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IDENTITIES WITH DERIVATIONS ON RINGS AND BANACH ALGEBRAS

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Abstract. In this paper we prove the following result. Let $m \geq 1, n \geq 1$ be integers and let R be a $2mn(m+n-1)!-$torsion free semiprime ring. Suppose there exist derivations $D, G : R \to R$ such that $D(x^m)x^n + x^n G(x^m) = 0$ holds for all $x \in R$. In this case both derivations D and G map R into its center and $D = -G$. We apply this purely algebraic result to obtain a range inclusion result of continuous derivations on Banach algebras.

1. Introduction

This research has been motivated by the work of Brešar [11], Lee [20] and Thaheem [29]. Throughout, R will represent an associative ring with center $Z(R)$. A ring R is $n-$torsion free, where $n > 1$ is an integer, in case $nx = 0$, $x \in R$ implies $x = 0$. As usual, the commutator $xy - yx$ will be denoted by $[x, y]$. We shall use the commutator identities $[xy, z] = [x, z]\, y + x\, [y, z]$ and $[x, yz] = [x, y]\, z + y\, [x, z]$ for all $x, y, z \in R$. Recall that a ring R is prime if for $a, b \in R$, $aRb = (0)$ implies that either $a = 0$ or $b = 0$, and is semiprime in case $aRa = (0)$ implies that $a = 0$. An additive mapping D is called a derivation if $D(xy) = D(x)y + xD(y)$ holds for all pairs $x, y \in R$. A mapping f of a ring R into itself is called centralizing on R if $[f(x), x] \in Z(R)$ holds for all $x \in R$; in the special case when $[f(x), x] = 0$ holds for all $x \in R$, the mapping f is said to be commuting on R. The history of commuting and centralizing mappings goes back to 1955 when Divinsky [15]

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proved that a simple Artinian ring is commutative if it has a commuting automorphism different from the identity mapping. Two years later Posner [25] has proved that the existence of a nonzero centralizing derivation on a prime ring forces the ring to be commutative (Posner's second theorem). Luh [21] generalized the Divinsky result, which has been mentioned, to arbitrary prime ring. Mayne [24] has proved that in case there exists a centralizing automorphism different from the identity mapping on a prime ring, then the ring is commutative. A lot of work has been done during the last thirty years in this field (see [2, 4, 6, 8, 9, 32, 33, 35] where further references can be found). A result of Brešar [8], which states that every additive commuting mapping f a prime ring R is of the form $f(x) = \lambda x + \zeta(x)$ where λ is an element of C, the extended centroid of R, and $\zeta : R \to C$ is an additive mapping, should be mentioned. For the explanation of the extended centroid of a semiprime ring and the symmetric Martindale ring of quotients, which will be denoted by Q_s, we refer the reader to [1]. A mapping $f : R \to R$ is called skew-centralizing on R if $f(x)x + xf(x) \in Z(R)$ holds for all $x \in R$; in particular, if $f(x)x+xf(x) = 0$ is fulfilled for all $x \in R$, then it is called skew-commuting on R. Brešar [7] has proved that if R is a 2-torsion free semiprime ring, and $f : R \to R$ is an additive skew-commuting mapping on R, then $f = 0$. Thaheem [29] has proved that in case D, G is a pair of derivations on a semiprime ring R satisfying the equation $D(x)x + xG(x) = 0$ for all $x \in R$, then D and G map R into $Z(R)$ and $G = -D$. Let us point out that the equation of the type $f(x)x + xg(x) = 0$ for a pair of operators f and g on von Neumann algebras and C^*-algebras appears in operator theory; in particular, in the study of elementary operators and other operator equations (see [30] and references therein for a detailed account of elementary operators and other operator equations). Banach algebras in this paper will be over the complex field. We denote by $\mathrm{rad}(A)$ the radical of a Banach algebra A and by $Q(A)$ the set of all quasinilpotent elements in A. The paper consists of two sections. Purely algebraic results from the first section are applied in the second section of the paper to obtain some results concerning derivations in Banach algebras.

2. Derivations on rings

Let us start with the result below, which has been recently proved by Thaheem [29].

Theorem A ([29] *Theorem 2.2*). *Let R be a semiprime ring and let $D, G : R \to R$ be derivations. Suppose that $D(x)x + xG(x) = 0$ holds for all $x \in R$. In this case D and G map R into $Z(R)$. Besides $G = -D$.*

Let R be a $2-$torsion free semiprime ring. Suppose there exist additive mappings $f, g : R \to R$, such that

$$f(x)x + xg(x) = 0, \qquad x \in R \tag{1}$$

holds for all $x \in R$. Thaheem [29] raised a question for a solution of the equation above.

In case we have a prime ring, the answer to Thaheem's question gives the following result proved by Brešar [11].

Theorem B ([11] *Corollary 4.9.*). *Let R be a prime ring and let $f, g : R \to R$ be additive mappings satisfying the relation (1) for all $x \in R$. In this case there exist $a \in Q_s$ and an additive mapping $\varsigma : R \to C$ such that*

$$f(x) = xa + \varsigma(x), g(x) = -ax - \varsigma(x)$$

is fulfilled for all $x \in R$.

Let us point out that the identity (1) generalizes both concepts, the concept of commuting and the concept of skew-commuting mappings. We shall use Theorem B in the sequel. It is our aim in this paper to prove the result below which, obviously, generalizes Theorem A.

Theorem 2.1. *Let $m \geq 1, n \geq 1$ be integers and let R be a $2mn(m+n-1)!-$torsion free semiprime ring. Suppose there exist derivations $D, G : R \to R$, such that*

$$D(x^m)x^n + x^n G(x^m) = 0$$

is fulfilled for all $x \in R$. In this case D and G map R into $Z(R)$ and $D = -G$.

In case $m = 1, G = -D$ the above theorem reduces to a result which can be compared with Theorem 2 in [14] (see also [20]).

In the proof of Theorem 2.1 we shall use the fact that any semiprime ring R and its maximal right ring of quotients Q satisfy the same differential identities which is very useful since Q contains the identity element (see Theorem 3 in [19]). For the explanation of differential identities we refer to [13].

Proof of Theorem 2.1. Using full linearization of the relation

$$D(x^m)x^n + x^n G(x^m) = 0, \qquad x \in R \tag{2}$$

one obtains

$$\begin{aligned} &\sum_{\pi \in S_{m+n}} D(x_{\pi(1)} \ldots x_{\pi(m)}) x_{\pi(m+1)} \ldots x_{\pi(m+n)} \\ &\qquad + x_{\pi(1)} \ldots x_{\pi(n)} G(x_{\pi(n+1)} \ldots x_{\pi(n+m)}) = 0 \end{aligned} \tag{3}$$

for all $x_1, x_2, \ldots, x_{m+n} \in R$. According to Theorem 3 in [19] the above relation holds for all $x_1, x_2, \ldots, x_{m+n} \in Q$ as well. Substituting $x_1 = x, x_2 = \ldots = x_{m+n} = 1$, where 1 denotes the identity element, and applying the fact that $D(1) = G(1) = 0$ we obtain $\alpha(D(x) + G(x)) = 0$, for all $x \in Q$, where α

stands for $m(m+n-1)!$. We have therefore $D=-G$ which proves a part of the proof and makes it possible to rewrite the relation (3) in the form

$$\sum_{\pi \in S_{m+n}} D(x_{\pi(1)}...x_{\pi(m)})x_{\pi(m+1)}...x_{\pi(m+n)} \\ -x_{\pi(1)}...x_{\pi(n)}D(x_{\pi(n+1)}...x_{\pi(n+m)})=0,$$

for all $x_1, x_2, ..., x_{m+n} \in Q$. Substituting $x_1 = x_2 = x$, $x_3 = ... = x_{m+n} = 1$ we obtain $\beta\,[D(x), x] = 0$, for all $x \in Q$, where β denotes $2mn(m+n-2)!$. We have therefore

$$[D(x), x] = 0, \tag{4}$$

for all $x \in R$. In other words R is commuting on R. It is well known that any commuting derivation of a semiprime ring maps the ring into its center. Besides, one can apply Theorem A, but we will proceed the proof for the sake of completeness. The linearization of the relation (4) gives

$$[D(x), y] + [D(y), x] = 0, \qquad x, y \in R. \tag{5}$$

Putting in the above relation xy for y and applying (4) and (5), we obtain

$$\begin{aligned} 0 &= [D(x), xy] + [D(xy), x] = [D(x), xy] + [D(x)y + xD(y), x] \\ &= [D(x), x]\,y + x\,[D(x), y] + [D(x), x]\,y + D(x)\,[y, x] + x\,[D(y), x] \\ &= D(x)\,[y, x]\,, \quad x, y \in R. \end{aligned}$$

We have therefore

$$D(x)\,[y, x] = 0, \qquad x, y \in R.$$

The linearization of the above relation gives $D(x)\,[y, z] + D(z)\,[y, x] = 0$ and in particular for $y = D(x)$

$$D(x)\,[D(x), y] = 0, \qquad x, y \in R.$$

The substitution zy for y in the above relation gives

$$D(x)z\,[D(x), y] = 0, \qquad x, y, z \in R. \tag{6}$$

Putting in the above relation first yz for z, then multiplying the relation (6) from the left side by y and then subtracting the relations so obtained one from another we arrive at $[D(x), y]\,z\,[D(x), y] = 0, x, y, z \in R$ whence it follows $[D(x), y] = 0, x, y \in R$. We have therefore proved that D maps R into $Z(R)$. Since $G = -D$ the same holds for G. The proof of the theorem is complete. ∎

Corollary 2.2. *Let $m \geq 1, n \geq 1$ be integers and let R be a noncommutative $2mn(m+n-1)!-$torsion free prime ring. Suppose there exist derivations $D, G : R \to R$, such that*

$$D(x^m)x^n + x^n G(x^m) = 0$$

is fulfilled for all $x \in R$. In this case $D = G = 0$.

PROOF. An immediate consequence of Theorem 2.1 and Posner's second theorem. ∎

A classical result of Posner [25] (Posner's first theorem), states that in case we have nonzero derivations $D, G : R \to R$, where R is a $2-$torsion free prime ring, then the mapping $x \mapsto D(G(x))$ cannot be a derivation. This result in general cannot be proved for semiprime rings (see [3] for the details). However, in case we have a semiprime ring, one can prove the following result.

THEOREM C ([16] *Lemma 1.1.9.*). *Let R be a $2-$torsion free semiprime ring and let $D, G : R \to R$ be derivations. Suppose that the relation $D^2(x) = G(x)$ holds for all $x \in R$. In this case $D = G = 0$.*

Theorem C was the motivation for the following result proved by Vukman [35]. Let R be a $2-$torsion free semiprime ring and let D,$G : R \to R$ be derivations, such that the mapping $x \mapsto D^2(x) + G(x)$ is centralizing on R. In this case both derivations $D$and G are commuting on R.

Our next result is in the spirit of Vukman's result we have just mentioned.

THEOREM 2.3. *Let R be a 2-torsion prime ring and $D_i, G_i : R \to R$, $i = 1, 2$ be derivations. Suppose that*

$$(D_1^2(x) + G_1(x))x + x(D_2^2(x) + G_2(x)) = 0 \tag{7}$$

is fulfilled for all $x \in R$. In this case $D_1 = D_2 = 0$, $G_1 = -G_2$. Derivations G_1 and G_2 map R into $Z(R)$.

For the proof of Theorem 2.3 we shall need Theorem B, Theorem C and the following lemma.

LEMMA 2.4 ([36, Lemma 1]). *Let R be a semiprime ring. Suppose the relation $axb + cxa = 0$ is fulfilled for all $x \in R$ and some $a, b, c \in R$. In this case $ax(b + c) = 0$ holds for all $x \in R$.*

PROOF OF THEOREM 2.3. Denoting $D_1^2(x) + G_1(x)$ and $D_2^2(x) + G_2(x)$ by $F_1(x)$ and $F_2(x)$, respectively the assumption of the theorem can be written in the form

$$F_1(x)x + xF_2(x) = 0, \qquad x \in R. \tag{8}$$

An easy calculation shows that

$$F_1(xy) = F_1(x)y + xF_1(y) + 2D_1(x)D_1(y), \tag{9}$$

holds for all pairs $x, y \in R$. Of course, we have also

$$F_2(xy) = F_2(x)y + xF_2(y) + 2D_2(x)D_2(y), \qquad x, y \in R. \tag{10}$$

The linearization of (8) gives

$$F_1(x)y + F_1(y)x + xF_2(y) + yF_2(x) = 0, \qquad x, y \in R. \tag{11}$$

By substituting yx for y in (11) and applying (9) and (10) we obtain

$$F_1(x)yx + F_1(y)x^2 + yF_1(x)x + 2D_1(y)D_1(x)x + xF_2(y)x \\ +xyF_2(x) + yxF_2(x) + 2xD_2(y)D_2(x) = 0, \quad x, y \in R.$$

The above relation reduces because of (8) and (11) to

$$xyF_2(x) - yF_2(x)x + 2D_1(y)D_1(x)x + 2xD_2(y)D_2(x) = 0, \quad x, y \in R. \tag{12}$$

Putting xy for y in the above relation we obtain

$$x^2yF_2(x) - xyF_2(x)x + 2D_1(x)yD_1(x)x + 2xD_1(y)D_1(x)x \\ +2xD_2(x)yD_2(x) + 2x^2D_2(y)D_2(x) = 0, \quad y \in R. \tag{13}$$

Left multiplication of the relation (12) by x gives

$$x^2yF_2(x) - xyF_2(x)x + 2xD_1(y)D_1(x)x + 2x^2D_2(y)D_2(x) = 0, \tag{14}$$

for all $x, y \in R$.

Subtracting the relation (14) from the relation (13) we arrive at

$$D_1(x)yD_1(x)x + xD_2(x)yD_2(x) = 0, \qquad x, y \in R. \tag{15}$$

From the relation (7) and Theorem B it follows that there exists $a \in Q_s$ and an additive mapping $\varsigma : R \to C$, such that

$$D_1^2(x) + G_1(x) = xa + \varsigma(x), \tag{16}$$

and

$$D_2^2(x) + G_2(x) = -ax - \varsigma(x) \tag{17}$$

holds for all $x \in R$. Combining the relation (16) with the relation (17) we obtain

$$D^2(x) + F(x) = 0, \qquad x \in R. \tag{18}$$

where $D(x)$ and $F(x)$ stand for $D_1(x) + D_2(x)$ and $G_1(x) + G_2(x) + [a, x]$, respectively. The mapping D is a derivation being the sum of two derivations. Note that the mapping $x \mapsto [a, x]$ is a derivation, which means that F is a derivation as well being the sum of three derivations. From the relation (18) we obtain, applying Theorem C, that $D = 0$. In other words we have $D_2 = -D_1$, which makes it possible to rewrite the relation (15) in the form

$$D_1(x)yD_1(x)x + xD_1(x)yD_1(x) = 0, \qquad x, y \in R.$$

According to Lemma 2.4 the above relation gives

$$D_1(x)y(D_1(x)x + xD_1(x)) = 0.$$

From the above relation it follows that either $D_1(x) = 0$ or $D_1(x)x+xD_1(x) = 0$. In any case

$$D_1(x)x + xD_1(x) = 0$$

holds for all $x \in R$. From the relation above it follows according to Theorem A or Theorem 2.1 that $D_1 = 0$, which means that $D_2 = 0$ as well. Now the relation (7) reduces to $G_1(x)x + xG_2(x) = 0$, whence it follows using again Theorem A or Theorem 2.1 that $G_1 = -G_2$ and that both derivations G_1 and G_2 map R into $Z(R)$. The proof of the theorem is complete. ∎

3. Derivations on Banach algebras

We start this section with the following theorem.

Theorem 3.1. *Let A be a Banach algebra and let $D, G : A \to A$ be continuous linear derivations. Suppose that*

$$D(x^m)x^n + x^n G(x^m) \in \operatorname{rad}(A),$$

holds for all $x \in A$ and some integers $m \geq 1, n \geq 1$. In this case both D and G map A into $\operatorname{rad}(A)$.

Let us explain in somewhat more details the background of the theorem above. In 1955 Singer and Wermer [28] proved that a continuous linear derivation on a commutative Banach algebra maps the algebra into its radical. Johnson and Sinclair [17] have proved that any linear derivation on a semisimple Banach algebra is continuous. According to these two results, one can conclude that there are no nonzero linear derivations on a commutative semisimple Banach algebras. Singer and Wermer conjectured in [28] that the continuity assumption in their result is superfluous. It took more than thirty years until this conjecture was finally proved by Thomas [31]. Obviously, from Thomas's result it follows directly that there are no nonzero linear derivations on a commutative semisimple Banach algebra. By our knowledge the first noncommutative extension of Singer-Wermer theorem has been proved by Yood [37] by showing that if for all pairs $x, y \in A$, where A is a noncommutative Banach algebra, the element $[D(x), y]$ lies in $\operatorname{rad}(A)$, then D maps A into $\operatorname{rad}(A)$. Brešar and Vukman [5] have generalized Yood's result by proving that in case $[D(x), x] \in \operatorname{rad}(A)$ for all $x \in A$, then D maps A into $\operatorname{rad}(A)$. The work of Mathieu and Murphy [22] and Runde [26] should be mentioned. Recently, Kim [18] has proved that in case $[D(x), x]\, D(x)\, [D(x), x] \in \operatorname{rad}(A)$ for any $x \in A$, then a continuous derivation D maps A into $\operatorname{rad}(A)$. Kim's result generalizes a result proved by Vukman [34]. For references concerning range inclusion results of continuous derivations on noncommutative Banach algebras we refer the reader to [10, 12] and [23].

Proof of Theorem 3.1. We have therefore

$$D(x^m)x^n + x^n G(x^m) \in \operatorname{rad}(A),$$

for any $x \in A$. By Lemma 3.2 in Sinclair's paper [27], every continuous linear derivation of a Banach algebra A leaves the primitive ideals invariant, which means that one can introduce for any primitive ideal $P \subset A$ derivations $D_P : A/P \to A/P$, $G_P : A/P \to A/P$, where A/P is the factor algebra, by $D_P(x^*) = D(x)$, $G_P(x^*) = G(x)$, $x^* = x + P$. Let us first assume that A/P

is noncommutative. In this case one can conclude from the assumptions of the theorem that $D_P(x^{*m})x^{*n} + x^{*n}G_P(x^{*m}) = 0$ holds for all $x^* \in A/P$, which gives $D_P = G_P = 0$ by Corollary 2.2 since A/P is prime. In case A/P is commutative we have $D_P = G_P = 0$ as well, since A/P is semisimple and since we know that there is no nonzero linear derivations on a commutative semisimple Banach algebra. Thus for any $x \in A$ we have $D(x) \in P$ and $G(x) \in P$, where P is any primitive ideal of A. Since $D(x)$ and $G(x)$, where x is any element from A, are in the intersection of all primitive ideals of A and since the intersection of all primitive ideals of A is the radical, one can conclude that $D(A) \subset \mathrm{rad}(A)$ and $G(A) \subset \mathrm{rad}(A)$ which was our intension to prove. The proof of the theorem is complete. $\square$

Most results in the field of range inclusion theory deal with one derivation, while in the theorem above we have a pair of derivation. The first result in this field with two derivations is, by our knowledge, the following result proved by Brešar and Vukman [5]. Let D and G be such continuous linear derivations on a noncommutative Banach algebra A, that $\left[D^2(x) + G(x), x\right] \in \mathrm{rad}(A)$ holds for all $x \in A$. In this case both derivations D and G map A into $\mathrm{rad}(A)$. In our next theorem, which generalize the result we have just mentioned, we have four derivations.

Theorem 3.2. *Let A be a Banach algebra and let $D_i, G_i : A \to A$, $i = 1, 2$ be continuous linear derivations. Suppose that*

$$(D_1^2(x) + G_1(x))x + x(D_2^2(x) + G_2(x)) \in \mathrm{rad}(A)$$

holds for all $x \in A$. In this case all derivations D_i, $G_i, i = 1, 2$ map A into $\mathrm{rad}(A)$.

Proof. One can apply Theorem 2.3 and the same arguments as in the proof of Theorem 3.1.

Brešar and Vukman [5] have proved that in case $[D(x), x]^2 \in \mathrm{rad}(A)$ for all $x \in A$, where D is a continuous linear derivation of a Banach algebra A, we have $D(A) \subset \mathrm{rad}(A)$. Brešar [10] fairly generalized this result by proving that in case $[D(x), x] \in Q(A)$ for all $x \in A$, then $D(A) \subset \mathrm{rad}(A)$. $\square$

This result leads to the following conjecture.

Conjecture. Let A be a Banach algebra and let $D, G : A \to A$ be continuous linear derivations. Suppose that $D(x^m)x^n + x^nG(x^m) \in Q(A)$ for all $x \in A$ and some integers $m \geq 1, n \geq 1$. In this case D and G map A into $\mathrm{rad}(A)$.

The question arises whether the results above can be proved without the continuity assumption. This question leads to the problem whether Sinclair's result [27], which states that continuous linear derivation on a Banach algebra leaves any primitive ideal of the algebra invariant, can be proved without the continuity assumption. By our knowledge this problem is still open. However, in a special case, when a Banach algebra is semisimple, one can prove some results without the continuity assumptions.

THEOREM 3.3. *Let A be a semisimple Banach algebra and let $D, G : A \to A$ be linear derivations. Suppose that*

$$D(x^m)x^n + x^nG(x^m) = 0$$

holds for all $x \in A$ and some integers $m \geq 1, n \geq 1$. In this case we have $D = G = 0$.

PROOF. The proof goes through in the same way as the proof of Theorem 3.1 with the only exception that at the beginning of the proof one has to use the fact that any linear derivation on a semisimple Banach algebra is continuous (see [17]). □

THEOREM 3.4. *Let A be a semisimple Banach algebra and let $D_i, G_i : A \to A$ $i = 1, 2$ be linear derivations. Suppose that*

$$(D_1^2(x) + G_1(x))x + x(D_2^2(x) + G_2(x)) = 0$$

holds for all $x \in A$. In this case we have $D_i = G_i = 0$, $i = 1, 2$.

PROOF. See the proof of Theorem 3.3. □

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