

# Evaluation of biomechanical effects of interocclusal surfaces on the mandible

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## ABSTRACT

**Introduction:** Only few studies in the literature employ a mathematical model in the evaluation of the stress which follows the application of loads and constraints onto the mandible. Therefore, new approaches are needed so that the study of this phenomenon can rely less on the clinical hypothesis and experience of the operator, while taking advantage of the many benefits that virtual representations and mathematical calculations present. Aim of the study is to determine, by means of the finite element method (FEM), the stress produced onto the mandible as a result of the application of a force on itself, in association or not to the perioral musculature and according to the dental support given by the positioning of an interocclusal surface at three different levels: mesial, intermediate and distal.

**Aim:** The aim is to allow a more objective evaluation of this phenomenon, its absolute repeatability, as well as to acquire important clinical informations concerning the role of orthodontic and gnathologic appliances.

**Materials and methods:** Starting from a 1:1 scale model of the mandible (human adult male), a virtual three - dimensional (3D) representation was first obtained thanks to a dedicate software; it was then imported into a second software in order to permit the discretization into finite elements of the virtual model and the attribution of its mechanical properties. Finally, thanks to a specific software, it was possible to simulate the presence of load and constraints and to evaluate the stress status by using pseudo - colors.

**Results:** The stress generated following the application of a force onto the mandible, undergoes significant variations in relation to the dental support and the presence or absence of the perioral musculature.

**Conclusions:** Following the results of our research, we consider FEM as a valid and interesting method for this purpose, however additional FEM conducted studies are necessary in order to assess this phenomenon in more detail and determine the role of the perioral musculature as well as the possible clinical implications.

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## INTRODUCTION

The mandible is subjected to forces produced by the masticatory muscles. Reaction forces are applied to the temporomandibular joints and the teeth. External loadings applied to the mandible produce stresses and strains whose range and distribution

depend on the nature of the external loading and on the material properties and geometry of the mandible. Loadings are thought to be factors that determine mandibular bone structure since they play an important role in the modeling and remodeling of bone. During biting and mastication, a combination of sagittal bending, corpus rotation and transverse bending occurs. In the longitudinal direction, the mandible is stiffer than in transverse directions, and the vertical cross - sectional dimension of the mandible is larger than its transverse dimension. These features enhance the resistance of the mandible to the relatively large vertical shear forces and bending moments that come into play in the sagittal plane. In addition, some clinical situations that alter loading conditions (e.g. tooth loss, orthodontic treatment,

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dental implants or maxillofacial surgery) would be able to affect mandibular architecture.<sup>1</sup>

The increasing interest towards the stomatognathic system led to scientific researches with the aim to understand the complexity of its functions and the onset mechanisms of its dysfunctions.<sup>1</sup> Engineering science, through the elaboration of mathematical models, can supply additional instruments to study the role of occlusal factors and the dynamics of inter-occlusal relationships more closely. In particular, an extensive knowledge of orthodontic or gnathologic devices is required as a correct approach whenever a certain treatment modifies the biomechanics of the patients' stomatognathic system.<sup>2</sup> However, most of the studies related to this topic do not go beyond an overall description of the appliances based on the hypothesis and the clinical experience of the therapist. The development and application of a mathematical model represents an attempt to determine the relationship between the different forces applied to the stomatognathic system and to understand more extensively the system's mechanisms of reaction whenever muscular and articular loads take place. The mathematical models have the advantage, compared to the *in vitro* or *in vivo* models, of being absolutely repeatable. Therefore, they permit comparison between different simulations, following the variation of a single parameter or in the case of calculations involving non measurable quantities (e.g. internal stress of the material).<sup>1</sup> This aspect is very important from the clinical perspective since it enables to establish the effects of the modification of such parameters during orthognathic surgery and gnathologic or orthodontic treatments.<sup>3-6</sup>

Among mathematical models, the most renowned one is the finite element method (FEM), which originates from the principles of electronic. After many enhancements, FEM is employed nowadays in the biomechanical analysis of the stomatognathic system contributing to the current knowledge despite the difficulties encountered in the evaluation of the mechanical properties of the biological tissues.<sup>7,8</sup> The creation of a finite element model requires the initial definition and comprehension of the system under study and the following geometrical representation with the attribution of its own mechanical properties. The next step is the partition of the geometrical shape into sufficiently small elements to guarantee an adequate precision of the calculations; a discretization is considered adequate if the reduction of the dimensions of the element does not produce a significant variation of the quantities calculated during the simulation. The decomposition into finite elements is followed by the application of loads and constraints and the evaluation of the stress ( $\delta$ ) and deformations originated by their application. Stress symbolizes the extent of the force (F) in relation to the area (A) and its represented by the formula  $\delta = F / A$ ; deformation ( $\epsilon$ ) describes the distortions following the application of a certain amount of stress; in the case of homogenous and isotropic

materials, a directly proportional relationship subsists between the stress and the resulting deformation. This is known as Hooke's law and it is expressed by the formula  $\epsilon = \delta / E$  (E= Young's elastic modulus). In order to obtain an immediate interpretation of the result, the presence of stress and its amount are represented as pseudo-color images. The great development of the methods for the elaboration of models representing the stomatognathic system led from self-created wooden mandibles to softwares for the realization of two-dimensional and three-dimensional models; additional instruments in the fields of medical imaging (computed tomography and nuclear magnetic resonance) and biomedical engineering (in vivo modern transducers)<sup>9</sup> were also made available to researchers due to recent technologic progresses. Many scientific works utilized finite element models of the mandible, obtained through computed tomography and nuclear magnetic resonance, to evaluate the function of masticatory muscles<sup>10</sup> and temporomandibular joint<sup>11,12</sup> during mandibular movements and execution of other functions. Other authors<sup>13-15</sup> used them to determine the stress occurring during occlusion or following the application of mono-lateral occlusal loads. In particular, Ishida et al.<sup>16</sup> observed during neutral occlusion the presence of a tension-type stress along the anterior margin of the ramus, a compressive stress on its angular portion, as well as a compressive and a tension-type stress on the outer and inner surface of the anterior portion of the mandibular body. Into the same areas, the authors measured an increase of the stress parameters during disto-occlusion; while in mesio-occlusion they discovered a stress increase at the level of the condylar heads. Finally, occlusion on the right side was associated to a tension-type stress on the surfaces of the left side and to a compressive stress on the right portion of the two ramus. Following these results, the authors supposed that the mandibular morphogenesis may be influenced by multiple stressful factors. Takayama et al.<sup>17</sup> conducted a study with FEM where a vertical and mono-lateral force is applied from the top downwards on the anterior, medial and posterior area of the mandible. Similarly, other researches<sup>18,19</sup> highlighted the directly proportional relationship between the entity and the direction of the muscular and/or occlusal forces, the intensity and the type of the stress (tensive or compressive) and the morpho-structural modifications of the mandible such as hypertrophy in certain areas of the cortical bone.<sup>20</sup>

The aim of the present study is to analyze, by mean of the finite element method, the distribution of stress related to the mandible following the application of different inter-occlusal surfaces all with the same interocclusal thickness and at three different levels: molar, premolar and canine position.

## MATERIALS AND METHODS

### *Study design*

The virtual computerized reconstruction of the mandible was performed with a parametric CAD named SolidWorks 2000

(Dassault Systèmes, Vélizy Villacoublay, France) and Microsoft 2000 (Microsoft Corporation, Redmond, WA, USA). Points, curves and the sections of interest of the mandibular geometry were obtained through reverse engineering process by means of a 3D touch probe of a coordinate measuring machine: MicroScribe 3D (Immersion Corporation, San José, CA, USA) in association with the analysis of a polymeric resin 1:1 scale model of a mandible (human adult male) fabricated referring to the mean values present in the international literature.<sup>20-24</sup> Starting from the geometrical evaluation of the space, the significant points were joined together in order to elaborate the sections from which a surface reproducing the external geometry of the mandible was initially obtained following the attainment of a three-dimensional mandibular solid (Figure 1). The solid model obtained with SolidWorks 2000 was the imported into a second software called Femap 7.01 (Structural Dynamics Research Corporation SDRC, Plano, TX, USA) which permitted to assign the biologic features of the material, the type and the size of the discretization into finite elements, the features of the constraints and of the loads. Based on the solid model, a mesh (Figure 2a) was formed by 19352 elements (10 nodes tetrahedral element with parabolic formulation) and 35568 nodes; material of the mandible was considered linear, isotropic and homogeneous and was used for cortical bone  $E=15000$  MPa and  $\nu=0.27$  and for spongy bone  $E=1500$  MPa and  $\nu=0.33$  ( $E$  is Young's modulus and  $\nu$  is Poisson's ratio achieved from international literature). Once the discretization in finite elements was concluded, we proceeded with the bilateral application of the constraints to the model along the muscular insertions of the anterior and posterior temporalis muscle,<sup>23</sup> the superficial and deep head of the masseter and of the medial and lateral heads of the pterygoid muscle so as to avoid translational movements along the X, Y and Z axes while allowing rotation around these same. Two additional vertical forces of 50 N each were also applied to the occlusal support in the premolar zone in order to simulate the forces developed during the masticatory activity. The model was enhanced with the insertion of two additional constraints at the condylar level with the purpose to prevent translation in respect of the X and Z axes and rotation around the Y and Z axes. The mesh, after the addition of the loads and constraints was then exported from Femap 7.01 and imported into Abaqus 5.8.1 (Dassault Systèmes Simulia Corporation, Providence, RI, USA) for the final solution; this software enables to run the mathematical calculation codes and therefore to evaluate the muscular forces into the three dimensions of space (Table 1). The such determined muscular forces were employed to create a second model (Figure 2b) where the constraints were replaced with these same forces but two constraints were still present at the condylar level to prevent the translation along the X and Z axes and along the Y and Z axes.

The model also presented two occlusal constraints which avoided translation in respect of X, Y, Z. The condyle and a distal dental support represented the constraints and once the new distribution of the forces was acquired, the results were then transferred from Abaqus 5.8.1 to Femap 7.01 in order to assess the mandibular strain status according to The von Mises criteria (von Mises criteria is used and accepted in the international literature as having information concerning stress and strain for a lot of biomechanical problems) which suggests that the yielding of materials begins when the second deviatoric stress invariant reaches a critical value.<sup>25</sup> Through the variation of the constraint's arm of reaction, three supports were considered: one distal to the third molar (Figure 3a), an intermediate one along to the second premolar (Figure 3b) and one mesial to the canine (Figure 3c), associated or not to the perioral musculature which was represented by a vertical force applied to the osseous pogonion.

Figure 1. Three dimensional mandibular solid model.



Figure 1a. With shade function.

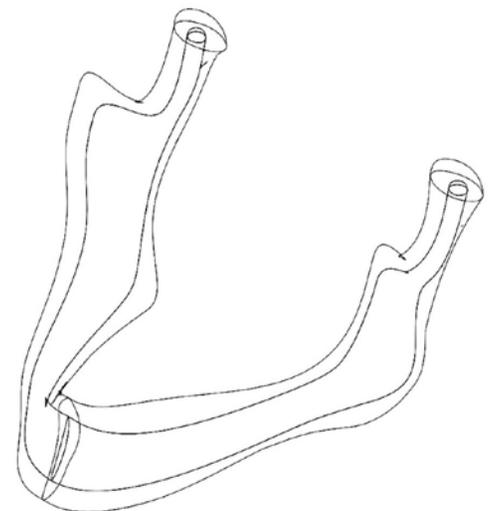


Figure 1b. As a network structure.

The only variable was to consider the presence of a constraint, that impeded translation related to the X, Y and Z axes and that was located into the perioral musculature's point of application of the force.

Following the application to the premolar area of two vertical forces of 50 N each, a new distribution of the muscular forces was calculated (Table 2) and after the utilization of the condyle and of the distal dental supports as constraints, the new resulting distribution of the stress was measured.

In order to obtain an immediate interpretation of the results, the presence and the quantity of stress were visualized as colored areas with the use of a red to violet scale, whereas the red color represented an area of important stress while violet stands for absence.

Figure 2. Three - dimensional mandibular model.



Figure 2a. Mandible with mesh.

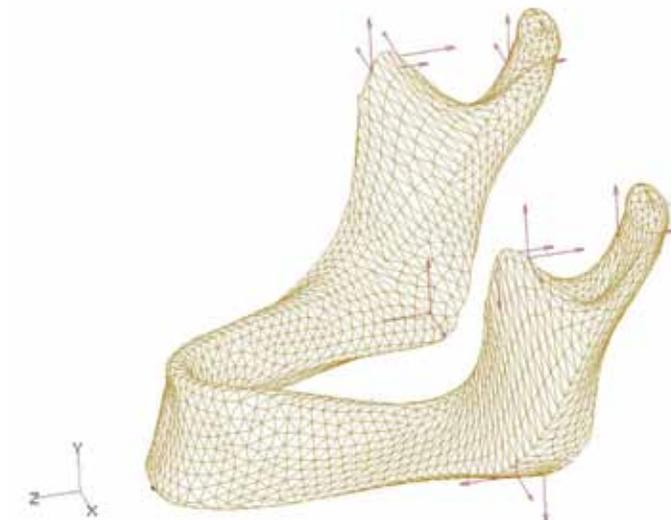


Figure 2b. Mandible with mesh and muscular forces.

## RESULTS

The use of finite elements and the application of loads and constraints permitted to realize and to establish quantitatively the strain status in six different mandibular models, characterized by a distal, mesial and intermediate dental support by the interposition of an interocclusal surface, without or not the association to the perioral musculature.

The results, displayed and analyzed basing to the colorimetric variations, pointed out the fact that the strain status of the mandible, calculated according to The von Mises theory, undergoes considerable modifications in relation to the dental support (Figure 4).

As for the models without the perioral musculature, especially those with a distal or intermediate support (Figure 4a, 4c), the stress status does not affect the mandible entirely; in the first case (Figure 4a) it is mostly located on the neck of the condyle, the coronoid apophysis and the sigmoid notch while in the second case (Figure 4c) it's mostly located along the coronoid process, the anterior margin and the mesial portion of the ramus. In the case of the model with the dental mesial support (Figure 4e), the mandible, except for the condylar head, the gonial angle and the para - symphyseal area, undergoes the maximum stress, represented by a diffuse red color gradient.

Table 1. X, Y and Z components of the LPT and RPT muscles (left and right posterior temporalis), LAT and RAT (left and right anterior temporalis), LDM and RDM (left and right deeper masseters), LSM and RSM (left and right superior masseters), LMP and RMP (left and right medial pterygoids) and LLP and RLP (left and right lateral pterygoids)

Muscle	Knot	FX (N)	FY (N)	FZ (N)
LPT	9403	72.385	131.340	-15.424
LAT	9348	10.162	-18.954	-73.186
LDM	13241	-60.269	-0.5927	58.276
LSM	13243	0.7540	126.100	98.377
LMP	13287	-0.3274	26.529	15.392
LLP	9468	56.493	31.901	-10.727
RPT	11839	-27.629	103.190	-73.765
RAT	11779	-38.585	-16.083	-14.383
RDM	10599	32.384	-20.348	45.667
RSM	10625	0.8832	97.834	76.475
RMP	10652	24.556	29.038	19.782
RLP	11705	-62.256	45.276	-12.038

Other modifications appear as we include into the three models the perioral musculature; indeed, the strain status extends mesial to the inferior portion of the symphyseal region, especially in the case of the model with a distal dental support (Figure 4b, 4d, 4f). In this situation, we also face a marked distribution of the stress to the body and to the mesial portion of the ramus of the mandible; the condyle instead, undergoes the same amount of stress which was evidenced in the absence of the perioral musculature as in the case of a mesial dental support (Figure 4f, 4e).

Finally, the model with an intermediate dental support and in association with perioral musculature presents an important reduction of the strain upon the entire mandible while stress persisted exclusively at the coronoid level and along the mesial portion of the ramus near the angle of the mandible.

## DISCUSSIONS

Within the limits of this study, finite elements mandibular models permitted to analyze and to obtain information about the strain status that is developed in relation to the mandible, calculated according to The von Mises theory, by varying the dental support (distal, intermediate and mesial). The results, evaluated on the basis of the colorimetric variations in relation to a scale from violet to red, have demonstrated the significant modifications into the distribution of the stress intensity, depending on the dental support and the presence of the perioral musculature. In particular, within the six models which were realized, the one with the mesial support and the absence of the perioral musculature is characterized by the major distribution of maximum stress areas (red color), whereas the one with an intermediate support and the presence of the perioral musculature instead, presented stress exclusively at the coronoid apophysis and the mesial portion of the ramus along the angle of the mandible. Apart from this last condition, in the case of other models without the association of perioral musculature, it's important to distinguish between the tension - type and compression - type stress in order to have a correct interpretation of the biomechanical effects of orthodontic/gnathologic devices with a distal support (e.g. Sears' pivot splint or distracting plaques), intermediate (e.g. interceptor of Schultze) or mesial (Hawley's retainer, ecc.). However, the exceptional and diffused stress, especially in the case of the model with an intermediate support and the presence of the perioral musculature, underlines the necessity to evaluate further, through more exhaustive studies, the relationship occurring between a uniform distal, intermediate and mesial distribution of the support and the related distribution of the stress (e.g. bite plane and the like). Nonetheless, the possibility of improving the distribution of the stress throughout the application of various orthodontic and/or gnathologic devices has to be considered. The role of the perioral musculature, although it is not yet clear on the basis of the results of the present study, may assume a certain importance in both areas of interest, orthodontic and

*Figure 3. Mandible with the three dental supports.*



*Figure 3a. Distal support A1.*



*Figure 3b. Support between the distal and the mesial A2.*



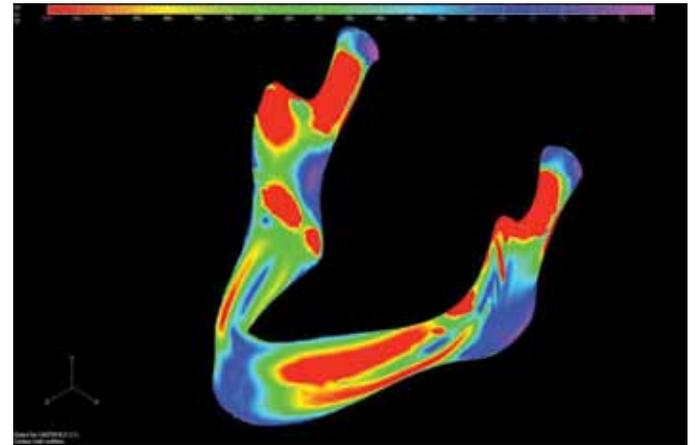
*Figure 3c. Mesial support A3.*

gnathologic, since the preliminary data seem to influence the mandible's strain distribution in association with the application of devices able to modify the dental area of support. Within the limit of the present study which fails to determine the probable influence of the joint on stress distribution, we consider the FEM analysis onto mandibular models, such as those employed in our research, as a starting approach that may lead to a major comprehension of the biomechanical consequences onto the stomatognathic system by the application of inter-occlusal devices. From the clinical point of view, this is an important opportunity that may allow the assessment of many parameters related to orthognathic surgery and orthodontic or gnathologic treatment that could not be assessed otherwise. Therefore, further studies are necessary to consider numerically, on finite element models with individualized data, the stress quantity and also to distinguish between tension-like or compression-like stress. Additional development of this study could not only let us explain more accurately the responses' mechanisms of the system following the application of inter-occlusal surface and also, thanks to these results, to be able to describe more efficient and precise therapeutic perspectives of removable inter-occlusal devices, especially in the case of young patients.

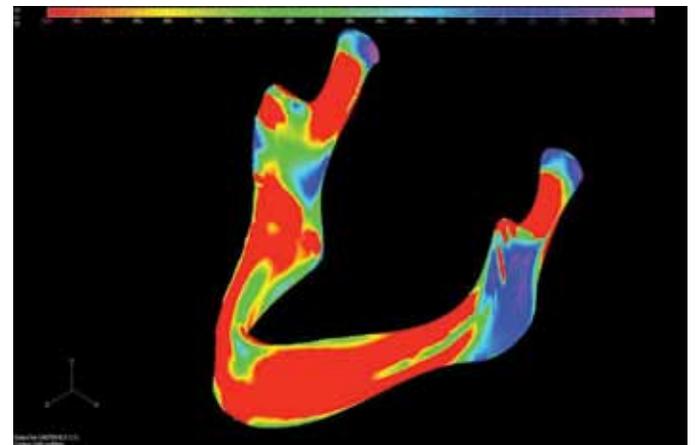
**Table 2.** X, Y and Z components of the LPT and RPT muscles (left and right posterior temporalis), LAT and RAT (left and right anterior temporalis), LDM and RDM (left and right deeper masseteres), LSM and RSM (left and right superior masseteres), LMP and RMP (left and right medial pterygoids), LLP and RLP (left and right lateral pterygoids) and PM (perioral musculature)

Muscle	Knot	FX (N)	FY (N)	FZ (N)
LPT	9403	40.092	77.528	-7.934
LAT	9348	0.4952	-11.641	-32.686
LDM	13241	-3.858	16.997	65.325
LSM	13243	-29.756	83.863	77.063
LMP	13287	-1.455	19.979	77.003
LLP	9468	31.173	18.207	-55.025
RPT	11839	-17.361	65.106	-38.599
RAT	11779	-2.144	0.4809	-71.818
RDM	10599	2.29	0.8035	50.336
RSM	10625	20.875	72.092	69.686
RMP	10652	3.342	21.081	10.172
RLP	11705	-34.478	28.271	-63.462
PM	8257	0.4714	22.363	-2.127

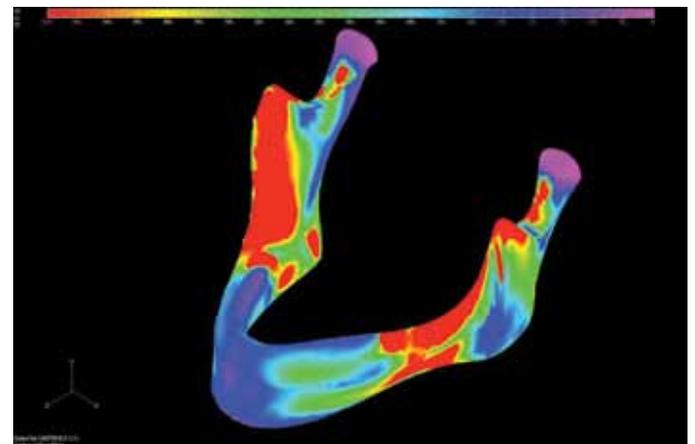
**Figure 4.** Strain distribution in each case.



**Figure 4a.** Load equally distributed on to the dental supports.



**Figure 4b.** Load equally distributed on to the dental supports in association to the perioral musculature.



**Figure 4c.** Distal support A1.

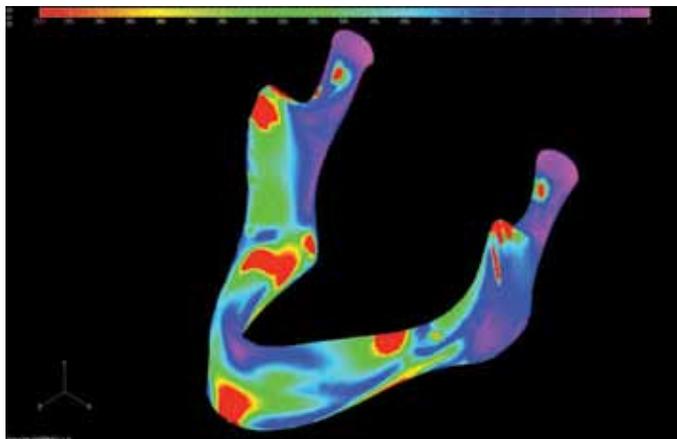


Figure 4d. Distal support A1 in association to the perioral musculature.

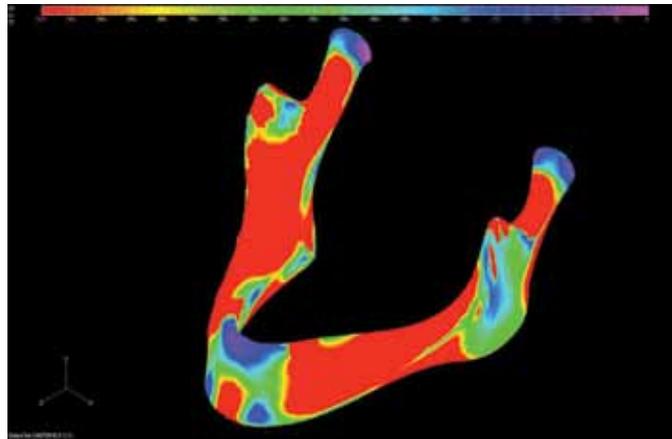


Figure 4f. Medium support A2 in association to the perioral musculature.



Figure 4e. Medium support A2.

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