Improved GPR-based Condition Assessment of Reinforced Concrete Bridge Decks Using Artificial Neural Network

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ABSTRACT - *Accounting for the effect of rebar depth variation is one of the most important and challenging tasks to accurately assess the condition of reinforced concrete elements using ground penetrating radar (GPR) technique. In current practices, this task is performed on the individual basis, as for each bridge deck a unique depth correction model is derived from the GPR data collected on it. It is found that such a practice has led to a limited capability of GPR in assessing the condition of highly-deteriorated concrete. Therefore, a generic model to account for the depth-amplitude effect is proposed in this research. Using artificial neural network (ANN) modeling, a model for depth correction was calibrated from extensive data collected for a group of bare concrete bridge decks. The obtained ANN for depth correction was then used to assess the condition of a bridge deck, and the attenuation map was compared with those using a traditional depth correction technique. Whereas the conventional approach only detected the relative difference in condition between local deck areas, the outputs using the proposed methodology clearly indicated its capability to assess deck deterioration in absolute terms.*

Keywords: *Nondestructive Testing (NDT), Ground Penetrating Radar (GPR), Concrete, Inspection, Condition Assessment, Artificial Neural Network (ANN).*

1. INTRODUCTION

The effect of asphalt and concrete cover thickness on the ground penetrating radar (GPR) rebar reflection amplitude, and the need for amplitude depth correction, have been investigated by many researchers/ practitioners in the evaluation of condition of concrete bridge decks. These variations are encountered as a result of deck design, inconsistent construction, deck repair or overlaying, and due to many other reasons. The main purpose of depth correction is to remove the signal loss due to depth-amplitude effects and to normalize rebar reflection amplitudes with respect to a common cover thickness [1, 2]. Once all rebar reflection amplitudes have been depth corrected and contour mapped, certain amplitude threshold values will be used to delineate areas of concrete at various severity levels of deterioration. The thresholds may vary from one bridge to another and usually are defined from comparisons with inspection results using other NDT methods, or using a statistically-based data interpretation [1, 2, 3, and 4].

In current practices, the depth correction analysis is usually performed for each individual bridge deck based on the GPR data collected for that same deck. Although the reported results have shown that the depth correction analysis significantly improves the accuracy of condition assessment of bridge decks [1, 2], one must note that the assessment based on such depth-corrected amplitudes is still a relative evaluation technique. Specifically, a rebar in a deck having the strongest reflection would always be considered being associated with sound/good concrete, whereas the ones with amplitude below the threshold would be considered as being in a deteriorated region. As a consequence, it was reported that a GPR survey alone usually provides results with grossly underestimated concrete repair quantities [5]. The reason, as explained by Dinh et al. [6], is that the area with the strongest reflection may have already been deteriorated and attenuated itself.

2. RESEARCH OBJECTIVES

The main goal of this research was to develop an analytical approach that can correct for the effects of signal loss due to the rebar depth variation, and at the same time can eliminate the relative nature of the current depth-correction approaches. In contrast to the traditional methods, a generic depth correction model is being implemented for an entire network of bridges of the same deck type. As the signal loss may be different between various materials, GPR antennas, as well as utilized frequency ranges, it was decided that only bare (without overlays) concrete bridge decks and a 1.5 GHz ground-coupled antenna would be investigated in this research. Since the absolute amplitude measured (voltages or data units) depends on the transmit power of each antenna, and the gain set during data collection, the amplitudes need to be normalized using a common basis/scale. Given these explanations, three research objectives were identified:

(i) To explore methods for rebar reflection amplitude normalization;
(ii) To better understand the impact of signal loss solely due to rebar depth variation; and
(iii) To develop a generic depth correction procedure.

3. BACKGROUND

The American Infrastructure Report Card in 2013 estimates that an annual investment of $20.5 billion would be needed to eliminate the United States' bridge deficient backlog by 2028 [7]. As many bridges are considered structurally deficient because of the deterioration of decks [8], a major portion of this investment would be allocated to maintenance, rehabilitation and replacement of bridge decks alone.

Bridge decks deteriorate as a result of various factors. However, rebar corrosion has been identified as one of the most common problems [9] and, thus, numerous research efforts have been directed toward development of inspection techniques that can detect signs of this deterioration mechanism, and to it related damage. In that context, the GPR stands out as one of the most commonly used NDT technologies.

Exploiting the principles and phenomena of electromagnetic wave propagation, the GPR has shown to be an effective NDT technology for bridge deck condition assessment. To acquire the condition information for a particular bridge deck location, a GPR antenna sends a short duration microwave into the deck and receives energy partially reflected from various interfaces. These reflections are produced as a result of difference in dielectric properties. The strength of reflections is proportional to the dielectric contrast between two adjacent media. However, in a concrete deck with a highly conductive environment caused by free chloride ions (Cl^-), pore moisture, along with products (Fe^{2+}) from rebar corrosion, the reflections tend to be attenuated and even diminish to zero. These phenomena have formed the basis for condition assessment of reinforced concrete bridge decks by GPR, and reinforced concrete elements in general. While a comprehensive review of literature regarding the application of GPR in the assessment of reinforced concrete elements is beyond the scope of this text, a detailed description of the most common practices in the GPR data analysis for bridge decks is provided in the subsequent paragraphs.

4. CURRENT PRACTICES FOR GPR DATA ANALYSIS

Current GPR data analysis practices for inspection of concrete bridge decks closely follow the guidance provided by ASTM [10]. With respect to ground-coupled antennas, it recommends the following procedure for GPR data analysis: (1) migration to focus the rebar reflection; (2) recording of rebar reflection amplitude from the migrated data; (3) conversion of reflection amplitude to decibels (dB); (4) definition of the deterioration threshold and contour mapping; and (5) calculation of the percentage of deteriorated deck area.

As a detailed guidance, some major assumptions have, however, rendered the standard inappropriate for the condition assessment of many bridge decks. The two major assumptions behind the standard are: the top concrete cover (rebar depth) is uniform throughout the deck, and there is always a portion of the deck area that is in a sound/good condition. Whereas the former assumption regarding the rebar depth uniformity has been criticized [1, 2, 11], the latter has barely been discussed.

To account for the signal loss due to rebar depth variation, Barnes et al. [1] proposed a statistical method that is based on the 90th percentile linear regression. The method was conceived when they observed the scatter plots (point cloud) of normalized rebar reflection amplitude versus two-way travel time of GPR signals for some bridge decks being studied. As the upper points in the semi-log plots appeared to form a straight line, they assumed that the relationship was linear and further assumption was made regarding the 90th percentile value. The reason for choosing this value has been to obtain an appropriate statistical reference when deterioration may have affected the reflected amplitude and created outliers.

In a comprehensive study, Romero et al. [2] summarized and compared three different methods for performing depth correction. In terms of implementation, one method is done manually by GPR experts and the other two are automated through computer software. Although each method was described as involving a different mathematical manipulation, they were all based on the idea previously explained. As a result, it was reported that the analysis outputs using the three approaches were similar. With respect to the deterioration threshold value, the research stated that it may vary regionally and that the values tend not to be disclosed.

5. RESEARCH METHODOLOGY

The main hypothesis in this research is that the effects of concrete cover thickness on the rebar reflection amplitude can be better studied on the network level, rather than for each bridge deck individually.

A generic relationship (if any) between the two factors can then be applied for the depth correction for all bridges in the network. If the current depth correction methods are used, highly-deteriorated bridge decks will appear to be in a better condition than they actually are, since all depth-corrected amplitudes will converge to a certain, very low value. Whereas the knowledge about true condition of the deck in such cases can be obtained by comparing the GPR results to those from other NDE techniques, it is not a favorable solution. Using a generic, network wide depth correction model can, therefore, ensure that different bridge decks on the network level be evaluated consistently by the GPR.

The research idea was enabled by the data collected within the scope of the Long-Term Bridge Performance (LTBP) Program, a research project funded by the Federal Highway Administration (FHWA). Specifically, as part of the project, a cluster of twenty-four bridge decks in the Mid-Atlantic region was surveyed by the team from the Center for Advanced Infrastructure and Transportation (CAIT) at Rutgers University, using a range of NDT technologies. All the decks were selected by the research team in coordination with the participating State Departments of Transportation (DOTs) to be representative samples of bridges of the same type. As the first cluster, untreated/bare cast-in-place concrete decks that rest either on steel or prestressed concrete girders were investigated in this study. Whereas the data collected for the cluster bridges were used to develop the depth-amplitude model, as shown in ***Figure 1***, two independent bridge decks of the same type were used for the validation of the proposed methodology.

As can be seen in ***Figure 1***, the depth-amplitude model was developed based on the GPR data collected from areas of sound/good concrete. These areas were identified for each bridge deck from the combined results of three NDT techniques, namely Half-Cell Potential (HCP), Electrical Resistivity (ER) and Impact Echo (IE). Specifically, the criteria for defining sound concrete from these techniques are as follows: (1) potential measurement greater than -200 mV for HCP; (2) resistivity greater than 100 kOhm•cm for ER; and (3) no signs of delamination for IE. After the sound concrete areas have been identified, the GPR data from these areas were processed to extract the rebar reflection amplitude and corresponding two way travel time. Finally, a total of 23,587 data points (rebars) were obtained, and used to study the effects of rebar depth on reflection amplitude.

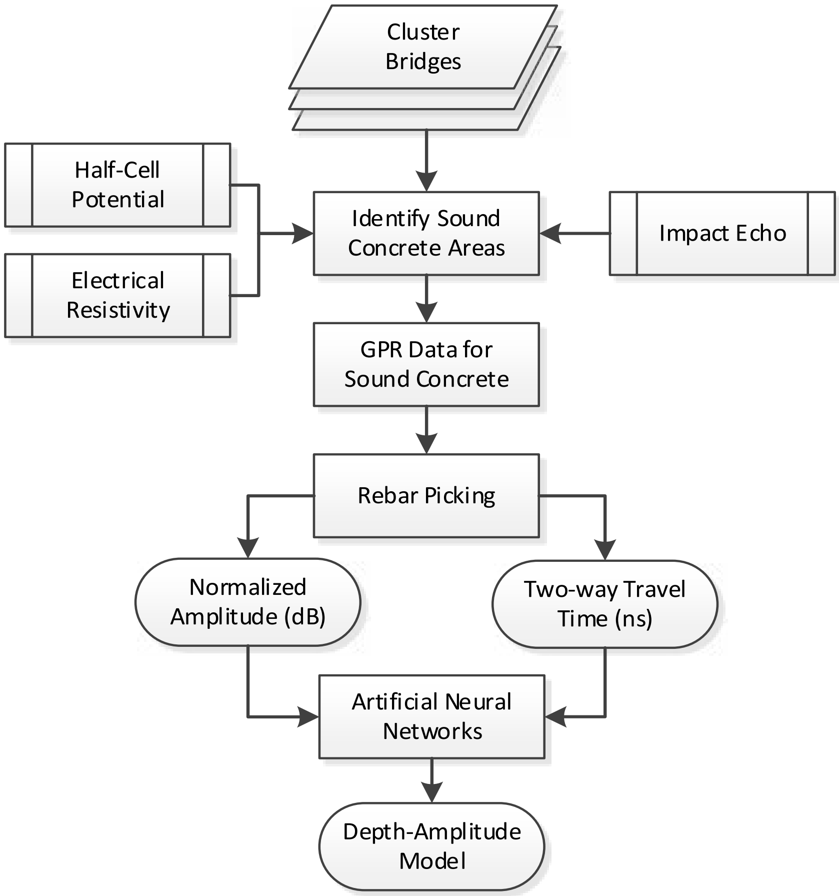

Figure 1 *Development of Depth-Amplitude Model*

5.1 Amplitude Normalization

There are situations that require the amplitude to be normalized when evaluating GPR data for different bridge decks. For example, for the same frequency antenna, the decks may be collected using different GPR units, or different gains may be set during the data collection. The comparison, in such cases, requires these data be normalized to a common basis/background. The ideal normalization would be to have the reflection amplitude measured using a metal plate. Such a data, unfortunately, does not usually exist during the most GPR data collections on bridge decks.

As a potential basis for amplitude normalization, direct-coupling is the effect in which the “air wave” and the “surface wave” merge when a GPR antenna is moved toward the surface of a bridge deck. Since having the air wave component, in comparison to the surface wave detected by an air-coupled (horn) antenna, the amplitude of this mixed reflection does not vary much with the local concrete condition. ***Figure 2*** is the illustration of this with two GPR waveforms collected on the same deck. As can be seen, while the rebar reflection amplitude is very sensitive to concrete deterioration, the direct-coupling amplitudes are almost identical for the two waveforms. This observation forms the basis for using direct-coupling as a normalization approach in this study.

For clarification, the normalization is done by dividing the amplitude from rebar reflection by the average direct-coupling amplitude measured in the corresponding profile. As can be imagined, the process will eliminate the difference in the transmit power of the antenna or gain set during the data collection, as long as the gain was set as a constant (one point gain). For the same radar unit, if a constant gain of 1 dB was used, the direct-coupling reflection amplitude would be amplified by 1 dB, as would be the reflection amplitude from a rebar. After the normalization through direct-coupling, the data for all sound concrete areas are converted to decibels to be investigated further for the effects of rebar depth variation.

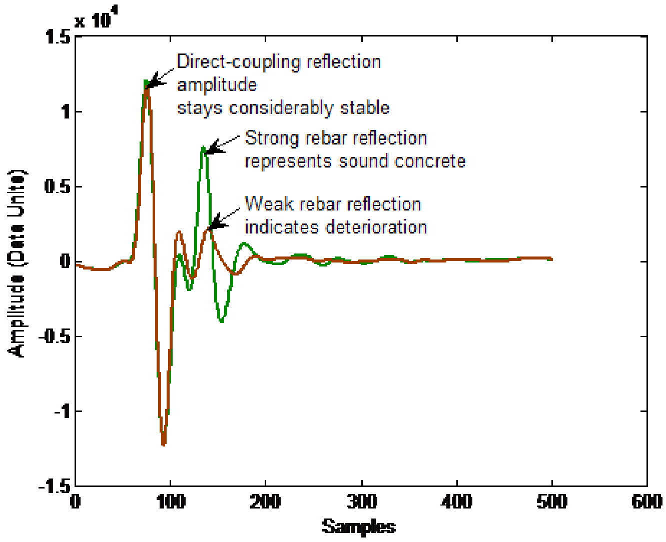

Figure 2 *Effects of deterioration on the direct-coupling amplitude.*

5.2 Artificial Neural Network (ANN)

Figure 3 shows a scatter plot illustrating the relationship between rebar depth and rebar reflection amplitude obtained in this study. In the literature, this relationship is assumed to be linear. The only rationale used has been the observation of the points in the upper part of scatter plots of the GPR data [1, 2]. As the linear regression may not represent the true relationship between the two factors, artificial neural network (ANN), an information-processing technique, is employed to better investigate such a dependency.

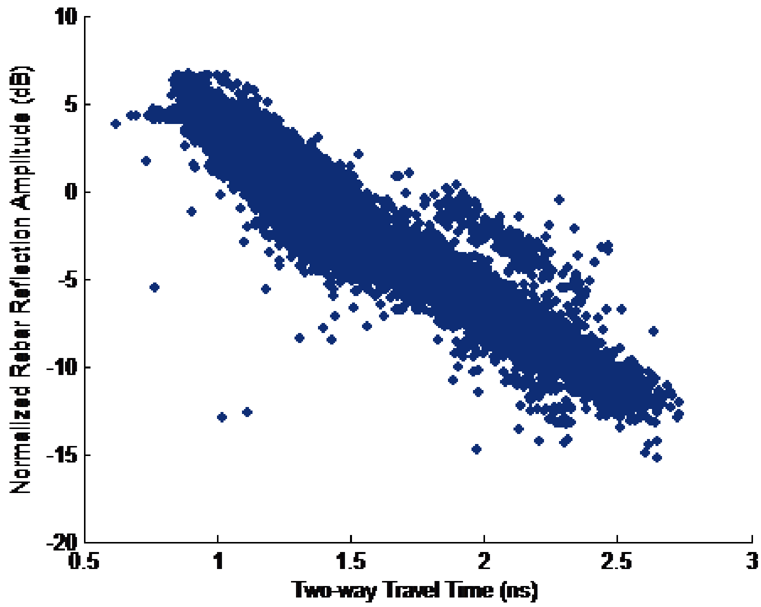

Figure 3 *Effects of deterioration on the direct-coupling amplitude.*

ANNs found beneficial applications in numerous research areas. Comprised of layers of parallel processing elements, or neurons, they simulate biological nervous systems to process acquired data and provide meaningful results/information. The strength of ANN models is that they are capable to learn from examples so as to extract essential characteristics or information without assuming the relationship between variables/factors. In comparison to the regression analysis, ANNs are much more appropriate for modeling problems in which the physical nature is too complex or not well understood [12,13].

Structurally, an ANN consists of an input layer, an output layer and one or several hidden layers in between. Each layer comprises one or more processing elements, also called neurons, which are connected as illustrated in ***Figure 4.*** As can be seen, each neuron in the hidden layer is connected to the neurons in neighboring layers by the so-called weighting factors. These factors are modifiable and will be adjusted during the training process when example input-output patterns are presented into the network. An ANN with this type of training algorithm is called a backpropagation (BP) neural network [14, 15].

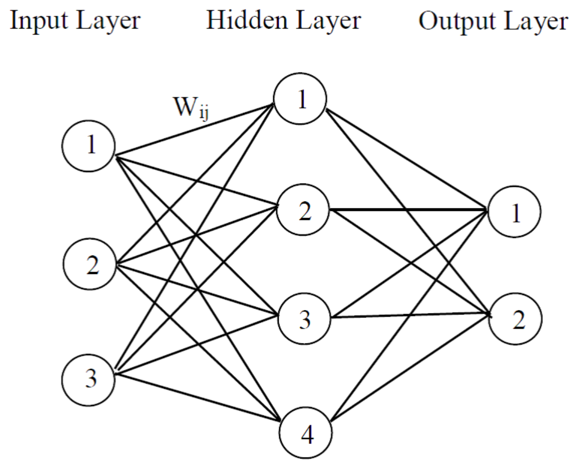

Figure 4 Typical ANN Structure.

As the name implies, a BP neural network trains itself from examples by propagating the errors of the output backward to the neurons in previous layers of the network. This task is iteratively implemented in two phases. In the first phase, or forward pass, the input signals propagate from input through hidden layer(s) to produce output signals that are calculated based on the initial weights set randomly during the network initialization. In the second phase, the errors, i.e., the difference between the actual and the desired output (target), are propagated backward to adjust the weighting factors. As described in Equation 1 below [16], the purpose of the adjustment is to reduce the errors corresponding to each input-output pattern. The process is repeated for all training data until the network stops improving. In other words, the training is completed when adjusting weighting factors does not result in reduced errors.

$$\Delta_p w_{ij} = \eta\left(t_{pj} - o_{pj}\right) i_{pi} \qquad (1)$$

Where:

t_{pj} is the target output for *j*th element of the output pattern *p*,

o_{pj} is the actual output for *j*th element of the output pattern *p*,

i_{pi} is the value of the *i*th element of the input pattern *p*,

$\Delta_p w_{ij}$ is the change to be made to the weight from the *i*th to *j*th neuron following the presentation of pattern *p*, and

η is the learning rate.

One of the issues with regression analysis and ANNs is problem “overfitting” [17]. It refers to the case when the regression or ANN model performs well for the training patterns, but has poor performance on new data sets presented to the model. In the literature, there have been several methods available to solve this problem, of which one is called “early stopping” [18]. In this technique, the available data is, basically, divided into three random data sets: training, validation, and test sets. While the training set is utilized to train the neural network and update weighting factors, the network performance (generalization) is monitored by observing the errors associated with the validation set patterns. After the training, the test set can be used to provide independent evaluation of the obtained model or to compare the performance of different networks. Initially in the training process, the errors for both training and validation sets normally decrease. However, when the overfitting occurs, the errors of the validation set will increase. Therefore, by stopping the training process at this point, a properly-trained neural network can be achieved.

With the theoretical background described above, ANNs are used to investigate the effects of concrete cover thickness on rebar reflection amplitude. Specifically, 23,587 data points (rebar peaks) were divided randomly into three sub sets with the following percentages: 70% for the training set plus 15% for both validation and test sets. With respect to the network topology, it consists of an input layer with one neuron representing concrete cover thickness (rebar depth); one hidden layer as recommended by Flood and Kartam [12] for one-input neural network; and an output layer of one neuron for predicting the amplitude of rebar reflection. As for the number of neurons in the hidden layer, since there is no specific rule for determining the appropriate number [12], trials were made to find acceptable values.

While the experiment has shown that a range of values can provide approximately the same accuracy, a hidden layer of 3 neurons was employed in this study.

Figure 5 depicts the performance of the obtained ANN through regression plots. Each data set in the plot shows the correlation between the target output and the one predicted by the network. As can be observed, a small difference in the correlation coefficients (R) between the three data sets indicates that the neural network has been properly trained. In addition, since the R2 value (coefficient of determination) for the entire data set is 0.91 (0.954272), one may say that the 91% variance of the rebar reflection amplitude can be attributed to the variation of the concrete cover thickness and therefore can be well predicted by the network. The small remaining variance can be caused by variables such as measuring errors, concrete properties, different sizes of reinforcing bar, or by other random factors.

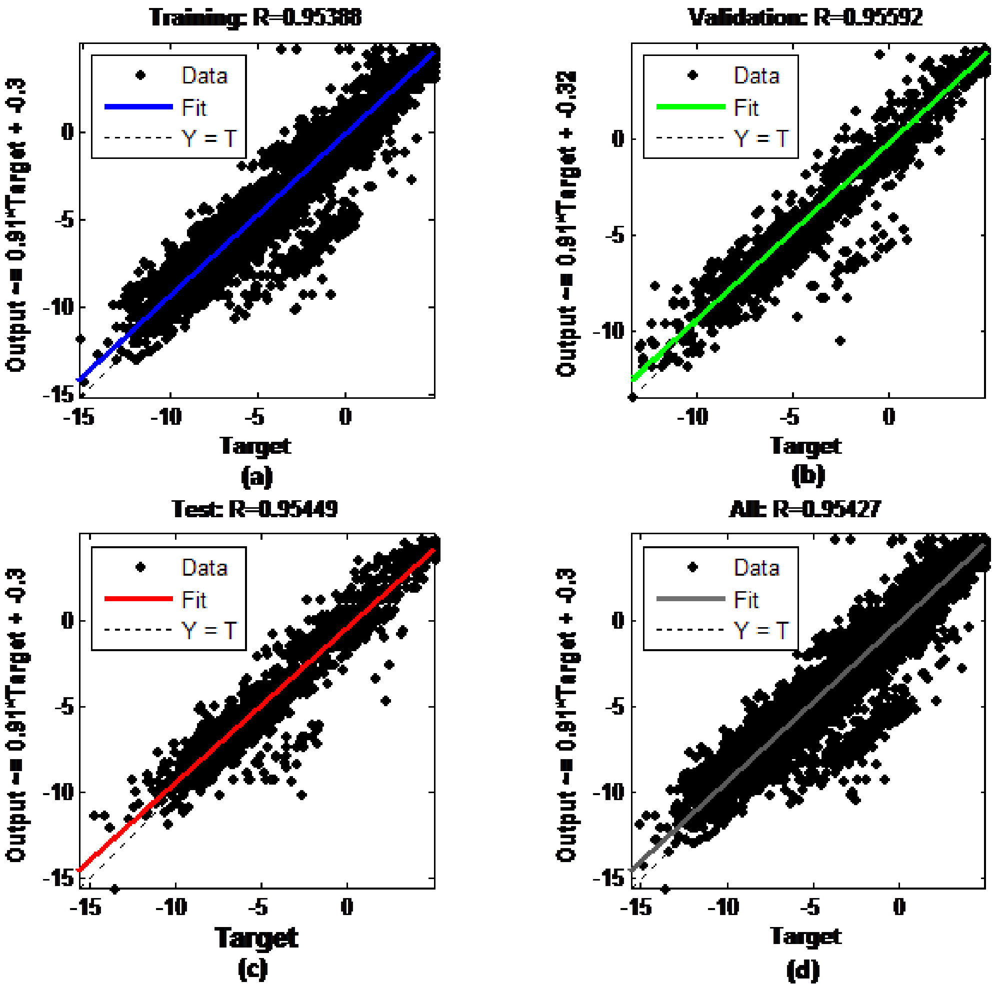

Figure 5 *Regression plots for (a) training set, (b) validation set, (c) test set, and (d) all data points.*

Figure 6 presents the fit function from the obtained ANN. As can be seen, the function does not exactly form a straight line, as expected by the conventional depth correction methodologies. It is especially more nonlinear in the region with a small concrete cover thickness. The reason is that, for a shallow reinforcing mat, the rebar reflection is blended with a portion of the direct-coupling signal, so that its amplitude is affected by the configuration of this mixture. In addition, Figure 6 also reveals that the difference in rebar reflection amplitude before the depth correction for sound concrete may be up to 18 dB. Clearly, this difference is significantly larger than the threshold of -6 to -8 dB specified in the ASTM standard.

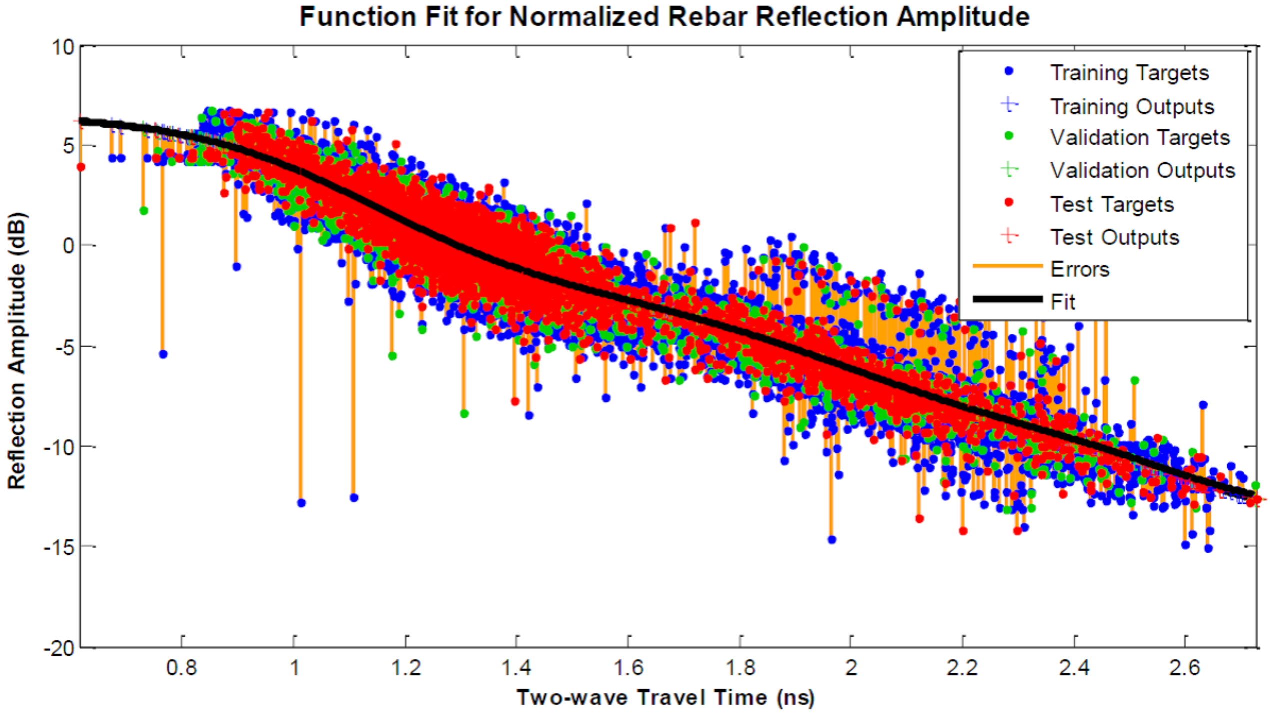

***Figure 6** Function fit with the artificial neural network (ANN).*

5.3 Depth Correction Procedure

As explained previously, it is clear that the relationship between concrete cover thickness and rebar reflection amplitude for sound concrete can be used for the depth correction of GPR data along with the "absolute" condition assessment of bridge decks. The proposed procedure for doing this is depicted in ***Figure 7.*** Specifically, for a concrete bridge deck that needs to be assessed, after time-zero correction, rebar locations (pixels) are picked on the profiles as in the conventional amplitude analysis. These locations are then used to extract rebar reflection amplitudes in data units and two-way travel time for implementing the next steps. While the purpose of the two-way travel time is to determine the reference amplitude from the ANN depth-amplitude model, the direct-coupling normalization is used to normalize the amplitude to the same background. Finally, the differences between the normalized and reference amplitudes are the depth-corrected amplitudes obtained using the proposed methodology. As can be realized, the more negative the depth-corrected amplitude, the more deteriorated the concrete.

6. CASE STUDY IMPLEMENTATION

The Pohatcong Bridge in Warren County, New Jersey, was built in 1978. It has a bare concrete deck resting on five single-span steel girders. The bridge is 36-m long and 11-m wide, and the deck is 25-cm thick. The bridge was scanned in August of 2014 using three NDE technologies, namely: GPR, ER, and IE.

For the comparison, GPR condition maps were created using both methodologies, i.e., the proposed and conventional depth correction techniques. As depicted in ***Figure 8,*** while the spatial distribution of the more deteriorated areas in the two maps appear to be at the same locations, the absolute level of deck deterioration (color spectrum) predicted by the two methods is completely different. To understand which method provides more reasonable results, the two GPR maps were compared to the results from the other NDT technologies. The results from those are provided in ***Figure 9.*** One can clearly observe from the ER results a very highly corrosive environment in the entire deck. Whereas this condition is the same as what was suggested by the proposed methodology, the conventional depth correction approach was unable to detect the global deterioration of the bridge deck.

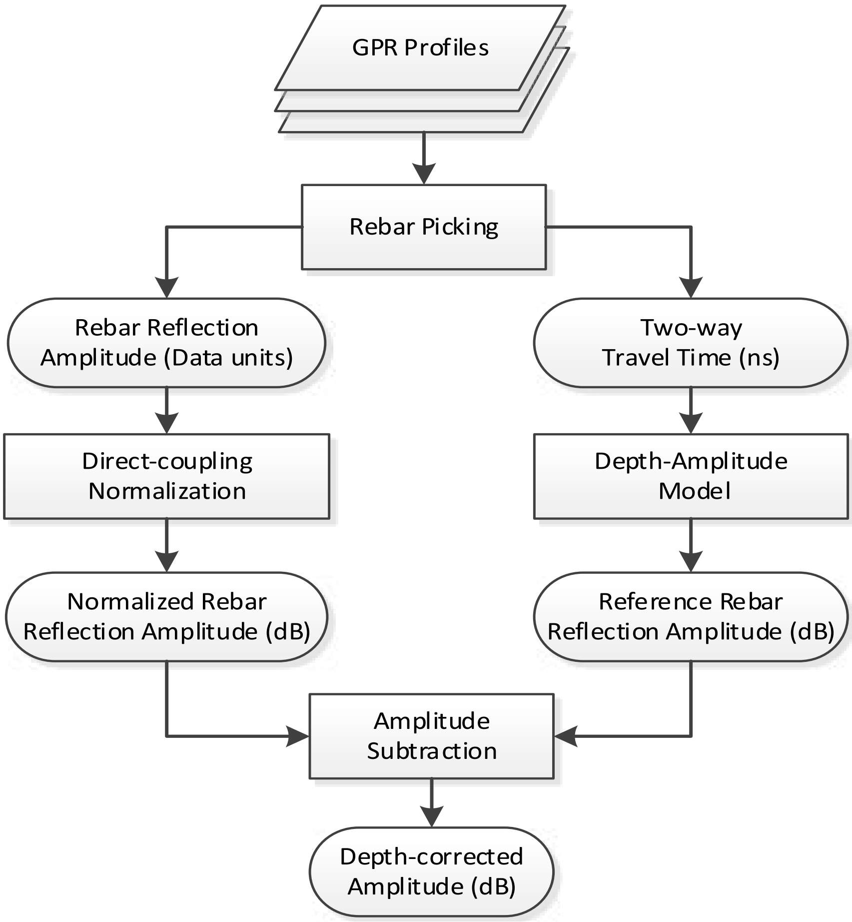

***Figure 7** Depth correction procedure.*

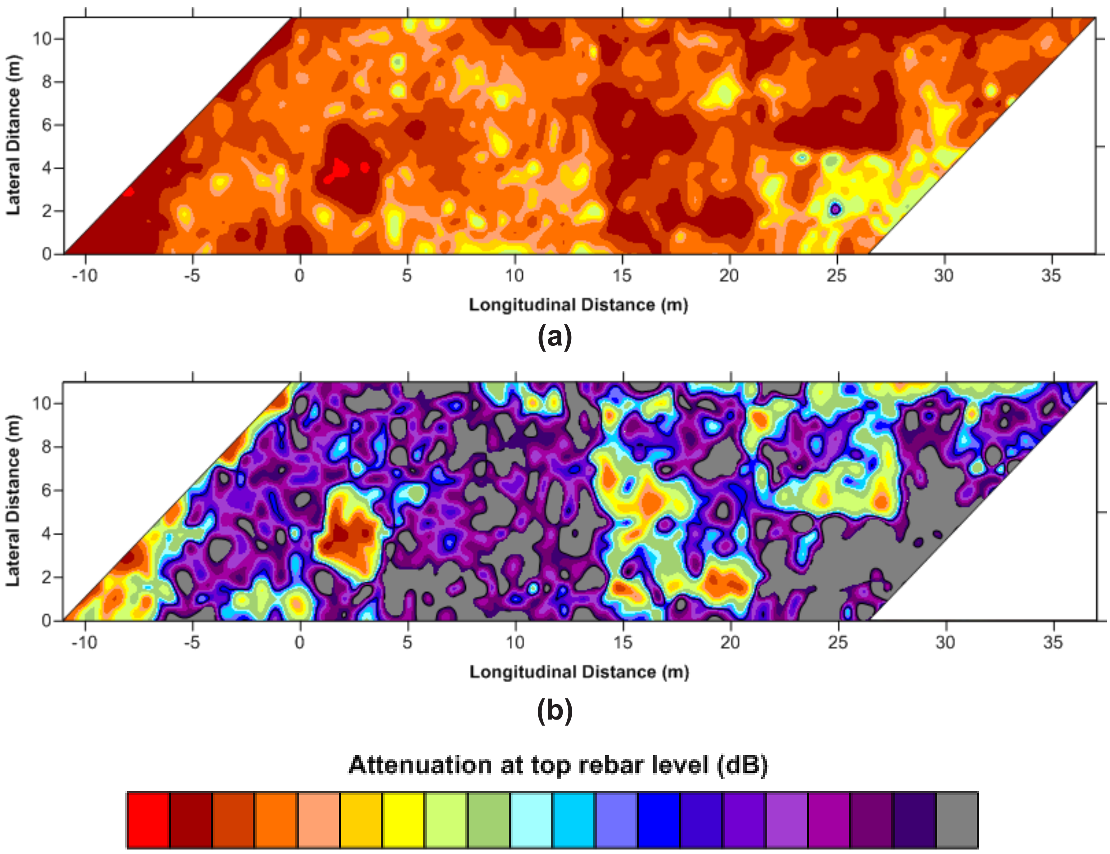

***Figure 8** GPR attenuation maps for the Pohatcong Bridge deck with (a) Proposed method and (b) Conventional depth-amplitude analysis.*

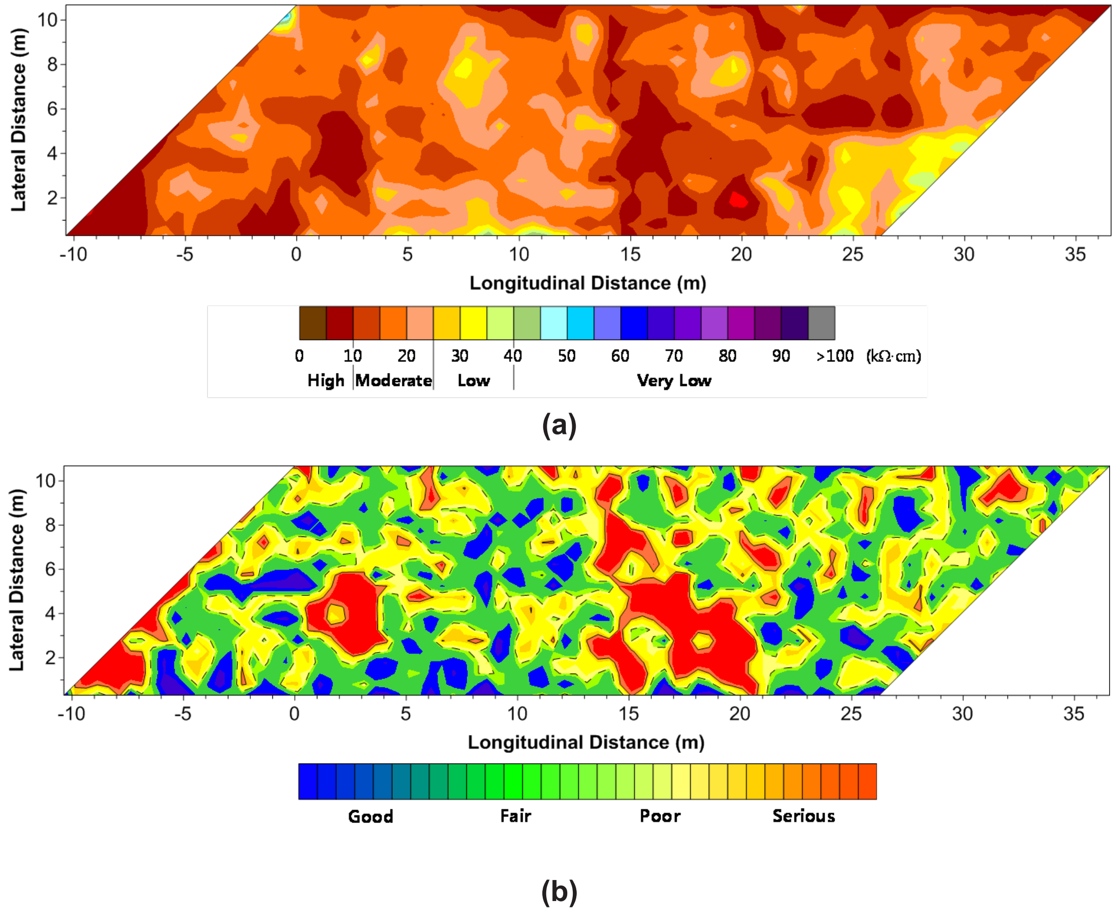

***Figure 9** Condition maps for the Pohatcong Bridge deck with (a) ER, and (b) IE.*

7. DISCUSSION

The implementation of the proposed method for the case study has proven that the GPR is an effective, powerful technology for condition assessment of bridge decks. As has been seen, through using a generic, network wide depth-amplitude model, the GPR has a capability to assess the condition of highly-deteriorated concrete. With the proposed methodology, it is anticipated that the GPR can be deployed as a tool for managing bridge deck assets in which the condition of different decks on the network level can be compared on the same basis. Specifically, the GPR can be used to estimate concrete repair quantity for each bridge deck, to develop bridge deck condition index, and to prioritize maintenance resources.

In order to achieve the above anticipated objective, a clear roadmap for model development is proposed. First, the ANN model should be expanded to include more types of bridge decks with different types of overlays, such as latex modified concrete (LMC), bituminous (asphalt), etc. These materials have properties different from a monolithic (bare) concrete and may respond differently to the propagating electromagnetic source waves. Second, the model should be improved/trained continuously by feeding it with the new data sets. Based on that, the effects of different factors on GPR data will be better understood, including the influence of the rebar depth investigated in this research.

8. CONCLUSIONS

The variation of rebar depth is the most visible factor affecting the condition assessment of bridge decks using the GPR technique. As has been demonstrated, a generic, network-wide depth correction model proposed in this research can minimize the effects of this variance. At the same time, the model can eliminate the relative nature of the overall deck condition when evaluating a depth-corrected data set. As a result, in comparison to the traditional depth correction techniques, the proposed model provides a better description of the absolute deterioration of bridge decks. It is anticipated that the current ANN will be expanded to include more types of bridge decks. In addition, its performance will be continuously improved with the addition of new data sets.

9. ACKNOWLEDGEMENTS

The authors sincerely acknowledge the support of the FHWA's LTBP Program for the support in the bridge deck data collection. The authors are also grateful to Warren County Department of Public Safety for their cooperation in providing access to the bridge in this study. Finally, the authors are grateful to Kenneth Lee, Shane Mott, and Insung Hwang from Rutgers' Center for Advanced Infrastructure and Transportation (CAIT) for their help during the data collection.

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