GLASNIK MATEMATIČKI
Vol. 38(58)(2003), 157 – 165

FUZZY P-SPACES GAMES AND METACOMPACTNESS

Sunil Jacob John
National Institute of Technology Calicut, India

Abstract. Fuzzy P-spaces are introduced and a characterization for the same in terms of a particular type of fuzzy topological game is obtained. Further some applications of fuzzy P-spaces in product α-metacompact spaces are also investigated.

1. Introduction

The concept of P-spaces was introduced by K. Morita and a characterisation for the same was given by Telgarsky in [10]. Just like the applications of P-spaces in general topology, fuzzy P-spaces help the study of covering properties in fuzzy topological spaces. In [8] and [9] the author introduced metacompactness for $[0, 1]$ and L-Fuzzy Topological Spaces respectively and in [8] it is shown that the product of two α-metacompact spaces need not be α-metacompact. But if we impose some conditions on one of these spaces, we can make the product α-metacompact. This is done in terms of fuzzy topological games and fuzzy P-spaces and this was the main motivation behind the study of fuzzy P-spaces. For this reason, we generalize the concept of P-spaces to fuzzy topological spaces (fts) and a characterization for the same in terms of some particular kind of fuzzy topological game is obtained. Some basic definitions and results regarding fuzzy topological games are given in [6] by the author.

2. Basic definitions and results

In this section we collect the basic definitions and results regarding metacompact spaces, fuzzy topological games and fuzzy P-spaces.

2000 *Mathematics Subject Classification.* 54D20, 54A40.
Key words and phrases. Fuzzy topological spaces, fuzzy topological games, P_α-spaces, shading families.

Definition 2.1. [1] *Let* (X,T) *be a fts and* $\alpha \in [0,1)$. *A collection* $\mathbf{U}$ *of fuzzy sets is called an* α*-shading (resp.* (α^**-shading) of* X *if for each* $x \in X$, *there exists* $g \in \mathbf{U}$ *with* $g(x) > \alpha$ *(resp.* $g(x) \geq \alpha$*).*

Definition 2.2. [3] *A family* $\{a_s : s \in S\}$ *of fuzzy sets in a fts* (X,T) *is said to be point finite if for each* x *in* X, $a_s(x) = 0$ *for all but at most finitely many* s *in* S *(or equivalently as* $a_s(x) > 0$ *for at most finitely many* s *in* S*). Where* S *is an indexing set.*

Definition 2.3. [3] *Let* (X,T) *be a fts and* $\alpha \in [0,1)$. *Let* $\mathbf{U}$ *and* $\mathbf{V}$ *be any two* α*-shadings (resp.* α^**-shading) of* X. *Then* $\mathbf{U}$ *is a refinement of* $\mathbf{V}(\mathbf{U} < \mathbf{V})$ *if for each* $g \in \mathbf{U}$ *there is an* $h \in \mathbf{V}$ *such that* $g \leq h$. *Also a refinement* $\{b_t : t \in T\}$ *of* $\{a_s : s \in S\}$ *is said to be precise if* $T = S$ *and* $a_s \leq b_s$ *for each* $s \in S$. *Where* S *and* T *are indexing sets.*

Definition 2.4. [1] *A fuzzy topological space* (X,T) *is* α*-compact (resp. countably* α*-compact) if every* α*-shading of* X *by open fuzzy sets has a finite (resp. countable)* α*-sub shading.*

Definition 2.5. [8] *A fuzzy topological space* (X,T) *is said to be* α *- metacompact if each* α*-shading (resp.* (α^**- shading) of* X *by open fuzzy sets has a point finite* α*-shading refinement by open fuzzy sets.*

Definition 2.6. [2] *A fuzzy topological space* (X,T) *is said to be fuzzy regular if and only if for every fuzzy point* p *in* X, *and for every open fuzzy set* U *containing* p, *there exists an open fuzzy set* W *such that* $p \leq W \leq cl\ W \leq U$. *Where* $p \leq W$ *means that* $p(x) \leq W(x)$, x *being the support of the fuzzy point* p.

Definition 2.7. [12] *Let* $\{X_i\}_{i \in I}$ *be a family of fuzzy topological spaces. Let* $\mathbf{X} = \prod_{i \in I} X_i$ *be the usual Cartesian product and let* P_i *be the projection from* X *on to* X_i *for each* $i \in I$. *The set* X *with fuzzy topology having the family* $F = \{P_i^{-1}(B) : B \in T_i, i \in I\}$ *as a subbase is called the product fuzzy topological space.*

Definition 2.8. *Let* $X \times Y$ *be a fuzzy product space. A subset of the form* $R = R_1 \times R_2$ *where* R_1 *and* R_2 *are projections of* R *in to* X *and* Y *respectively is called a fuzzy rectangle in* $X \times Y$.

As a generalization of Topological Game $G(\mathbf{K}, X)$ introduced by Telgarsky [10], the author [6] introduced the Fuzzy Topological Game $G'(\mathbf{K}, X)$ in the following way.

Definition 2.9. [6] *Let* K *be a non empty family of fuzzy topological spaces, where all spaces are assumed to be* T_1 *(fuzzy singletons are fuzzy closed).* $\underline{I^x}$ *denote the family of all fuzzy closed subsets of* X. *Also* $X \in \mathbf{K}$ *implies* $\underline{I_x} \subseteq \mathbf{K}$. *Let* $X \in \mathbf{K}$. *Then the fuzzy topological game* $G'(\mathbf{K}, X)$ *is* *defined as follows. There are two players Player I and Player II. They alternatively choose consecutive terms of the sequence $(E_1, F_1, E_2, F_2, \ldots)$ of fuzzy subsets of X. When each player chooses his term he knows $\mathbf{K}$, X and their previous choices. A sequence $(E_1, F_1, E_2, F_2, \cdots)$ is a play for $G'(\mathbf{K}, X)$ if it satisfies the following conditions for each $n \geq 1$.*

1. *E_n is a choice of Player I*
2. *F_n is a choice of Player II*
3. $E_n \in \underline{I_x} \cap \mathbf{K}$
4. $F_n \in \underline{I_x}$
5. *$E_n \vee \overline{F_n} < F_{n-1}$ where $F_0 = X$*
6. $E_n \wedge F_n = 0$

Player I wins the play if $\inf_{n \geq 1} F_n = 0$. Otherwise Player II wins the Game. A finite sequence $(E_1, F_1, E_2, F_2, \ldots, E_m, F_m)$ is admissible if it satisfies conditions (1) – (6) for each $n \leq m$.

Definition 2.10. *Let S' be a crisp function defined as follows*

$$S' : \cup(\underline{I_x})^n \ \underrightarrow{into}\ \underline{I^x} \cap \mathbf{K} \quad n > 1 \tag{2.1}$$

Let $S_1 = \{X\}$, $S_2 = \{F \in \underline{I_x} : (S'(X), F)$ is admissible for $G'(\mathbf{K}, X)\}$. Continuing like this inductively we get $S_n = \{(F_1, F_2, F_3, \ldots, F_n) : (E_1, F_1, E_2, F_2, \ldots E_n, F_n)$ is admissible for $G(\mathbf{K}, X)$ where $F_0 = X$ and $E_i = S'(E_1, F_1, E_2, F_2, \ldots F_{i-1})$ for each $i < n\}$. Then the restriction S of S' to $\cup_{n>1} S_n$ is called a fuzzy strategy for Player I in $G'(\mathbf{K}, X)$. If Player I wins every play $(E_1, F_1, E_2, F_2, \ldots E_n, F_n, \ldots)$ such that $E_n = S(F_1, F_2, \ldots, F_{n-1})$, then we say that S is a fuzzy winning strategy.

Definition 2.11. *$S : \underline{I^x}\ \underrightarrow{into}\ \underline{I^x} \cap \mathbf{K}$ is called a fuzzy stationary strategy for Player I in $G'(\mathbf{K}, X)$ if $S(F) < F$ for each $F \in I_x$. We say that S is a fuzzy stationary winning strategy if he wins every play $(S(X), F_1, S(F_1), F_2, \ldots)$*

Definition 2.12. *A collection $\{U_i : i = 1, 2, 3, \ldots\}$ of fuzzy subsets of a set X is called an increasing family if $U_i < U_{i+1}$ for every $i = 1, 2, 3, \ldots$.*

As a generalization of P-spaces defined by K. Morita, Fuzzy P-spaces are defined as follows.

Definition 2.13. *A fts X is said to be a P_α-space if for every increasing family $\mathbf{U} = \{U(a_1, a_2, \ldots, a_i)/a_1, a_2, \ldots, a_i \in A,\ i = 1, 2, 3, \ldots\}$ of open fuzzy sets in X, there exists a precise refinement*

$$\mathbf{F} = \{F(a_1, a_2, \ldots, a_i)/a_1, a_2, \ldots, a_i \in A,\ i = 1, 2, 3 \ldots\}$$

by closed fuzzy sets satisfying the condition that if $\mathbf{U}$ is an α-shading of X, then $\mathbf{F}$ is also an α-shading of X where $\alpha \in [0, 1)$.

Theorem 2.14. *A fts X is a P_α-space if and only if there exists a crisp function*

$$p : \cap \mathbf{G}^n \to \mathbf{F}$$

such that

1. *If $(G_1, G_2, G_3, \ldots, G_n) \in \mathbf{G}^n$, $n \in N$ then $p(G_1, G_2, G_3, \ldots) < \sup\{G_k : 1 \leq k \leq n\}$*
2. *If $\{G_1, G_2, G_3, \ldots\}$ is an α-shading of X, then so is $\{p(G_1), p(G_1, G_2), p(G_1, G_2, G_3), \ldots\}$. Where $\mathbf{G}$ and $\mathbf{F}$ represent the family of all open and closed fuzzy subsets of X respectively.*

Proof. Let X be a P_α-space. Let $(G_1, G_2, G_3, \ldots G_n) \in \mathbf{G}^n$ and take $a_i = G_i$ in the definition of P_α-spaces and define

$$U(a_1, a_2, \ldots a_n) = U(G_1, G_2, G_3, \ldots G_n) = \sup\{G_i : 1 \leq i \leq n\}.$$

Then clearly $U(G_1, G_2, G_3, \ldots G_n) < U(G_1, G_2, G_3, \ldots G_{n+1})$. Then from the definition of P_α-spaces the remaining follows.

Conversely let $\mathbf{U} = \{U(a_1, a_2, \ldots a_i)\ a_i \in A,\ i = 1, 2, 3 \ldots\}$ be an increasing family of open fuzzy sets in X. Now corresponding to each $U(a_1, a_2, a_3, \ldots a_i)$ in $\mathbf{U}$, we define

$$\begin{aligned} F(a_1, a_2, \ldots a_i) &= p(U(a_1), U(a_1, a_2,)U(a_1, a_2, a_3), \ldots U(a_1, a_2, a_3, \ldots a_n)) \\ &< \sup\{U(a_1, a_2, \ldots a_i) : 1 \leq i \leq n\} \\ &= U(a_1, a_2, \ldots a_n) \text{ since } \mathbf{U} \text{ is increasing.} \end{aligned}$$

Now if $\mathbf{U}$ is an α-shading of X, for every $x \in X$, there exists a $U(a_1,\ a_2,\ a_3, \ldots a_k)$ such that $U(a_1, a_2, a_3, \ldots a_k)(x) > \alpha$. Now clearly by definition, we have $F(a_1, a_2, a_3, \ldots a_k)(x) > \alpha$ and hence $\{F(a_1, a_2, a_3, \ldots a_i) : a_i \in A, i = 1, 2, 3, \ldots\}$ is an α-shading of X. Hence X is a P_α-space. □

From the definition of P_α-Spaces and Theorem 2.14, next theorem follows clearly.

Theorem 2.15. *A fuzzy topological space X is a P_α-space if and only if there is a crisp function p defined from the family of all increasing finite sequences of open fuzzy sets $\mathbf{G}$ to the collection of all closed fuzzy sets $\mathbf{F}$ with $p(G_1, G_2, G_3, \ldots G_n) < G_n$ where $(G_1, G_2, G_3, \ldots G_n) \in \mathbf{G}_n$ and if $G_n < G_{n+1}$ for each $n \in N$ and if $\{G_1, G_2, G_3 \ldots G_n\}$ is an α-shading then so is $\{p(G_1), p(G_1, G_2), p(G_1, G_2, G_3), \ldots\}$.*

Theorem 2.16. *A fts X is a P_α-space if and only if there exists a crisp function $p : \cup(\mathbf{F})^n \to F$ such that*

i) *For each $(F_0, F_1, \ldots F_n) \in (\mathbf{F})^n$, $n \geq 0$*

$$p(F_0, F_1, \ldots F_n) \wedge \inf_{i \leq n} F_i = 0.$$

ii) *For each* $(F_0, F_1, \dots) \in (\mathbf{F})^\infty$ *with* $\inf_{n\geq 1} F_n = 0$*, the collection* $\{p(F_0, F_1, \dots F_n) : n \geq 0\}$ *is an* α*-shading of* X.

PROOF. Let $(F_1, \dots F_n) \in (\mathbf{F})^n$. Then F_1^c, $F_1^c \wedge F_2^c$, $F_1^c \wedge F_2^c \wedge F_3^c, \dots$ is an increasing family of open sets. Take $U(a_1) = F_1^c$, $U(a_1, a_2) = F_1^c \wedge F_2^c \dots U(a_1, a_2 \dots, a_n) = F_1^c \wedge F_2^c \wedge \dots \wedge F_n^c$. Now since X is a P_α-space, there exists a collection $\{F(a_1), F(a_1, a_2) \dots\}$ such that $F(a_1, a_2, \dots a_i) < U(a_1, a_2, \dots a_i)$ for each $i = 1, 2, 3, \dots$

Now define

$$p(F_1, \dots F_n) = \begin{cases} 0, & \text{if } \inf_{i\leq n} F_i \neq 0, \\ F(a_1, a_2, \dots a_n), & \text{otherwise.} \end{cases}$$

Clearly p has properties (i) and (ii).

Conversely let $(G_1, G_2, \dots G_n) \in \mathbf{G}^n$. Then $F_1 = G_1^c$, $F_2 = G_2^c, \dots F_n = G_n^c$ are all closed and hence there exists a function $p' : (\mathbf{F})^n \to \mathbf{F}$ such that

$$p'(F_1, \dots F_n) \wedge \inf_{i\leq n} F_i = 0.$$

Take $p(G_1, G_2, \dots G_n) = p'(F_1, \dots F_n)$ in Theorem 2.14, then

$$p(G_1, G_2, \dots G_n) \wedge \inf_{i\leq n} F_i = 0.$$

Therefore

$$\begin{aligned} p(G_1, G_2, \dots G_n) &< (\inf_{i\leq n} F_i^c) \\ &= \sup_{i\leq n} F_i^c \\ &= \sup_{i\leq n} G_i \end{aligned}$$

and hence p satisfies (i) and (ii) of Theorem 2.14 and hence X is a P_α-space. □

THEOREM 2.17. *If a fuzzy topological space* X *has a* σ*- closure preserving fuzzy closed* α*-shading by countably* α*-compact sets, then* X *is a* P_α*-space.*

PROOF. Let $\mathbf{F} = \cup\{\mathbf{F}_n : n \in N\}$ be an α-shading of X such that each $\mathbf{F}_n$ is closure preserving and every $F_n(\mathbf{F}_n$ is countably α-compact. Let $\{U(a_1, a_2, \dots a_n) : a_i \in A, i = 1, 2, 3, \dots\}$ be an increasing sequence of open fuzzy sets. Now corresponding to each $U(a_1, a_2, \dots a_n)$ we define

$$F(a_1, a_2, \dots a_n) = \sup\{F : F < U(a_1, a_2, \dots a_n), F \in \cup_{i=1}^n \mathbf{F}_i\}$$

Since $\cup_{i=1}^n \mathbf{F}_i$ is closure preserving it follows that $F(a_1, a_2, \dots a_n)$ is fuzzy closed and $F(a_1, a_2, \dots a_n) < U(a_1, a_2, \dots a_n)$ for each $n \geq 1$.

Again let $\{U(a_1, a_2, \dots a_i) : i = 1, 2, 3 \dots\}$ be an α-shading of X. Let $x \in X$. Now since $\mathbf{F}$ is an α-shading of X, there exists an $F_0 \in \mathbf{F}$ such that $F_0(x) > \alpha$. Let $F_0 \in \mathbf{F}_k$ for some k. Since F_0 is countably α-compact, and

$U(a_1, a_2, \ldots)$'s are increasing we can find out some $j \in N$ such that $j \geq k$ and $F_0 < U(a_1, a_2, \ldots, a_j)$.

Now

$$F(a_1, a_2, \ldots, a_j)(x) = \sup_{F < U(a_1, \ldots a_j)} \{F(x) : F \in \cup_{i=1}^{j} \mathbf{F}_i\} \geq F_0(x) > \alpha.$$

Thus $\{F(a_1, a_2, \ldots, a_j) : a_i \in A, i = 1, 2, 3 \ldots\}$ is also an α-shading of X. This completes the proof. □

3. A characterisation of P_α-spaces using the game $G_\alpha(X)$

In this section we describe a game associated with P_α-spaces. Here $G_\alpha(X)$ denote the following infinite positional fuzzy topological game. Let $\mathbf{G}$ and $\mathbf{F}$ denote the collection of all open (resp. closed) fuzzy subsets of an fts X. There are two players Player I and Player II. Players alternatively choose fuzzy subsets of X so that each player knows X and first k elements when he is choosing the $(k+1)$th element.

We say that a sequence $(G_1, F_1, \ldots G_n, F_n)$ is a play for $G_\alpha(X)$ if for each $n \geq 1$, we have

i. $G_n \in \mathbf{G}$ is a choice of Player I.

ii. $F_n \in \mathbf{F}$ and $F_n < \sup\{G_k : 1 \leq k \leq n\}$ is a choice of Player II.

Player I wins the play $(G_1, F_1, G_2, F_2 \ldots)$ if $\{G_n : n \in N\}$ is an α-shading of X and $\{F_n : n \in N\}$ is not. And Player II wins if $\{F_n : n \in N\}$ or both $\{G_n : n \in N\}$ and $\{F_n : n \in N\}$ are α-shadings of X.

A strategy for Player I is a crisp function $s : \{0\} \cup_{n=1}^{\infty} \mathbf{F}^n \to \mathbf{G}$ and that of Player II is t: $\mathbf{G}^n \to \mathbf{F}$ such that $t(G_1, G_2, \ldots G_n) < \sup\{G_i : 1 \leq i \leq n\}$ for each $(G_1, G_2, \ldots G_n) \in \mathbf{G}^n$ and $n \geq 1$.

Now clearly for each pair of strategies (s, t) there exists a unique Play $(G_1, F_1, G_2, F_2, \ldots)$ of $G_\alpha(X)$ defined as follows.

Take $G_1 = s(0)$, $F_1 = t(G_1)$, $G_2 = s(F_1)$, $F_2 = t(G_1, G_2)$ and so on.

A strategy s (resp. t) is winning for Player I (resp. Player II) if he wins every play of $G_\alpha(X)$ using it.

From Theorem 2.17 and definition of $G_\alpha(X)$, we get the following game theoretic characterization of P_α-spaces.

Theorem 3.1. *A fuzzy topological space X is a P_α-space if and only if Player II has a winning strategy in $G_\alpha(X)$.*

4. Applications in metacompact spaces

Theorem 4.1. *Let X be a fuzzy regular α-metacompact P_α-space and Player I has a winning strategy in $G'(\mathbf{DC}, X)$, then $X \times Y$ is α-metacompact for every α-metacompact space Y. Where $\mathbf{DC}$ denote the class of all fts which have a discrete fuzzy closed α-shading by members of $\mathbf{C}$. Where C is the collection of all α-compact spaces.*

PROOF. We use the following notations. If $a = (a_1, a_2 \ldots, a_n)$ then $a \oplus \zeta = (a_1, a_2 \ldots a_n, \zeta)$, $a/k = (a_1, a_2, \ldots a_k)$ and $a- = a/n-1$. Also $'$ and $''$ represents the projections on X and Y respectively.

Given that Player I has a fuzzy winning strategy in $G'(\mathbf{DC}, X)$. Therefore by Theorem 2.4 of [8] it follows that Player I has a stationary winning strategy and let this be s. Let p be a function defined as in 2.16. We will prove that every α-shading $\mathbf{G}$ of $X \times Y$ by open fuzzy sets has a point finite α-shading refinement by open fuzzy rectangles.

Let $\mathbf{U}_0 = \{0\}$, $\mathbf{A}_0 = \{0\}$ and $R(0) = H(0) = X \times Y$. For each $n \geq 1$, we shall construct a collection $\mathbf{U}_n$ of open fuzzy rectangles and a collection $\{\{R(a), H(a)\} : a = (a_1, a_2, \ldots a_n) \in A_n\}$ of pairs consisting of fuzzy closed $\times$ open rectangle $R(a)$ and open rectangle $H(a)$ satisfying the following conditions.

For each $n \geq 1$

(i) $\mathbf{U}_n$ is a point finite collection in $X \times Y$.
(ii) For every $U \times V(\mathbf{U}_n$, there is a $G \in G$ such that $U \times V < G$.
(iii) $\{H(a) : a \in A_n\}$ is point finite in $X \times Y$.
(iv) $\sup\{U : U(\mathbf{U}_n\} < \sup\{H(a) : a \in A_{n-1}\}$.
(v) $a_- \in A_{n-1}$.
(vi) $R(a) < R(a-)$ and $R(a) < H(a) < H(a-)$.
(vii) $S(R(a-)') \wedge R(a)' = 0$.
(viii) $R(a) \backslash \sup\{U : U \in \mathbf{U}_{n+1}\} < \sup\{R(a^+ \zeta) \; ; \; \{a^+ \zeta) \in A_{n+1}\}$.
(ix) $p(R(a/1)', \ldots, R(a/n-1), \, R(a)') \wedge H(a)'' = 0$.

Assume that for each $i \leq n$, the collections $\mathbf{U}_i$ and $\{R(a), H(a); \, a \in A_i\}$ have been constructed.

Now for any $a \in A_n$, let $\{C_\gamma : \gamma \in \Gamma(a)\}$ be a discrete collection of α-compact sets whose supremum is $S(R(a)')$. From the fact that X is fuzzy regular α-metacompact it follows that there exists point finite collections $\{W_\gamma : \gamma \in \Gamma(a)\}$ and $\{O_\gamma : \Gamma(a)\}$ of open fuzzy sets such that $C_\gamma < W_\gamma < clW_\gamma < O_\gamma < H(a) | \sup\{C_\beta : \beta \in \Gamma(a), \beta \neq \gamma\}$ for each $\gamma \in \Gamma(a)$. Now Y is α-metacompact and $R''(a)$ is open in Y. Now $R''(a)$ is α-metacompact (Since α-metacompact is hereditary with respect to open subsets) and hence for each $\gamma \in \Gamma(a)$, there exists a collection $\mathbf{U}_\gamma = U_{\delta,j} \times V_\delta : j$ $j = 1, 2, 3 \ldots m_\delta$ and $\delta \in \Delta(\gamma)\}$ such that

i) $C_\gamma < U_\delta = \sup_{i \leq \delta} U_{\delta,j} < W_\delta$ for each $\delta \in \Delta(\gamma)$.
ii) Each $U_{\delta,j} \times V_\delta$ is contained in some $G \in \mathbf{G}$.
iii) $\{V_\delta : \delta \in \Delta(\gamma)\}$ is point finite α-shading of $R(a)''$.

Set $\mathbf{U}_{n+1} = \cup\{\mathbf{U}_\gamma : \Gamma(a)$ and $a \in A_n\}$ and

$$A_{n+1} = \{a \oplus \delta : \delta \in \Delta(\gamma), \gamma \in \Gamma(a), a \in A_n\} \cup \{a + \theta : a \in A_n\}$$

Take any $a + \zeta \in A_{n+1}$. Then observe that $a/i \in A_i$ for all $i \in n$.

If $\zeta = \delta$ for some $\delta \in \Delta(\gamma)$ and, $\gamma \in \Gamma(a)$ put $R(a \oplus \delta) = [(clW_\gamma \backslash V_\delta) \wedge R(a)'] \times V_\delta$ and $H(a \oplus \delta) = O_\gamma \backslash p(R(a/1)', \ldots R(a/n)', R(a+\theta)') \times V_\delta$.

If $\zeta = \theta$, put $R(a+\theta) = R(a)`\backslash \sup_{\gamma \in \Gamma(a)} W \times R(a)''$ and

$$H(a+\theta) = H(a)'\backslash p(R(a/1)', \ldots R(a/n)', R(a+\theta)')\theta H(a)''$$

Then clearly U_{n+1} and $\{R(a), H(a) : a \in A_{n+1}\}$ satisfies conditions (i) – (ix).

Now take $\mathbf{U} = \cup_{n \geq 1} \mathbf{U}_n$. Now it can be shown that U is an α-shading of X and we will prove that $\mathbf{U}$ is also point finite. Also by (ii) $\mathbf{U}$ is a collection of open fuzzy rectangles in $X \times Y$ and any $U \times V \in \mathbf{U}$ is contained in some $G \in \mathbf{G}$.

Proceeding in a similar manner as in the proof of Theorem 2.4 in [7], we get if $\{a_n\}$ is a sequence such that $a_n \in A_n$ and $(a_n)- = a_{n-1}$ for each $n \geq 1$ where $a_0 = 0$, then

$$\inf_{n \geq 1} H(a_n)' = 0. \tag{4.2}$$

Again we claim that $\inf_{n \geq 1}(\sup \mathbf{H}_n) = 0$. Where $\mathbf{H}_n = \{H(a) : a \in A_n\}$. For if possible let there be an z_0 such that $\inf_{n \geq 1}(\sup \mathbf{H}_n)(z_0) > \eta =$ for some $\eta > 0$. Take $A_n(z_0) = \{a \in A_n : H(a)(z_0) \geq \eta\}$. By (iii) we get $A_n(z_0)$ is finite and by (v) and (vi) $a \in A_n(z_0) \Rightarrow a- \in A_{n-1}(z_0)$. Then by Konings Lemma, there exists $(\beta_1, \beta_2, \beta_3, \ldots)$ such that $a_n \in (\beta_1, \beta_2, \ldots \beta_n) \in A_n(z0)$ for each $n \geq 1$. Then $H(a_n)(z0) \geq \eta$ for each $n \geq 1$. Hence $\inf_{n \geq 1} H(a_n)(z_0) \geq \eta$. This is a contradiction to our claim.

Let $Z \in X \times Y$ then by claim above we can find an $m \geq 1$ such that $\sup H_m(z) = 0$. Now from (v) and (vi) it follows that $\sup \mathbf{H}_{n+1} < \sup \mathbf{H}_n$ for each $n \geq 1$. Since $\sup \mathbf{H}_n(z) = 0$ for each $n \geq m$, from (iv) we get that $\sup \mathbf{U}_n(z) = 0$ whenever $n > m$. Hence it follows from (i) that $\mathbf{U}$ is point finite in $X \times Y$. This completes the proof. ∎

THEOREM 4.2. [5] *If a fts X has a σ-closure preserving α-shading by fuzzy closed α-compact sets, then Player I has fuzzy winning strategy in $G'(\mathbf{DK}, X)$.*

From Theorems 2.17, 4.1, and 4.2 next corollary follows easily.

COROLLARY 4.3. *If X is a fuzzy regular α-metacompact space with a α-closure preserving α-shading by α-compact sets, then $X \times Y$ is α-metacompact for every α-metacompact space Y.*

ACKNOWLEDGEMENTS.

The author is very much indebted to Prof.T.Thrivikraman, Department of Mathematics, Cochin University of Science and Technology, India for his valuable guidance. Author is also thankful to the Council of Scientific and Industrial Research, India for the financial support throughout the preparation of this paper.

References

[1] T.E. Gantner, R.C. Steinlage and R.H. Warren, *Compactnesss in Fuzzy Topological Spaces*, J. Math. Anal. Appl. **62** (1978), 547–562.
[2] M.H. Ghanim, E.E. Kerre and A.S. Mashour, *Separation Axioms, Subspaces and Sums in Fuzzy Toplology*, J. Math. Anal. Appl. **102** (1984), 189–202.
[3] S.R. Malghan and S.S. Benchalli, *On Fuzzy Topological Spaces*, Glasnik Mathematički **16(36)** (1981), 313–325.
[4] J. Nagata, Modern General Topology, North Holland Pub., 1974.
[5] S.J. John, *Countable α-compactness, α-metacompactness and the fuzzy topological game*, $G'(DK, X)$ [Communicated]
[6] S.J. John, *Fuzzy Topological Games I*, Far East J. Math. Sci., Special Vol. (1999) Part III (Geometry and Topology), 361–371.
[7] S.J. John, *Fuzzy Topological Games II*, [Communicated].
[8] S.J. John, *Metacompactness in Fuzzy Topological Spaces through α-shadings*, [Communicated].
[9] S.J. John, *Metacompactness in the L-Fuzzy Context*, The Journal of Fuzzy Mathematics **8** (2000) 661–668.
[10] R.A. Telgarsky, *Characterisation of P-Spaces*, Proc. Japan Acad. **51** (1975), 802–807.
[11] R. Telgarsky, *Spaces defined by Topological Games*, Fund. Math. **88** (1975), 193–223.
[12] C.K. Wong, *Fuzzy Topology: Product and Quotient Theorems*, J. Math. Annal. Appl. **45** (1973), 512–521.
[13] Y. Yajima, *Topological Games and Applications*, in: Topics in General Topology, eds: K. Morita and J. Nagata, Elsevier Science Pub. 1989, 524–562.

S. J. John
Department of Mathematics
National Institute of Technology Calicut
Calicut – 673 601, Kerala
India
E-mail: sunil@nitc.ac.in

Received: 26.07.2000.