freedom. To obtain the necessary conditions of the optimality, the Hamiltonian function is defined as $\mathrm{H}(X, U, \Psi, t) = L(X,U) + \psi^T(t) F(X,U)$ and the conditions of the optimal trajectory planning result in the following equations:

$$\dot{X}^*(t) = \frac{\partial \mathrm{H}}{\partial \psi}(X^*, U^*, \psi^*, t) \tag{5}$$

$$\dot{\psi}^*(t) = -\frac{\partial \mathrm{H}}{\partial X}(X^*, U^*, \psi^*, t) \tag{6}$$

$$\mathrm{H}(X^*, U^*, \psi^*, t) \le \mathrm{H}(X^*, \psi^*, \overline{U}, t) \tag{7}$$

where the symbol (*) refers to the extremals of *X(t)*, *U(t)* and *ψ(t)*. In addition to the boundary condition equations, which are initial and final configurations of the system, these equations established a two-point boundary value problem. There exist numerical techniques for solving such problems. In this study, BVP4C command in MATLAB, which is based on the collocation method, is used to solve the obtained problem. The details of this numerical technique are given in [18].

Furthermore, to determine the maximum payload of the system, some important parameters, such as accuracy of placing the final target point in a specified period of time and torque bounds imposed to the actuators, must be satisfied in the problem solving procedure. In this study, in order to consider the accuracy of the final configuration of the system, the desired accuracy (*e*) is defined as:

$$e = \left\| X(t_f) - X_f \right\| \tag{8}$$

4. Simulation results

In the study, the motion analysis of a single link flexible manipulator is simulated. The robot moves from the initial conditions at $\theta_1 = -45^\circ$ and $\dot{\theta}_1 = 0^\circ$. The physical parameters of the model used in this simulation study are: $m_1=5$ kg/m, $L_1=1$ m, $I_1=5E\text{-}9$ kg.m^2, $E=20$ Gpa and the execution time is $t_f=2s$. In the simulations the effects of the number of link elements on the dynamic motion of the system are highlighted. Table 1 shows a comparison between the reference values resulting from the rigid manipulator and the data resulted from the simulation of the flexible manipulator with 1, 2, 3 and 4 elements.

As it can be seen from the results, the response of the flexible system is in good agreement with the rigid manipulator model, which validates the method studied in this paper. Also, it is evident that the higher the number of elements, the farther the response of the resulting model from the rigid manipulator model.

Table 1 Data resulting from comparison between rigid and flexible single link manipulator

Parameter	Max data of rigid manipulator	Difference between rigid and flexible manipulator			
		1 element	2 elements	3 elements	4 elements
Horizontal position of end effector	0.7071	0.0024	0.0027	0.0029	0.0029
Vertical position of end effector	1.0000	0.0014	0.0016	0.0017	0.0018
Angular displacement of joint	2.3552	0.0071	0.0069	0.0068	0.0068
Angular velocity of joint	2.9344	0.3280	0.3495	0.3539	0.3572
Maximum mode shape amplitude	0	0.0085	0.0087	0.0088	0.0088
Maximum mode shape slope	0	0.0152	0.0150	0.0149	0.0148
Maximum derivative of mode shape amplitude	0	0.3910	0.4062	0.4129	0.4234
Maximum derivative of mode shape slope	0	0.6718	0.7018	0.7223	0.7423

Table 1 show that the dynamics of the flexible manipulators are influenced by the number of elements selected for the study, so that by increasing the number of elements the accuracy and compatibility of the results increase. Also, the results demonstrate that increasing the number of elements generally leads to convergence of the resulting data.

However, the robot characteristics with more fluctuating feature such as the velocity of joints or derivate of the mode shapes do not converge completely. In fact, these features require additional increases in the number of elements to converge. So, it is concluded that the satisfactory modelling and analysis of the manipulator dynamics can lead to the use of advanced control techniques to solve complex problems associated with the flexure of elastic manipulator arms.

An additional simulation study is performed to investigate the trajectory optimization of a single link flexible manipulator with one and two elements. It is assumed that the robot moves from the initial conditions at $\theta_1 = 0^\circ$, $\dot{\theta}_1 = 0$ rad/s to the final conditions at $\theta_1 = 90^\circ$, $\dot{\theta}_1 = 0$ rad/s in its point-to-point motion with the execution time of $t_f = 1.5$ second. A permanent magnet DC motor, which is commonly used for medium and small size manipulators, is employed as a robot actuator. The limits of control for such a motor are obtained as:

$$\begin{aligned} U_{max} &= k_1 - k_2\,\dot{\theta}_1 \\ U_{min} &= -k_1 - k_2\,\dot{\theta}_1 \end{aligned} \tag{9}$$

In the simulation carried out in this study, the actuator constants are chosen as k_1=40, k_2=1.12. Also, the weighting matrix of the speed of the system is assumed to be W=diag(1), and the weighting matrix of the torque of the system is R=diag(1). Other parameters of the robot are the same as in the previous simulation. The results are shown as follows:

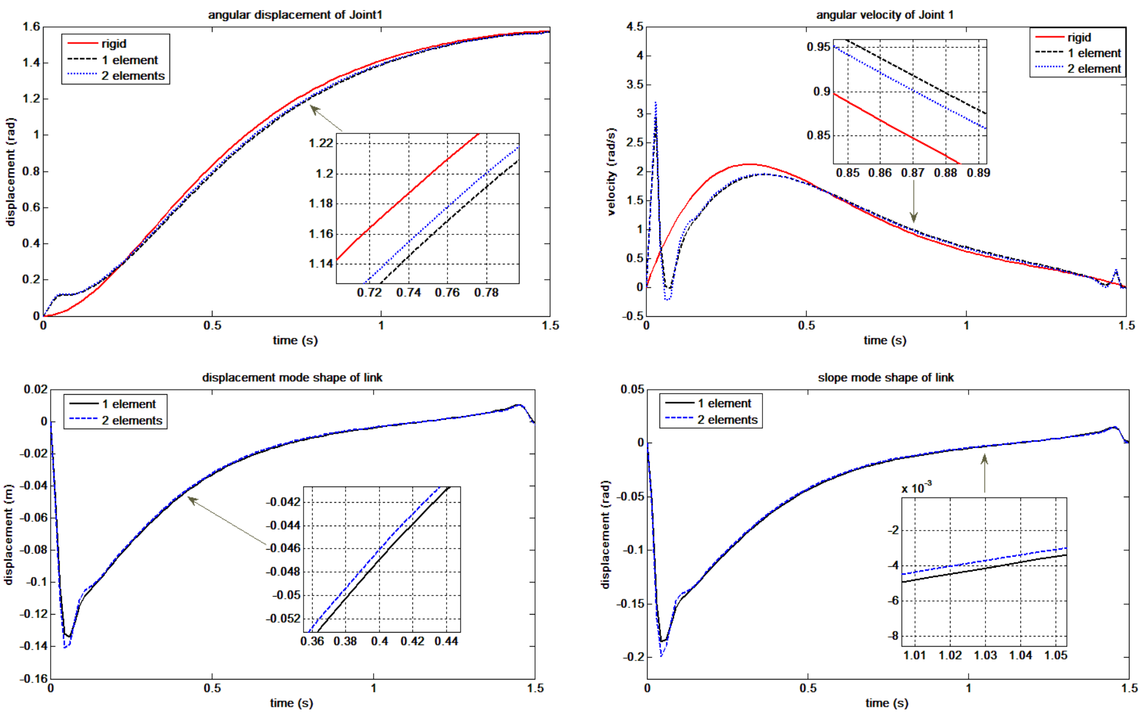

Fig. 1 Results for the optimal path of the flexible manipulator

As it can be seen from Fig. 1, with the use of optimal control, which is compatible with nonlinear systems, the fluctuating motion around the rigid mode disappeares and the nonlinear behavior of the dynamic response of the system is reduced. Also, it is evident from the figure that the use of the optimal control can reduce the effects of increasing the number of elements. Therefore, it can be concluded that with employing a suitable control, designers can decrease the calculation procedure while considering the accurate dynamics of the system.

In the final simulation the effect of the number of mode shapes on the calculating of the maximum payload of the flexible manipulators is investigated. Moreover, corresponding optimal trajectories in the point-to-point motion are illustrated for different values of load. Note that in the previous simulation, the payload mass of the system was assumed to be m_p=0 kg. This simulation investigated the maximum allowable payload for the flexible link manipulator with one and two elements and compared the results with the rigid manipulator. The torque curve graphs for the case with no end effector load and for the simulation with a range of maximum payloads given in Table 2 are shown in Figs. 2 and 3.

Table 2 Number of elements and the corresponding maximum payloads

Case	**1**	**2**	**3**
Number of elements	Rigid	1	2
M_{pMax} /kg	1.68	1.46	1.39

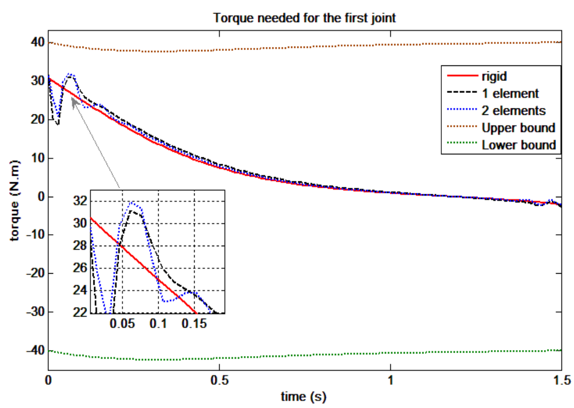

Fig. 2 Optimal torque of the flexible manipulator for mp=0 kg

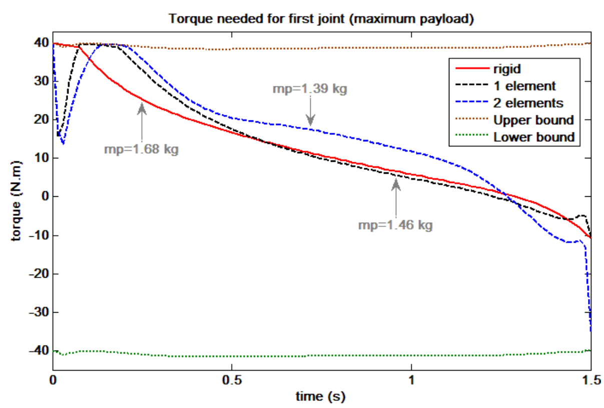

Fig. 3 The results for optimal torque of the flexible manipulator for maximum payload

The comparison between Figs. 2 and 3 shows that increasing the payload leads to an increase in the required torque until the maximum payload is obtained. So that in this case the torque curve lies in its limits. Thus, it is the most possible value of the torque and increasing the payload is impossible, because it leads to the violation of the boundary conditions. Also, it is illustrated in Fig. 3 that the rigid manipulator can carry more payload in comparison with the flexible manipulator. In addition, the figure shows that increasing the number of elements causes the value of the maximum payload to be reduced.

5. Conclusion

In this paper, the dynamic modelling and control of a flexible manipulator has been studied. Each flexible arm of a robot has been treated as a finite number of elements and the displacement of elements has been presumed to be a combination of rigid motion and flexible displacement. Employing the FE method, the kinetic and potential energies of the manipulator have been developed and the nonlinear dynamic equations of the elastic robot have been derived. Reflecting the variation in the effective inertia of the system in addition to the interaction between the rigid dynamics and the vibration models of the links, the final model is a highly nonlinear and complex system. Initially, a check of the validity of the model has been performed and the simulation results have been presented considering variant elements for a single link elastic robot. The results show that the FE method is flexible in modelling elastic manipulators providing different levels of model accuracy by selecting an adequate number of elements into which each link was divided. Then, the application of the optimal control strategy on the trajectory optimization of the system has been investigated. Accordingly, the problem of the dynamic motion planning of the system has been formulated by using the open loop optimal control theory. Then, an indirect approach based on Pontryagin's minimum principle has been employed to solve the problem. Using the proposed method, the complete nonlinear states and control constraints were treated without any simplifications such as linearization of the dynamics equations, discretization of the robot's workspace or parameterization of the solution. It was observed that the proposed control strategy can compensate for the effect of nonlinearities, and it meets the need of increasing the number of elements. After that, the effect of increasing the number of elements per link in the dynamic maximum payload capacity of the flexible manipulators has been studied and the results have been compared with the rigid manipulators. The comparison showed that increasing the number of elements in analyzing the flexible manipulators gradually causes the maximum allowable dynamic payload of the system to be reduced.

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