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Constrained Reference Tracking based on Homothetic Sets

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In this paper, we consider the problem of constrained tracking of piecewise constant references for nonlinear dynamical systems. In the considered problem we assume that an existing controller satisfies constraints in a corresponding positive-invariant set of the system. To solve the problem we propose the use of homothetic transformations of the positive-invariant set to modify the existing control law. The proposed approach can be implemented as a tracking model predictive control or as a reference governor. Simulation and experimental results are provided, showing the applicability of the proposed approach to a class of nonlinear systems.

Key words: Reference tracking, Model predictive control, Reference governor

Slijeđenje reference s ograničenjima zasnovano na homotetičnim skupovima. U radu se razmatra problem slijeđenja reference s ograničenjima za nelinearne dinamičke sustave. Polazna je pretpostavka da postojeći zakon upravljanja zadovoljava ograničenja u pripadnom invarijantom skupu sustava. Uz takvu pretpostavku u radu se predlaže primjena homotetične transformacije invarijantnih skupova kako bi se izmjenio postojeći zakon upravljanja. Predloženi pristup se može primjeniti u sklopu modelskog prediktivnog upravljanja za slijeđenje reference ili samostalno za oblikovanje reference. Dani su simulacijski i eksperimentalni rezultati koji pokazuju primjenjivost predložene metode za klasu nelinearnih sustava.

Ključne riječi: Slijeđenje reference, Modelsko prediktivno upravljanje, Oblikovanje reference

1 INTRODUCTION

In the last two decades the interest in advanced control approaches which can explicitly take into account constraints on control input and states has significantly increased. Among such control approaches the most popular one is Model Predictive Control (MPC). An alternative approach for constraint satisfaction is to design a controller which does not consider constraints and to add an additional device called a reference governor to ensure constraint satisfaction by modifying the reference signal of the primary controller. Such approaches often rely on the concepts of invariant sets for guaranteeing the closed-loop stability and constraint satisfaction.

In the MPC case, a terminal cost and a terminal set constraint are typically employed for guaranteeing the closed-loop stability and recursive feasibility [1]. In that way the MPC algorithm provides guarantees for a set of initial states $\mathcal{X}_F$, the so-called region of attraction of the MPC algorithm, from which it is possible to reach the terminal set in N steps. However, $\mathcal{X}_F$ can be undesirably small in some cases. A trivial approach for enlarging the region of attraction is to increase the prediction horizon length N. However, this may not be acceptable in real time applications due to an additional computational burden. An alternative way of enlarging the region of attraction of the MPC is the usage of a sequence of contractive sets as a stabilizing constraint [2–4].

Another possibility of increasing the region of attraction is to modify the control objective to steer the system states to a non-zero equilibrium state rather than to the origin. In that case the stabilizing ingredients become reference-dependent. In the literature a reference-dependent terminal set is often used, being recalculated for each reference signal or calculated in the augmented state-reference space [5, 6].

For linear systems, the reference-dependent terminal set can be obtained simply by translating and scaling the terminal set calculated for the regulation problem [7, 8]. The main benefit of such an approach is in a less complex terminal set comparing to the one calculated in the augmented state-reference space. Unfortunately, for general non-linear systems, translation and scaling of the terminal set calculated for a regulation problem is not always possible.

On the other hand, the reference governor approach is generally considered as a simpler but less-performing alternative to MPC [9, 10], even though a reference governor can be implemented in a receding horizon manner [11]. A reference governor is a non-linear device that has been extensively used in control systems to modify the behavior of the primary controller so as to ensure the constraints satisfaction [12], [13], [14], [9]. The action of the primary controller is typically modified by means of changing its reference signal when necessary, based on the current state, set-point and system constraints. In order to guarantee the constraint satisfaction at all times, typically the maximum output admissible set $\mathcal{O}_\infty$ [15] is used, forming an additional set membership constraint. The goal of a reference governor is to find an admissible reference signal that is as close as possible to the desired one.

In this paper we propose a novel approach to constrained reference tracking suitable to a class of non-linear, parameter-varying systems, based on a collection of sets homothetic to a positive-invariant set, calculated for a regulation problem. The proposed approach is intended to be applicable as a reference tracking MPC or as a reference governor. Unlike the reference governor, the proposed approach assumes that a primary controller (e.g. MPC controller or conventional feedback controller) is designed for a regulation problem and satisfies the input/state constraints, however with a possibly conservative associated initial feasible set $\mathcal{X}_F$. In order to enlarge the feasible set of the primary controller its active reference signal is modified using a reference governor or by altering the original controller. Both cases are investigated by means of simulations and experiments.

The main contributions of the paper are as follows: (i) deriving the conditions for which a collection of sets homothetic to a positive-invariant set can be used for constrained piecewise constant reference tracking for a special class of non-linear parameter-varying systems, (ii) connecting the proposed approach to a reference governor and an MPC tracking approach. In addition the paper shows simulation and experimental results demonstrating the applicability of the proposed approach.

The rest of the paper is organized as follows. In Section II basic definitions and preliminaries are given. In Section III the standard approaches to a constrained reference tracking are reviewed. Conditions for using a constrained reference tracking based on a homothetic transformation of positive-invariant sets for non-linear systems are proposed in Section IV, while simulation and experimental results are given in Section V. Section VI concludes the paper.

2 PRELIMINARIES

2.1 Basic notation

The following notation is used throughout the paper. Let $\mathbb{R}$, $\mathbb{R}_+$, $\mathbb{Z}$, $\mathbb{Z}_+$ denote the field of real numbers, non-negative real numbers, integers and non-negative integers. The Minkowski sum of two sets $\mathcal{A}$ and $\mathcal{B}$ is denoted as $\mathcal{A} \oplus \mathcal{B} = \{a + b : a \in \mathcal{A}, b \in \mathcal{B}\}$, while the Minkowski difference is denoted as $\mathcal{A} \ominus \mathcal{B} = \{x \in \mathbb{R}^n : x + b \in \mathcal{A}, \forall b \in \mathcal{B}\}$. For a set $\mathcal{S} \in \mathbb{R}^n$, we use the notation $\mathrm{int}(\mathcal{S})$ to denote the interior of the set, while notation $\lambda\mathcal{S}$ denotes a scaled set $\lambda\mathcal{S} = \{\lambda x : x \in \mathcal{S}\}$.

2.2 System description

In this paper we consider a non-linear parameter-varying system given as:

$$\begin{aligned} x_{k+1} &= f(x_k, u_k, p_k), k \in \mathbb{Z}_+, \\ y_k &= h(x_k, u_k), \end{aligned} \tag{1}$$

subject to constraints

$$u_k \in \mathcal{U}, x_k \in \mathcal{X}, p_k \in \mathcal{P}, \tag{2}$$

where $x_k \in \mathbb{R}^n$ is a state of the system, $u_k \in \mathbb{R}^r$ is a control input and $p_k \in \mathbb{R}^l$ is a known time-varying parameter at time step k. Given a controller

$$u_k = \kappa(x_k, v_k, p_k), \tag{3}$$

the system (1) can be rewritten in the following closed-loop form

$$\begin{aligned} x_{k+1} &= f_{cl}(x_k, v_k, p_k),\ k \in \mathbb{Z}_+, \\ y_k &= h_{cl}(x_k, v_k), \end{aligned} \tag{4}$$

with $v_k \in \mathbb{R}^m$ being an active-reference signal. In the case of a regulation problem, for the sake of simplicity, we omit v from the notation. The set of constraints can be rewritten as

$$y_k \in \mathcal{Y}, p_k \in \mathcal{P}, \tag{5}$$

where y_k can represent any combination of the state and input constraints.

2.3 Some concepts of invariant sets

Definition 1 *The maximal robust output admissible set $\mathcal{O}_\infty$ for the closed-loop system* (4) *subject to constraints* (5) *is defined as follows.*

$$\begin{aligned} \mathcal{O}_\infty = \{(\bar{v}, x_k) :& y_{k+j} \in \mathcal{Y}, (\bar{v}, x_{k+j}) \in \mathcal{O}_\infty, \\ & \forall p_{k+j} \in \mathcal{P}, v_{k+j} = \bar{v}, j \in \mathbb{Z}_+\}. \end{aligned} \tag{6}$$

Definition 2 (Robust positive-invariant set) *Given a constant active reference signal $v_{k+j} = \bar{v}$, a set $\mathcal{X}_T \subseteq \mathcal{X}$ is said to be robust positive-invariant set subject to the system representation* (1) *with the controller* (3) *and the constraints* (2), *if $\forall x \in \mathcal{X}_T$ and $\forall p \in \mathcal{P}$, $\kappa(x, v, p) \in \mathcal{U}$ and $f(x, \kappa(x, \bar{v}, p), p) \in \mathcal{X}_T$,*

Definition 3 (Homothetic sets) *Sets $\mathcal{X} \subset \mathbb{R}^n$ and $\mathcal{Y} \subset \mathbb{R}^n$ are called **homothetic** if and only if*

$$\mathcal{Y} = z \oplus \alpha\mathcal{X} \text{ for some } z \in \mathbb{R}^n \text{and } \alpha \in \mathbb{R}_+. \tag{7}$$

2.4 Other definitions

Definition 4 (Limit of the sequence) *We say that* $\lim_{n\to\infty} x_n = L$ *if for every* $\epsilon > 0$ *there is an integer* N *such that* $|x_n - L| < \epsilon$ *for all* $n > N$.

Definition 5 (Equilibrium point stability) *Let* $\bar{x}$ *be a stationary fixed point of an autonomous parameter-varying system* $x_{k+1} = f(x_k, p_k), p_k \in \mathcal{P}$, *then* $\bar{x}$ *is stable equilibrium point if for every neighborhood* $\mathcal{X}_\epsilon$ *of* $\bar{x}$, *there exist a neighborhood* $\mathcal{X}_\delta$ *such that* $x_0 \in \mathcal{X}_\delta \Rightarrow x_k \in \mathcal{X}_\epsilon, \forall k \geq 0$.

Definition 6 (Local asymptotic stability) *Let* $\bar{x}$ *be a stationary fixed point of an autonomous parameter-varying system* $x_{k+1} = f(x_k, p_k)$, *where* $p_k \in \mathcal{P}$, *then* $\bar{x}$ *is locally asymptotically stable equilibrium point if it is stable and there exists a neighborhood* $\mathcal{X}_\epsilon$ *of* $\bar{x}$, *such that for all* $x_k \in \mathcal{X}_\epsilon$, $\lim_{k\to\infty} x_k \to \bar{x}$.

3 CONSTRAINED REFERENCE TRACKING

This section provides an overview of the main existing approaches to a constrained reference tracking: (i) a reference governor approach and (ii) a tracking MPC approach.

3.1 Reference governor approach

A reference governor approach considers an asymptotically stable closed-loop discrete-time system (4) subject to constraints (5). In order to ensure the constraint satisfaction an active reference v_k has to be suitably modified using a system called reference governor. First a scalar reference governor ($m = 1$) is considered, followed by a vector reference governor $m > 1$.

3.1.1 Scalar reference governor

The scalar reference governor modifies the active reference signal v_k using the following update law:

$$v_k = v_{k-1} + \beta_k(r_k - v_{k-1}), \tag{8}$$

where $r_k \in \mathbb{R}^m$ represents the desired reference signal and $\beta_k \in [0, 1]$ is a positive adjustable scalar. At each time step k, an admissible active reference signal v_k is chosen so as to guarantee the constraint satisfaction at all times. To achieve that, usually a set-membership constraint

$$(v_k, x_k) \in \mathcal{T}, \tag{9}$$

is used, with $\mathcal{T} \subseteq \mathcal{O}_\infty$ being a positive-invariant subset of the maximal output admissible set.

The objective of a reference governor is to find an admissible reference signal v_k as close as possible to the desired reference signal r_k, which can be formulated as the following optimization problem.

Problem 1 (Scalar reference governor)

$$\begin{aligned} &\max \beta_k \\ \text{subject to } &0 \leq \beta_k \leq 1 \\ &v_k = v_{k-1} + \beta_k(r_k - v_{k-1}) \\ &(x_k, v_k) \in \mathcal{T} \end{aligned} \tag{10}$$

Problem 1 is recursively feasible due to the positive-invariance of the set $\mathcal{T}$, which guarantees that $\beta_k = 0$ is always a feasible solution. In addition it guarantees the stability and constraint satisfaction at all times.

3.1.2 Vector reference governor

In the case when $m > 1$ the scalar reference governor can be easily extended by using a diagonal matrix $B_k = diag(\beta_{1,k}, \beta_{2,k}, \ldots, \beta_{m,k})$ instead of scalar β_k. Similarly the elements of the matrix B_k are chosen to minimize the distance between an (multidimensional) active reference and a desired reference signal, given by the following optimization problem [9].

Problem 2 (Vector reference governor) *Given the matrix* $Q = Q^T \succ 0$, *solve the following problem*

$$\begin{aligned} &\min\, (v_k - r_k)^T Q (v_k - r_k) \\ \text{subject to } &0 \leq \beta_{i,k} \leq 1, i = 1, \ldots, m \\ &v_k = v_{k-1} + B_k(r_k - v_{k-1}) \\ &(x_k, v_k) \in \mathcal{T}. \end{aligned} \tag{11}$$

3.2 Reference tracking MPC

As an alternative to the reference governor for constrained reference tracking problems, a tracking MPC approach can be used. Before describing the tracking MPC, a standard MPC problem that uses a terminal set and a terminal cost [1] is formulated.

Problem 3 (MPC with a terminal set and a terminal cost) *Given a stage cost* $l(\cdot)$, *a terminal cost* $\psi(\cdot)$, *a terminal set* $\mathcal{X}_T$ *and a measured state* x_k *at time step* k, *at every sampling instant solve the following optimization problem:*

$$\begin{aligned} &\min_{u,x} \psi(x_{k+N}) + \sum_{i=0}^{N-1} l(x_{k+i}, u_{k+i}), \\ &\text{s.t } x_{k+1+i} = f(x_{k+i}, u_{k+i}, p_{k+i}), i = 0, \ldots, N-1 \\ &\quad x_{k+i} \in \mathcal{X}, i = 0, \ldots N \\ &\quad u_{k+i} \in \mathcal{U}, i = 0, \ldots N-1 \\ &\quad x_{k+N} \in \mathcal{X}_T, \end{aligned} \tag{12}$$

and apply the control input $u = u_k^*$, *where* u_k^* *is the first element of the optimal control sequence.*

The terminal set $\mathcal{X}_T$ is a robust positive-invariant set under the control law $u_k = \kappa_T(x_k, p_k)$ and contains the origin in its interior. The stage cost $l(x_k, u_k)$ is a positive definite function, and a terminal cost $\psi(x_k)$ is a local Lyapunov function which satisfies $\psi(x_{k+1}) - \psi(x_k) + l(x_k, \kappa(x_k, p_k)) \leq 0, \forall x_k \in \mathcal{X}_T$.

If the reference signal $r_k \neq 0$, then the system generally converges to a non-zero steady state $(\bar{x}, \bar{u})$, corresponding to the reference signal. The standard MPC formulation given with Problem 3 guarantees the stability and the recursive feasibility for a fixed reference signal. However if the reference signal is not constant but rather time-varying the recursive feasibility can be lost. In order to solve the aforementioned problem in [16] a so-called dual-mode MPC for tracking a piecewise constant references for constrained linear systems is proposed. The proposed controller consists of a two modes: a feasibility recovery mode which closely resembles a reference governor, and a predictive controller which is employed once the feasibility is recovered. An alternative approach is proposed in [5], where the predictive controller minimizes a cost function with respect to an artificial steady-state, while at the same time minimizes the distance between the artificial steady-state and the desired setpoint using a so called offset cost function. The stability of the proposed approach is guaranteed by the virtue of an augmented terminal set calculated for the tracking MPC. The conditions on the offset-cost function under which a locally optimal controller is recovered inside the terminal set are given in [17]. The proposed approach is extended for tracking of non-linear systems in [6] and for robust tube-based MPC in [18]. The presented approaches for the tracking MPC require calculating the terminal set in the extended state-reference space, which increases the complexity of the corresponding terminal set. Additionally, for non-linear systems such set can be rather difficult to calculate or the resulting set is overly conservative approximation of the true terminal set. To overcome the problem of the increased complexity of the terminal set an alternative approach is presented in [7, 8] which utilizes the idea of scaling and translating the terminal set computed for the regulation problem to track a given reference signal. In this paper we combine the ideas of the reference governor [9] and the tracking MPC [7, 8, 16] to develop a predictive reference management solution to constrained reference tracking problem for a class of non-linear, parameter-varying systems, applicable in the form of both reference governor and tracking MPC.

The approach [7, 8] assumes a unique (invertible) mapping between a fixed point $(\bar{x}, \bar{u})$ and an active reference signal v:

$$(\bar{x}, \bar{u}) = \phi(v), \tag{13}$$

and formulates a tracking MPC problem as follows:

Problem 4 (Tracking MPC) *Given a positive definite stage cost $l(\cdot)$, a terminal cost $\psi(\cdot)$, a terminal set $\mathcal{X}_T$ and a controller* (3)*, solve the following optimization problem:*

$$\begin{aligned}
\min_{\mathbf{u}_k, \mathbf{x}_k, \lambda_k, \bar{x}_k, \bar{u}_k, v_k} \; & \psi(x_{k+N} - \bar{x}_k) + \alpha(v_k - r_k) + \\
& + \sum_{i=0}^{N-1} l(x_{k+i} - \bar{x}_k, u_{k+i} - \bar{u}_k), \\
\text{s.t}\; & x_{k+1+i} = f(x_{k+i}, u_{k+i}, p_{k+i}), \\
& x_{k+i} \in \mathcal{X}, \\
& u_{k+i} \in \mathcal{U}, \\
& x_{k+N} \in \bar{x}_k \oplus \lambda_k \mathcal{X}_T, \\
& (\bar{x}_k, \bar{u}_k) = \phi(v_k), \\
& \bar{u}_k + \kappa(x_k - \bar{x}_k, p_k) \in \bar{U}, \forall x_k \in \bar{x}_k \oplus \lambda_k \mathcal{X}_T \\
& \bar{x}_k \oplus \lambda_k \mathcal{X}_T \subseteq \mathcal{X},
\end{aligned} \tag{14}$$

where $\alpha(\cdot)$ represents a positive definite offset function which penalizes deviation of the active reference v_k from the desired reference signal r_k and $\lambda_k \in (0, 1]$ is a scaling factor.

In order to ensure the stability of the tracking MPC, defined by Problem 4, the terminal set need to preserve its invariance under translation and scaling transformations. For linear time-invariant systems:

$$f(x_{k+i}, u_{k+i}, p_{k+i}) = Ax_{k+i} + Bu_{k+i}, \tag{15}$$

$$h(x_{k+i}, u_{k+i}, p_{k+i}) = Cx_{k+i} + Du_{k+i}, \tag{16}$$

with

$$\phi(v_k) = \begin{bmatrix} A - I & B \\ C & D \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ \bar{v}_k \end{bmatrix}, \tag{17}$$

the required property is guaranteed, which is exploited in [7, 8] to show the system stability.

In the sequel we extend this idea to a class of non-linear parameter-varying systems.

4 PREDICTIVE REFERENCE MANAGEMENT

We consider the system described by (1), with an asymptotically stabilizing controller (3), designed such that it satisfies the constraints (2) inside the corresponding positive-invariant set $\mathcal{X}_F$ which contains the origin in its interior. For an MPC problem the set $\mathcal{X}_F$ is typically calculated as a N-step controllable set $\mathcal{X}_N$ to a given terminal set $\mathcal{X}_T$, i.e. the set of states from which it is possible to reach the terminal set in N steps.

The main goal of the paper is to extend the region of attraction of the controller without recomputing a robust output admissible set in the extended reference-state space.

The idea behind the proposed approach is to use a family of admissible sets homothetic to the positive-invariant initial feasible set $\mathcal{X}_F$ to drive the system states to an equilibrium point which corresponds to a given reference signal, while satisfying input/state constraints. We consider the family of sets:

$$S(\mathcal{X}_F) = \{\bar{x} \oplus \lambda\mathcal{X}_F, \bar{x} \in \mathbb{R}^n, \lambda \in (0,1]\} \tag{18}$$

where each set is parameterized by its center (sometimes referred to as an orientation vector) $\bar{x}$, and a scaling factor λ. In order to ensure the constraint satisfaction it is necessary to calculate the admissible set of pairs $(\bar{x}, \lambda)$ such that the robust-positive invariance of the transformed set $S(\mathcal{X}_F)$ is preserved. We define a set of admissible homothetic transformations as follows:

Definition 7 *Given a system* (1) *and a controller* (3)*, subject to constraints* (2)*, we define a set with the following properties:*

$$\Gamma = \left\{ (\bar{x}, \bar{u}, \lambda) \;\middle|\; \begin{array}{l} \exists v \text{ such that} \\ \text{i) } \forall x \in \bar{x} \oplus \lambda\mathcal{X}_F, \forall p \in \mathcal{P}, \\ \bar{x} \oplus \lambda\mathcal{X}_F \in \mathcal{X}, \\ \bar{u} + \kappa(x - \bar{x}, p) \in \mathcal{U}, \\ \text{ii) } \forall x \in \bar{x} \oplus \lambda\mathcal{X}_F, \forall p \in \mathcal{P}, \\ f(x, \bar{u} + \kappa(x - \bar{x}, p), p) \in \bar{x} \oplus \lambda\mathcal{X}_F, \\ \text{iii) } \bar{x} \text{ is locally asymptotically stable} \\ \text{ equilibrium point in the set } \bar{x} \oplus \lambda X_F \\ \text{under control law} \\ \bar{u} + \kappa(x - \bar{x}, p) \in \mathcal{U}, \\ \text{iv) } (\bar{x}, \bar{u}) = \phi(v), \end{array} \right. \tag{19}$$

to be a set of admissible homothetic transformations of the positive-invariant set $\mathcal{X}_F$.

According to Definition 7, if the system states lies in the set $\bar{x} \oplus \lambda\mathcal{X}_F$, where the pair $(\bar{x}, \lambda)$ belongs to Γ then it is guaranteed that there exists a feasible control action $\bar{u} + \kappa(x - \bar{x}, p)$ that keeps the system states within the set $\bar{x} \oplus \lambda\mathcal{X}_F$. In order to ensure steering the system states towards a desired fixed point the homothetic transformation of the set $\mathcal{X}_F$ has to be performed dynamically, at every time instant k (see Fig. 1). To select the optimal homothetic transformation at time instant k, we define the following optimization problem.

Problem 5 *Given a constant reference signal r, we denote the corresponding steady state as $(\bar{x}^*, \bar{u}^*)$. Given a state x_k, at time instant k, a positive-invariant set $\mathcal{X}_F$, a set of admissible homothetic transformations Γ, and a strictly convex positive definite objective function $\bar{J}(\bar{x})$, with $\bar{J}(\bar{x}^*) = 0$, find a feasible homothetic transformation $(\bar{x}_k, \bar{u}_k, \lambda_k)$, by solving the following optimization problem*

$$\begin{aligned} (\bar{x}_k^\star, \bar{u}_k^\star, \lambda_k^\star) = \text{argmin}\, &\bar{J}(\bar{x}_k) \\ \text{subject to } &x_k \in \bar{x}_k \oplus \lambda_k\mathcal{X}_F \\ &(\bar{x}_k, \bar{u}_k, \lambda_k) \in \Gamma \\ &\bar{x}_k \oplus \lambda_k\mathcal{X}_N \subseteq \mathcal{X} \end{aligned} \tag{20}$$

and use the control law

$$u_k = \bar{u}_k^\star + \kappa(x_k - \bar{x}_k^\star, p_k). \tag{21}$$

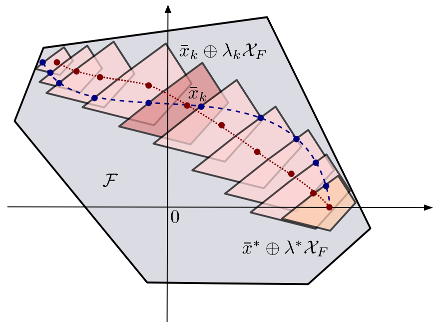

Fig. 1. The idea of the proposed reference governor

Assumption 1 *(1) There exist a number $\lambda_{min} \in (0,1]$ such that for all $(\bar{x}, \bar{u}) \in \text{Proj}_{(\bar{x},\bar{u})}\Gamma$, there exists a $\lambda > \lambda_{min}$ such that $(\bar{x}, \bar{u}, \lambda) \in \Gamma$.*

(2) $\text{Proj}_{\bar{x}}\Gamma$ is a convex set such that $\bar{x}^* \in \text{Proj}_{\bar{x}}\Gamma$.

(3) $\mathcal{X}_F$ is a convex set with a non-empty interior.

Theorem 1 *If an asymptotically stabilizing controller* (3) *which satisfies the constraints* (2) *inside a positive-invariant set $\mathcal{X}_F$ is given and Assumption 1 holds, then solving Problem 5 at every time step k, will steer the system states to the fixed point $\bar{x}^*$ corresponding to a desired constant reference signal r_k, for all $x \in \mathcal{F}$ where $\mathcal{F} \subseteq \mathcal{X}$ is the set of initial states for which Problem 5 is feasible.*

Proof: Suppose that Assumption 1 holds. Given the states of the system $x_k \in \mathcal{F}$ at a time step k, according to Definition 7 there exists a solution of Problem 5 given by the triplet $(\bar{x}_k^*, \bar{u}_k^*, \lambda_k^*) \in \Gamma$ such that $x_k \in \bar{x}_k^* \oplus \lambda_k^*\mathcal{X}_F$. This guaranties that a feasible control action $u_k = \bar{u}_k^* + \kappa(x_k - \bar{x}_k^*, p_k) \subseteq \mathcal{U}$ will keep the state x_{k+1} within the set $\bar{x}_k^* \oplus \lambda_k^*\mathcal{X}_F$ and thus in the set $\mathcal{F}$. Therefore Problem 5 is recursively feasible.

As a result of the recursive feasibility the value of the objective function is non-increasing $\bar{J}(\bar{x}^*_{k+1})-\bar{J}(\bar{x}^*_k) \leq 0$.

Due to the positive definiteness of the objective function, the value of the objective function is either positive or zero. Let's consider these two cases separately.

(i) Suppose that $\bar{J}(\bar{x}^*_k) = 0$, then the optimal solution of Problem 5 is $\bar{x}^*_{k+j} = \bar{x}^*$, $\forall j \in \mathbb{Z}_+$. By Definition 7 the control law (21) is asymptotically stabilizing subject to the translated origin $\bar{x}^*$, thus solving Problem 5 at every time step steers the system states to the fixed point $\bar{x}^*$.

(ii) Suppose that $\bar{J}(\bar{x}^*_k) > 0$. Since the value of the objective function in non-increasing, $\lim_{j\to\infty} \bar{J}(\bar{x}_{k+j}) = L$, where L can be either zero or some positive value.

a) Suppose that $L = 0$, then due to the strict convexity and the positive definiteness of the objective function $\lim_{j\to\infty} \bar{x}_{k+j} = \bar{x}^*$.

b) Suppose that $L > 0$, then there exists a time step k such that the triplet $(\bar{x}^*_k, \bar{u}^*_k, \lambda^*_k)$ is the optimal solution of Problem 5 for all future time steps. Since the control law (21) is asymptotically stabilizing subject to the translated origin $\bar{x}^*_k$, by Definition 6, for every $\rho \in (0,1)$, there exists a time step $k_\rho \in \mathbb{Z}_+$, such that $x_{k+k_\rho} \in \bar{x}^*_k \oplus \rho\lambda^*_k\mathcal{X}_F$. Given $\rho : \rho\lambda^*_k < \lambda_{min}$ and the state of the system x_{k+k_ρ} it is always possible to find a new equilibrium point $\bar{x}_{k+k_\rho} = \bar{x}^*_k + \delta_{\bar{x}}$, where $\delta_{\bar{x}} \in (\rho - \bar{\rho})\lambda^*_k\mathcal{X}_F \bigcap \mathrm{Proj}_{\bar{x}}\Gamma$ and $\bar{\rho} \in (\rho, 1]$. Since by Assumption 1 the sets $\mathrm{Proj}_{\bar{x}}\Gamma$ and $\mathcal{X}_F$ are convex, the set $\bar{x}^*_k \oplus (\rho - \bar{\rho})\lambda^*_k\mathcal{X}_F$ contains a part of the line segment connecting $\bar{x}^*_k$ and $\bar{x}^*$. Therefore there exists $\gamma \in [0,1)$ such that $\bar{x}_{k+k_\rho} = \gamma\bar{x}^*_k + (1-\gamma)\bar{x}^*$ is a feasible equilibrium point for Problem 5. Due to strict convexity and the positive-definiteness of the objective function it follows that $\bar{J}(\bar{x}_{k+k_\rho}) \leq \gamma\bar{J}(\bar{x}^*_k)$ and $\lim_{j\to\infty} \bar{J}(\bar{x}_{k+j}) = 0$, which is a contradiction and therefore $L = 0$.

By Definition 7, there exists a triplet $(\bar{x}^*, \bar{u}^\star, \lambda^\star) \in \Gamma$ which represents the optimal solution of Problem 5, $\forall x \in \bar{x}^* \oplus \lambda^\star\mathcal{X}_F$. Since $\lim_{j\to\infty} \bar{x}_{k+j} = \bar{x}^*$, by Definition 4, there exist a finite time step $k_{\lambda^\star}$ such that $\bar{x}_{k+k_{\lambda^\star}} \in \mathrm{int}(\bar{x}^* \oplus \lambda^\star\mathcal{X}_F)$. Furthermore, by Definition 6 there exist a time step $k_{x^\star}$ such that $x_{k+k_{x^\star}} \in \bar{x}^* \oplus \lambda^\star\mathcal{X}_F$ and therefore the feasible equilibrium point $\bar{x}_{k+j}$ converges to $\bar{x}^*$ in a finite time. This corresponds to the case (i) of the proof and therefore solving Problem 5 at every time step steers the system states to the fixed point $\bar{x}^*$.

■

Remark 1 *Once the set of admissible homothetic transformations Γ is calculated we can obtain the output admissible set as follows*

$$\mathcal{T} = \{(v,x) : \exists\lambda \in (0,1], (\phi(v),\lambda) \in \Gamma, x \in \mathcal{F}\}, \quad (22)$$

and use the standard reference governor, as given in Problem 1 or 2.

The idea of utilizing a family of homothetic transformations $S(\mathcal{X}_F)$ can be further used to extend the tracking MPC proposed in [7,8] to the non-linear parameter-varying case. Given a positively-invariant terminal set $\mathcal{X}_T$ for the system (1) under control law (3) and a suitable terminal cost $\psi(\cdot)$ we pose the tracking MPC problem as follows.

Problem 6 *Given a set of admissible homothetic transformations Γ, a stage cost $l(\cdot)$, a terminal set $\mathcal{X}_T$, a terminal cost $\psi(\cdot)$ and a controller* (3)*, the tracking MPC problem can be written as follows*

$$\begin{aligned}
\min_{\mathbf{u},\mathbf{x},\lambda_k,\bar{x}_k,\bar{u}_k,v_k} \quad & \psi(x_{k+N} - \bar{x}_k) + \alpha(v_k - r_k) + \\
& + \sum_{i=0}^{N-1} l(x_{k+i} - \bar{x}_k, u_{k+i} - \bar{u}_k), \\
s.t\; & x_{k+1+i} = f(x_k, u_k, p_k), \\
& x_{k+i} \in \mathcal{X}, \\
& u_{k+i} \in \mathcal{U}, \\
& x_{k+N} \in \bar{x}_k \oplus \lambda_k\mathcal{X}_T, \\
& (\bar{x}_k, \bar{u}_k) = \phi(v_k), \\
& (\bar{x}_k, \bar{u}_k, \lambda_k) \in \Gamma, \\
& \bar{u}_k + \kappa(x_k - \bar{x}_k, p_k) \in \mathcal{U}, \forall x_k \in \bar{x}_k \oplus \lambda_k\mathcal{X}_T \\
& \bar{x}_k \oplus \lambda_k\mathcal{X}_T \subseteq \mathcal{X},
\end{aligned} \quad (23)$$

where $l(\cdot)$ represents a positive definite stage cost and $\alpha(\cdot)$ represents a positive definite offset function.

4.1 Application of the proposed method to mechatronic systems

Earlier in this section we have provided sufficient conditions to use the homothetic set transformation in a tracking MPC and a reference governor control of non-linear parameter-varying systems. However, the existence of a set of admissible homothetic transformations is not guaranteed and even if exists it may be far from trivial to calculate in the general case.

For a class of non-linear systems consisting of a non-linear, possibly parameter-varying, dynamics and an integrator:

$$\begin{aligned}
\dot{x}_1 &= f(x_1, u, p), \\
\dot{x}_2 &= x_1,
\end{aligned} \quad (24)$$

where $f(0,0,p)=0$, it is straightforward to identify the associated set of admissible homothetic transformations. Denoting $x=(x_1\,x_2)^T$ the system has the non-linear part which does not depend on the subspace defined by the state x_2 and consequently every point $(0,x_2)$ is an equilibrium point with $\bar{u}=0$. Thus the set Γ can be readily identified as follows:

$$\begin{aligned}\Gamma=\{(\bar{x},\bar{u},\lambda):\bar{u}=0,\bar{x}=(0,x_2),\\ \bar{x}\oplus\lambda\mathcal{X}_F\subseteq\mathcal{X},\lambda=1\}.\end{aligned}\tag{25}$$

If the set $\mathcal{X}_F$ preserves its positive-invariance under scaling transformation, which is the case for linear or linear-parameter varying systems the set Γ can be calculated as:

$$\begin{aligned}\Gamma=\{(\bar{x},\bar{u},\lambda):\bar{u}=0,\bar{x}=(0,x_2),\\ \bar{x}\oplus\lambda\mathcal{X}_F\subseteq\mathcal{X},\lambda=(0,1]\}.\end{aligned}\tag{26}$$

It is important to note that many mechatronic systems belong to the class of non-linear systems (24) e.g. most of the position and velocity controlled drives, crane systems, robotic systems, etc. In the following section the method will be applied to a 3D tower crane system. Additionally the proposed method will be compared to other reference tracking methods on a double integrator example.

5 THE RESULTS

In this section we compare the proposed reference governor based on a set of admissible homothetic transformations to the standard scalar reference governor and the tracking MPC. Additionally the comparison between the tracking MPC proposed in [8] and the one that uses a set of admissible homothetic transformations is performed. As a first example we consider a double integrator system, while in the second example we consider an application of the proposed method to a laboratory model of a tower crane which represents a non-linear parameter-varying system.

5.1 Application to a double-integrator system

A double-integrator system, used in the first example, is adopted from [9] and described by the following equation:

$$\begin{aligned}\dot{x}_1&=x_2\\ \dot{x}_2&=u.\end{aligned}\tag{27}$$

The system (27) is further converted to a discrete-time system with a sampling time $T_s=0.1s$. A nominal controller for a regulation problem (i.e. zero setpoint) is defined as an LQR controller that minimizes the infinite horizon cost function:

$$J_{LQR}(u)=\sum_{i=0}^{\infty}x_{k+i}^TQx_{k+i}+u_{k+i}^TRu_{k+i},\tag{28}$$

with $Q=I$ and $R=1$. The obtained controller is given as follows:

$$u=-0.917x_1-1.636x_2.\tag{29}$$

The constraints on control input and states are:

$$-1\leq x_1\leq 1,-0.1\leq x_2\leq 0.1,-1\leq u\leq 1.\tag{30}$$

Given an initial state x_0, the control goal is to steer the system states to a new equilibrium point $(\bar{x}_1\,0)^T$, where $\bar{x}_1=r$ and r represents the desired set-point.

In this paper we have compared five different controllers for the constrained reference tracking problem of the system (27)-(30), briefly described below.

i) **Scalar reference governor with LQR.** The first controller, presented in [9], combines a scalar-reference governor with an LQR controller and it requires an offline computation of the maximal output-admissible set.

ii) **Scalar reference governor with homothetic transformation.** Instead of calculating the maximal output admissible set, the second controller combines a scaling and translation of the positive-invariant set calculated for the LQR controller. Instead of the maximal output-admissible set we use a set of admissible homothetic transformations according to (26) and choose Γ as follows:

$$\begin{aligned}\Gamma=\{(\bar{x},\bar{u},\lambda):-1\leq\bar{x}_1\leq 1,\bar{x}_2=0,\\ \bar{u}=0,\lambda=(0,1]\},\end{aligned}\tag{31}$$

while the mapping between the active reference signal and the corresponding steady state is given as follows:

$$\phi(v)=(v\,0)^T.\tag{32}$$

iii) **MPC with a reference governor based on the homothetic transformation.** The third controller combines an MPC controller which uses a terminal cost and a terminal set constraint to guarantee the stability in a positive-invariant initial feasible set around the equilibrium, with the reference governor based on the homothetic transformation of the corresponding positive-invariant initial feasible set, using a set of admissible homothetic transformations according to (31). The MPC controller is synthesized to minimize a finite horizon cost function:

$$\begin{aligned}J_{MPC}(x_k)=x_{k+N}^TPx_{k+N}+\sum_{i=0}^{N-1}x_{k+i}^TQx_{k+i}\\ +u_{k+i}^TRu_{k+i},\end{aligned}\tag{33}$$

with $N = 5$ and P chosen as the solution of the discrete-time algebraic Ricatti equation:

$$P = \begin{bmatrix} 17.8349 & 10.0125 \\ 10.0125 & 17.8566 \end{bmatrix}, \tag{34}$$

while the corresponding terminal set is chosen as a maximum admissible positive-invariant set for the corresponding LQR controller.

iv) **Tracking MPC with translation and scaling of the terminal set.** The fourth controller employs a tracking MPC controller proposed in [8] and given in Problem 4 with:

$$\psi(x) = x^T Px,\ l(x,u) = x^T Qx + u^T Ru, \\ \alpha(x) = \|x\|_\infty. \tag{35}$$

v) **Tracking MPC based on homothetic transformation.** The fifth controller employs a tracking MPC controller given in Problem 6 using Γ given in (31).

The system responses for all the controllers together with the pseudo-reference signal are shown in Fig 2. The pseudo-reference signal v starts from 0.109 for the LQR controller (i) and from 0.2107 in the case of the MPC controllers ((iii)-(v)) as a result of the larger positive-invariant initial feasible sets of the MPC controllers. The value of infinite-horizon objective function is approximated using the cost function

$$\hat{J}(x_k) = x^T_{k+N_{sim}} P x_{k+N_{sim}} + \\ + \sum_{i=0}^{N_{sim}-1} x^T_{k+i} Q x_{k+i} + u^T_{k+i} R u_{k+i}. \tag{36}$$

The achieved values of the cost function are $J_1 = 21.4741$, $J_2 = 21.4772$, $J_3 = 21.4245$, $J_4 = 21.4048$, $J_5 = 21.4048$ for the controllers (i)-(v), respectively. The results show that a constrained reference tracking can be achieved by using either a reference governor or a tracking MPC with the reference governor approach being more conservative comparing to a tracking MPC approach. In addition, the comparison between the controller (iv) and (v) show that the proposed conservative approximation of a set of admissible homothetic transformations in the case of a double-integrator system gives the same results as the method proposed in [8]. However such the approximation can immediately be applied to a class of non-linear parameter-varying systems as shown in the sequel.

5.2 Application to a 3D tower crane system

In order to show the applicability of the proposed approach to a real non-linear parameter varying system, it has been tested both by means of simulations and experimentally on the 3D tower crane laboratory model.

The equations of the system motion of the tower crane can be derived via Lagrange equations, by defining the total potential and kinetic energy of the system as functions of generalized coordinates: jib angular position θ, swing angle ϕ, trolley position x, swing angle α and cable length L, according to Fig 3. If the swing angles are assumed to be small during the normal operation of the crane $(\sin\alpha \approx \alpha, \cos\alpha \approx 1, \sin\phi \approx \phi, \cos\phi \approx 1)$ and the swing is assumed to be slow especially in the case of a long cable length $(\dot{\alpha} \approx 0, \dot{\phi} \approx 0)$, the equations of motion can be approximated as

$$\ddot{L} + \left(\frac{1}{\tau_L}\right)\dot{L} - mg = \frac{K_L}{\tau_L}u_L, \tag{37}$$

$$\ddot{x}_t + \left(\frac{1}{\tau_x}\right)\dot{x}_t - m_t g\alpha = \frac{K_x}{\tau_x}u_x, \tag{38}$$

$$L\ddot{\alpha} + g\alpha + \ddot{x}_t = 0, \tag{39}$$

$$(1 + M_r x_t^2)\ddot{\theta} + \left(\frac{1}{\tau_\theta}\right)\dot{\theta} - m_r g x_t \phi = \frac{K_\theta}{\tau_\theta}u_\theta, \tag{40}$$

$$L\ddot{\phi} + g\phi + x_t\ddot{\theta} = 0, \tag{41}$$

where K_L, K_x, K_θ represents the gains of the corresponding DC motors of the crane system, and τ_L, τ_x, τ_θ represents the corresponding time-constants. In addition m represents the mass of the payload, m_t represents a relative mass of the payload compared to the trolley, while M_r represents a relative mass of the trolley compared to the moment inertia of the jib. The parameters of the crane are given in Tab. 1. The crane is considered as a three dynamically coupled subsystems while the coupling among them is treated as a change in the system parameters

Table 1. Identified parameters of the tower crane

Parameter	Value
K_L	0.0108 $\frac{\text{m}}{\text{Vs}}$
K_x	0.0154 $\frac{\text{m}}{\text{Vs}}$
K_θ	0.11 $\frac{\text{rad}}{\text{Vs}}$
τ_L	0.02 s
τ_x	0.02 s
τ_θ	0.055 s
m	0.32 kg
m_t	0.0852
m_r	0.0419
M_r	0.0917

$$\dot{x}^{(1)} = A^{(1)}x^{(1)} + B^{(1)}u^{(1)}, \tag{42}$$

$$\dot{x}^{(2)} = A^{(2)}(x^{(1)})x^{(2)} + B^{(2)}(x^{(1)})u^{(2)}, \tag{43}$$

$$\dot{x}^{(3)} = A^{(3)}(x^{(1)}, x^{(2)})x^{(3)} + B_d^{(3)}(x^{(1)}, x^{(2)})u^{(3)}, \tag{44}$$

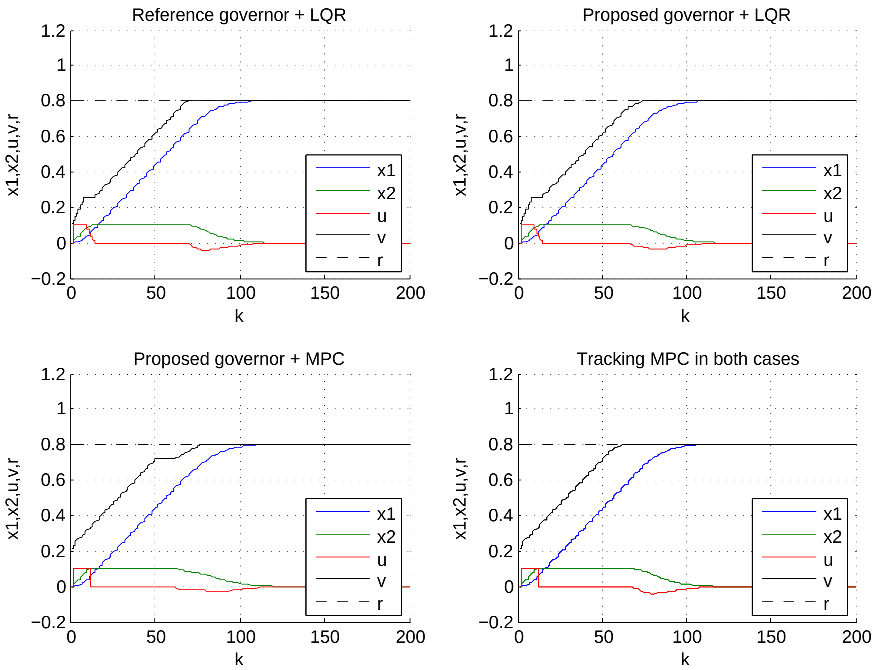

Fig. 2. System response for the double integrator system

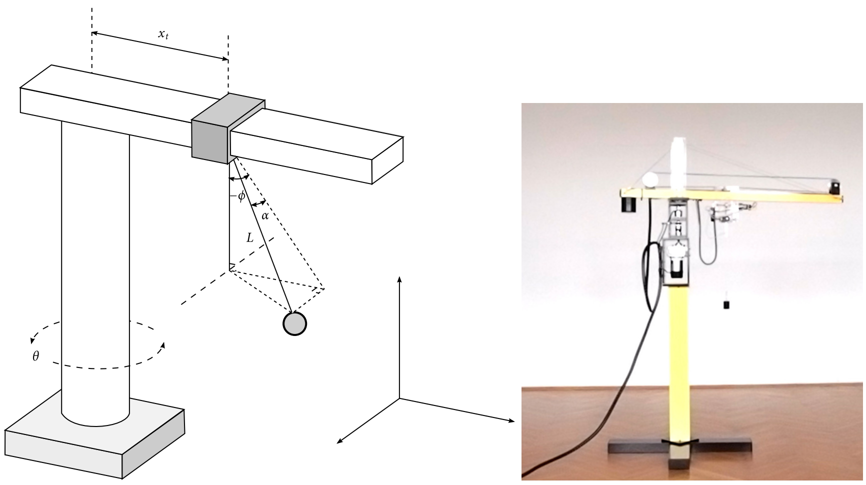

Fig. 3. Experimental model of a 3D Tower crane

where $\left(A^{(i)}(x^{(1)},\ldots,x^{(i-1)}), B^{(i)}(x^{(1)},\ldots,x^{(i-1)})\right) \in Co\left(A^{(i)}_{j^{(i)}}, B^{(i)}_{j^{(i)}}\right)$ where Co denotes the convex hull defined by the vertices $A^{(i)}_{j^{(i)}}$ and $B^{(i)}_{j^{(i)}}$, $j^{(i)} = 1,\ldots,R^{(i)}$ and $R^{(i)}$ denotes the number of vertices for the corresponding subsystem. The corresponding vertices are obtained by sampling the original model in a dense grid in a work-space of the crane system defined as $[L_{min}, L_{max}] = [0.2, 0.8]$ and $[x_{min}, x_{max}] = [0.24, 0.6]$, by using the tensor product model transformation [19]. Such polytopic state-space linear parameter varying (LPV) model is suitable for controller synthesis using linear matrix (LMI) inequalities [20]. In a similar way a discrete-time polytopic LPV model can be obtained

Two different controllers, have been designed for the crane system subject to input constraints

$$-12 \leq u^{(i)} \leq 12. \tag{45}$$

The first controller is a continuous-time controller calculated offline, for each subsystem of the crane, by solving a set of LMIs using the continuous-time LPV model to represent the system dynamics. The controller is designed to

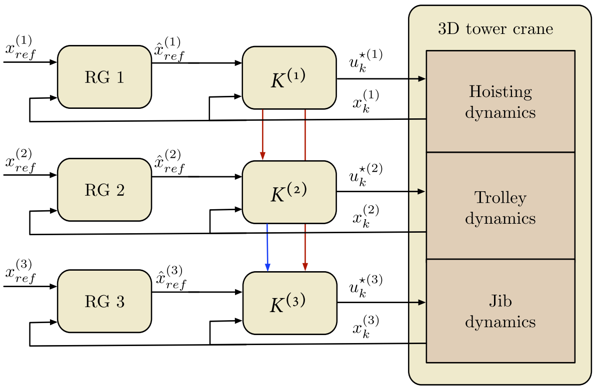

Fig. 4. Reference governor as a way to enlarge the feasibility region of decentralized/distributed control algorithms based on control invariant sets

maximize the decay rate α of a parameter-dependent Lyapunov function for each subsystem of the crane (for details see [21]). Each controller guarantees the constraint satisfaction in a positive-invariant set which corresponds to a sublevel set of the corresponding parameter-dependent Lyapunov function.

Since the first subsystem of the crane is linear and time-invariant only the second and third subsystems will be considered in the analysis in the sequel. Starting from the resting position, the corresponding positive invariant sets allows for the maximum admissible reference signals to be $x_{\mathrm{LMI},ref}^{(2)} \leq 0.15\,\mathrm{m}$ and $x_{\mathrm{LMI},ref}^{(3)} \leq 0.4\,\mathrm{rad}$.

The second controller is a distributed dual-mode MPC algorithm which uses a terminal set and a terminal cost to guarantee the closed-loop stability and the constraint satisfaction (for details see [22]). The terminal set and the terminal cost are calculated using the LMI conditions based on the discrete-time LPV model of the crane as presented in [22], while the corresponding positive-invariant sets are calculated using LMI conditions similar to the ones presented in [23].

Starting from the resting position, the maximum allowed reference signal is defined by the corresponding positive-invariant sets. The obtained controllers guarantees the constraint satisfaction for the reference signals $x_{\mathrm{MPC},ref}^{(2)} \leq 0.133\,\mathrm{m}$ and $x_{\mathrm{MPC},ref}^{(3)} \leq 1.08\,\mathrm{rad}$ starting from the resting position of the crane.

Using a reference governor with homothetic transformations of the corresponding positive-invariant sets, the maximum allowed reference signal becomes the whole workspace of the crane since the homothetic transformation is performed dynamically.

The proposed reference governor is tested experimentally for both LMI and MPC controllers. In both cases the system is tested by starting from the initial state that is outside the positive-invariant initial feasible sets associated to the primary controllers, i.e. $x_{ref}^{(2)} = 0.36$. The simulation and experimental results for the trolley subsystem are shown in Fig. 5. As a result of the reference governor action, it can be seen that in both cases the reference signals (black lines) are altered. They start from the maximum admissible reference of the primary controller and converge to the desired reference signal. The complete crane operations, for LMI based controller and MPC controller, are shown in Fig 6 and Fig 7, respectively.

6 CONCLUSION

In this paper a constrained reference tracking is considered based on the homothetic transformation of positive-invariant sets. Conditions for using the proposed method to a special class of parameter-varying non-linear systems are presented. The presented class of non-linear systems includes many mechatronic systems, e.g. most of the position and velocity controlled drives, crane systems, robotic systems, etc. In order to show the applicability of the proposed approach, a 3D tower crane model is used together with controllers which utilize positive-invariant sets to satisfy constraints on the control input and states. The benefits of the proposed approach are shown using two different controllers: (i) a continuous-time LMI based controller and (ii) a distributed MPC controller, showing generality of the proposed method. In addition the proposed method is compared to the standard reference tracking methods on a double integrator system. The main advantage of the proposed method is ability to extend the region of attraction of the controllers based on positive-invariant sets for a special class of non-linear parameter-varying systems, still providing the guarantees for the constraint satisfaction.

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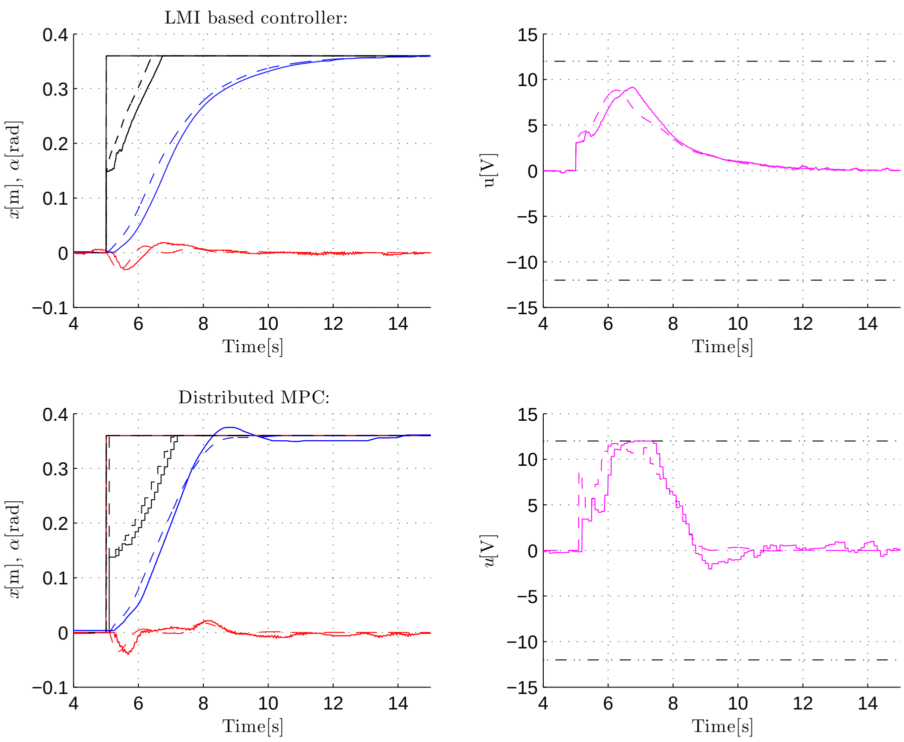

Fig. 5. Reference governor with an LMI based controller and distributed MPC for the trolley system, dashed - simulation results, solid - experimental results, black - active reference signal, blue - the position of the trolley subsystem, red - the corresponding swing angle, magenta - control input

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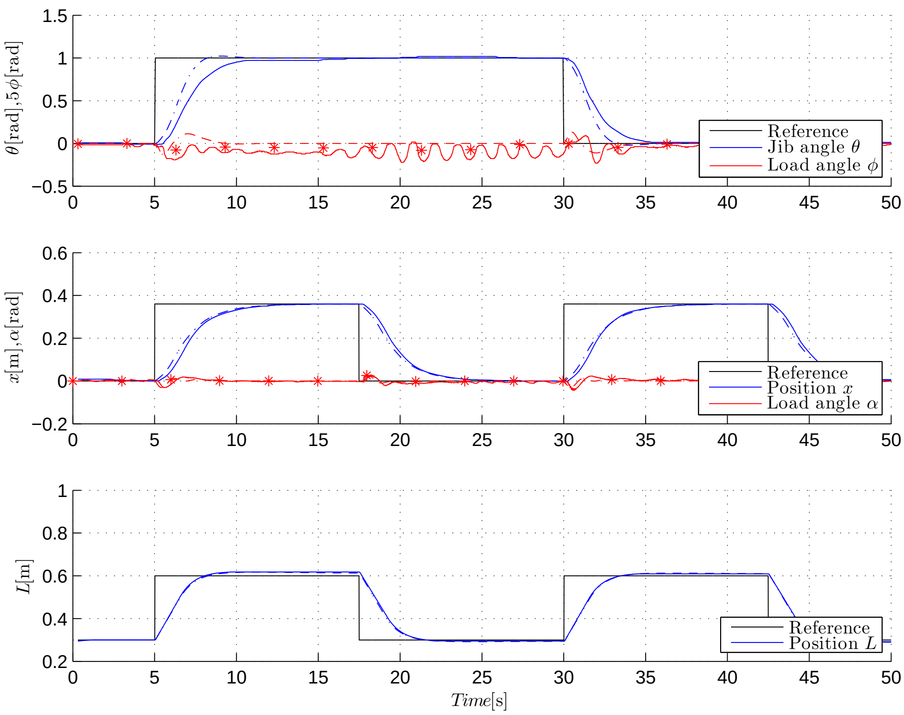

Fig. 6. Complete crane operation: LMI based controller with a reference governor

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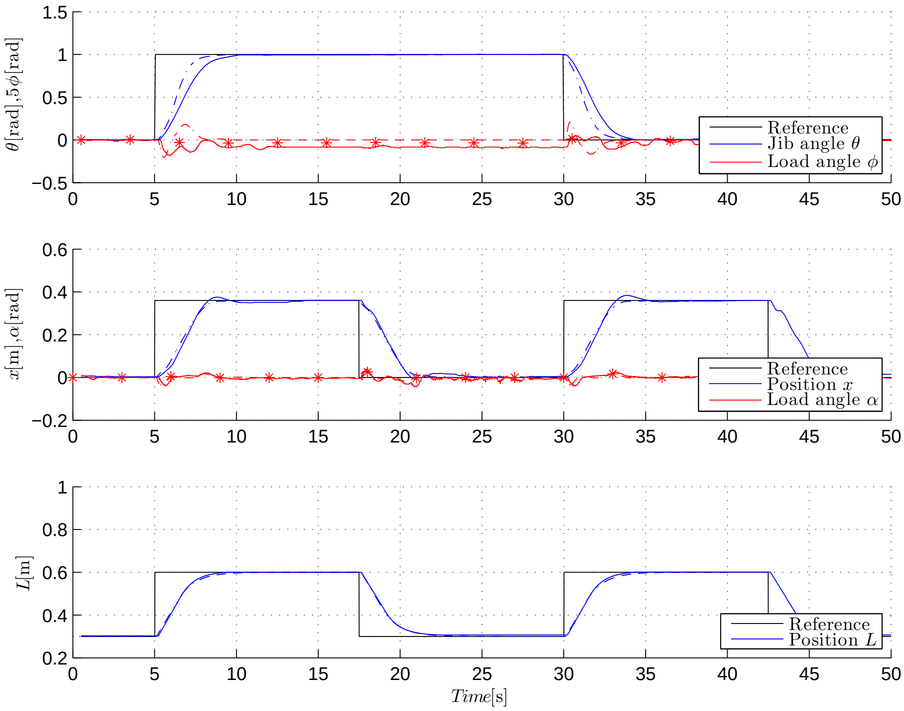

Fig. 7. Complete crane operation: MPC based controller with a reference governor

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