GLASNIK MATEMATIČKI
Vol. 38(58)(2003), 1 – 9

SOME SYMMETRIC (47,23,11) DESIGNS

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Abstract. Up to isomorphism there are precisely fifty-four symmetric designs with parameters $(47, 23, 11)$ admitting a faithful action of a Frobenius group of order 55. From these fifty-four designs one can construct 179 pairwise nonisomorphic 2-(23,11,10) designs as derived and 191 pairwise nonisomorphic 2-(24,12,11) designs as residual designs. We have determined full automorphism groups of all constructed designs. One of 2-(24,12,11) designs has full automorphism group of order 15840, isomorphic to the group $M_{11} \times Z_2$, acting transitively on the set of points.

1. Introduction and Preliminaries

A 2-(v, k, λ) design is a finite incidence structure $(\mathcal{P}, \mathcal{B}, I)$, where $\mathcal{P}$ and $\mathcal{B}$ are disjoint sets and $I \subseteq \mathcal{P} \times \mathcal{B}$, with the following properties:

1. $|\mathcal{P}| = v$,
2. every element of $\mathcal{B}$ is incident with exactly k elements of $\mathcal{P}$,
3. every pair of distinct elements of $\mathcal{P}$ is incident with exactly λ elements of $\mathcal{B}$.

The elements of the set $\mathcal{P}$ are called points and the elements of the set $\mathcal{B}$ are called blocks.

Given two designs $\mathcal{D}_1 = (\mathcal{P}_1, \mathcal{B}_1, I_1)$ and $\mathcal{D}_2 = (\mathcal{P}_2, \mathcal{B}_2, I_2)$, an isomorphism from $\mathcal{D}_1$ onto $\mathcal{D}_2$ is a bijection which maps points onto points and blocks onto blocks preserving the incidence relation. An isomorphism from a design $\mathcal{D}$ onto itself is called an automorphism of $\mathcal{D}$. The set of all automorphisms of the design $\mathcal{D}$ forms a group; it is called the full automorphism group of $\mathcal{D}$ and denoted by $Aut\mathcal{D}$. A symmetric (v, k, λ) design is a 2-(v, k, λ) design with $|\mathcal{P}| = |\mathcal{B}|$.

2000 *Mathematics Subject Classification.* 05B05, 05E20.

Key words and phrases. Symmetric design, Hadamard design, orbit structure, automorphism group, Mathieu group.

Let $\mathcal{D} = (\mathcal{P}, \mathcal{B}, I)$ be a symmetric (v, k, λ) design and $G \leq Aut\mathcal{D}$. The group action of G produces the same number of point and block orbits (see [8, Theorem 3.3]). We denote that number by t, the point orbits by $\mathcal{P}_1, \ldots, \mathcal{P}_t$, the block orbits by $\mathcal{B}_1, \ldots, \mathcal{B}_t$, and put $|\mathcal{P}_r| = \omega_r$ and $|\mathcal{B}_i| = \Omega_i$. We shall denote the points of the orbit $\mathcal{P}_r$ by $r_0, \ldots, r_{\omega_r - 1}$, (i.e. $\mathcal{P}_r = \{r_0, \ldots, r_{\omega_r-1}\}$). Further, we denote by γ_{ir} the number of points of $\mathcal{P}_r$ which are incident with a representative of the block orbit $\mathcal{B}_i$. For those numbers the following equalities hold (see [3]):

$$\sum_{r=1}^{t} \gamma_{ir} = k \,, \tag{1.1}$$

$$\sum_{r=1}^{t} \frac{\Omega_j}{\omega_r} \gamma_{ir}\gamma_{jr} = \lambda\Omega_j + \delta_{ij} \cdot (k - \lambda) \,. \tag{1.2}$$

Definition 1.1. *A $(t \times t)$-matrix (γ_{ir}) with entries satisfying conditions* (1.1) *and* (1.2) *is called an orbit structure for the parameters (v, k, λ) and orbit lengths distributions $(\omega_1, \ldots, \omega_t)$, $(\Omega_1, \ldots, \Omega_t)$.*

The algorithm used for constructing symmetric (v, k, λ) designs $(\mathcal{P}, \mathcal{B}, I)$ admitting presumed automorphism group G is described in details in [3]. More about the elimination of isomorphic structures using elements of the normalizer of the group G one can find in [2]. Here we give a short explanation of the algorithm.

The first step - when constructing designs for given parameters and orbit lengths distributions - is to find all compatible orbit structures (γ_{ir}). Mutually isomorphic orbit structures would lead to mutually isomorphic symmetric designs. Therefore, during the construction of orbit structures we use elements of the normalizer of the group G in the group $S = S(\mathcal{P}) \times S(\mathcal{B})$ for elimination of isomorphic orbit structures, taking from each $N_S(G)$-orbit of orbit structures the representative which is first in the reverse lexicographical order.

The next step, called indexing, consists in determining exactly which points from the point orbit $\mathcal{P}_r$ are incident with a fixed representative of the block orbit $\mathcal{B}_i$ for each number γ_{ir}. In this step we also use the elements of the group $N_S(G)$ for elimination of mutually isomorphic structures, applying permutations from $N_S(G)$ which induce automorphisms of the orbit structure (γ_{ir}). At the end of this step, all symmetric designs with given parameters admitting an automorphism group G acting with presumed orbit lengths distributions will be constructed.

Symmetric designs described in this article are obtained by using the computer programs written in programming language **C** which we have developed following the above algorithm.

The authors will send the details on the algorithm and programs to the interested readers.

Definition 1.2. *The set of those indices of points of the orbit $\mathcal{P}_r$ which are incident with a fixed representative of the block orbit $\mathcal{B}_i$ is called the index set for the position (i, r) of the orbit structure and the given representative.*

A Hadamard matrix of order m is an $(m \times m)$-matrix $H = (h_{i,j})$, $h_{i,j} \in \{-1, 1\}$, satisfying $HH^T = H^T H = mI$, where I is the unit matrix. From each Hadamard matrix of order m one can obtain a symmetric $(m-1, \frac{1}{2}m-1, \frac{1}{4}m-1)$ design (see [8]). Also, from any symmetric $(m-1, \frac{1}{2}m-1, \frac{1}{4}m-1)$ design we can recover a Hadamard matrix. Symmetric designs with parameters $(m-1, \frac{1}{2}m-1, \frac{1}{4}m-1)$ are called Hadamard designs.

It is known that Hadamard matrices of order 48 could be obtained from Hadamard matrices of order 12 or 24, using Kronecker product. As far as we know, symmetric $(47, 23, 11)$ designs constructed from such Hadamard matrices of order 48 are not investigated and classified. According to [9] only one symmetric $(47, 23, 11)$ design has been constructed so far, and that had been obtained via cyclic difference set (see [5]). Applying the [7, Teorem 4.1] one gets that there are at least $\dfrac{23!}{47 * 23^2 * 11^2}$ designs, but this article doesn't give us an information about structure of designs and automorphism groups.

The aim of this article is to construct all symmetric $(47, 23, 11)$ designs having Frobenius group of order 55 as an automorphism group and to determine full automorphism groups of these designs and their derived and residual designs. It turns out that Mathieu group M_{11} acts as an automorphism group on one of obtained residual designs.

For further basic definitions and construction procedures we refer the reader to [4] and [13].

2. A Frobenius Group of Order 55 Acting on a Symmetric (47, 23, 11) Design

Let $\mathcal{D}$ be a symmetric (47,23,11) design and G a Frobenius group of order 55, further denoted by $Frob_{55}$. Since there is only one isomorphism class of nonabelian groups of order 55 we may write

$$G = \langle \rho, \sigma \mid \rho^{11} = 1,\, \sigma^5 = 1,\, \rho^\sigma = \rho^3 \rangle.$$

Lemma 2.1. *Let $\mathcal{D}$ be a symmetric $(47, 23, 11)$ design and let $\langle \rho \rangle$ be a subgroup of $Aut\mathcal{D}$. If $|\langle \rho \rangle| = 11$, then $\langle \rho \rangle$ fixes precisely three points and three blocks of $\mathcal{D}$.*

Proof. By [8, Theorem 3.1], the group $\langle \rho \rangle$ fixes the same number of points and blocks. Denote that number by f. Obviously, $f \equiv 47 \pmod{11}$, i.e., $f \equiv 3 \pmod{11}$. Using the formula $f \leq k + \sqrt{k - \lambda}$ of [8, Corollary 3.7], we get $f \in \{3, 14, 25\}$. For $f = 14$ or $f = 25$ one cannot solve the equations (1.1) and (1.2) for block orbits of the length 11.

For $f=3$, solving equations (1.1) and (1.2) by the method described in [3], one can get up to isomorphism and duality exactly two orbit structures:

OS1	1	1	1	11	11	11	11
1	1	0	0	11	11	0	0
1	0	1	0	11	0	11	0
1	0	0	1	11	0	0	11
11	1	1	1	5	5	5	5
11	1	0	0	5	5	6	6
11	0	1	0	5	6	5	6
11	0	0	1	5	6	6	5

OS2	1	1	1	11	11	11	11
1	1	0	0	11	11	0	0
1	0	1	0	11	0	11	0
1	0	0	1	11	0	0	11
11	1	1	0	5	5	5	6
11	1	0	1	5	5	6	5
11	0	1	1	5	6	5	5
11	0	0	0	5	6	6	6

Orbit structure OS1 is self-dual, and OS2 is not self-dual. □

Let G be an automorphism group of a symmetric design $\mathcal{D}$, acting with orbit lengths distributions $(\omega_1,\dots,\omega_t)$, $(\Omega_1,\dots,\Omega_t)$. Automorphism group G is said to be semistandard if, after possibly renumbering orbits, we have $\omega_i=\Omega_i$, for $i=1,\dots,t$.

LEMMA 2.2. *Let $G=\langle\rho,\sigma\rangle$ be the Frobenius group of order 55 defined above, $\mathcal{D}$ a symmetric $(47,23,11)$ design, and $G\le Aut\mathcal{D}$. Then G acts semistandardly on $\mathcal{D}$ with orbit lengths distribution $(1,1,1,11,11,11,11)$.*

PROOF. The Frobenius kernel $\langle\rho\rangle$ of order 11 acts on $\mathcal{D}$ with orbit lengths distribution $(1,1,1,11,11,11,11)$. Since $\langle\rho\rangle\lhd G$, the element σ of order 5 maps $\langle\rho\rangle$-orbits onto $\langle\rho\rangle$-orbits. Therefore, the only possibility for orbit lengths distribution is $(1,1,1,11,11,11,11)$.

The stabilizer of each block from a block orbit of length 11 is conjugate to $\langle\sigma\rangle$. Therefore, the entries in the orbit structures corresponding to point and block orbits of length 11 must satisfy the condition $\gamma_{ir}\equiv 0,1 \pmod 5$. Both orbit structure OS1 and OS2 satisfy this condition. □

3. Construction of the Symmetric Designs

THEOREM 3.1. *Up to isomorphism there are precisely fifty-four symmetric designs with parameters $(47,23,11)$ admitting a faithful action of a Frobenius group of order 55. Six of these designs have $Frob_{55}$ as a full automorphism group, thirty-nine designs have $Frob_{55}\times Z_2$ and six designs have $Frob_{55}\times S_3$ as a full automorphism group. Full automorphism groups of three designs are isomorphic to the group $L_2(11)\times S_3$ of order 3960.*

PROOF. We denote the points by $1_0,2_0,3_0,4_i,5_i,6_i,7_i$, $i=0,1,\dots,10$ and put $G=\langle\rho,\sigma\rangle$, where the generators for G are permutations defined as follows:

$\rho=(1_0)(2_0)(3_0)(I_0,\dots,I_{10}),\ I=4,5,6,7,$
$\sigma=(1_0)(2_0)(3_0)(K_0)(K_1,K_3,K_9,K_5,K_4)(K_2,K_6,K_7,K_{10},K_8),$
$K=4,5,6,7.$

Indexing the fixed part of an orbit structure is a trivial task. Therefore, we shall consider only the right-lower part of the orbit structures of order 4. To eliminate isomorphic structures during the indexing process we have used the permutation which - on each $\langle\rho\rangle$-orbit of length 11 of points - acts as $x \mapsto 2x \pmod{11}$, and - in addition - automorphisms of our orbit structures $OS1$ and $OS2$.

As representatives for the block orbits of length 11 we chose blocks fixed by $\langle\sigma\rangle$. Therefore, the index sets - numbered from 0 to 3 - which could occur in the designs constructed from OS1 and OS2 are among the following:

$$0 = \{1,3,4,5,9\},\ 1 = \{2,6,7,8,10\},\ 2 = \{0,1,3,4,5,9\},\ 3 = \{0,2,6,7,8,10\}.$$

The indexing process of the orbit structure OS1 led to the eighteen designs, denoted by $\mathcal{D}_1, \ldots, \mathcal{D}_{18}$. Using a computer program by V. Krčadinac (see [6] and [10]) we get that these designs are pairwise nonisomorphic. Pairs of mutually dual designs are $(\mathcal{D}_2, \mathcal{D}_9)$, $(\mathcal{D}_4, \mathcal{D}_{12})$, $(\mathcal{D}_5, \mathcal{D}_{14})$, $(\mathcal{D}_6, \mathcal{D}_{11})$, $(\mathcal{D}_7, \mathcal{D}_{17})$, $(\mathcal{D}_8, \mathcal{D}_{18})$ and $(\mathcal{D}_{10}, \mathcal{D}_{13})$. The designs $\mathcal{D}_1$, $\mathcal{D}_3$, $\mathcal{D}_{15}$ and $\mathcal{D}_{16}$ are self-dual. Full automorphism groups of designs $\mathcal{D}_{10}$ and $\mathcal{D}_{13}$ are isomorphic to $Frob_{55}$, full automorphism groups of $\mathcal{D}_2$, $\mathcal{D}_3$, $\mathcal{D}_4$, $\mathcal{D}_5$, $\mathcal{D}_6$, $\mathcal{D}_7$, $\mathcal{D}_9$, $\mathcal{D}_{11}$, $\mathcal{D}_{12}$, $\mathcal{D}_{14}$, $\mathcal{D}_{15}$, $\mathcal{D}_{16}$ and $\mathcal{D}_{17}$ are isomorphic to $Frob_{55} \times Z_2$, $Aut\mathcal{D}_8$ and $Aut\mathcal{D}_{18}$ are isomorphic to $Frob_{55} \times S_3$, and $Aut\mathcal{D}_1$ is isomorphic to the group $L_2(11) \times S_3$ of order 3960.

Orbit structure OS2 led to the eighteen designs, denoted by $\mathcal{D}_{19}, \ldots, \mathcal{D}_{36}$. Full automorphism groups of designs $\mathcal{D}_{20}$ and $\mathcal{D}_{30}$ are isomorphic to $Frob_{55}$, full automorphism groups of $\mathcal{D}_{19}$, $\mathcal{D}_{21}$, $\mathcal{D}_{24}$, $\mathcal{D}_{25}$, $\mathcal{D}_{26}$, $\mathcal{D}_{27}$, $\mathcal{D}_{28}$, $\mathcal{D}_{29}$, $\mathcal{D}_{31}$, $\mathcal{D}_{32}$, $\mathcal{D}_{33}$, $\mathcal{D}_{34}$ and $\mathcal{D}_{35}$ are isomorphic to $Frob_{55} \times Z_2$, $Aut\mathcal{D}_{23}$ and $Aut\mathcal{D}_{36}$ are isomorphic to $Frob_{55} \times S_3$, and $Aut\mathcal{D}_{22}$ is isomorphic to the group $L_2(11) \times S_3$.

Orbit structure OS2 is not self-dual. Therefore, dual structure of OS2 must produce also eighteen designs, which are dual to designs obtained from OS2. Those designs are denoted by $\mathcal{D}_{37}, \ldots, \mathcal{D}_{54}$.

We present symmetric designs $\mathcal{D}_1$ and $\mathcal{D}_{22}$ by (4×4)-matrices of index sets as follows:

$$\mathcal{D}_1 \qquad \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 3 \\ 0 & 3 & 0 & 3 \\ 0 & 3 & 3 & 0 \end{pmatrix} \qquad\qquad \mathcal{D}_{22} \qquad \begin{pmatrix} 0 & 0 & 0 & 3 \\ 0 & 0 & 3 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 3 & 3 & 3 \end{pmatrix}$$

From these matrices it is easy to obtain incidence matrices of designs. A computer program by Vladimir D. Tonchev [12] computes the orders as well as the generators of the full automorphism groups of these designs. The structures of these automorphism groups are determined by using GAP [11]. ∎

REMARK 3.2. Application of the computer program by Tonchev [12] yields the following assertion about the 2-ranks:

1. Designs with 2-rank equals 13 are $\mathcal{D}_1$, $\mathcal{D}_2$, $\mathcal{D}_5$, $\mathcal{D}_8$, $\mathcal{D}_9$, $\mathcal{D}_{14}$ and $\mathcal{D}_{18}$.
2. Designs with 2-rank equals 14 are $\mathcal{D}_{19}$, $\mathcal{D}_{22}$, $\mathcal{D}_{23}$, $\mathcal{D}_{26}$, $\mathcal{D}_{27}$, $\mathcal{D}_{33}$, $\mathcal{D}_{36}$, $\mathcal{D}_{37}$, $\mathcal{D}_{40}$, $\mathcal{D}_{41}$, $\mathcal{D}_{44}$, $\mathcal{D}_{45}$, $\mathcal{D}_{51}$ and $\mathcal{D}_{54}$.
3. Designs with 2-rank equals 23 are $\mathcal{D}_3$, $\mathcal{D}_4$, $\mathcal{D}_6$, $\mathcal{D}_7$, $\mathcal{D}_{10}$, $\mathcal{D}_{11}$, $\mathcal{D}_{12}$, $\mathcal{D}_{13}$, $\mathcal{D}_{15}$, $\mathcal{D}_{16}$, $\mathcal{D}_{17}$.
4. Designs with 2-rank equals 24 are $\mathcal{D}_{20}$, $\mathcal{D}_{21}$, $\mathcal{D}_{24}$, $\mathcal{D}_{25}$, $\mathcal{D}_{28}$, $\mathcal{D}_{29}$, $\mathcal{D}_{30}$, $\mathcal{D}_{31}$, $\mathcal{D}_{32}$, $\mathcal{D}_{34}$, $\mathcal{D}_{35}$, $\mathcal{D}_{38}$, $\mathcal{D}_{39}$, $\mathcal{D}_{42}$, $\mathcal{D}_{43}$, $\mathcal{D}_{46}$, $\mathcal{D}_{47}$, $\mathcal{D}_{48}$, $\mathcal{D}_{49}$, $\mathcal{D}_{50}$, $\mathcal{D}_{52}$ and $\mathcal{D}_{53}$.

4. 2-(23,11,10) AND 2-(24,12,11) DESIGNS

Excluding the block x and all points that do not belong to that block from a symmetric (47,23,11) design $\mathcal{D}$, one can obtain its derived design $\mathcal{D}_x$, i.e. 2-(23,11,10) design. Also, excluding the block x and all points belonging to that block from the design $\mathcal{D}$, one can obtain its residual design $\mathcal{D}^x$, i.e. 2-(24,12,11) design (see [8]).

Our aim is to investigate full automorphism groups of all pairwise nonisomorphic derived and residual designs that can be obtained from symmetric designs $\mathcal{D}_1, \ldots, \mathcal{D}_{54}$. Because of the following corollary, it is enough to consider derived and residual designs with respect to the block orbits representatives of full automorphism groups of constructed (47,23,11) designs.

COROLLARY 4.1. *Let $\mathcal{D} = (\mathcal{P}, \mathcal{B}, I)$ be a symmetric design, $x, x' \in \mathcal{B}$ and $G \leq Aut\mathcal{D}$. If $x' \in xG$, then $\mathcal{D}_x \cong \mathcal{D}_{x'}$ and $\mathcal{D}^x \cong \mathcal{D}^{x'}$.*

PROOF. [1, Corollary 1]. □

We obtained 179 such pairwise nonisomorphic derived designs. Further investigation of constructed 2-(23,11,10) designs led us to the results in Table 1.

Among constructed 2-(23,11,10) designs there are three quasi-symmetric designs: one with full automorphism group of order 660 isomorphic to the group $L_2(11)$ and two with full automorphism group of order 55 isomorphic to the group F_{55}.

Investigating all residual designs with respect to the block orbits representatives, we have constructed all pairwise nonisomorphic 2-(24,12,11) designs that can be obtained from constructed symmetric (47,23,11) designs $\mathcal{D}_1, \ldots, \mathcal{D}_{54}$. We obtained 191 such pairwise nonisomorphic designs. Further investigation of constructed 2-(24,12,11) designs led us to the results in Table 2.

Among constructed 2-(24,12,11) designs there are three quasi-symmetric designs: one with full automorphism group of order 15840 isomorphic to the group $M_{11} \times Z_2$ and two with full automorphism group of order 110 isomorphic to the group $F_{55} \times Z_2$.

Order of the full automorphism group	*The full automorphism group is isomorphic to the group*	*Number of designs*
1320	$L_2(11) \times Z_2$	1
720	$A_5 : D_{12}$	2
660	$L_2(11)$	1
360	$A_5 \times S_3$	1
240	$A_5 : E_4$	2
120	$A_5 \times Z_2$	1
110	$F_{55} \times Z_2$	7
60	$S_3 \times D_{10}$	1
55	F_{55}	13
40	$Z_2 \times (Z_5 : Z_4)$	2
30	$S_3 \times Z_5$	4
20	D_{20}	1
10	Z_{10}	80
5	Z_5	63

TABLE 1. The derived designs 2-(23,11,10).

Order of the full automorphism group	*The full automorphism group is isomorphic to the group*	*Number of designs*
15840	$M_{11} \times Z_2$	1
1320	$L_2(11) \times Z_2$	1
360	$A_5 \times S_3$	3
120	$A_5 \times Z_2$	3
110	$F_{55} \times Z_2$	8
60	$S_3 \times D_{10}$	2
55	F_{55}	22
30	$S_3 \times Z_5$	4
20	D_{20}	4
10	Z_{10}	80
5	Z_5	63

TABLE 2. Residual designs 2-(24,12,11).

5. 2-(24,12,11) Design Having M_{11} as an Automorphism Group

As an example of constructed 2-(24,12,11) designs we present, according to our judgment, the most interesting design: a quasi-symmetric design $\mathcal{D}$ having full automorphism group G isomorphic to the group $M_{11} \times Z_2$. It can be constructed from presented design $\mathcal{D}_1$ as residual design with respect to any of three blocks fixed by $Frob_{55}$. We also present generators of G.

Blocks of a quasi-symmetric design $\mathcal{D}$ are:

0 2 3 4 5 6 7 8 9 10 11 12
1 13 14 15 16 17 18 19 20 21 22 23
0 1 3 5 6 7 11 14 16 17 18 22
0 1 4 6 7 8 12 15 17 18 19 23
0 1 2 5 7 8 9 13 16 18 19 20
0 1 3 6 8 9 10 14 17 19 20 21
0 1 4 7 9 10 11 15 18 20 21 22
0 1 5 8 10 11 12 16 19 21 22 23
0 1 2 6 9 11 12 13 17 20 22 23
0 1 2 3 7 10 12 13 14 18 21 23
0 1 2 3 4 8 11 13 14 15 19 22
0 1 3 4 5 9 12 14 15 16 20 23
0 1 2 4 5 6 10 13 15 16 17 21
2 4 8 9 10 12 13 15 19 20 21 23
2 3 5 9 10 11 13 14 16 20 21 22
3 4 6 10 11 12 14 15 17 21 22 23
2 4 5 7 11 12 13 15 16 18 22 23
2 3 5 6 8 12 13 14 16 17 19 23
2 3 4 6 7 9 13 14 15 17 18 20
3 4 5 7 8 10 14 15 16 18 19 21
4 5 6 8 9 11 15 16 17 19 20 22
5 6 7 9 10 12 16 17 18 20 21 23
2 6 7 8 10 11 13 17 18 19 21 22
3 7 8 9 11 12 14 18 19 20 22 23
0 3 5 6 7 11 13 15 19 20 21 23
0 4 6 7 8 12 13 14 16 20 21 22
0 2 5 7 8 9 14 15 17 21 22 23
0 3 6 8 9 10 13 15 16 18 22 23
0 4 7 9 10 11 13 14 16 17 19 23
0 5 8 10 11 12 13 14 15 17 18 20
0 2 6 9 11 12 14 15 16 18 19 21
0 2 3 7 10 12 15 16 17 19 20 22
0 2 3 4 8 11 16 17 18 20 21 23
0 3 4 5 9 12 13 17 18 19 21 22
0 2 4 5 6 10 14 18 19 20 22 23
1 2 4 8 9 10 12 14 16 17 18 22
1 2 3 5 9 10 11 15 17 18 19 23
1 3 4 6 10 11 12 13 16 18 19 20
1 2 4 5 7 11 12 14 17 19 20 21
1 2 3 5 6 8 12 15 18 20 21 22
1 2 3 4 6 7 9 16 19 21 22 23
1 3 4 5 7 8 10 13 17 20 22 23
1 4 5 6 8 9 11 13 14 18 21 23
1 5 6 7 9 10 12 13 14 15 19 22
1 2 6 7 8 10 11 14 15 16 20 23
1 3 7 8 9 11 12 13 15 16 17 21

Generators of G are:

g_1=(4,7)(6,9)(8,10)(11,12)(15,18)(17,20)(19,21)(22,23)
g_2=(3,4)(5,12)(6,7)(8,11)(14,15)(16,23)(17,18)(19,22)
g_3=(2,3)(6,9)(8,11)(10,12)(13,14)(17,20)(19,22)(21,23)
g_4=(0,1)(2,13)(3,14)(4,15)(5,16)(6,17)(7,18)(8,19)(9,20)(10,21)(11,22)(12,23)
g_5=(0,2)(1,13)(5,6)(7,12)(8,11)(16,17)(18,23)(19,22)

The group G acts transitively on the set of points of the design $\mathcal{D}$. Excluding generator g_4 one can obtain generators of the group $G' \cong M_{11}$. The group G', isomorphic to the Mathieu group M_{11}, acts on the set of points of $\mathcal{D}$ in two orbits of the length 12.

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Received: 12.03.2002.
Revised: 28.10.2002.