GLASNIK MATEMATIČKI
Vol. 38(58)(2003), 201 – 210

2-(56,12,3) DESIGNS AND THEIR CLASS GRAPHS

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Abstract. There exist exactly 1122 pairwise non-isomorphic 2-(56,12,3) designs being the residual designs of the known symmetric (71,15,3) designs. Six pairwise non-isomorphic strongly regular graphs with parameters (35,16,6,8) were constructed as class graphs of the obtained residual designs. Orders of the full automorphism groups of these graphs are 12, 32, 96, 192, 288 and 40320.

1. Introduction and preliminaries

A balanced incomplete block design (BIBD) with parameters (v, b, r, k, λ) (or $2-(v, k, \lambda)$ design) is a pair (V, B), where V is a v-set and B is a collection of b k-subsets (blocks) such that each element of V is contained in exactly r blocks and any 2-subset of V is contained in exactly λ blocks. Elements of the set V are called points. A BIBD with $b = v$ (or equivalently, $r = k$) is a symmetric (v, k, λ) design.

Let $\mathcal{D} = (V, B)$ be a symmetric (v, k, λ) design. A residual design $\mathcal{D}^x$ with respect to a block x is a BIBD whose points are those points of $\mathcal{D}$ which are not incident with x and whose blocks are the sets $y - x$, where y ranges over all blocks of $\mathcal{D}$ other than x. It is a $2 - (v - k, k - \lambda, \lambda)$ design. A BIBD with $r = k + \lambda$ is called a quasi-residual design. If such a design is a residual design of some symmetric design, then it is said to be embeddable; otherwise, nonembeddable.

First symmetric (71,15,3) designs have been constructed from an embeddable 2-(56,12,3) design. More precisely, in 1980 W.H. Haemers (see [5]) constructed eight symmetric (71,15,3) designs and proved the existence of at least four embeddable 2-(56,12,3) designs. No other constructions of 2-(56,12,3) designs were reported to [3].

2000 *Mathematics Subject Classification.* 05B05, 05E30.
Key words and phrases. Residual design, automorphism group, class graph.

All known symmetric designs with parameters (71,15,3) admit an action of the cyclic group of order six. Recently, a classification of all symmetric (71,15,3) designs admitting an action of the cyclic group of order six was completed ([2, 4, 9]) and it is known that up to isomorphism there are 146 such designs. Among them, there are 10 self-dual designs and 68 pairs of dually isomorphic designs.

Let $\mathcal{D}$ be a $2-(v,k,\lambda)$ design with just three distinct intersection numbers $k-r+\lambda$, ρ_1 and ρ_2 ($\rho_1 > \rho_2$). Then it is possible to construct a strongly regular graph, called the class graph of $\mathcal{D}$. A class graph of $\mathcal{D}$ is a graph whose vertices are equivalence classes (two blocks B_1 and B_2 are equivalent if $|B_1 \cap B_2| \in \{k, k-r+\lambda\}$), two vertices being adjacent if two blocks representing the corresponding classes have ρ_1 points in common. For further facts about class graphs of a design we refer the reader to [5].

Two non-isomorphic strongly regular graphs with parameters (35,16,6,8), being the class graphs of some 2-(56,12,3) designs, were constructed in [5].

2. 2-(56,12,3) DESIGNS

After eliminating isomorphic copies among the residual designs of all known symmetric (71,15,3) designs, we obtained 1122 pairwise non-isomorphic 2-(56,12,3) designs. Further investigation of the so constructed 2-(56,12,3) designs led us to the following results.

The order of the full automorphism group	*The full automorphism group structure*	*Number of designs*
336	$E_8 \cdot F_{21} \times Z_2$	2
168	$E_8 \cdot F_{21}$	1
48	$E_4 \times A_4$	18
42	$F_{21} \times Z_2$	6
24	$A_4 \times Z_2$	137
24	$E_4 \times S_3$	12
21	F_{21}	1
16	E_{16}	20
12	A_4	20
12	D_{12}	32
8	E_8	134
6	Z_6	120
4	E_4	156
3	Z_3	101
2	Z_2	246
1	*trivial group*	116

Investigating the intersection numbers of blocks of the constructed 2-(56,12,3) designs we have obtained that there are 83 designs with exactly three distinct intersection numbers and that among the constructed designs there are no designs with exactly two distinct intersection numbers. Therefore, the constructed designs are not quasi-symmetric and for 83 designs we could investigate the existence of their class graphs.

3. Class graphs of 2-(56,12,3) designs

Theorem 3.1. *Let $\mathcal{D}$ be a symmetric $(71, 15, 3)$ design admitting an action of the cyclic group of order six and let $\mathcal{D}^x$ be its residual design with respect to some block x. If $\mathcal{D}^x$ has exactly three distinct intersection numbers, then the class graph G of the design $\mathcal{D}^x$ exists. Moreover, G is isomorphic to one of the graphs G_i $(i = 1, 2, 3, 4, 5, 6)$ presented below.*

Proof. A class graph of a 2-(56,12,3) design with exactly three distinct intersection numbers exists if one of the intersection numbers is equal to the number $k - r + \lambda = 0$. All 83 designs from the previous section are designs with intersection numbers 0, 2 and 3. Therefore, for each of these designs it is possible to construct a corresponding class graph, being a strongly regular graph on 35 vertices (see [5, Theorem 3.2.4]), whose vertices are equivalence classes (two blocks B_1 and B_2 are equivalent if $|B_1 \cap B_2| = 0$), two vertices being adjacent if two blocks representing the corresponding classes have three points in common.

After eliminating isomorphic copies (using the Nauty by B.D. McKay [8]), we have obtained six pairwise non-isomorphic strongly regular graphs G_i $(i = 1, 2, 3, 4, 5, 6)$ with parameters (35,16,6,8). These graphs are presented below. We denote by Gr_i $(i = 1, 2, 3, 4, 5, 6)$ the full automorphism groups of the constructed graphs G_i $(i = 1, 2, 3, 4, 5, 6)$.

We shall assume that the vertices of the graphs G_i are notated by 0, . . . , 34. The graphs G_i are given in the form of the adjacency lists, ordered lexicographically.

G_1
1 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34
0 5 6 7 8 9 10 11 12 13 29 30 31 32 33 34
9 10 12 13 15 16 18 19 21 22 24 25 30 31 33 34
8 10 11 13 14 16 17 19 20 22 23 25 29 31 32 34
8 9 11 12 14 15 17 18 20 21 23 24 29 30 32 33
1 9 10 12 13 15 16 18 19 20 23 26 27 28 29 32
1 8 10 11 13 14 16 17 19 21 24 26 27 28 30 33
1 8 9 11 12 14 15 17 18 22 25 26 27 28 31 34
1 3 4 6 7 9 13 15 19 20 21 22 23 26 30 34
1 2 4 5 7 8 10 14 16 21 22 23 24 27 29 31
1 2 3 5 6 9 11 15 17 22 23 24 25 28 30 32
1 3 4 6 7 10 12 16 18 20 23 24 25 26 31 33

1 2 4 5 7 11 13 17 19 20 21 24 25 27 32 34
1 2 3 5 6 8 12 14 18 20 21 22 25 28 29 33
3 4 6 7 9 13 15 19 24 25 27 28 29 31 32 33
2 4 5 7 8 10 14 16 20 25 26 28 30 32 33 34
2 3 5 6 9 11 15 17 20 21 26 27 29 31 33 34
3 4 6 7 10 12 16 18 21 22 27 28 29 30 32 34
2 4 5 7 11 13 17 19 22 23 26 28 29 30 31 33
2 3 5 6 8 12 14 18 23 24 26 27 30 31 32 34
0 3 4 5 8 11 12 13 15 16 22 24 27 28 30 31
0 2 4 6 8 9 12 13 16 17 23 25 26 28 31 32
0 2 3 7 8 9 10 13 17 18 20 24 26 27 32 33
0 3 4 5 8 9 10 11 18 19 21 25 27 28 33 34
0 2 4 6 9 10 11 12 14 19 20 22 26 28 29 34
0 2 3 7 10 11 12 13 14 15 21 23 26 27 29 30
0 5 6 7 8 11 15 16 18 19 21 22 24 25 29 32
0 5 6 7 9 12 14 16 17 19 20 22 23 25 30 33
0 5 6 7 10 13 14 15 17 18 20 21 23 24 31 34
0 1 3 4 5 9 13 14 16 17 18 24 25 26 30 34
0 1 2 4 6 8 10 15 17 18 19 20 25 27 29 31
0 1 2 3 7 9 11 14 16 18 19 20 21 28 30 32
0 1 3 4 5 10 12 14 15 17 19 21 22 26 31 33
0 1 2 4 6 11 13 14 15 16 18 22 23 27 32 34
0 1 2 3 7 8 12 15 16 17 19 23 24 28 29 33

The group Gr_1 is a nonabelian group of order 192 isomorphic to the group $G9642D : Z_2$. Gr_1 is presented in terms of generators and relations as follows.

$$\begin{aligned} Gr_1 \quad = \quad & \Big\langle a,b,c,d \mid a^2 = b^2 = 1, \big(cd^{-1}\big)^2 = 1, \big(bc^{-1}\big)^2 = 1, ad^{-1}bd^{-2} = 1, \\ & \big(abc^{-1}\big)^2 = 1, \big(abd^{-1}\big)^2 = 1, adcbd^{-1}c^{-1} = 1 \Big\rangle \end{aligned}$$

G_2
10 11 13 14 15 16 17 18 20 21 22 23 25 26 27 28
9 12 13 14 15 16 19 22 23 24 25 26 29 30 31 32
9 10 11 12 17 18 19 20 21 24 27 28 29 30 31 32
5 6 7 8 9 12 13 14 17 18 22 23 27 28 31 32
5 6 7 8 9 10 11 12 15 16 20 21 25 26 31 32
3 4 19 20 21 22 23 24 25 26 27 28 31 32 33 34
3 4 10 11 13 14 15 16 17 18 19 24 31 32 33 34
3 4 9 10 11 12 17 18 22 23 25 26 29 30 33 34
3 4 9 12 13 14 15 16 20 21 27 28 29 30 33 34
1 2 3 4 7 8 10 14 16 17 19 21 22 24 25 28
0 2 4 6 7 9 13 15 23 24 25 27 28 29 31 34

0 2 4 6 7 12 14 16 19 22 26 27 28 30 32 34
1 2 3 4 7 8 11 13 15 18 19 20 23 24 26 27
0 1 3 6 8 10 12 17 21 24 25 26 27 30 32 34
0 1 3 6 8 9 11 18 19 20 25 26 28 29 31 34
0 1 4 6 8 10 12 17 19 20 22 23 28 30 31 33
0 1 4 6 8 9 11 18 21 22 23 24 27 29 32 33
0 2 3 6 7 9 13 15 19 20 21 22 26 29 32 33
0 2 3 6 7 12 14 16 20 21 23 24 25 30 31 33
1 2 5 6 9 11 12 14 15 17 21 23 25 27 33 34
0 2 4 5 8 12 14 15 17 18 22 24 25 29 32 34
0 2 4 5 8 9 13 16 17 18 19 23 26 30 31 34
0 1 3 5 7 9 11 15 16 17 20 24 27 30 31 34
0 1 3 5 7 10 12 15 16 18 19 21 28 29 32 34
1 2 5 6 9 10 12 13 16 18 20 22 26 28 33 34
0 1 4 5 7 9 10 13 14 18 19 20 27 30 32 33
0 1 4 5 7 11 12 13 14 17 21 24 28 29 31 33
0 2 3 5 8 10 11 12 13 16 19 22 25 29 31 33
0 2 3 5 8 9 10 11 14 15 23 24 26 30 32 33
1 2 7 8 10 14 16 17 20 23 26 27 31 32 33 34
1 2 7 8 11 13 15 18 21 22 25 28 31 32 33 34
1 2 3 4 5 6 10 14 15 18 21 22 26 27 29 30
1 2 3 4 5 6 11 13 16 17 20 23 25 28 29 30
5 6 7 8 15 16 17 18 19 24 25 26 27 28 29 30
5 6 7 8 10 11 13 14 19 20 21 22 23 24 29 30

The group Gr_2 is a nonabelian group of order 40320 isomorphic to the symmetric group S_8. Gr_2 acts on the vertices of the graph G_2 transitively.

G_3
10 11 13 14 15 16 17 18 20 21 22 23 25 26 27 28
3 4 9 12 13 14 17 18 19 22 23 24 27 28 29 30
3 4 9 10 11 12 15 16 19 20 21 24 25 26 29 30
1 2 5 6 10 11 17 18 20 21 27 28 29 30 31 32
1 2 5 6 13 14 15 16 22 23 25 26 29 30 31 32
3 4 19 20 21 22 23 24 25 26 27 28 31 32 33 34
3 4 10 11 13 14 15 16 17 18 19 24 31 32 33 34
9 11 12 13 16 17 20 23 25 28 29 30 31 32 33 34
9 10 12 14 15 18 21 22 26 27 29 30 31 32 33 34
1 2 7 8 10 14 16 17 19 21 22 24 25 28 31 32
0 2 3 6 8 9 13 15 19 20 22 23 28 29 31 34
0 2 3 6 7 12 14 16 21 22 23 24 27 30 31 34
1 2 7 8 11 13 15 18 19 20 23 24 26 27 31 32
0 1 4 6 7 10 12 17 19 20 21 22 26 30 32 34

0 1 4 6 8 9 11 18 20 21 23 24 25 29 32 34
0 2 4 6 8 10 12 17 23 24 25 27 28 30 32 33
0 2 4 6 7 9 11 18 19 22 26 27 28 29 32 33
0 1 3 6 7 9 13 15 21 24 25 26 27 29 31 33
0 1 3 6 8 12 14 16 19 20 25 26 28 30 31 33
1 2 5 6 9 10 12 13 16 18 21 23 25 27 33 34
0 2 3 5 7 10 12 13 14 18 22 24 25 29 32 33
0 2 3 5 8 9 11 13 14 17 19 23 26 30 32 33
0 1 4 5 8 9 10 11 13 16 20 24 27 30 31 33
0 1 4 5 7 10 11 12 14 15 19 21 28 29 31 33
1 2 5 6 9 11 12 14 15 17 20 22 26 28 33 34
0 2 4 5 7 9 14 15 17 18 19 20 27 30 31 34
0 2 4 5 8 12 13 16 17 18 21 24 28 29 31 34
0 1 3 5 8 11 12 15 16 17 19 22 25 29 32 34
0 1 3 5 7 9 10 15 16 18 23 24 26 30 32 34
1 2 3 4 7 8 10 14 16 17 20 23 26 27 33 34
1 2 3 4 7 8 11 13 15 18 21 22 25 28 33 34
3 4 5 6 7 8 9 10 11 12 17 18 22 23 25 26
3 4 5 6 7 8 9 12 13 14 15 16 20 21 27 28
5 6 7 8 15 16 17 18 19 20 21 22 23 24 29 30
5 6 7 8 10 11 13 14 19 24 25 26 27 28 29 30

The group Gr_3 is a nonabelian group of order 288 isomorphic to the group $(E_{16} : E_9) : Z_2$. Gr_3 is presented in terms of generators and relations as follows.

$$\begin{aligned} Gr_3 \quad = \quad & \Big\langle a, b, c, d \mid b^2 = c^2 = d^2 = 1, a^3 = 1, (ab)^2 = 1, (ac)^2 = 1, \\ & \big(ca^{-1}d\big)^2 = 1, (bc)^3 = 1, \big(ada^{-1}d\big)^2 = 1, (bd)^4 = 1, \\ & adbda^{-1}da^{-1}bd = 1, ada^{-1}cbcdbdcb = 1\Big\rangle \end{aligned}$$

G_4
10 11 13 14 15 16 17 18 20 21 22 23 25 26 27 28
3 4 9 12 13 14 17 18 19 22 23 24 27 28 29 30
3 4 9 10 11 12 15 16 19 20 21 24 25 26 29 30
1 2 5 6 13 14 15 16 22 23 25 26 29 30 31 32
1 2 5 6 10 11 17 18 20 21 27 28 29 30 31 32
3 4 19 20 21 22 23 24 25 26 27 28 31 32 33 34
3 4 10 11 13 14 15 16 17 18 19 24 31 32 33 34
9 11 12 13 16 17 20 23 25 28 29 30 31 32 33 34
9 10 12 14 15 18 21 22 26 27 29 30 31 32 33 34
1 2 7 8 10 14 16 17 19 20 23 24 26 27 31 32
0 2 4 6 8 9 13 15 19 20 22 23 28 29 32 34

0 2 4 6 7 12 14 16 21 22 23 24 27 30 32 34
1 2 7 8 11 13 15 18 19 21 22 24 25 28 31 32
0 1 3 6 7 10 12 17 19 20 21 22 26 30 31 34
0 1 3 6 8 9 11 18 20 21 23 24 25 29 31 34
0 2 3 6 8 10 12 17 23 24 25 27 28 30 31 33
0 2 3 6 7 9 11 18 19 22 26 27 28 29 31 33
0 1 4 6 7 9 13 15 21 24 25 26 27 29 32 33
0 1 4 6 8 12 14 16 19 20 25 26 28 30 32 33
1 2 5 6 9 10 12 13 16 18 21 23 25 27 33 34
0 2 4 5 7 9 10 13 14 18 22 24 25 30 31 33
0 2 4 5 8 11 12 13 14 17 19 23 26 29 31 33
0 1 3 5 8 10 11 12 13 16 20 24 27 29 32 33
0 1 3 5 7 9 10 11 14 15 19 21 28 30 32 33
1 2 5 6 9 11 12 14 15 17 20 22 26 28 33 34
0 2 3 5 7 12 14 15 17 18 19 20 27 29 32 34
0 2 3 5 8 9 13 16 17 18 21 24 28 30 32 34
0 1 4 5 8 9 11 15 16 17 19 22 25 30 31 34
0 1 4 5 7 10 12 15 16 18 23 24 26 29 31 34
1 2 3 4 7 8 10 14 16 17 21 22 25 28 33 34
1 2 3 4 7 8 11 13 15 18 20 23 26 27 33 34
3 4 5 6 7 8 9 12 13 14 15 16 20 21 27 28
3 4 5 6 7 8 9 10 11 12 17 18 22 23 25 26
5 6 7 8 15 16 17 18 19 20 21 22 23 24 29 30
5 6 7 8 10 11 13 14 19 24 25 26 27 28 29 30

The group Gr_4 is a nonabelian group of order 32 isomorphic to the group $(D_8 \times Z_2) : Z_2$. Gr_4 is presented in terms of generators and relations as follows.

$$Gr_4 = \left\langle a, b, c, d \mid a^2 = b^2 = c^2 = d^2 = 1, (ac)^2 = 1, adcd = 1, (bd)^2 = 1, (cab)^2 = 1, (ab)^4 = 1 \right\rangle$$

G_5
1 8 9 10 11 12 13 14 15 16 29 30 31 32 33 34
0 5 6 7 23 24 25 26 27 28 29 30 31 32 33 34
9 10 12 13 18 19 21 22 24 25 27 28 30 31 33 34
8 10 11 13 17 19 20 22 23 25 26 28 29 31 32 34
8 9 11 12 17 18 20 21 23 24 26 27 29 30 32 33
1 8 11 14 15 16 18 19 21 22 24 25 27 28 29 32
1 9 12 14 15 16 17 19 20 22 23 25 26 28 30 33
1 10 13 14 15 16 17 18 20 21 23 24 26 27 31 34
0 3 4 5 12 13 15 16 20 21 25 26 27 28 30 34

0 2 4 6 11 13 14 16 21 22 23 26 27 28 31 32
0 2 3 7 11 12 14 15 20 22 24 26 27 28 29 33
0 3 4 5 9 10 15 16 18 19 23 24 25 26 31 33
0 2 4 6 8 10 14 16 17 19 23 24 25 27 29 34
0 2 3 7 8 9 14 15 17 18 23 24 25 28 30 32
0 5 6 7 9 10 12 13 18 19 20 21 25 26 29 32
0 5 6 7 8 10 11 13 17 19 21 22 23 27 30 33
0 5 6 7 8 9 11 12 17 18 20 22 24 28 31 34
3 4 6 7 12 13 15 16 21 22 24 25 29 31 32 33
2 4 5 7 11 13 14 16 20 22 23 25 29 30 33 34
2 3 5 6 11 12 14 15 20 21 23 24 30 31 32 34
3 4 6 7 8 10 14 16 18 19 27 28 30 31 32 33
2 4 5 7 8 9 14 15 17 19 26 28 29 31 33 34
2 3 5 6 9 10 15 16 17 18 26 27 29 30 32 34
1 3 4 6 7 9 11 12 13 15 18 19 27 28 29 34
1 2 4 5 7 10 11 12 13 16 17 19 26 28 30 32
1 2 3 5 6 8 11 12 13 14 17 18 26 27 31 33
1 3 4 6 7 8 9 10 11 14 21 22 24 25 30 34
1 2 4 5 7 8 9 10 12 15 20 22 23 25 31 32
1 2 3 5 6 8 9 10 13 16 20 21 23 24 29 33
0 1 3 4 5 10 12 14 17 18 21 22 23 28 30 31
0 1 2 4 6 8 13 15 18 19 20 22 24 26 29 31
0 1 2 3 7 9 11 16 17 19 20 21 25 27 29 30
0 1 3 4 5 9 13 14 17 19 20 22 24 27 33 34
0 1 2 4 6 10 11 15 17 18 20 21 25 28 32 34
0 1 2 3 7 8 12 16 18 19 21 22 23 26 32 33

The group Gr_5 is a nonabelian group of order 96 isomorphic to the group $G9633D$. Gr_5 is presented in terms of generators and relations as follows.

$$Gr_5 = \left\langle a, b, c, d \mid a^3 = c^3 = 1, a^{-1}b^{-2} = 1, (ac)^2 = 1, (bc)^4 = 1, \left(bcb^{-1}c^{-1}\right)^2 = 1 \right\rangle$$

G_6
1 11 12 13 17 18 19 20 21 22 26 27 28 32 33 34
0 5 6 7 23 24 25 26 27 28 29 30 31 32 33 34
9 10 12 13 15 16 21 22 24 25 27 28 30 31 33 34
8 10 11 13 14 16 20 22 23 25 26 28 29 31 32 34
8 9 11 12 14 15 20 21 23 24 26 27 29 30 32 33
1 9 10 11 15 16 17 18 19 20 24 25 26 30 31 32
1 8 10 12 14 16 17 18 19 21 23 25 27 29 31 33
1 8 9 13 14 15 17 18 19 22 23 24 28 29 30 34

3 4 6 7 11 12 13 15 18 19 25 26 27 30 31 34
2 4 5 7 11 12 13 16 17 19 23 27 28 29 31 32
2 3 5 6 11 12 13 14 17 18 24 26 28 29 30 33
0 3 4 5 8 9 10 18 19 21 23 24 28 31 33 34
0 2 4 6 8 9 10 17 19 22 24 25 26 29 32 34
0 2 3 7 8 9 10 17 18 20 23 25 27 30 32 33
3 4 6 7 10 17 19 20 21 22 24 27 28 30 31 32
2 4 5 7 8 17 18 20 21 22 25 26 28 29 31 33
2 3 5 6 9 18 19 20 21 22 23 26 27 29 30 34
0 5 6 7 9 10 12 13 14 15 20 21 23 26 31 34
0 5 6 7 8 10 11 13 15 16 21 22 24 27 29 32
0 5 6 7 8 9 11 12 14 16 20 22 25 28 30 33
0 3 4 5 13 14 15 16 17 19 24 25 27 29 33 34
0 2 4 6 11 14 15 16 17 18 23 25 28 30 32 34
0 2 3 7 12 14 15 16 18 19 23 24 26 31 32 33
1 3 4 6 7 9 11 13 16 17 21 22 24 25 26 33
1 2 4 5 7 10 11 12 14 18 20 22 23 25 27 34
1 2 3 5 6 8 12 13 15 19 20 21 23 24 28 32
0 1 3 4 5 8 10 12 15 16 17 22 23 27 28 30
0 1 2 4 6 8 9 13 14 16 18 20 24 26 28 31
0 1 2 3 7 9 10 11 14 15 19 21 25 26 27 29
1 3 4 6 7 9 10 12 15 16 18 20 28 32 33 34
1 2 4 5 7 8 10 13 14 16 19 21 26 32 33 34
1 2 3 5 6 8 9 11 14 15 17 22 27 32 33 34
0 1 3 4 5 9 12 13 14 18 21 22 25 29 30 31
0 1 2 4 6 10 11 13 15 19 20 22 23 29 30 31
0 1 2 3 7 8 11 12 16 17 20 21 24 29 30 31

The group Gr_6 is a nonabelian group of order 12 isomorphic to the group A_4. ∎

REMARK 3.2. Nonabelian groups $G9642D$ and $G9633D$ are presented in [6].

REMARK 3.3. It is obvious that the constructed class graphs include those ones which have been constructed by Haemers.

REMARK 3.4. From a strongly regular graph G with parameters $(35, 16, 6, 8)$ one can automatically construct a symmetric $(35, 17, 8)$ design D. More precisely, replacing zeros on the main diagonal of the adjacency matrix of the graph G with ones one gets an incidence matrix of the design D. It is a well known fact that $\mathrm{Aut}G \leq \mathrm{Aut}D$. We denote by D_i $(i = 1, 2, 3, 4, 5, 6)$ the designs obtained in such a way from the constructed graphs G_i $(i = 1, 2, 3, 4, 5, 6)$ and investigate what are the automorphism groups of them. In this particular case the full automorphism groups remain the same. So, we have obtained six symmetric $(35, 17, 8)$ designs D_i $(i = 1, 2, 3, 4, 5, 6)$ having the full automorphism groups Gr_i $(i = 1, 2, 3, 4, 5, 6)$.

Remark 3.5. All designs and graphs described in this paper are available from `ftp://polifem.pefri.hr/matematika/sanja`.

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Received: 19.11.2001.
Revised: 06.11.2002. & 05.03.2003.