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TRANSRAPID AND THE TRANSITION CURVE AS SINUSOID

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Professional paper
The transition curves in the modern highway and railway construction are route elements equally crucial as alignment and curve (circular). In order to prevent a sudden change of the centrifugal force, the transition curve must be applied due to the impact of the motion in a sharp curve. Over the years the application of the clothoid has become widespread in many countries in the world. However, in this study, in order to eliminate the problems concerning the road dynamics, created by clothoid for vehicles at high speed, sinusoid is examined. The possibility of using sinusoid in defining transition curves during design of transportation facilities is analysed in this paper. Basic properties of sinusoid, their fundamental mathematical expression, calculation of point coordinates, driving-dynamic characteristics of sinusoid are described (especially for Transrapid using), and the function of change in curvature and lateral impact along the sinusoid is presented. Sinusoid is dealt with as ideal curvature diagram which has curve and superelevation ramp in the articles.

Keywords: *clothoid; sinusoid; transition curves; transrapid*

Transrapid i krivulja prijelaza kao sinusoida

Stručni članak
Krivulje prijelaza pri konstrukciji moderne glavne ceste i željeznice su elementi jednako važni kao i smjer trase i zakrivljenost (kružnost). Kako bi se spriječila nagla promjena centrifugalne sile, mora se primijeniti krivulja prijelaza zbog djelovanja gibanja u oštrom zavoju. Tijekom godina raširila se primjena klotoida u mnogim zemljama svijeta. Međutim, u ovoj analizi, u svrhu eliminiranja problema povezanih s dinamikom puta koje je stvarao klotoid kod vozila pri visokim brzinama, istražuje se sinusoid. U radu se analizira mogućnost primjene sinusoida kod definiranja krivulja prijelaza tijekom projektiranja prijevoznih sredstava. Opisuju se osnovna svojstva sinusoida, njihov fundamentalni matematički izraz, izračun koordinata, pogonsko-dinamičke karakteristike sinusoida (naročito za uporabu Transrapida) i prezentira se funkcija promjene zakrivljenosti puta i bočni udar duž sinusoida. Sinusoid se predstavlja kao dijagram s idealnom zakrivljenošću koji ima rampu krivulje i superelevacije.

Ključne riječi: *klotoid; krivulja prijelaza; sinusoid; transrapid*

1 Introduction

Straight (tangent) sections of most types of transportation routes, such as highways, railroads, and pipelines, are connected by curves in both the horizontal and vertical planes. An exception is a transmission line, in which a series of straight lines is used with abrupt angular changes at tower locations if needed. Curves used in horizontal planes to connect two straight tangent sections are called horizontal curves. Two types are used: circular arcs and spirals (transition curves). Both are readily laid out in the field with surveying equipment. A simple curve is a circular arc connecting two tangents. It is the type most often used. A compound curve is composed of two or more circular arcs of different radii tangent to each other, with their centres on the same side of the alignment. The combination of a short length of tangent connecting two circular arcs that have centres on the same side is called a broken-back curve. A reverse curve consists of two circular arcs tangent to each other, with their centres on opposite sides of the alignment. Compound, broken-back, and reverse curves are unsuitable for modern high-speed highway, rapid transit, and railroad traffic and should be avoided if possible. However, they are sometimes necessary in mountainous terrain to avoid excessive grades or very deep cuts and fills. Compound curves are often used on exit and entrance ramps of interstate highways and expressways, although easement curves are generally a better choice for these situations. Easement curves are desirable, especially for railroads and rapid transit systems, to lessen the sudden change in curvature at the junction of a tangent and a circular curve. A spiral makes an excellent easement curve because its radius decreases uniformly from infinity at the tangent to that of the curve it meets. Spirals are used to connect a tangent with a circular curve, a tangent with a tangent (double spiral), and a circular curve with a circular curve [1÷ 4, 6÷12].

Spirals are used to provide gradual transitions in horizontal curvature. Their most common use is to connect straight sections of alignment with circular curves, thereby lessening the sudden change in direction that would otherwise occur at the point of tangency. Since spirals introduce curvature gradually, they afford the logical location for introducing superelevation to offset the centrifugal force experienced by vehicles entering curves. The effect of centrifugal force on a vehicle passing around a curve can be balanced by superelevation, which raises the outer rail of a track or outer edge of a pavement. Correct transition into superelevation on a spiral increases uniformly with the distance from the beginning of the spiral and is in inverse proportion to the radius at any point. Properly superelevated spirals ensure smooth and safe riding with less wear on equipment. As noted, spirals are used for railroads and rapid-transit systems. This is because trains are constrained to follow the tracks, and thus a smooth, safe, and comfortable ride can only be assured with properly constructed alignments that include easement curves. On highways, spirals are less frequently used because drivers are able to overcome abrupt directional changes at circular curves by steering a spiralled path as they enter and exit the curves [1, 3, 5, 7, 8, 10, 12, 13].

A track transition curve, or spiral easement, is a mathematically calculated curve on a section of highway, or railroad track, where a straight section changes into a curve. It is designed to prevent sudden changes in centrifugal force. In plan the start of the transition of the horizontal curve is at infinite radius and at the end of the transition it has the same radius as the curve itself, thus forming a very broad spiral. At the same time, in the vertical plane, the outside of the curve is gradually raised until the correct degree of bank is reached. If such easement were not applied, the lateral acceleration of a rail vehicle would change abruptly at one point – the tangent point where the straight track meets the curve – with undesirable results. With a road vehicle the driver naturally applies the steering alteration in a gradual manner and the curve is designed to permit this, using the same principle. While railroad track geometry is intrinsically three-dimensional, for practical purposes the vertical and horizontal components of track geometry are usually treated separately [5, 13, 14, 15, 17].

2 Transrapid

Transrapid is a German high-speed monorail train using magnetic levitation. Based on a patent from 1934, planning of the Transrapid system started in 1969, see Fig. 1. The test facility for the system in Emsland, Germany was completed in 1987. In 1991, the technical readiness for application was approved by the Deutsche Bundesbahn in cooperation with renowned universities. Its current application-ready version, the Transrapid 09, has been designed for 500 km/h (311 mph) cruising speed and allows acceleration and deceleration of approx. 1 m/s^2 (3,28 ft/s^2), see Fig. 1 [14, 15, 16].

Figure 1 Transrapid

In 2004, the first commercial implementation was completed. The Shanghai Maglev Train connects the rapid transit network 30,5 km (18,95 mi) to the Shanghai Pudong International Airport. The Transrapid system has not yet been deployed on a long-distance intercity line. The system is developed and marketed by Transrapid International, a joint venture of Siemens and ThyssenKrupp [14, 15. 16].

At the end of 2011, the operation license has expired, and the test track has been closed. Early 2012, the demolition and reconversion of the entire Emsland site, including the factory, has been approved. The super-speed Transrapid maglev system has no wheels, no axles, no gear transmissions, no steel rails, and no overhead electrical pantographs (Fig. 1). The maglev vehicles do not roll on wheels; rather, they hover above the track guide way, using the attractive magnetic force between two linear arrays of electromagnetic coils − one side of the coil on the vehicle, the other side in the track guide way − which functions together as a magnetic dipole. During levitation and travelling operation, the Transrapid maglev vehicle floats on a frictionless magnetic cushion with no physical contact whatsoever to the track guide way. On-board vehicle electronic systems measure the dipole gap distance 100 000 times per second, to guarantee the clearance between the coils attached to the underside of the guide way and the magnetic portion of the vehicle wrapped around the guide way edges. With this precise, constantly updated electronic control, the dipole gap remains nominally constant at 10 mm (0,39 inches). When levitated, the maglev vehicle has about 15 centimetres (5,91 inches) of clearance above the guide way surface [14, 15, 16].

The Transrapid maglev vehicle requires less power to hover than it needs to run its on-board air conditioning equipment. In Transrapid vehicle versions TR08 and earlier, when travelling at speeds below 80 km/h (50 mph), the vehicle levitation system and all on-board vehicle electronics were supplied power through physical connections to the track guide way. At vehicle speeds above 80 km/h, all on-board power was supplied by recovered harmonic oscillation of the magnetic fields created from the track's linear stator (since these oscillations are parasitic, they cannot be used for vehicle propulsion). A new energy transmission system has since been developed for Transrapid vehicle version TR09, in which the maglev vehicle now requires no physical contact with the track guide way for these on-board power needs, regardless of the maglev vehicle speed. This feature helps to reduce on-going maintenance and operational costs. In case of power failure of the track's propulsion system, the maglev vehicle can use on-board backup batteries to temporarily power the vehicle's levitation system [6, 12, 14, 15].

3 Clothoid

The simplest and most commonly used form of transition curve is that in which the superelevation and horizontal curvature both vary linearly with distance along the track. Cartesian coordinates of points along this spiral are given by the Fresnel integrals. The resulting shape matches a portion of an Euler spiral, which is also commonly referred to as a clothoid, and sometimes Cornu spiral. A transition curve can connect a track segment of constant non-zero curvature to another segment with constant curvature that is zero or non-zero of either sign. The Euler spiral has two advantages. One is that it is easy for surveyors because the coordinates can be looked up in Fresnel integral tables. The other is that it provides the shortest transition subject to a given limit on the rate of change of the track superelevation (i.e. the twist of the track). However, as has been recognized for a long time, it has undesirable dynamic characteristics due to the large (conceptually infinite) roll acceleration and rate of change of centripetal acceleration at each end. Because of the capabilities of personal computers it is now practical to employ spirals that have dynamics better than those of the Euler spiral [1÷7, 10, 12÷15].

An Euler spiral is a curve whose curvature changes linearly with its curve length (the curvature of a circular curve is equal to the reciprocal of the radius). Euler spirals are also commonly referred to as spiros, clothoids or Cornu spirals. Euler spirals have applications to diffraction computations. They are also widely used as transition curve in railroad engineering/highway engineering for connecting and transiting the geometry between a tangent and a circular curve. The principle of linear variation of the curvature of the transition curve between a tangent and a circular curve defines the geometry of the Euler spiral [1, 3, 5, 6, 7, 17].

Its curvature begins with zero at the straight section (the tangent) and increases linearly with its curve length. Where the Euler spiral meets the circular curve, its curvature becomes equal to that of the latter. An object travelling on a circular path experiences a centripetal acceleration. When a vehicle travelling on a straight path approaches a circular path, it experiences a sudden centripetal acceleration starting at the tangent point; and thus centripetal force acts instantly causing much discomfort (causing jerk). On early railroads this instant application of lateral force was not an issue since low speeds and wide-radius curves were employed (lateral forces on the passengers and the lateral sway was small and tolerable). As speeds of rail vehicles increased over the years, it became obvious that an easement is necessary so that the centripetal acceleration increases linearly with the travelled distance. Given the expression of centripetal acceleration V^2/R, the obvious solution is to provide an easement curve whose curvature, $1/R$, increases linearly with the travelled distance. This geometry is an Euler spiral [1, 6÷13, 17]

The clothoid spiral is the most commonly used spiral type. The clothoid spiral is used worldwide in both highway and railway track design. First investigated by the Swiss mathematician Leonard Euler, the curvature function of the clothoid is a linear function chosen such that the curvature is zero (0) as a function of length where the spiral meets the tangent. The curvature then increases linearly until it is equal to the adjacent curve at the point where the spiral and curve meet. Such an alignment provides for continuity of the position function and its first derivative (local azimuth), just as a tangent and curve do at a Point of Curvature (PC). However, unlike the simple curve, it also maintains continuity of the second derivative (local curvature), which becomes increasingly important at higher speeds spiral [1, 6, 7, 8, 9, 10, 11, 12, 13, 17].

Clothoid spirals can be expressed as:

Flatness of spiral:

$$A=\sqrt{LR} \tag{1}$$

Total angle subtended by spiral:

$$\tau_{\mathrm{s}}=\frac{L}{2R} \tag{2}$$

Taking the origin at the ÜA (start point of clothoid), the point where the tangent ends and the spiral begins, the coordinates of a point on the spiral a distance L from the ÜA are X and Y (ÜE, the final point of clothoid). The angle τ is the angle of the tangent to the spiral at (X, Y). The line shows that τ is proportional to the square of the distance L from the ÜA. Now we can express X and Y in terms of integrals, in fact taking τ as a parameter. These are the Fresnel integrals familiar in physical optics, which are tabulated and can be computed easily. They can also be expressed as integrals of Bessel functions of order 1/2, and this can be worked into expressions for X and Y as infinite series of Bessel functions. The coordinates of clothoid are obtained by using Fresnel integrals [1, 3, 4, 5, 7, 8, 9, 10, 11, 15, 17]

$$\mathrm{d}y=\mathrm{d}L\sin\tau,\quad \mathrm{d}x=\mathrm{d}L\cos\tau$$

$$X=\int_{L=o}^{L}\cos\tau\,\mathrm{d}L$$

$$X=L\left(1-\frac{L^2}{40R^2}+\frac{L^4}{3456R^4}-\dots\right) \tag{3}$$

$$Y=\int_{L=o}^{L}\sin\tau\,\mathrm{d}L$$

$$Y=\frac{L^2}{6R}\left(1-\frac{L^2}{56R^2}+\frac{L^4}{7040R^4}-\dots\right) \tag{4}$$

4 Sinusoidal curves

These curves represent a consistent course of curvature and are applicable to transition from 0° through 90° of tangent deflections, see Fig. 2. However, sinusoidal curves are not widely used because they are steeper than a true spiral and are therefore difficult to tabulate and stake out [1, 6, 7, 8, 9, 10, 15, 16, 17].

Sinusoidal curves can be expressed as:

$$\theta=\frac{l^2}{2RL}+\left(\frac{L}{4\pi^2R}\right)\left[\cos\left(\frac{2\pi l}{L}\right)-1\right] \tag{5}$$

Differentiating with l we get an equation for l/r, where r is the radius of curvature at any given point:

$$r=\frac{2\pi LR}{2\pi l-L\sin\left(\frac{2\pi l}{L}\right)} \tag{6}$$

The curvature equation of sinusoid is obtained:

$$k=\frac{1}{R}\left[\frac{L}{L_{\mathrm{E}}}-\frac{1}{2\pi}\sin\left(\frac{2\pi L}{L_{\mathrm{E}}}\right)\right] \tag{7}$$

The differential change in spiral angle dτ is obtained:

$$\mathrm{d}\tau=\frac{1}{r}\mathrm{d}L=\frac{1}{R}\left[\frac{L}{L_{\mathrm{E}}}-\frac{1}{2\pi}\cdot\sin\left(\frac{2\pi L}{L_{\mathrm{E}}}\right)\right]\mathrm{d}L \tag{8}$$

Spiral angle of any point of the sinusoid τ is obtained:

$$\tau = \int_{L=0}^{L} \mathrm{d}\tau = \frac{1}{R}\left[\frac{L^2}{2L_\mathrm{E}} + \frac{L_\mathrm{E}}{4\pi^2}\cdot\left(\cos\left(\frac{2\pi L}{L_\mathrm{E}}\right)-1\right)\right] \tag{9}$$

L_E − the length of sinusoid
R − the radius of circular curve.

The coordinates of sinusoid are obtained by using Fresnel integrals [1, 6, 7, 8, 9, 10, 15, 16, 17]

$$\mathrm{d}y = \mathrm{d}L\sin\tau,\quad \mathrm{d}x = \mathrm{d}L\cos\tau$$
$$Y = \int_{L=0}^{L}\sin\tau\,\mathrm{d}L,\quad X = \int_{L=0}^{L}\cos\tau\,\mathrm{d}L \tag{10}$$

The basic component values of sinusoid are calculated with the following equations [1, 6, 7, 8, 9, 10, 15, 16, 17]:

Spiral angle of the final point (UE) of sinusoid

$$\tau_\mathrm{E} = \frac{L_\mathrm{E}}{2R}\frac{200}{\pi} \tag{11}$$

The shift of the circular curve

$$\Delta R = Y_\mathrm{E} - R(1-\cos\tau_\mathrm{E}) \tag{12}$$

The coordinates of the centre of the circular curve are:

$$X_\mathrm{E} = X_\mathrm{E} - R\cos\tau_\mathrm{E}$$
$$Y_\mathrm{E} = R + \Delta R \tag{13}$$

The other components are calculated with the following equations:

$$T_k = \frac{Y_\mathrm{E}}{\sin\tau_\mathrm{E}} \tag{14}$$

$$T_\mathrm{U} = X_\mathrm{E} - Y_\mathrm{E}\tan\tau_\mathrm{E} \tag{15}$$

$$N = \frac{Y_\mathrm{E}}{\cos\tau_\mathrm{E}} = T_k\tan\tau_\mathrm{E} \tag{16}$$

$$T = T_\mathrm{U} + \sqrt{T_k^2 + N^2} = X_\mathrm{E} + Y_\mathrm{E}\tan\tau_\mathrm{E} \tag{17}$$

The sinusoid with its continuous progression of curvature, cant, acceleration, and jerk shows clear advantages for high-speed transportation systems in comparison with the clothoid, see Fig. 2 [1, 6, 7, 8, 9, 10, 15, 16, 17]:

- At the ends of the sinusoid ramp no "kinks" develop within the gradient of the guide way edges.
- The passenger will find the effects of riding quality - lateral acceleration and lateral jerk - at the same amplitudes significantly more comfortable through the "gentle" increase and decrease in sinusoids.

However, this improvement of riding quality results in that the sinusoid of a transition curve must have double the length of a clothoid under the same initial alignment conditions, if the resulting maximum lateral jerk is to be identical in both transition curves. The alignment of reverse-curve is realized by one single reverse sinusoid each, because the riding quality becomes significantly more comfortable and smooth by this process in contrast to the use of two separate colliding sinusoids. Another characteristic of the Maglev guide way is the so-called axle rotation around the space curve in the cant development, see Figure 2 [1, 6÷10, 15÷17].

The space curve follows the alignment in horizontal and upright projection in the centre of the guide way; the edges of the guide way are rotated around this axle with the cant [1, 6÷10, 15÷17].

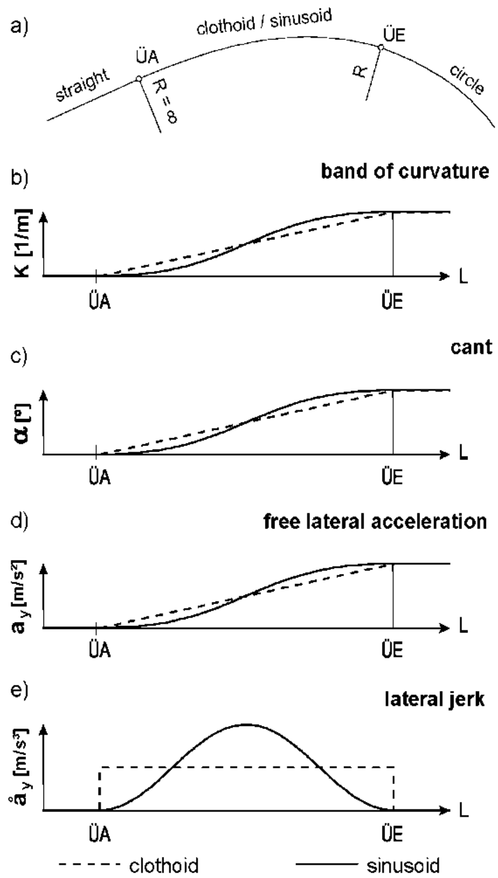

Figure 2 Diagram of clothoid and sinusoid: a) situation, b) curvature, c) superelevation, d) lateral acceleration, e) the rate of change of radial acceleration [6, 15, 16]

Differences in riding quality approach to other transportation systems. The effects of accelerations and jerks on the passenger determine the main parameters for the route alignment of the Maglev guideway. In addition to the imperative compliance with the limits, effort has to be made to achieve an alignment optimised in riding quality. This includes for example an almost balanced lateral acceleration in curves and a quality of acceleration and jerk that is as tempered as possible. For the riding quality examination as well as for preliminary studies of riding quality during alignment works, the speed is a critical initial parameter. There is a very high interdependency, because the parameter speed influences the resulting accelerations to its second power, and the resulting jerks even to its third power, see Fig. 2 [4, 6, 9, 11, 15, 16].

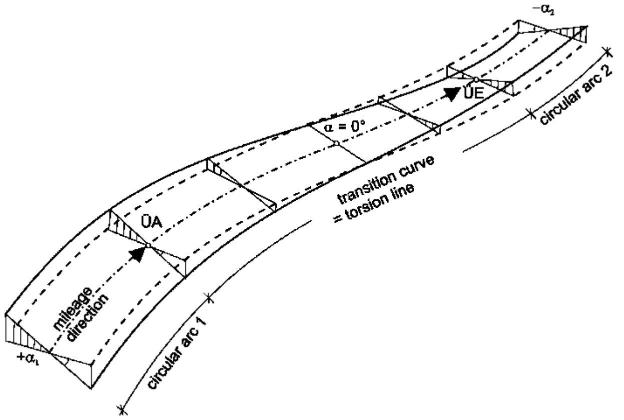

Figure 3 The super elevation diagram of sinusoid [6, 15, 16]

This combination of functional dependency with the great range of speed as well as with the great accelerating power of the Maglev system has various results: not only are precise mathematical simulations necessary for a most exact evaluation of vehicle dynamics, but also there should be a correspondingly well approximated database of the (future) operational speed profiles. The planning of operational speed profiles is itself dependent on the alignment as well, because here aspects of economy (energy consumption, travel times…) and possible local constraint points have to be considered, where the route alignment might require speed reductions and which will therefore have to be taken into account within the path of the guide way. This necessitates an interdisciplinary iterative process, which will result in the "fine tuning" of the alignment and in the optimisation of the riding quality on the basis of "operational" speed profiles, see Fig. 3 [4, 6, 9, 10, 15, 16, 17].

Table 1 The coordinates and the basic component values of the clothoid and sinusoid (R = 1000 m, L = 250 m) [1, 4, 6, 9, 10, 15]

	Sinusoid		Clothoid	
km	X (m)	Y (m)	X (m)	Y (m)
0	0,000	0,000	0,000	0,000
0+050	50,000	0,006	50,000	0,083
0+100	100,000	0,182	99,996	0,667
0+150	149,989	1,152	149,970	2,250
0+200	199,914	3,826	199,872	5,331
0+250	249,658	8,823	249,610	10,405
τ_E	$7{,}9577^g$		$7{,}9577^g$	
ΔR	1,021 m		2,603 m	
X_M	124,983 m		124,935 m	
T_K	70,769 m		83,458 m	
T_U	179,442 m		166,803 m	

The alignment is examined and evaluated with the help of simulation programmes. These evaluate all geometrical limits on the basis of horizontal and upright projections of the alignment, the transverse cant band and the speed profile (incl. acceleration and deceleration areas); they establish the riding quality values along the whole route and compare these values with the corresponding limits. The individual values are calculated taking into account the complex three-dimensional alignment, incl. "operational" speed profile; all possible deviations are recorded completely. The determined values are tabulated at desired intervals and are displayed in a graph, see Figs. 2 and 3 [4, 6, 9, 10, 15, 16, 17].The sinusoid and clothoid coordinate values are obtained with 50 m intervals in Tab. 1 [1, 4, 6, 9, 10, 15].

5 Conclusions

In order to come to an optimal route for the guide way of the Maglev (Transrapid) system, the particular parameters of riding quality have to be taken into account in addition to the economic and ecological conditions. Because the Maglev system is capable of relatively high speeds in comparison to other guided transportation systems, the application of sinusoids with their advantages in riding quality is an obvious element of alignment in horizontal projection. With regard to the riding quality the big advantage of the sinusoid - compared to other traditionally used transition curves - is the continuity in the lateral acceleration band and in the jerking curve. The alignment in upright projection is designed comfortably through the integration of clothoids between the straight-line gradients or descents respectively and the curves.

The alignment of the whole Maglev route is continuously screened for compliance with technical and riding quality limits using a realistic evaluation base resulting from riding quality simulations. This way all criteria of riding quality, taking into account the effects of deviations, can be established and evaluated completely during the precise three-dimensional examination of the space curve. The aim of the simulations is not only the compliance with the limits of the system and the riding quality, but also specifically the riding quality optimization of the tracks in order to increase the already high riding quality, taking into consideration operational aspects like travel times, propulsion, and energy consumption.

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