GEOFIZIKA VOL. 32 2015

DOI: 10.15233/gfz.2015.32.10
Original scientific paper
UDC 551.242

Kinematic models of recent motion of the Earth's crust on the territory of Croatia, Slovenia and Bosnia and Herzegovina

Nevio Rožić

University of Zagreb, Faculty of Geodesy, Geomatic Department

Received 13 April 2015, in final form 17 September 2015

Based on relative height displacement grid models of the Earth's crust, related to the territory of the Croatia, Slovenia and Bosnia and Herzegovina, which were created by modelling of the benchmark height data contained in levelling networks of the Austrian precise levelling (APN), and First (INVT) and Second (IINVT) levelling of high accuracy, possibility of creation of uniformly accelerated or decelerated motion model and uniform motion model of Earth's crust is analyzed. Kinematic laws of straight-line benchmark height motion has been applied to the values of Earth's crust height displacements that are associated with the nodes in the grid models of relative height displacements created between explicit epochs APN and INVT, and APN and IINVT. This application enabled determination of motion kinematic parameters associated with the grid nodes. Kinematic parameter's determination, structuring and including in a separate grid models, in analogy to the relative height displacement grid models, along with definition of basic kinematic equations of uniformly accelerated or decelerated motion and uniform motion of Earth's crust, allowed for the territory of the Croatia, Slovenia and Bosnia and Herzegovina kinematic models creation.

Keywords: height displacement, displacement models, Earth's crust kinematics, Croatia

1. Introduction

In the territory of Croatia, Slovenia and Bosnia and Herzegovina mathematical models of relative height displacements of the Earth's crust have been initially created and presented in the papers Rožić and Razumović (2010) and Rožić et al. (2011). These specific models have been created in the form of a grid models using a combination of regression modelling and minimum curvature surface modelling. The basis of each model makes a rectangular grid of homogeneous resolution with cells of approximately square shape that is referenced and fixed to the body of the Earth. To each grid node values of relative height displacement of the Earth's crust were assigned, which are determined by modelling. Each individual grid node is defined by ellipsoidal position (ellipsoidal longitude λ, ellipsoidal latitude φ) on the Bessel reference ellipsoid with the zero meridian in Greenwich, and the associated value of the relative height displacement of the Earth's crust. Each model can be defined as an ordered set of grid nodes (λ, φ, $\Delta\overline{H}$), which in the associated spatial rectangular coordinate system defines continuous spatial model surface. Use of bi-linear interpolation allows, along the model coverage area, prediction of the relative height displacements of the Earth's crust for all points with known ellipsoidal positions that are not matched model grid nodes. It is assumed that changes of displacement values between nodes of the corresponding grid cells are linear in both specific directions, i.e. in the direction of the ellipsoidal longitude and latitude.

In fact, on the basis of the results initially published in Rožić and Razumović (2010) and according to the Rožić et al. (2011), three distinct relative height displacement models were created, which are linked consequently to the three different and explicit epochs. Considering that empirical data for the creation of these models served surveying levelling measurements of geometric levelling networks of the highest order of accuracy in the state territory of Croatia, Slovenia and Bosnia and Herzegovina, epochs were determined like mean moments of their field survey. These are the levelling network of the Austrian precise levelling (APN) from the period of Austro-Hungarian Empire and the First levelling of high accuracy (INVT) and Second levelling of high accuracy (IINVT) from the period of Yugoslavia, Rožić (2001). The models of relative height displacements between epochs: APN and INVT, INVT and IINVT, and APN and IINVT were made. Particularly important are models between epochs APN and INVT, and APN and IINVT, because they contain relative height displacements in relation to the oldest of the three epochs in the time series (APN epoch).

As a basis for described grid modelling, the data of absolute height positioning of benchmarks contained in levelling networks APN, INVT and IINVT were used, i.e. the same benchmarks are explicitly surveyed by repeated height positioning in different networks (epochs). While benchmarks, like specific geodetic points, originally are not designed for determination of the height displacements, they show to be qualitatively very usable. They have durable building construction on the topographic surface and they can be considered as discrete material points of the Earth's crust, along with the fact that the quality of their relative and absolute height positioning is very high. But, they are useful only if they are changing height position primarily and exclusively like the consequence of changes in the Earth's crust, and not their own instability. Absolute positioning of benchmarks included in the levelling networks APN, INVT and IINVT and determination of their absolute heights was made in a specially designed height reference system and by applying separate adjustment of each of the levelling network with the use of indirect measurements and the least squares method according to the Pelzer (1985). Empirical values of the relative height displacements of the same benchmarks from different epochs, which were the starting point for the creation of displacement models, are direct differences of benchmark absolute heights. The relative displacements are determined according to the Rožić and Razumović (2010) by subtracting the absolute benchmark heights of older epochs from absolute heights of younger epochs. Signs of benchmark displacement always express the natural direction of height position change relative to the starting position (+ sign denote raising, – sign denote sinking).

All other relevant and specific details of the previously explained initial determination of the relative height displacements and creation of the displacement models, starting with the quantity and quality of benchmarks available empirical data and processing, with the concept, principles, hypotheses and applied modelling methods, and to a series of favourable and unfavourable properties of these models, in details are presented in the papers Rožić and Razumović (2010) and Rožić et al. (2011).

Indeed, availability of relative height displacement models presented in the Rožić et al. (2011), for territory of the Croatia, Slovenia and Bosnia and Herzegovina, and primarily height displacement models between epochs APN and INVT, and APN and IINVT, logically points to the possibility of solving the following important task related to the kinematics of the Earth's crust. It is a creation of kinematic models of height motion, which would describe in mathematically appropriate and explicit way, define and quantify the law of height motion of Earth's crust and allow their application to solve various scientific and practical tasks. From the standpoint of geodesy, rather than going into a wide range of geo-sciences were kinematics and dynamics of the Earth's crust are in the primary or secondary focus, one of the important areas of potential application of kinematic model could be related to the quality increase of the national height reference systems. Expected increase in quality could be built on the systematic errors reduction from the measured height differences in levelling networks that are consequences of height motion of the Earth's crust. The kinematic model of applicable quality could enable the reduction of geometric levelling measurements associated with different surveying epochs to the same and unique epoch, or so-called reference epoch.

Therefore, in this paper the results of the verification and re-modeling of the initially created height displacement models originally shown in the paper Rožić et al. (2011) are elaborated and presented, first of all in an effort to improve their quality. Moreover, using these qualitatively improved models the result of the unique Earth's crust height kinematic model creation in form of specific kinematic parameter grid models is presented, related to the paper Rožić and Razumović (2014). In doing so, it is necessary to emphasize the fact that in this specific case and as a result of a combination of objective circumstances the geographic area to which the height displacements, displacement models and kinematic models refer is defined by the state territorial boundaries of the Croatia, Slovenia and Bosnia and Herzegovina, rather than natural boundaries of some structural or tectonic regions of the Earth's crust which are integral parts of the Eurasian tectonic plate with homogeneous geological and geomorphological origin or other relevant characteristics. That is explained by the fact that the fundamental precondition for the application of geodetic methods for quantifying and qualifying height displacements and creating a kinematic model is the availability of adequate and homogeneous height data sets that are the basis for processing and analysis. In this case, the corresponding height data or better to say state geometric leveling network data on the territory of Croatia, Slovenia and Bosnia and Herzegovina neighborhood states were not available.

Therefore, height displacement models and kinematic models are related, in the contact zone of the Eurasian and African tectonic plate, to the geographic area that encompasses part of the Eastern Alps (Slovenia), Dinarides (Croatia and Bosnia and Herzegovina) and the Pannonian Plane (Croatia), that are integral parts of the so-called Pannonian basin. This situation in the application of geodetic methods and leveling data is common because design and realization of the state leveling networks coincides with the territorial borders of each state, where Kontny and Bogusz (2012) can be mentioned like explicit example. Of course, especially in more recent time and in accordance with the geodetic globalization processes Earth's crust height kinematics take the trans-national character. Restricted to the Europe, i.e. to the continental part of the Eurasian plate, as a relevant example can be cited for example Fuhrman and al. (2014). This line of research and cooperation, not just geodetic but geo-interdisciplinary, it is particularly marked in the activities of the EUREF (IAG Sub-commission: European Reference Frame), Ihde and Augath (2002) and UELN (United Europen Levelling Network) whose focus is directed to the definition and realization of the European height reference frame and system, based on the integration of European state levelling networks data and gravimetric and tide gauge data. Also, it should be noted that very interesting correlation may arise from the interdisciplinary analysis of the geodetic results presented in this paper taking into account research results of other geoscience disciplines that relate to the same or at least partially same observed area, for example Horváth (1993), Lorinczi and Houseman (2010).

2. New models creation of relative height displacement of the Earth's crust

Because the models of relative height displacements of the Earth's crust on the territory of Croatia, Slovenia and Bosnia and Herzegovina, which are presented in Rožić et al. (2011), are the starting point for the creation of kinematic models, their verification need to be performed and possibility of improving quality investigated. As in the meantime, it was not possible to find and include in the process of modelling new benchmarks and how the original application of combining regression modelling (trend modelling) and minimal curvature surface modelling (residuals modelling) provided a viable and acceptable results, main attention is directed to the modification of the model grid parameters originally applied. The resolution of the model grid, as an important parameter within the modelling, should be adequately correlated with the total number of benchmarks and their positional distribution along the area of models coverage. Therefore, it is reasonable to vary the grid resolution, especially if the benchmarks positional distribution is quite inhomogeneous, to examine the possibility of improving the model quality. Often identifying the best grid resolution is an empirical process, which can be in the simplest way defined like "trial and error" process. Improvement of the model quality can be diagnosed and expressed by the appropriate criteria of models' internal accuracy because independent benchmarks reference data for determining models external accuracy are not available. Accordingly, models verification and attempt to improve quality is reasonable to perform in a single step, i.e. by repeating the modelling process using the same benchmark empirical data, applying the same modelling methods and principles, but modifying the grid resolution.

As in the Rožić et al. (2011), like a basis for repeated modelling empirical height positioning data set of 390 benchmarks that were contained in networks APN and INVT, and 1287 completely different benchmarks that were contained in networks INVT and IINVT, were used. These sets include only 49 benchmarks that are under the circumstances included by levelling survey in all three levelling networks. It should be emphasized that these are indeed the first benchmarks whoever were established in the observed area. They were included in the first systematic levelling survey within the network APN during the last quarter of the 19th century and functionally preserved to the epoch of the IINVT network, Rožić (1999). Epochs associated with all levelling networks and belonging benchmarks height positioning data are presented in Tab. 1.

Table 1. Epochs of levelling networks APN, INVT and IINVT.

Epoch	Levelling network	Surveying interval [year]	Mean surveying epoch [year]	Epoch difference [year]
t_1	APN	1874–1909	1892.8	0.0
t_2	INVT	1946–1963	1949.0	56.2
t_3	IINVT	1970–1973	1971.1	22.1

Based on testing several variants of different grid resolutions ultimately as the best variant the grid with twice the resolution compared to its original resolution is accepted. A rectangular grid fixed at origin grid node $\lambda = 13°\ 12'\ 00''$ and $\varphi = 42°\ 12'\ 00''$, with the length $\Delta\lambda = 6°\ 40'\ 00''$ and width $\Delta\varphi = 4°\ 48'\ 00'$, and the size of the grid cells in the amount of $2'\ 00'' \times 1'\ 30''$ (approximately 2.7 km × 2.7 km) is adopted. Grid contains 201 × 193 lines and 38 793 nodes. In the model coverage area, i.e. in the territory of Croatia, Slovenia and Bosnia and Herzegovina, there are 19 975 grid nodes. Original grid had a cell size of 4' × 3' (approximately 5.4 km × 5.4 km), contain 101 × 97 lines with 9797 nodes, out of which 4267 were within the model coverage area. It is clear that on the observed territory the grid of twice as large resolution containing practically four times more nodes is developed, containing notably greater number of triplets $(\lambda, \varphi, \Delta\overline{H})$ compared to the original grid.

The results of new modelling of relative height displacements $\Delta H_{INVT-APN}$ and $\Delta H_{IINVT-INVT}$, made in accordance with the recipe described in Rožić et al. (2011), are shown in Fig. 1 and Fig. 2. A model of relative height displacements $\Delta H_{IINVT-APN}$ is shown in Fig. 3, generated indirectly by mutual addition of $\Delta H_{INVT-APN}$ and $\Delta H_{IINVT-INVT}$ model. Specifically, this model is not possible to determine directly since in APN and IINVT networks there are only 49 of the same benchmarks that are insufficient for meaningful modelling.

Models presented in Figs. 1–3 are directly comparable with the models presented in Figs. 15–17 of Rožić et al. (2011). It can be seen that between the cited models high level of compatibility exist, but with certain differences which are very mild and moderate. They are not surprising because they are results of changes within the grid resolution used during repeated modelling.

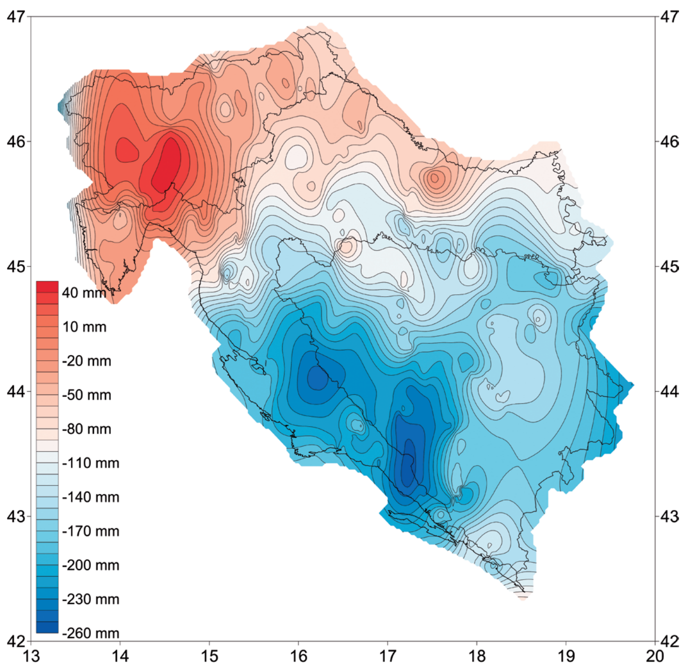

Figure 1. Model of relative height displacements between APN and INVT epoch.

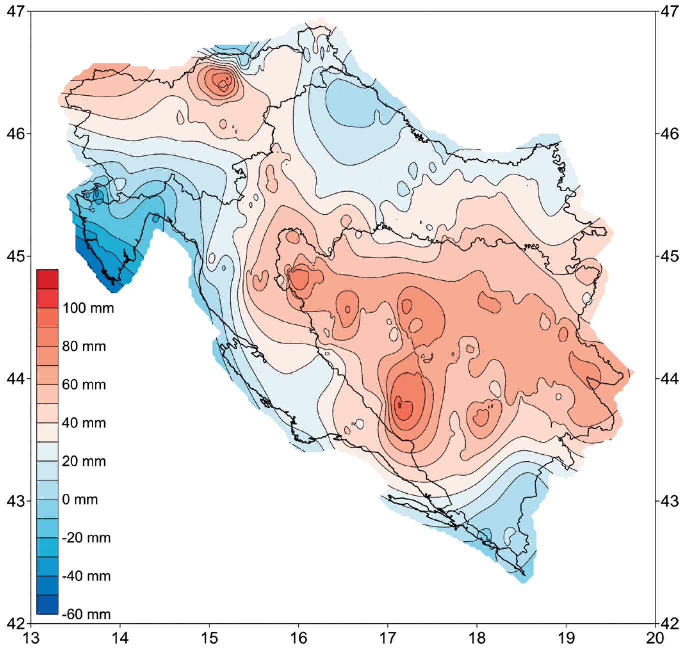

Figure 2. Model of relative height displacements between INVT and IINVT epoch.

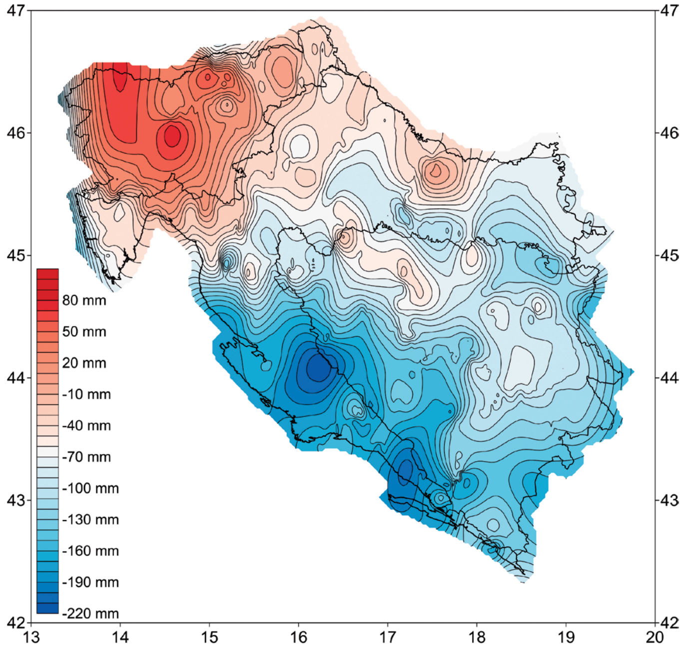

Figure 3. Model of relative height displacements between APN and IINVT epoch.

The relationship between original and new models are explicitly outlined in the descriptive statistical indicators contained in Tab. 2.

Indicators are generated separately for the set of empirical data of benchmark relative displacements ΔH that were the basis for modelling and for sets of model data from the original $\Delta \overline{H}_0$ and new model $\Delta \overline{H}_N$. Because of understandable reasons in the case of displacements between epoch APN and IINVT only 49 benchmarks were used. In addition, Tab. 2 contains statistical indicators regarding discrepancy data, i.e. the differences between the empirical displacement values ΔH and associated model values $\Delta \overline{H}_0$ and $\Delta \overline{H}_N$.

Without going into detailed comments of the statistical indicators contained in Table 2, they show that the original models from Rožić et al. (2011) and new models of height displacements are not mutually significantly different, although some modest differences are evident. The correspondence level indicates the consistency of modelling method application and absence of gross errors during the modelling process. Standard deviations derived from original and new models, based on discrepancies, show that favourable adaptation of model displacement values to the corresponding empirical values achieved slightly higher level of internal quality within new models. New displacement model for the epoch APN and INVT has standard deviation 5.2 mm compared to 6.9 mm in original model, model for the epoch INVT and IINVT has 3.1 mm compared to 3.9 mm and model for the epoch APN and IINVT 6.8 mm compared to 9.0 mm. Although not directly expressing the inner quality of the models, discrepancies dispersion range in the new models also has a somewhat more favourable amount.

Based on the presented results it can be concluded that the increase in grid resolution moderately contributed to the internal quality improvement of new models and therefore it is advisable to apply them as a starting point for the kinematic models creation of Earth's crust height motion on the territory of Croatia, Slovenia and Bosnia and Herzegovina.

3. Kinematic models of benchmark height motion

Height motion of any benchmark, considering it like a discrete material point of the Earth's crust with known ellipsoidal position, can be discussed fully as a straight-line motion. With the hypothesis that over time the benchmark continuously retains the same ellipsoidal position, its height or vertical movement occurs exclusively along the straight line that is defined by the vertical axis of the height reference system. Therefore, kinematic law of benchmark motion can be determined by using benchmark known height positions in specific moments of time, i.e. epochs. Generally, for this purpose a number of mutually different and successive epochs t_1, t_2, ..., t_k and belonging benchmark positions expressed by absolute heights H_1, H_2, ..., H_k (k = total number of epochs) are at

Table 2. Statistical indicators of displacement models.

Relative height displacements	Empirical data	Modelled data		Discrepancy data	
	ΔH [mm]	$\Delta\overline{H}_0$ [mm]	$\Delta\overline{H}_N$ [mm]	$\varepsilon = \Delta H - \Delta\overline{H}_0$ [mm]	$\varepsilon = \Delta H - \Delta\overline{H}_N$ [mm]
Epochs & benchmarks	APN and INVT, 390 benchmarks				
Mean value	–127.4	–127.5	–127.5	0.2	0.1
Standard deviation	68.2	67.6	67.8	6.9	5.2
Median	–136.1	–138.5	–137.5	0.3	0.0
Minimum	–251.6	–250.8	–251.2	–29.2	–24.7
Maximum	29.7	27.7	29.4	32.3	32.4
Range	281.3	278.6	280.6	61.5	57.1
Epochs & benchmarks	INVT and IINVT, 1287 benchmarks				
Mean value	36.2	36.2	36.2	0.0	0.0
Standard deviation	26.4	26.0	26.1	3.9	3.1
Median	37.0	36.3	36.8	0.3	0.1
Minimum	–39.1	–39.4	–38.8	–21.4	–22.0
Maximum	103.0	98.7	100.1	18.7	16.6
Range	142.1	138.1	138.9	40.1	38.6
Epochs & benchmarks	APN and IINVT, 49 benchmarks				
Mean value	–73.0	–73.3	–73.5	0.3	0.5
Standard deviation	43.8	44.8	43.2	9.0	6.8
Median	–73.5	–78.1	–76.4	0.6	0.8
Minimum	–169.1	–168.2	–169.6	–29.6	–26.6
Maximum	41.1	47.8	35.5	21.0	14.2
Range	210.2	216.0	205.1	50.6	40.8

disposal. Since benchmark during the time intervals Δt between epochs travels along the height axis in intervals ΔH (height displacement) it is evident that the benchmark absolute height H generally can be expressed as an explicit function of time t, i.e.

$$H_i = H(t_i), \qquad (i = 1, 2, ..., k). \tag{1}$$

In doing so implies correctness of essentially and entirely hypothetical assumption, that over time the height reference system in which the benchmark height positioning is performed does not change its spatial orientation and position relative to the body of the Earth. In other words, the height reference surface or so-called "zero height surface" during the total period of benchmark positions tracking $H(t_i)$ retains its shape and spatial position completely fixed. As stated in Rožić et al. (2011), in the particular case of the levelling networks APN, INVT and IINVT a unique normal-orthometric height reference system was introduced, with a height reference surface determined by equipotential surface of the Earth's gravity field. This surface, referring to the epoch 1971.5, is fixed at the location of the tide gauge in Bakar with mean sea level derived from the period of 18.6 years of continuous sea level observations.

According to the Pelzer (1985) if it is in an initial or so-called zero epoch t_0 benchmark height position

$$H_0 = H(t_0) \tag{2}$$

the function given by the expression (1) can be developed into a Taylor series keeping only the linear terms and the square term

$$H_i = H(t_i) = H(t_0) + \frac{H'(t_0)}{1!}(t_i - t_0) + \frac{H''(t_0)}{2!}(t_i - t_0)^2. \tag{3}$$

After introduction of substitution

$$\Delta t_i = t_i - t_0, \qquad (i = 1, 2, ..., k), \tag{4}$$

expression (3) takes the form

$$H_i = H_0 + \Delta t_i v_0 + \tfrac{1}{2}\Delta t_i^2 a, \qquad (i = 1, 2, ..., k), \tag{5}$$

taking into account that the first and second derivative of height position (distance) equals to the benchmark motion velocity v_0 and acceleration a. From kinematic standpoint, expression (5) defines the fundamental kinematic equation of law of uniformly accelerated or decelerated benchmark motion along the vertical axis of the height reference system. According to this equation, it is evident that in the zero epoch t_0 benchmark has an absolute height H_0, velocity v_0 and acceleration a. Concerning the expression (5) the following kinematic equation is also easily defined

$$\nu_i = \nu(t_i) = \nu_0 + \Delta t_i a, \qquad (i = 1, 2, ..., k), \tag{6}$$

that shows the principle of benchmark velocity change. Namely, due to the action of constant acceleration, benchmark velocity continuously changes its amount. In other words, v_i is a benchmark present velocity in the epoch t_i as opposed to its initial speed v_0 in the epoch t_0, while during the total period of motion acceleration is constant.

Expressions (5) and (6) are basic expressions that define the kinematic model of uniformly accelerated or decelerated benchmark height motion were the height H_0, velocity v_0, and acceleration a, are the belonging kinematic parameters of motion, Gladding (2012). Although the expression (5) in the theoretically presented form includes at the same time uniformly accelerated and uniformly decelerated motion the empirical realization gives easy possibility to distinguish whether motion is accelerated or decelerated. If the velocity and acceleration in an epoch have the same sign, benchmark moves uniformly accelerated, and if they are mutually different, he moves uniformly decelerated.

Empirical realization of kinematic model defined by expressions (5) and (6) is possible by means of empirical data of benchmark absolute heights H_1, H_2, ..., H_k and the corresponding epoch's t_1, t_2, ..., t_k. If the number of available epochs exceeds the number of unknown kinematic parameters, their unambiguous determination is only possible with the use of regression modelling based on the regression function given by the expression (5) and the method of least squares, Seeber and Lee (2003). However, in case of the levelling networks APN, INVT and IINVT empirical determination of the benchmark kinematic parameters assumes quite simpler solution. Considering that for every benchmark only three epochs' t_1, t_2 and t_3 are available kinematic parameters can be unambiguously determined without using regression modelling. Number of unknown parameters is equal to the number of benchmark height data, so it is possible to set up the system of three equations with three unknowns:

$$\left.\begin{aligned} H_1 &= H_0 + \nu_0 \Delta t_1 + \tfrac{1}{2}\Delta t_1^2 a, \\ H_2 &= H_0 + \nu_0 \Delta t_2 + \tfrac{1}{2}\Delta t_2^2 a, \\ H_3 &= H_0 + \nu_0 \Delta t_3 + \tfrac{1}{2}\Delta t_3^2 a, \end{aligned}\right\} \tag{7}$$

that directly leads to the solution. The solution of the system can be quite effectively determined using different methods, but application of the Cramer's rule with the introduction of appropriate matrix coefficients and their determinants can be very useful, i.e.

$$\left.\begin{aligned} D &= \begin{bmatrix} 1 & \Delta t_1 & \frac{1}{2}\Delta t_1^2 \\ 1 & \Delta t_2 & \frac{1}{2}\Delta t_2^2 \\ 1 & \Delta t_3 & \frac{1}{2}\Delta t_3^2 \end{bmatrix} \qquad D_{H_0} = \begin{bmatrix} H_1 & \Delta t_1 & \frac{1}{2}\Delta t_1^2 \\ H_2 & \Delta t_2 & \frac{1}{2}\Delta t_2^2 \\ H_3 & \Delta t_3 & \frac{1}{2}\Delta t_3^2 \end{bmatrix} \\ D_{\nu_0} &= \begin{bmatrix} 1 & H_1 & \frac{1}{2}\Delta t_1^2 \\ 1 & H_2 & \frac{1}{2}\Delta t_2^2 \\ 1 & H_3 & \frac{1}{2}\Delta t_3^2 \end{bmatrix} \qquad D_a = \begin{bmatrix} 1 & \Delta t_1 & H_1 \\ 1 & \Delta t_2 & H_2 \\ 1 & \Delta t_3 & H_3 \end{bmatrix} \end{aligned}\right\} \tag{8}$$

therefore, the general solution of the system is:

$$H_0 = \frac{|D_{H_0}|}{|D|} = \frac{\Delta t_2 \Delta t_3 (\Delta t_2 - \Delta t_3) H_1 + \Delta t_1 \Delta t_3 (\Delta t_3 - \Delta t_1) H_2 + \Delta t_1 \Delta t_2 (\Delta t_1 - \Delta t_2) H_3}{(\Delta t_1 - \Delta t_2)(\Delta t_1 - \Delta t_3)(\Delta t_2 - \Delta t_3)}, \tag{9}$$

$$\nu_0 = \frac{|D_{\nu_0}|}{|D|} = \frac{\Delta t_3^2 (H_2 - H_1) + \Delta t_2^2 (H_1 - H_3) + \Delta t_1^2 (H_3 - H_2)}{(\Delta t_1 - \Delta t_2)(\Delta t_1 - \Delta t_3)(\Delta t_3 - \Delta t_2)}, \tag{10}$$

$$a = \frac{|D_a|}{|D|} = \frac{2\Delta t_3 (H_1 - H_2) + 2\Delta t_2 (H_3 - H_1) + 2\Delta t_1 (H_2 - H_3)}{(\Delta t_1 - \Delta t_2)(\Delta t_1 - \Delta t_3)(\Delta t_3 - \Delta t_2)}. \tag{11}$$

In accordance with the specific epochs data taken from Tab. 1, i.e. $t_0 = 1874.0$, $t_1 = 1892.8$, $t_2 = 1949.0$, $t_3 = 1971.1$ and $\Delta t_0 = 0.0$, $\Delta t_1 = 18.8$, $\Delta t_2 = 75.0$, $\Delta t_3 = 97.1$, the exact solution can be specified, noting that the zero epoch t_0 that was adopted is the beginning year of levelling network APN field survey:

$$H_0 = 1.65494 H_1 - 1.46977 H_2 + 0.81483 H_3, \tag{12}$$

$$v_0 = -0.03911 H_1 + 0.09332 H_2 - 0.05421 H_3, \tag{13}$$

$$a = 0.00045 H_1 - 0.00161 H_2 + 0.00116 H_3. \tag{14}$$

Empirically determined kinematic parameters enable the exact specification of equation (5) and (6):

$$H_i = H_0 + (t_i - 1874.0)v_0 + \tfrac{1}{2}(t_i - 1874.0)^2 a, \qquad (15)$$

$$v_i = v_0 + (t_i - 1874.0)a, \qquad (16)$$

and enable determining the absolute height and velocity of explicit benchmark in any epoch. Determination of kinematic parameters by the equations (12), (13) and (14) and the use of equations defined by expressions (15) and (16) are unambiguous, but different for each benchmark. This implies that, regardless of the ease of kinematic parameters determination and kinematic equations specification, analogous computational procedure should be carried out for each benchmark separately and independently from all other benchmarks.

Kinematic equation given by expression (15) can be easily visualized in a plane rectangular co-ordinate system (t, H), Fig. 4.

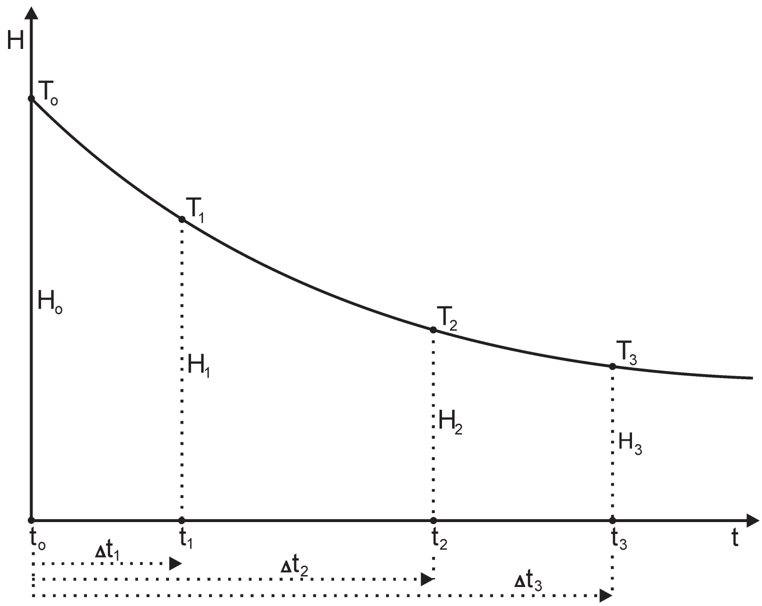

Figure 4. The trajectory of benchmark motion – parabola.

Second-order polynomial or simply a parabola defines the trajectory of benchmark motion. It is clear that the parabola contains all benchmark height positions during motion in accordance with the number of epochs in which positioning's are conducted. The initial benchmark position in the epoch t_0 is presented with point $T_0(t_0, H_0)$, while the positions in epochs t_1, t_2 and t_3 are presented with points $T_1(t_1, H_1)$, $T_2(t_2, H_2)$ and $T_3(t_3, H_3)$.

Based on the expression (5), but omitting the square term, a simpler kinematic model of benchmark motion can be defined, i.e. the model of benchmark uniform motion

$$H_i = H_0 + \Delta t_i v_0, \qquad (i = 1, 2, 3). \tag{17}$$

In this model, absolute height position H_0 in zero epoch t_0 and constant velocity v_0 is the parameter of benchmark height motion while a trajectory of motion is a straight-line. In accordance with this modification, the expression (6) takes a new form

$$v_i = v(t_i) = v_0, \qquad (i = 1, 2, 3). \tag{18}$$

The empirical determination of kinematic parameters when explicitly three epochs are available requires the application of regression modelling based on the regression function given by expression (17) and the method of least squares. Namely, a system of three equations with two unknown kinematic parameters can be structured, which does not lead to a unique solution directly since one benchmark height is redundant:

$$\begin{aligned} H_1 &= H_0 + \Delta t_1 v_0, \\ H_2 &= H_0 + \Delta t_2 v_0, \\ H_3 &= H_0 + \Delta t_3 v_0. \end{aligned} \tag{19}$$

In this particular case, the regression modelling leads to a unique general solution that is given by the expressions:

$$\begin{aligned} \bar{H}_0 = {} & \frac{\Delta t_3^2 (H_1 + H_2) + \Delta t_2^2 (H_1 + H_3) + \Delta t_1^2 (H_2 + H_3) - \Delta t_2 \Delta t_3 (H_2 + H_3)}{2(\Delta t_1^2 + \Delta t_2^2 + \Delta t_3^2 - \Delta t_2 \Delta t_3 - \Delta t_1 \Delta t_2 - \Delta t_1 \Delta t_3)} - \\ & - \frac{\Delta t_1 \Delta t_2 (H_1 + H_2) + \Delta t_1 \Delta t_3 (H_1 + H_3)}{2(\Delta t_1^2 + \Delta t_2^2 + \Delta t_3^2 - \Delta t_2 \Delta t_3 - \Delta t_1 \Delta t_2 - \Delta t_1 \Delta t_3)}, \end{aligned} \tag{20}$$

$$\bar{v}_0 = - \frac{\Delta t_3 (H_1 + H_2 - 2H_3) + \Delta t_2 (H_1 - 2H_2 + H_3) + \Delta t_1 (-2H_1 + H_2 + H_3)}{2(\Delta t_1^2 + \Delta t_2^2 + \Delta t_3^2 - \Delta t_2 \Delta t_3 - \Delta t_1 \Delta t_2 - \Delta t_1 \Delta t_3)} \tag{21}$$

and additionally, after the introduction of specific epoch's data, by the expressions:

$$\bar{H}_0 = 1.20866 H_1 + 0.11141 H_2 - 0.32007 H_3, \tag{22}$$

$$\bar{v}_0 = -0.01376 H_1 + 0.00349 H_2 + 0.01027 H_3. \tag{23}$$

Based on the determined kinematic parameters, expressions (17) and (18) can be written in the form:

$$\bar{H}_i = H_0 + (t_i - 1874.0)\bar{v}_0, \tag{24}$$

$$\bar{v}_i = \bar{v}_0 = const. \tag{25}$$

and used for the determination of benchmark height $\bar{H}_i$ in any epoch t_i in accordance with the fact that benchmark is constantly moving at a uniform rate. As in the previous case, the kinematic equation given by expression (24) can be easily visualized, Fig. 5.

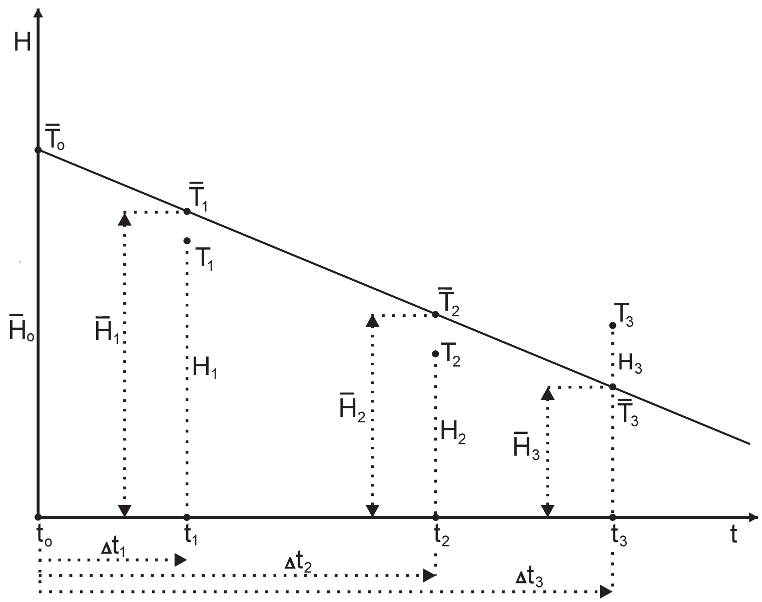

Figure 5. The trajectory of benchmark motion – regression line.

On regression straight-line all specific modelled benchmark positions during the motion are contained, as set out in point $\bar{T}_0(t_0, \bar{H}_0)$ and points $\bar{T}_1(t_1, \bar{H}_1)$, $\bar{T}_2(t_2, \bar{H}_2)$, $\bar{T}_3(t_3, \bar{H}_3)$. Due to the application of regression modelling these points are not matched points $T_1(t_1, H_1)$, $T_2(t_2, H_2)$, $T_3(t_3, H_3)$ as they are determined by benchmark height empirical data. Therefore, the differences between the heights H_i and $\bar{H}_i$ in explicit epoch's t_1, t_2 and t_3 are the residuals of the regression model.

In view of the above, it is clear that determination of the kinematic parameters and kinematic equations of benchmarks uniformly accelerated or decelerated height motion and of uniform motion, in the case of explicit three epochs related to the levelling networks APN, INVT and IINVT, in essence is not too demanding. It can be done separately for each benchmark where a basic prerequisite is the availability of the benchmark absolute heights in all three explicit epochs. Also, the kinematic model of benchmarks uniform motion can be applied in the case of only two available epochs. In such a case, the determination of kinematic parameters and equations is not based on regression modelling, but also on direct solution of the system of two equations with two unknown kinematic parameters.

4. Kinematic models of grid nodes height motion

Despite the mathematical exactness and simplicity of the above presented method of height motion determination, in particular case of networks APN, INVT and IINVT there is a significant aggravating circumstance. Specifically, all three levelling networks contain only 49 benchmarks with known absolute height position and those benchmarks have a very unfavourable positional distribution along the territory of Croatia, Slovenia and Bosnia and Herzegovina. Such small number of benchmarks evidently is not sufficient for adequate modelling purposes and making any relevant conclusions about the motion of the Earth's crust on a rather large observed area.

Therefore, it is reasonable to take advantage of the availability of the relative height displacement models of the Earth's crust instead of direct application of benchmarks absolute height data, Rožić et al. (2011), Rožić and Razumović (2014). The models of relative height displacements between epochs of levelling networks APN and INVT, and APN and IINVT contain displacements of the Earth's crust that are associated with the locations of nodes contained in grid models in relation to the oldest epoch. In such a case the oldest displacements grid model, assigned to the epoch of network APN, can be defined like zero-model, i.e. in the APN epoch relative height displacements on all nodes of the associated grid are defined to be zero. It is important to emphasize the fact that each node contained in the displacement grid models of the Earth's crust, in any epoch, has no available absolute heights, but just the relative height displacement created by modelling of the benchmarks relative displacements. Theoretically, based on absolute height data belonging to the grid nodes H_1, H_2 and H_3, in epoch's t_1, t_2 and t_3, relative height displacements in relation to the epoch t_1 can be formulated:

$$\Delta H_1 = H_1 - H_1 = 0, \tag{26}$$

$$\Delta H_2 = H_2 - H_1, \tag{27}$$

$$\Delta H_3 = H_3 - H_1. \tag{28}$$

The relative displacements ΔH_1 are associated with the epoch t_1 (APN), the relative displacements ΔH_2 derived from the APN and INVT networks are associated with the epoch t_2 (INVT) and relative displacements ΔH_3 derived from the APN and IINVT networks with the epoch t_3 (IINVT). Accordingly, kinematic models of the Earth's crust motion can be created by using the relative height displacements associated to the grid nodes, separately for each node, and using almost the same process which previously is applied in the case of benchmark kinematics. Although the procedure is generally congruent, in the determination of kinematic parameters of uniformly accelerated or decelerated motion direct use of the expression (5) is no longer appropriate, but it is necessary to be modified

$$\Delta H_i(t_i) = \Delta H_i = \Delta H_0 + \Delta t_i v_0 + \tfrac{1}{2}\Delta t_i^2 a, \qquad (i = 1, 2, 3). \tag{29}$$

With this setup, in relation to the system of equations given by (7), there is a system:

$$\Delta H_1 = \Delta H_0 + \Delta t_1 v_0 + \tfrac{1}{2}\Delta t_1^2 a = 0, \tag{30}$$

$$\Delta H_2 = \Delta H_0 + \Delta t_2 v_0 + \tfrac{1}{2}\Delta t_2^2 a, \tag{31}$$

$$\Delta H_3 = \Delta H_0 + \Delta t_3 v_0 + \tfrac{1}{2}\Delta t_3^2 a, \tag{32}$$

whose general solution is:

$$\Delta H_0 = \frac{|D_{\Delta H_0}|}{|D|} = \frac{\Delta t_1 \Delta t_2 (\Delta t_2 - \Delta t_1)\Delta H_3 + \Delta t_1 \Delta t_3 (\Delta t_1 - \Delta t_3)\Delta H_2}{(\Delta t_1 - \Delta t_2)(\Delta t_1 - \Delta t_3)(\Delta t_3 - \Delta t_2)}, \tag{33}$$

$$\nu_0 = \frac{|D_{\nu_0}|}{|D|} = \frac{\Delta H_3 (\Delta t_2^2 - \Delta t_1^2) + \Delta H_2 (\Delta t_1 - \Delta t_3)(\Delta t_1 + \Delta t_3)}{(\Delta t_1 - \Delta t_2)(\Delta t_1 - \Delta t_3)(\Delta t_2 - \Delta t_3)}, \tag{34}$$

$$a = \frac{|D_a|}{|D|} = \frac{2\Delta H_3 (\Delta t_1 - \Delta t_2) + 2\Delta H_2 (\Delta t_3 - \Delta t_1)}{(\Delta t_1 - \Delta t_2)(\Delta t_1 - \Delta t_3)(\Delta t_2 - \Delta t_3)} \tag{35}$$

and the solution with concrete epoch's data introduced:

$$\Delta H_0 = -1.46977\Delta H_2 + 0.81483\Delta H_3, \tag{36}$$

$$v_0 = 0.09332\Delta H_2 - 0.05421\Delta H_3, \tag{37}$$

$$a = -0.00161\Delta H_2 + 0.00116\Delta H_3. \quad (38)$$

The kinematic parameters of the grid nodes motion are relative height displacement ΔH_0 and motion velocity v_0 in the zero epoch t_0 and acceleration a, which allow specification of kinematic equations for each grid node:

$$\Delta H_i = \Delta H_0 + v_0(t_i - 1874.0) + \tfrac{1}{2}a(t_i - 1874.0)^2, \quad (39)$$

$$v_i = v_0 + a(t_i - 1874.0). \quad (40)$$

In analogy with the determination of kinematic parameters of uniform benchmark motion, but starting from the expression (29) and with the elimination of its square term, for the case of explicit three epochs in the case of application of regression modeling unique general solution is determined:

$$\Delta\bar{H}_0 = \frac{\Delta H_3(\Delta t_1^2 + \Delta t_2^2 - \Delta t_3(\Delta t_1 + \Delta t_2)) + \Delta H_2(\Delta t_1^2 - \Delta t_1\Delta t_2 + \Delta t_3(\Delta t_3 - \Delta t_2))}{2(\Delta t_1^2 + \Delta t_2^2 + \Delta t_3^2 - \Delta t_2\Delta t_3 - \Delta t_1\Delta t_2 - \Delta t_1\Delta t_3)}, \quad (41)$$

$$\bar{v}_0 = -\frac{\Delta H_3(\Delta t_1 + \Delta t_2 - 2\Delta t_3) + \Delta H_2(\Delta t_1 - 2\Delta t_2 + \Delta t_3)}{2(\Delta t_1^2 + \Delta t_2^2 + \Delta t_3^2 - \Delta t_2\Delta t_3 - \Delta t_1\Delta t_2 - \Delta t_1\Delta t_3)} \quad (42)$$

and the solution with concrete epoch's data introduced:

$$\Delta\bar{H}_0 = 0.11141\Delta H_2 - 0.32007\Delta H_3, \quad (43)$$

$$\bar{v}_0 = 0.00349\Delta H_2 + 0.01027\Delta H_3. \quad (44)$$

The kinematic parameters of grid node motion are relative height displacement ΔH_0 and motion velocity in the zero epoch. They allow the specification of kinematic equations for each grid node:

$$\Delta\bar{H}_i = \Delta H_0 + v_0(t_i - 1874.0), \quad (45)$$

$$\bar{v}_i = \bar{v}_0 = const. \quad (46)$$

As with the visualization of the kinematic equations given by the expression (15) and (24) in Figs. 4 and 5, the kinematic equations given by the expressions (39) and (45) can be also visualized, Fig. 6. Parabola directly contain points

$T_1(t_1, H_1)$, $T_2(t_2, H_2)$ and $T_3(t_3, H_3)$, while the regression straight-line contain points $\overline{T}_1$ (t_1, $\overline{H}_1$), $\overline{T}_2$(t_2, $\overline{H}_2$) and $\overline{T}_3$(t_3, $\overline{H}_3$).

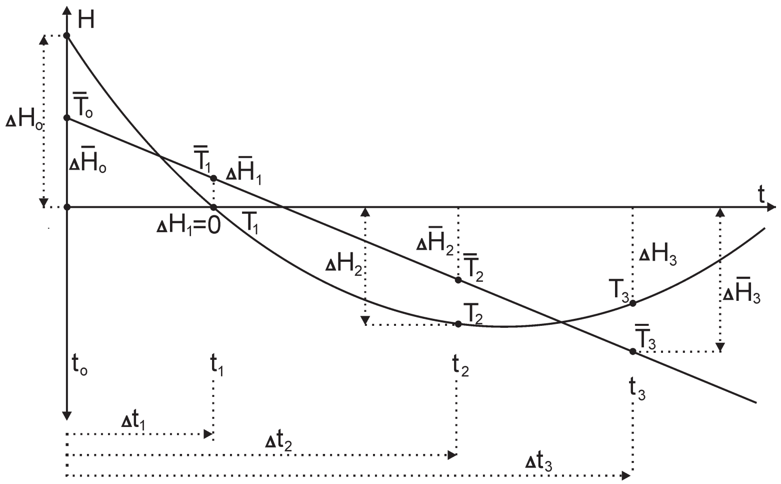

Figure 6. Trajectories of grid node motion – parabola and regression straight-line.

It is evident that on the basis of displacements, which are associated with the nodes in the grid models of the relative height displacements between epochs of the APN and INVT, and APN and IINVT levelling networks, the kinematic parameters of height motion of each node can be determined by applying quite simple mathematical expressions, as for more complex uniformly accelerated or decelerated motion, as for more simple uniform motion.

The kinematic parameters determination of each grid node, in accordance with the presented procedure, is completely independent from kinematic parameters determination of all the other nodes included in the grid. Since the kinematic parameters are associated with grid nodes it is simple to structure separate grid models for each kinematic parameter, i.e. grid model with kinematic parameters ΔH_0, grid model with the parameters v_0 and the grid model with the parameters a, in the case of uniformly accelerated or decelerated motion, and grid models with parameters $\Delta\overline{H}_0$ and parameters $\overline{v}_0$ in the case of uniform motion. Based on these models, it is possible to predict kinematic parameters for any point of known ellipsoidal position which is located on the territory of Croatia, Slovenia and Bosnia and Herzegovina by applying bi-linear interpolation. As the grid cells are relatively small, i.e. approximately 4.7 km × 4.7 km, it can be assumed that the kinematic parameters from node to node are linearly changing along direction of the ellipsoid longitude and latitude.

Kinematic parameter grid models are the basis of height motion models of the Earth's crust. Based on the predicted kinematic parameters from the respective grid models it is easy to determine the direction and amount of relative height displacement of any point of the Earth's crust and the speed of motion for any epoch. In the case of uniformly accelerated or decelerated motion height displacements in arbitrarily selected epoch are determined by expression (39) and the current velocity by expression (40). In the case of uniform motion, relative height displacements are determined by the expression (45) and the velocity by expression (46). In both cases, the relative height displacements are referred with the zero epoch t_0. Therefore, kinematic models of the Earth's crust motion on the territory of Croatia, Slovenia and Bosnia and Herzegovina are defined by the appropriate kinematic parameter grid models and the associated kinematic equations. Kinematic parameters prediction for the epochs which are outside the interval $(t_1 - t_3)$ can be quite unreliable and the level of uncertainty increases with the epoch shift size in relation to the interval limits.

5. The realization of kinematic parameters grid models of the Earth's crust motion

According to the expressions (36), (37) and (38), and based on the relative height displacement grid models between epochs of the APN and INVT, and APN and IINVT networks, Fig. 1 and Fig. 3, the kinematic parameters grid models are created related to uniformly accelerated or decelerated motion of the Earth's crust on the territory of Croatia, Slovenia and Bosnia and Herzegovina. Grid models are presented in Figs. 7–9 for the kinematic parameters ΔH_0, v_0 and a.

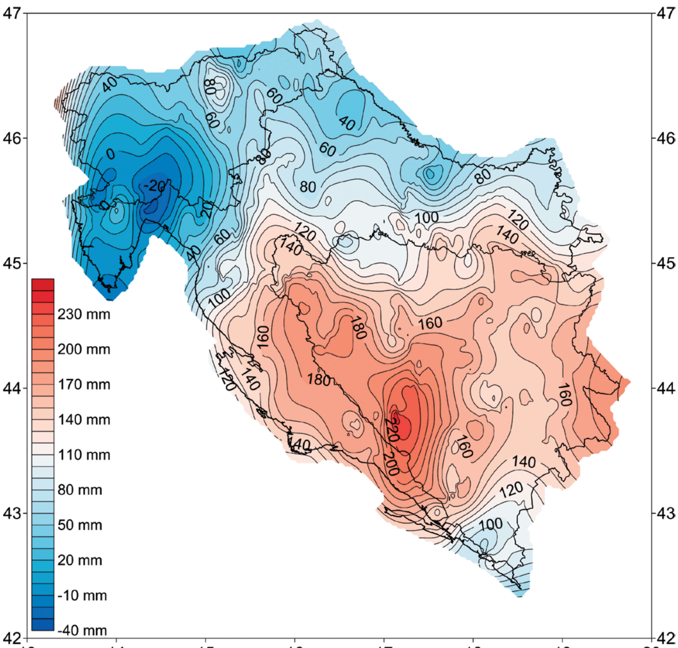

Figure 7. Uniformly accelerated or decelerated motion – parameter ΔH_0.

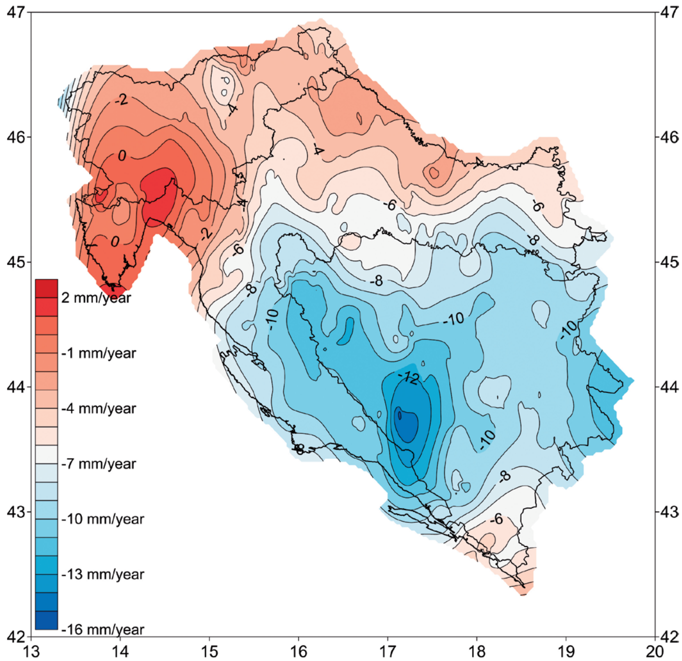

Figure 8. Uniformly accelerated or decelerated motion – parameter v_0.

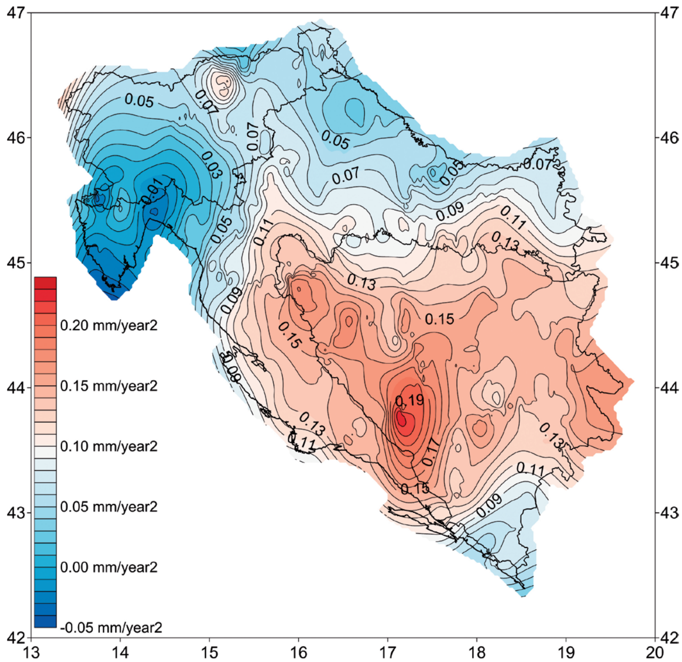

Figure 9. Uniformly accelerated or decelerated motion – parameter a.

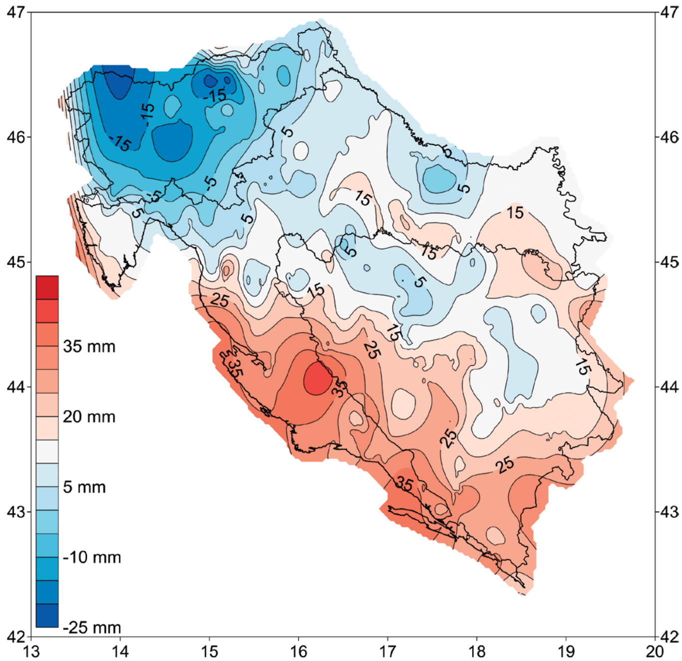

Figure 10. Uniform motion – parameter $\Delta\overline{H}_0$.

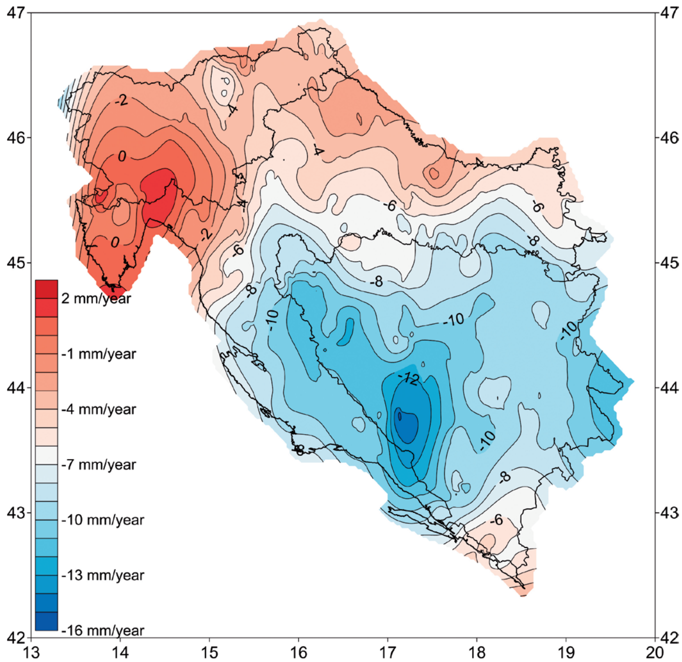

Figure 11. Uniform motion – parameter $\overline{v}_0$.

According to the equations (43) and (44), and based on the same displacement grid models, the kinematic parameters grid models related to the uniform motion of the Earth's crust for the territory of Croatia, Slovenia and Bosnia and Herzegovina are created. Grid models are presented in Figs. 10 and 11 for the kinematic parameters $\Delta\overline{H}_0$ and $\overline{v}_0$.

6. The quality and use of the Earth's crust kinematic model

Quality evaluation of the kinematic models is a special problem given by the fact that an appropriate set of benchmarks reference empirical data completely independent of the relative height displacement models, which could be used for testing and evaluation quality, is not available. Also, it is quite obvious that in this specific case the qualitative advantage of a more complex kinematic model exists, i.e. the model of uniformly accelerated or decelerated motion of the Earth's crust. This is based on the fact that this mathematical model at the observed territory more efficiently describes the empirical reality, namely the fact that relative height displacements between the epochs of APN and INVT are mostly negative (sinking crust), while between epochs INVT and IINVT are mostly positive (raising crust) and with somewhat more moderate amounts. In contrast, the kinematic model of uniform motion of the Earth's crust, determined by using regression modeling and least squares method, shows reasonably lower level of empirical data accommodation to the regression line compared to the parabola. This relationship between regression line and parabola is essentially very clearly illustrated in Fig. 6, noting that theoretical content of the figure is indeed quite congruent with exact empirical situation for a considerable number of benchmarks and their empirical data on the observed territory. Also, it is quite clear that the quality of kinematic models significantly depends on the quality of the relative height displacement models of the Earth's crust, in accordance with all favorable and unfavorable properties related to quality of empirical data and hypotheses, methods and procedures applied during their creation, initially declared in the Rožić and Razumović (2010) and Rožić et al. (2011).

Focusing primarily on the kinematic model of uniformly accelerated or decelerated motion, it is possible to express the inner accuracy of the kinematic parameters grid models by comparing set of directly determined kinematic parameters with corresponding set of modeled parameters for the 49 benchmarks contained in all epochs. Based on a set of discrepancies between the empirical and modelled values corresponding standard deviation may be used as quality criteria. Statistical indicators relating to this comparison are given in Tab. 3. It can be seen that for all kinematic parameters standard deviations calculated from discrepancies have a favorable amount.

Also, statistical indicators regarding comparison between empirical values of the relative height displacements and modelled values, for same benchmarks and for all three specific epochs, are given in Tab. 4. Modelled values are calculated using the expression (39) with kinematic parameters predicted from the kinematic parameter grid models presented in Figs. 7–9. As in the previous case, the standard deviations determined from the discrepancies between empirical and modelled values are of very favorable amounts, which are for all three epochs under 1 cm. In other words, the application of the kinematic model of uniformly accelerated or decelerated motion of the Earth's crust on the territory of Croatia, Slovenia and Bosnia and Herzegovina lead to the quality of the centimeter level.

Table 3. Kinematic parameters statistical indicators.

Kinematic parameters	Empirical data			Modelled data			Discrepancy data		
	ΔH_0 [mm]	v_0 [mm/year]	a [mm/year^2]	ΔH_0 [mm]	v_0 [mm/year]	a [mm/year^2]	ε [mm]	ε [mm/year]	ε [mm/year^2]
Benchmarks	49								
Mean value	104.1	–6.4	0.09	104.2	–6.4	0.09	–0.1	0.0	0.00
Standard deviation	51.4	3.2	0.05	50.7	3.1	0.05	3.7	0.2	0.00
Median	104.1	–6.1	0.10	97.5	–6.0	0.09	–0.7	0.0	0.00
Minimum	–19.8	–11.4	–0.03	–20.6	–11.5	–0.03	–6.0	–0.8	–0.01
Maximum	186.0	1.3	0.16	185.7	1.3	0.17	12.9	0.3	0.01
Range	205.8	12.7	0.19	206.3	12.8	0.19	18.9	1.1	0.02

Table 4. Relative height displacements statistical indicators.

Relative height displacements	Empirical data			Modelled data			Discrepancy data		
	t_1	t_2	t_3	t_1	t_2	t_3	t_1	t_2	t_3
	ΔH [mm]	ΔH [mm]	ΔH [mm]	ΔH [mm]	ΔH [mm]	ΔH [mm]	ε [mm]	ε [mm]	ε [mm]
Benchmarks	49								
Mean value	0.0	–111.3	–73.0	–0.1	–112.6	–75.1	0.1	1.3	2.1
Standard deviation	0.0	56.3	43.8	0.0	56.0	44.2	0.0	5.7	6.8
Median	0.0	–107.1	–73.5	–0.1	–113.3	–78.1	0.1	1.7	3.1
Minimum	0.0	–212.7	–169.1	–0.1	–211.6	–173.3	0.0	–20.9	–24.4
Maximum	0.0	4.3	41.1	0.0	7.3	36.2	0.1	13.9	17.4
Range	0.0	217.0	210.2	0.1	218.9	209.5	0.1	34.8	41.9

Based on these results one can conclude that at the moment kinematic model of uniformly accelerated or decelerated motion presented in this paper is the fully digital, most complex and most accurate model of Earth's crust height motion regarding the observed area, especially in relation to the previous results of height displacements, kinematic parameters and models of the Earth crust determination, Klak (1954), Jovanović (1968), Jovanović (1971).

At the level of the model use, without a detailed examination of its full potential, it is reasonable to draw attention to two elements of interest from the geodetic point of view.

First of all, the relative height displacement of the same point between two arbitrarily selected epochs can be easily defined. If the relative height displacements of some point P_1 for the epochs t_A and t_B, are

$$\Delta H_A = \Delta H_0 + v_0(t_A - 1874.0) + \tfrac{1}{2}a(t_A - 1874.0)^2, \tag{47}$$

$$\Delta H_B = \Delta H_0 + v_0(t_B - 1874.0) + \tfrac{1}{2}a(t_B - 1874.0)^2, \tag{48}$$

then in respect to the time interval

$$\Delta t_{AB} = t_B - t_A, \tag{49}$$

relative height displacement of point P_1 is

$$\Delta H_{AB} = \tfrac{1}{2}\Delta t_{AB}\left(2v_0 + a(t_A + t_B - 3748.0)\right), \tag{50}$$

noting that t_A is older and t_B is younger epoch.

Second of all, on the basis of preceding expression the reduction of height differences defined by two points, from some initial to another completely different epoch, can be also easily defined. Namely, if height difference Δh between points P_1 and P_2 in the epoch t_A is known, it is possible to reduce it to the epoch t_B. In analogy with the expression (50), relative height displacement of point P_1 between epochs t_A and t_B is

$$\Delta H_{P_1} = \tfrac{1}{2}\Delta t_{AB}\left(2v_{0P_1} + a_{P_1}(t_A + t_B - 3748.0)\right), \tag{51}$$

and relative height displacement of point P_2 between same epochs is

$$\Delta H_{P_2} = \tfrac{1}{2}\Delta t_{AB}\left(2v_{0P_2} + a_{P_2}(t_A + t_B - 3748.0)\right). \tag{52}$$

Based on the expressions (51) and (52) the reduced height difference in the epoch t_B is

$$\Delta h_R = \Delta h + (\Delta H_{P_2} - \Delta H_{P_1}), \tag{53}$$

provided that the height difference is determined in the direction from the point P_1 to point P_2.

The potential application of the presented procedure of height differences reduction to a precisely determined and conveniently selected epoch, the so-called reference epoch, is in geodesy extremely interesting and important. The application could potentially enable the elimination of systematic impact of the benchmark height motion from the measured height differences, i.e. reducing the height differences measured in different survey epochs to conveniently selected reference epoch. In this context, as a logical reference epoch directly connected with the national reference height system, the height datum epoch can be adopted. At this point, the sustainability of the kinematic model application for this purpose and qualitative acceptability of the height differences reduction in the state levelling networks has not yet been examined. Based on the kinematic model centimeter-level accuracy in relation to decimeter level magnitude of relative height displacements of the Earth's crust, the expectation could be optimistic. Although in favor of greater optimism is not the fact that the millimeters are significant for measured height differences in state levelling networks, not centimeters.

7. Conclusion

It can be concluded that based on the relative height displacement grid models referred to the territory of Croatia, Slovenia and Bosnia and Herzegovina two standard kinematic models of height motion of Earth's crust were created. These are the model of uniformly accelerated or decelerated motion and model of uniform motion of the Earth's crust. The models have been realized like a combination of kinematic parameters grid models and associated kinematic equations. Grid models allow prediction of the kinematic parameters for any point or group of points on the Earth's crust, according to known ellipsoidal position on the Bessel ellipsoid. On the basis of predicted kinematic parameters, the relative height position determination of the points for any epoch with respect to zero epoch is enabled, determination of relative height position change of the same point between two different and arbitrarily selected epochs is enabled and reduction of height differences from the measurements epoch in a suitable selected reference epoch is also enabled.

Internal quality of kinematic model of uniformly accelerated or decelerated motion point to a satisfactory level which in terms of determining the relative height positions of points on the Earth's crust reach reliable centimeter-level. However, the specified level of quality must be received with considerable caution especially taking into account a series of unfavorable elements connected with the process of grid models creation. A more realistic view on the model quality could not be tested since corresponding reference set of appropriate data quality which would be completely independent of the data used in the models creation it is not available.

References

Fuhrmann, T., Westerhaus, M., Zippelt, K. and Heck, B. (2014): Vertical displacement rates in the Upper Rhine Graben area derived from precise levelling, *J. Geodesy*, **88**, 773–787, DOI: 10.1007/s00190–014–0721–0.

Gladding, G., Selen, M. and Stelzer, T. (2011): *Classical mechanics*. Freeman and Co., New York, ISBN 1-429-29502-3, 303 pp.

Horváth, F. (1993): Towards a mechanical model for the formation of the Pannonian basin, *Tectonophysics*, **226**, 333–357, DOI: 10.1016/0040–1951(93)90126–5.

Ihde, J. and Augath, W. (2002): The European Vertical Reference System (EVRS), its relation to a World Height System and to the ITRS. International Association of Geodesy Symposia, Springer, **125**, 78–83.

Jovanović, P. (1968): Pregledna karta brzina savremenih vertikalnih pomeranja zemljine kore u Jugoslaviji iz podataka ponovljenog nivelmana u mreži nivelmana visoke točnosti. IV. Kongres geodetskih inženjera i geometara Jugoslavije, Savezna geodetska uprava, Beograd, 17 pp.

Jovanović, P. (1971): Isljedovanije savremenih vertikaljnih dviženij zemnoj kori v Jugoslaviji. Savezna geodetska uprava, Beograd, 13 pp.

Klak, S. (1954): Prilog proučavanju sekularnih pomaka zemljine kore pomoću geometrijskog nivelmana, *Geodetski list*, 49–62.

Kontny, B. and Bogusz, J. (2012): Models of vertical movements of the Earth crust surface in the area of Poland derived from levelling and GNSS data, *Acta Geodyn. Geomater.*, **9**, 331–337.

Lorinczi, P. and Houseman, G. (2010): Geodynamical models of lithospheric deformation, rotation and extension of the Pannonian Basin of Central Europe, *Tectonophysics*, **492**, 73–87, DOI: 10.1016/j.tecto.2010.05.007.

Pelzer, H. (1985): *Geodätische Netze in Landes– und Ingenieurvermessung II*. Verlag Konrad Wittwer KG, Stuttgart ISBN 3-87919-140-9, 846 pp.

Rožić, N. (1999): Erhaltung der Reperepunkte im Österreichischen Präzisionsnivellement auf dem Gebiet der Republik Kroatien, Österreichische Zeitschrift für Vermessung & Geoinformation, **4**, 188–195.

Rožić, N. (2001): Fundamental levelling networks and height datums at the territory of the Republic of Croatia. *Acta Geod. Geophys. Hu.*, **2**, 231–243, DOI: 10.1556/AGeod.36.2001.2.10.

Rožić, N. and Razumović, I. (2010): Vertical crustal movements on the territory of the Croatia, Bosnia and Hercegovina, and Slovenia, *Bulletin of Geodesy and Geomatics*, **2**–**3**, 195–209.

Rožić, N. and Razumović, I. (2014): Kinematic model of recent crustal movements for the territory of the Republic of Croatia, in Conference proceedings of *14th International multidisciplinary scientific geoconference SGEM 2014*, 17–26 June 2014, Albena, Bulgaria, **Vol. II**, 329–336.

Rožić, N., Razumović I. and Nazifovski, I. (2011): Modelling of the recent crustal movements at the territory of Croatia, Slovenia and Bosnia and Hercegovina, *Geofizika*, **28**, 183–213.

Seber, G. A. F. and Lee, A. J. (2003): *Linear regression analysis*. John Wiley – Interscience, 2nd ed., New Jersey, ISBN 0–471–41540–5, 565 pp.

SAŽETAK

Kinematički modeli recentnih gibanja zemljine kore na teritoriju Hrvatske, Slovenije i Bosne i Hercegovine

Nevio Rožić

Na temelju grid modela relativnih visinskih pomaka zemljine kore, koji se odnose na teritorij Hrvatske, Slovenije i Bosne i Hercegovine, koji su kreirani modeliranjem visinskih podataka repera obuhvaćenih nivelmanskim mrežama Austrijskog preciznog nivelmana – APN te I. nivelmana visoke točnosti – INVT i II. nivelmana visoke točnosti – IINVT iz razdoblja Jugoslavije, analizirana je mogućnost kreacije kinematičkih modela jednoliko ubrzanog ili usporenog gibanja te jednolikog gibanja zemljine kore. Kinematička zakonitost visinskog pravocrtnog gibanja repera primijenjena je na vrijednosti pomaka zemljine kore koji su pridruženi čvorovima grida u modelima relativnih visinskih pomaka određenih između eksplicitnih epoha: APN i INVT te APN i IINVT. Ova primjena je omogućila određivanje kinematičkih parametara gibanja pridruženih čvorovima grida. Određivanje, strukturiranje i uvrštenje kinematičkih parametara u zasebne grid modele, u analogiji s grid modelima relativnih visinskih pomaka, uz definiranje temeljnih kinematičkih jednadžbi jednoliko ubrzanog ili usporenog gibanja, odnosno jednolikog gibanja zemljine kore, omogućilo je kreiranje kinematičkih modela za teritorij Hrvatske, Slovenije i Bosne i Hercegovine.

Ključne riječi: visinski pomaci, modeli pomaka, kinematika zemljine kore, Hrvatska

Corresponding author's address: Nevio Rožić, University of Zagreb, Faculty of Geodesy, Geomatic Department, Kačićeva 26, Zagreb 10000, Croatia; tel: +385 1 463 9224, fax: +385 1 482 8081; e–mail: nrozic@geof.hr