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Robust bearings-only tracking algorithm using structured total least squares-based Kalman filter

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Original scientific paper

A nonlinear approach called the robust structured total least squares kalman filter (RSTLS-KF) algorithm is proposed for solving tracking inaccuracy caused by outliers in bearings-only multi-station passive tracking. In that regard, the robust extremal function is introduced to the weighted structured total least squares (WSTLS) location criterion, and then the improved Danish equivalent weight function is built on the basis, which can identify outliers automatically and reduce the weight of the polluted data. Finally, the observation equation is linearized according to the RSTLS location result with the structured total least norm (STLN) solution. Hence location and velocity of the target can be given by the Kalman filter. Simulation results show that tracking performance of the RSTLS-KF is comparable or better than that of conventional algorithms. Furthermore, when outliers appear, the RSTLS-KF is accurate and robust, whereas the conventional algorithms become distort seriously.

Key words: Passive tracking, Robust estimation, Nonlinear system, Equivalent weight function

Robusni algoritam praćenja mjerenjem smjera pomoću strukturiranog potpunog Kalmanovog filtra zasnovanog na metodi najmanjih kvadrata. U ovome radu predložen je nelinearni pristup za rješavanje netočnosti uzrokovanih netipčnim vrijednostima kod praćenja mjerenjem smjera pasivnim senzorima s više stanica. Pristup je zasnovan na robusnom strukturiranom potpunom Kalmanovom filtru zasnovanom na metodi najmanjih kvadrata. Pomoću predložene metode moguće je estimirati položaj i brzinu praćenog objekta. Simulacijski rezultati pokazuju da je učinkovitost predloženog algoritma jednaka ili bolja od konvencionalnih algoritama. Nadalje, u prisustvu netipčnih vrijednosti mjerenja, predloženi algoritam zadržava točnost i robusnost, dok konvencionalni algoritmi pokazuju pogreške u estimaciji.

Ključne riječi: praćenje pasivnim senzorima, robusna estimacija, nelinearni sustav, ekvivalentna težinska funkcij

1 INTRODUCTION

Bearings-only tracking plays an important role in control system, navigation field, military applications, etc. [1-3]. It is a typical nonlinear filtering problem to estimate the location and velocity of a target only using angle measurement from observation station [4-6]. It can be divided into single-station tracking and multi-station tracking. Multi-station tracking is widely used because more information can be obtained to improve tracking accuracy.

Various algorithms have been proposed for the multi-station bearings-only tracking problem. The particle filter (PF) [7] can provide sufficient precision but it suffers from enormous computational demands which limit its applications in practice. The extended Kalman filter (EKF) handles the problem through linearizing measurement model [8, 17]. Unfortunately, it often leads to poor accuracy and tracking divergence. The unscented Kalman filter (UKF) selects a set of sigma points to approximate the posterior probability density and performs superior accuracy [9].

Actually, the nonlinear measurement model of bearings-only tracking can be transformed into a pseudo-linear equation without linearization errors where the coefficient matrix $\boldsymbol{H}$ and the observation vector $\boldsymbol{Z}$ are both polluted by noise [10]. To make use of the correlation of errors between $\boldsymbol{H}$ and $\boldsymbol{Z}$, the constrained total least squares (CTLS) algorithm was introduced [11-12]. [10] discussed the structured total least squares (STLS) algorithm with the inverse iteration method which reduced the computational cost. The equivalence of the STLS and CTLS was also proved [13]. According to the STLS solution, the location and velocity of the target can be obtained by Kalman filter (KF), which is called (STLS-KF) algorithm. However, there are also some problems in the STLS-KF algorithm. On the one hand, the measurement data is easily polluted by outliers in practice, especially in military application or under other bad conditions. [14] shows that the probability of outliers in the samples is about 1%∼10%. If we do nothing about outliers, the tracking result will be distorted seriously. On the other hand, parameter settings are difficult and complex in inverse iteration method and the computational cost still needs to reduce.

The contribution of this paper is to derive a new robust STLS-KF (RSTLS-KF) tracking algorithm with structured total least norm (STLN) solution. Firstly, the multi-station bearings-only location problem is transform into the weight STLS (WSTLS) problem, and then the robust location criterion is introduced to build equivalent weight function which can reduce weights of outliers automatically. According to the RSTLS solution with STLN method [15], we linearize the measurement equation and obtain accurate and robust tracking result by means of the Kalman filter.

The paper is organized as follows. In section 2, we build the tracking model. In section 3, the robust STLS criterion is introduced. Solution to the new filtering algorithm called the RSTLS-KF is discussed in section 4. section 5 yields the performance comparison of the algorithms. Conclusions of the paper are drawn in section 6.

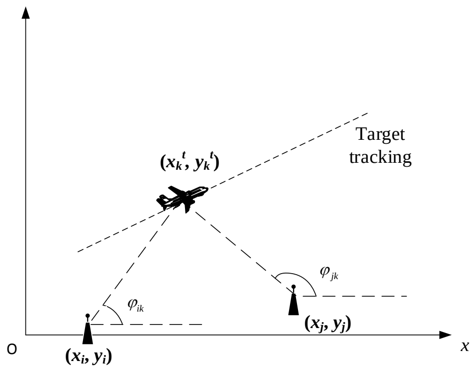

Fig. 1. Tracking model

2 TRACKING MODEL

As shown in Fig.1, we assume that the target is in uniform linear motion with acceleration disturbance. The location and velocity vector at time index k is given by $\boldsymbol{X}_k = (x_k^t, y_k^t)$ and $\dot{\boldsymbol{X}}_k = (\dot{x}_k^t, \dot{y}_k^t)$, respectively. So that the state vector of the target can be expressed by $\boldsymbol{X}_k^t = [x_k^t, \dot{x}_k^t, y_k^t, \dot{y}_k^t]^T$. where $k = 1, 2, \cdots, n$, n is measurement times. m is the number of observation stations, which can be devoted as $\boldsymbol{X}_i^s = (x_i, y_i)$, where $i = 1, 2, \cdots, m$.

The dynamics model can be written as:

$$\boldsymbol{X}_k^t = \boldsymbol{F}\boldsymbol{X}_{k-1}^t + \boldsymbol{\Gamma}\boldsymbol{U}_{k-1,k}, \tag{1}$$

where F is the state transition matrix, Γ is noise drive matrix, and U is process noise matrix. They can be given by:

$$\boldsymbol{F} = \begin{bmatrix} 1 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \Delta t \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \boldsymbol{\Gamma} = \begin{bmatrix} \frac{(\Delta t)^2}{2} & 0 \\ \Delta t & 0 \\ 0 & \frac{(\Delta t)^2}{2} \\ 0 & \Delta t \end{bmatrix},$$

where Δt is the sampling interval.

ϕ_{ik} is the azimuth of i^{th} observation station to the target at time index k. So that measurement equation can be written as:

$$\phi_{ik}^* = \arctan\frac{y_k^t - y_i}{x_k^t - x_i}. \tag{2}$$

Considering the additive noise in azimuth:

$$\phi_{ik} = \arctan\frac{y_k^t - y_i}{x_k^t - x_i} + e_{ik}, \tag{3}$$

where e_{ik} is the measurement error of azimuth and it is generally assumed to be Gaussian distributed with zero mean.

3 ROBUST STRUCTURED TOTAL LEAST SQUARES PASSIVE LOCATION ALGORITHM

3.1 Location Model and the WSTLS Problem

Assume that $z_i^* = x_i \sin\phi_{ik}^* - y_i \cos\phi_{ik}^*$ is a transformed quantity at time index k and then the measurement equation (2) can be rewritten as:

$$\boldsymbol{Z}^* = \boldsymbol{H}^*\boldsymbol{X}, \tag{4}$$

where

$$\boldsymbol{Z}^* = [z_1^*, z_2^*, \cdots, z_m^*]^T, \boldsymbol{H}^* = \begin{bmatrix} \sin\phi_{1k}^* & -\cos\phi_{1k}^* \\ \sin\phi_{2k}^* & -\cos\phi_{2k}^* \\ \ldots & \ldots \\ \sin\phi_{mk}^* & -\cos\phi_{mk}^* \end{bmatrix}.$$

where $\boldsymbol{Z}^*$ is exact observation vector and $\boldsymbol{H}^*$ is coefficient matrix. (3) shows that $\boldsymbol{Z}^*$ and $\boldsymbol{H}^*$ are both polluted by noise, that is:

$$\boldsymbol{Z} = \boldsymbol{Z}^* + \Delta\boldsymbol{Z}, \boldsymbol{H} = \boldsymbol{H}^* + \Delta\boldsymbol{H}, \tag{5}$$

where $\Delta\boldsymbol{Z}$ and $\Delta\boldsymbol{H}$ are error matrices of observation vector and coefficient matrix, respectively. Both $\Delta\boldsymbol{Z}$ and $\Delta\boldsymbol{H}$ come from the measurement noise $\boldsymbol{E} = [e_{1k}, e_{2k}, \cdots, e_{mk}]^T$, and they are not statistically independent. Assume that $\boldsymbol{W}$=diag $[w_1, \cdots, w_m]$ is the weight matrix, to make use of their relations, the passive location can be render into the weight STLS problem [6]:

$$\begin{cases} \min\limits_{E \in R^m} \boldsymbol{E}^T\boldsymbol{W}\boldsymbol{E} \\ s.t. \begin{cases} \boldsymbol{D}(\boldsymbol{E})\boldsymbol{Z} = 0 \\ \boldsymbol{Z}^T\boldsymbol{Z} = 1 \end{cases} \end{cases}, \tag{6}$$

where $\boldsymbol{D}(\boldsymbol{E}) = \boldsymbol{D}_0 + \boldsymbol{D}_1 e_{1k} + \boldsymbol{D}_2 e_{2k} + \cdots + \boldsymbol{D}_m e_{mk}$, augmented matrix of coefficient matrix and observation vector can be expressed by $\boldsymbol{D}_0 = \boldsymbol{D} = [\boldsymbol{H}, \boldsymbol{Z}]$, $\boldsymbol{D}_i = \frac{1}{\sqrt{w_i}}[\boldsymbol{G}_1(:,i).\ \boldsymbol{G}_2(:,i), \boldsymbol{G}_3(:,i)]$, where $\boldsymbol{G}_1, \boldsymbol{G}_2$ and $\boldsymbol{G}_3$ are the diagonal matrix of column elements of $\boldsymbol{H}$ and $\boldsymbol{Z}$. $\boldsymbol{G}_1 = diag[\cos\phi_{1k}, \cdots, \cos\phi_{mk}]$, $\boldsymbol{G}_2 = diag[\sin\phi_{1k}, \cdots, \sin\phi_{mk}]$, $\boldsymbol{G}_3 = diag[x_1\cos\phi_{1k} + y_1\sin\phi_{1k}, \cdots, x_m\cos\phi_{mk} + y_m\sin\phi_{mk}]$.

3.2 The robust STLS Algorithm

The WSTLS algorithm will be accurate and consistent when azimuth noise are Gaussian exactly. However, the azimuth is easily polluted by outliers in practice. At that time, the WSTLS result will be far away from the real value. Hence, we introduce the pollution distribution [16]:

$$G_d = (1-\varepsilon)F_d + \varepsilon Q_d, \tag{7}$$

where F_d is the dominated distribution, Q_d is the interference distribution and ε represents the pollution rate which is the rate of the polluted data in all the measurement data. In order to obtain accurate and robust result, both of F_d and Q_d should be considered.

The residuals of location estimation $\hat{\boldsymbol{X}}$ is:

$$\boldsymbol{S} = \boldsymbol{H}\hat{\boldsymbol{X}} - \boldsymbol{Z} = [\boldsymbol{h}_{1k}, \cdots, \boldsymbol{h}_{mk}]^T \hat{\boldsymbol{X}} - [z_1, ..., z_m]^T, \tag{8}$$

where the residual vector $\boldsymbol{S} = [s_1, s_2, \cdots, s_m]^T$ contains the errors both in $\boldsymbol{H}$ and $\boldsymbol{Z}$. Consequently, the extremal function of (6) can be written as:

$$\sum_{i=1}^{n} w_i \rho(\Delta h_{i1}, \Delta h_{i2}, \Delta z_i) = \min. \tag{9}$$

where $\rho(\cdot)$ is score function which is generally convex and symmetrical. Make the derivative of (9) to $\boldsymbol{X}$, the extremal function can be obtained by:

$$\begin{aligned}\sum_{i=1}^{n} w_i\psi = \sum_{i=1}^{n} w_i[\frac{\partial(\Delta h_{i1})}{\partial X}\psi_1(\Delta h_{i1}) + \\ \frac{\partial(\Delta h_{i2})}{\partial \boldsymbol{X}}\psi_2(\Delta h_{i2}) + \frac{\partial(\Delta z_i)}{\partial \boldsymbol{X}}\psi_3(\Delta z_i)] = 0.\end{aligned} \tag{10}$$

where $\psi(\cdot)$ is the derivative of $\rho(\cdot)$. Substituting $(\Delta h_{i1})^2 + (\Delta h_{i2})^2 + (\Delta z_i)^2$ for $\rho(\Delta h_{i1}, \Delta h_{i2}, \Delta z_i)$ in (10), the derivative is:

$$\sum_{i=1}^{n} w_i[\frac{\partial(\Delta h_{i1})}{\partial \boldsymbol{X}}\Delta h_{i1} + \frac{\partial(\Delta h_{i2})}{\partial \boldsymbol{X}}\Delta h_{i2} + \frac{\partial(\Delta z_i)}{\partial \boldsymbol{X}}\Delta z_i] = 0. \tag{11}$$

In order to reduce the effect of outliers, we make ψ function bounded and non-negative, and then the effect of outliers will also be bounded. Accordingly, when $\boldsymbol{W}_b = diag(w_{b1}, \cdots w_{bm})$, $\psi_1(\Delta h_{i1}) = \Delta h_{i1} w_{bi}$, $\psi_2(\Delta h_{i2}) = \Delta h_{i2} w_{bi}$, $\psi_3(\Delta z_i) = \Delta z_i w_{bi}$, (10) can be stated as:

$$\sum_{i=1}^{n} \overline{w}_i[\frac{\partial(\Delta h_{i1})}{\partial \boldsymbol{X}}\Delta h_{i1} + \frac{\partial(\Delta h_{i2})}{\partial \boldsymbol{X}}\Delta h_{i2} + \frac{\partial(\Delta z_i)}{\partial \boldsymbol{X}}\Delta z_i] = 0. \tag{12}$$

where $\overline{w}_i = w_i w_{bi}$. The only difference of (12) and (11) is the weight matrix, and (12) will equivalent to the WSTLS problem if $\rho(\Delta h_{i1}, \Delta h_{i2}, \Delta z_i) = (\Delta h_{i1})^2 + (\Delta h_{i2})^2 + (\Delta z_i)^2$. Substituting equivalent weight matrix $\overline{\boldsymbol{W}} = diag[\overline{w}_1, \cdots, \overline{w}_m]$ for $\boldsymbol{W}$ in (6), the robust STLS (RSTLS) criterion can be given by:

$$\begin{cases} \min\limits_{\boldsymbol{E}\in R^n} \boldsymbol{E}^T\overline{\boldsymbol{W}}E \\ s.t. \begin{cases} \overline{\boldsymbol{D}}(\boldsymbol{E})\boldsymbol{Z} = 0 \\ \boldsymbol{Z}^T\boldsymbol{Z} = 1 \end{cases} \end{cases}, \tag{13}$$

where $\overline{\boldsymbol{D}}(E) = \boldsymbol{D}_0 + \overline{\boldsymbol{D}}_1 e_{1k} + \overline{\boldsymbol{D}}_2 e_{2k} + \cdots + \overline{\boldsymbol{D}}_m e_{mk}$, $\overline{\boldsymbol{D}}_i = \frac{1}{\sqrt{\overline{w}_i}}[\boldsymbol{G}_1(:,i), \boldsymbol{G}_2(:,i), \boldsymbol{G}_3(:,i)]$, $i = 1, 2, \cdots, m$.

That is, we convert the RSTLS problem to design of equivalent weight function $\overline{\boldsymbol{W}}$. It generally consists of two parts: the division of measurement data and the corresponding weight of each interval.

Two piecewise weight method divides the measurement data into normal data and abnormal data and then the weights of abnormal data will be reduced, such as Huber method and Danish method [14, 18]. On the other hand, three piecewise weight method such as the Institute of Geodesy and Geophysics III (IGGIII) method [14] further divides abnormal data into available data and harmful data. It is assumed that the available data are polluted by small outliers whose weights should be reduced, and the harmful data are polluted by large outliers whose weights should be taken to zero.

In the RSTLS location, if $\overline{w}_i$ is zero or close to it, equivalent weight matrix $\overline{W}$ will become singular and then the RSTLS result will be inaccurate. Thus we build nonzero weight function. i.e., Danish weight function:

$$\overline{w}_i = \begin{cases} w_i & |\tilde{s}_i| < r_0 \\ w_i \cdot \exp\left[1 - \left(\frac{|\tilde{s}_i|}{r_0}\right)^2\right] & |\tilde{s}_i| \geq r_0 \end{cases}. \tag{14}$$

Actually, the influence of outliers to location is limited because azimuth is bounded, so we also bound the value of weight to avoid $\overline{W}$ being close to singular. Therefore, improved Danish weight function is given by:

$$\overline{w}_i = \begin{cases} w_i & |\tilde{s}_i| < r_0 \\ w_i \cdot \exp\left[1 - \left(\frac{|\tilde{s}_i|}{r_0}\right)^2\right] & r_0 \leq |\tilde{s}_i| < r_1 \\ w_i \cdot \exp\left[1 - \left(\frac{r_1}{r_0}\right)^2\right] & |\tilde{s}_i| \geq r_1 \end{cases}, \tag{15}$$

where $\tilde{s}_i$ is standard residual. r_0 and r_1 are related to the sensitivity of the weight function to outliers. In general, r_0 tunes both accuracy and robustness of the algorithm. Appropriate r_0 is related to the true pollution distribution and the confidence coefficient of $\tilde{s_i}$. If less prior information can be utilized, the experience value $r_0 \in [1.5, 2]$ is available, which can also make the result robust [14].

[6] has presented the location mean square error for the WSTLS algorithm, and it is expressed as:

$$\boldsymbol{P}_{WSTLS} = \sigma^2 \left[\boldsymbol{H}^T(\boldsymbol{H}_X \boldsymbol{W}^{-1}\boldsymbol{H}_X^T)^{-1}\boldsymbol{H}\right]^{-1}. \quad (16)$$

Therefore, the location mean square error of the RSTLS algorithm can be shown as:

$$\boldsymbol{P}_{RSTLS} = \sigma^2 \left[\boldsymbol{H}^T(\boldsymbol{H}_X \overline{\boldsymbol{W}}^{-1}\boldsymbol{H}_X^T)^{-1}\boldsymbol{H}\right]^{-1}. \quad (17)$$

where σ^2 represents measurement covariance.

4 THE RSTLS-KF PROBLEM AND SOLUTION

According to the RSTLS location result, the observation equation can be linearized without linearization error, that is:

$$\hat{\boldsymbol{X}}_{\text{RSTLS},k} = \boldsymbol{G}_k \boldsymbol{X}_k^t + \boldsymbol{N}_k, \quad (18)$$

where $\boldsymbol{G}_k = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}^T$, N_k is the estimation error vector of the RSTLS location algorithm, and its covariance matrix is given by (17). Therefore, the RSTLS-KF solution can be described as follows:

$$\begin{cases} \hat{\boldsymbol{X}}_{k,k-1}^t = \boldsymbol{F}\hat{\boldsymbol{X}}_{k-1}^t \\ \boldsymbol{P}_{k,k-1} = \boldsymbol{F}\boldsymbol{P}_{k-1}\boldsymbol{F}^T + \boldsymbol{Q}_{k-1} \\ \boldsymbol{K}_k = \boldsymbol{P}_{k,k-1}\boldsymbol{G}_k^T\left[\boldsymbol{G}_k\boldsymbol{P}_{k,k-1}\boldsymbol{G}_k^T + \boldsymbol{P}_{\text{RSTLS},k}\right]^{-1} \\ \hat{\boldsymbol{X}}_k^t = \hat{\boldsymbol{X}}_{k,k-1}^t + \boldsymbol{K}_k\left[\hat{\boldsymbol{X}}_{\text{RSTLS},k} - \hat{\boldsymbol{X}}_{k,k-1}\right] \\ \boldsymbol{P}_k = \left[\boldsymbol{I} - \boldsymbol{K}_k\boldsymbol{G}_k\right]\boldsymbol{P}_{k,k-1} \end{cases}. \quad (19)$$

There is an important practical issue which is called wraparound problem of KF based method in bearings-only tracking. The azimuth measurements are in the interval $[0, 2\pi)$, but when two or more measurements are obtained and be operated (addition, average, etc.), the result may out of the bound which may cause degradation performance of the RSTLS-KF. Hence, we must keep the operation in the interval and the track estimates will more accurate.

Now, we discuss the solution of the RSTLS algorithm with STLN method. Defining a matrix $\boldsymbol{C} \in \boldsymbol{R}^{m\times q}$ such that:

$$\boldsymbol{C}\boldsymbol{\alpha} = \boldsymbol{E}_r\boldsymbol{X} \quad (20)$$

where $\boldsymbol{E}_r \in R^{m\times 2}$ is an error matrix to be determined and $\boldsymbol{\alpha} \in \boldsymbol{R}^{q\times 1}$ represents the corresponding elements of $\boldsymbol{E}_r$. q is the number of different such element. In bearings-only location model, $q = 2m$. The construction of $\boldsymbol{C}$ is carried out according to the following rule:

If $\boldsymbol{\alpha}_l$ is the $(i,j)^{th}$ element of $\boldsymbol{E}_r$, then c_j is the $(i,l)^{th}$ element of $\boldsymbol{C}$, where $i = 1, \cdots, m$, $j = 1, 2$ and $l = 1, \cdots, q$.

Thus, the solution of the RSTLS problem with the STLN method can be given as follow.

1) Initialization:

Input $\boldsymbol{H}$, $\boldsymbol{Z}$, and tolerance ε_s.Set equivalent weight matrix $\overline{\boldsymbol{W}}_{STLN} = diag(\overline{w}_1, \overline{w}_1, \cdots, \overline{w}_m, \overline{w}_m) = \boldsymbol{I}_{2m}$, $\boldsymbol{E}_r$=$\boldsymbol{O}$, and $\boldsymbol{\alpha}$= 0.

2) Calculate residuals $\boldsymbol{S} = \boldsymbol{H}\hat{\boldsymbol{X}}_{k,k-1} - \boldsymbol{Z}$, and then normalize them, and calculate equivalent weight $\overline{w}_i$ according to (15).

3) Repeat

(a) $\underset{\Delta x, \Delta\alpha}{\text{minimize}} \left\| \begin{bmatrix} \boldsymbol{C} & \boldsymbol{H}+\boldsymbol{E}_r \\ \overline{\boldsymbol{W}}_{STLN} & \boldsymbol{O} \end{bmatrix} \begin{pmatrix} \Delta\boldsymbol{\alpha} \\ \Delta\boldsymbol{X} \end{pmatrix} + \begin{pmatrix} -\boldsymbol{S} \\ \overline{\boldsymbol{W}}_{STLN}\boldsymbol{\alpha} \end{pmatrix} \right\|_2$

(b) Set $\boldsymbol{X} = \boldsymbol{X} + \Delta\boldsymbol{X}$, $\boldsymbol{\alpha} = \boldsymbol{\alpha} + \Delta\boldsymbol{\alpha}$

(c) Construct $\boldsymbol{E}_r$ from $\boldsymbol{\alpha}$, and $\boldsymbol{C}$ from $\boldsymbol{X}$.

Until $(\|\Delta\boldsymbol{X}\|, \|\Delta\boldsymbol{\alpha}\| \le \varepsilon_s)$.

The Newton's method can be used to deal with step3(a). From the above discussion, we know that the exact and robust location result can be obtained in the RSTLS solution, which guarantees the accuracy and stability of the Kalman filter. Compared with the inverse iteration method, the STLN method is simple and direct which reduces the calculation in bearings-only tracking problem.

5 SIMULATION

As shown in Fig.1, the target is in uniform linear motion and origin state is $\boldsymbol{X}_0^t = [0, 8, 50, 10]^T$. Measurement times is 60, and interval t is 1 second. L is the number of Monte-Carlo runs, here $L = 200$. The root mean squares error (RMSE) in location at time index k is defined as:

$$\text{RMSE}_k = \sqrt{\frac{1}{L}\sum_{i=1}^{L}(\hat{x}_k^i - x_k^i)^2 + (\hat{y}_k^i - y_k^i)^2}. \quad (21)$$

In theory, the larger the number of observation stations is, the more accurate the filter would be. Moreover, the bad effect of single outlier will be reduced accordingly. However, the number of stations is generally limited in practical applications. Measurement delay is another problem which can affect the accuracy of the multi-station tracking. In this paper, we focus on the robust performance of algorithms, and thus measurement delay problem is neglected and fixed number of observation stations $m = 4$ is set.

The motion of the target is subject to a process noise scalar of $w = 10^{-4}$, and the measurement noise follows Gaussian distribution with mean value 0, standard deviation $\sigma = 0.6°$. Process noise and measurement noise are independent and identically distributed. Set r_0= 1.5, r_1= 6.

For fair comparison, the algorithms are set as the same initial conditions. The initial location is estimated by the least squares method at the beginning of tracking and the estimate velocity follows $\hat{v} \sim N(v, \sigma_v^2)$, where v is the true velocity, $\sigma_v = 10m/s$.

Simulation experiment 1: No outliers in measurement data. The RMSEs of algorithms are shown in Fig.2.

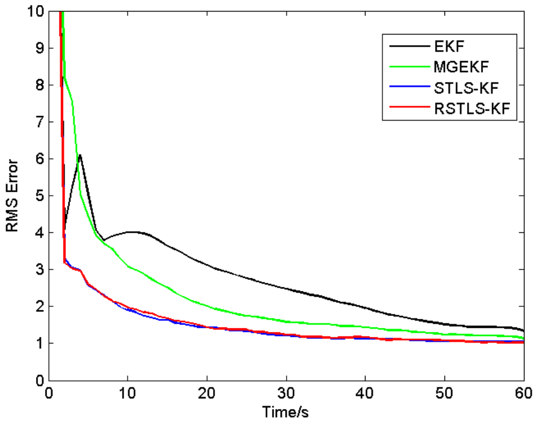

Fig. 2. The RMSEs when no outliers exist

As shown in Fig.2, when no outliers in measurement data, all algorithms are close to convergence. Towards the end of the simulation time, the RSTLS-KF and STLS-KF obviously outperform the EKF and MGEKF algorithms. This is because there are no linearization error in the RSTLS-KF and STLS-KF and the RSTLS and STLS location result is reliable. The computational complexity of the RSTLS-KF algorithm is a bit higher than that of the STLS-KF algorithm due to equivalent weight calculation.

Simulation experiment 2: There are some outliers in the measurement data of one of stations. Continuous outliers appear from 41s to 45s, whose intensities are 5 . Large outlier appears at 50s, whose intensity is 50. Simulation results are shown in Fig.3.

As shown in Fig.3, when measurement data is polluted by outliers, the performance of the EKF, MGEKF, and STLS-KF algorithms are degraded significantly, whereas the result of the RSTLS-KF algorithm is still close to the true value. When continuous small outliers appear (41$\sim$45s), the RSTLS-KF algorithm is still robust but the curve of the other algorithms become incredible. When large outlier appears (50s), the RSTLS-KF algorithm is

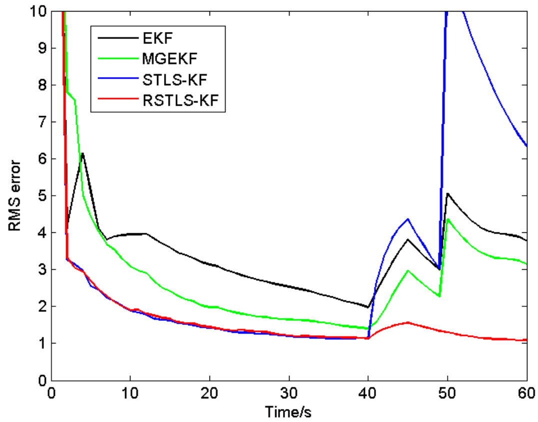

Fig. 3. The RMSEs when there are some outliers

hardly affected, however, the result of the other algorithms are distorted seriously.

6 CONCLUSION

We have proposed an efficient method called the RSTLS-KF algorithm for solving nonlinear problem and inaccuracy caused by outliers in bearings-only tracking. The main new features of this algorithm are that it linearizes the measurement equation without linearization error and has better tracking performance over traditional algorithms. Furthermore, it can identify the outliers automatically according to improved Danish weight function we presented and reduce the weight of the polluted data effectively. Simulation results show that the RSTLS-KF is convergent and stable whether outliers exist. Of course, to calculate the equivalent weight function increases the computational complexity, which will be addressed in future work by developing fast versions of the RSTLS-KF.

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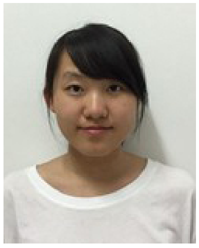

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