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TRUST MODEL FOR SOCIAL NETWORK USING SINGULAR VALUE DECOMPOSITION

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ABSTRACT

For effective interactions to take place in a social network, trust is important. We model trust of agents using the peer to peer reputation ratings in the network that forms a real valued matrix.

Singular value decomposition discounts the reputation ratings to estimate the trust levels as trust is the subjective probability of future expectations based on current reputation ratings.

Reputation and trust are closely related and singular value decomposition can estimate trust using the real valued matrix of the reputation ratings of the agents in the network.

Singular value decomposition is an ideal technique in error elimination when estimating trust from reputation ratings. Reputation estimation of trust is optimal at the discounting of 20 %.

KEY WORDS

singular value decomposition, reputation, trust, social network, discounting

CLASSIFICATION

JEL: C65

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INTRODUCTION

Social networks permeate our lives as no agent life in a vacuum; it must interact with other agents to achieve its goals. With so much user interactions, the question of whom to trust has become an increasingly important question. We rely on trust in our day to day interactions and activities with each other [1-3]. Social networks are central to transmission of information as these networks embed dynamic, stochastic and interdependency behaviour of the agents [3-5]. An agent is likely to encounter agent generated content, some of which the agent uses to make decisions and develop context within a community with respect to whom to trust and why. For agents to interact, they require information on whom to trust [1].

The history of past interactions forms a basis for the agents' abilities and dispositions that can be aggregated together through a good reputation system [6]. Reputation is a collective measure of trustworthiness and an important factor in performing trust decisions. Reputation is what is generally said or believed about a person's or thing's character or standing [3]. In social networks, reputation is a quantity derived from the underlying network and the agent's reputation is visible to all agents [7]. Each agent rates all the other agents in the network to form a real valued matrix of the agents' reputation ratings. These peer to peer reputation ratings are used to estimate trust levels of the agents in the network using SVD technique.

TRUST

A number of methods and models for computing trust have been developed [1, 5, 8-11]. The works agree that trust levels play a central role in interactions of agents. Social interaction structures formed over time tend to separate agents into small interaction groups [10]. These social interactions on networks affect agent activities that should be incorporated in social network models to develop optimal models [12].

In the work of [8], agents optionally expressed some level of trust for the other agents. The expressions become entries for a real valued matrix that is used to predict a known trust value between any two users. [1] computes trust using a path probability in a random graph. For each pair of users (x, y), they placed an edge between them with some probability that depends on the direct trust between them denoted by $t_{x,y}$. Trust is represented in the interval (0, 1) by [9]. A value of 1 indicates that the agent is highly trusted and hence blind trust. This model reflects members of social network and differentiates them according to their disposition to trusting somebody.

The basic criteria for judging the quality and soundness of reputation computation engines are highlighted by [13]. Reputation is a perception that an agent has of another's intentions and norms [14]. It is a social quantity calculated based on actions by a given agent and observations made by others in an "embedded social network". Trust is a particular level of subjective probability with which an agent assesses that another agent of group of agents will perform a particular action both before he can monitor such action [15]. We can conclude that trust is a subjective probability or expectation an agent has about another's future behaviour.

We use singular value decomposition (SVD) to extract trust levels from the real valued matrix of the agents' reputation ratings. We discount the reputation ratings by estimating the approximation errors of the matrix. SVD is a matrix approximation technique that is widely researched and a common tool used heavily in recommendation systems, bioinformatics, computer vision and text processing among other applications [3, 16-18]. SVD has an efficient algorithm for its computations; a stable and effective method to split the system into a set of linearly independent components, each of them bearing its own energy contribution [19].

We model the trust levels of agents in the social network using the SVD method which has wide appeal, highly versatile and used extensively in many applications in different areas of research. No known research work that has used SVD to estimate trust from reputation ratings of a real valued matrix. Section three introduces social network and rank application based on SVD. The results are analyzed in section 4 and conclusions are in section 5.

SOCIAL NETWORKS

We assume that reputation ratings about current interactions are captured and distributed and agents are willing to provide the ratings. Consider a set $\mathbf{N} = \{1, 2, \ldots, N\}$ of agents whose state and interactions in a social network evolve in discrete time t. We assume that the agents are connected to each other at any given time $t \in [0, T]$ and thus we have a peer to peer review system for the agent's reputation ratings in the network. Let $R_i = \{r_{1i}, r_{2i}, \ldots, r_{(n-1)i}\}$ be the reputation ratings agent i receives from the other $N-1$ agents in the social network. This peer to peer reputation rating is based on the five star scale: 1- lowest, 2 - low, 3 - medium, 4 - good and 5 - high, that is, $R \in [1, 2, 3, 4, 5]$:

$$R = \begin{cases} r_{ij} = 5, & if \quad i = j, \\ 1 \le r_{ij} \le 5, & if \quad i \ne j. \end{cases} \tag{1}$$

Each agent is expected to rate the other $N-1$ agents. As would be ideal in real life situations, if we were to rate ourselves, we would likely give ourselves a maximum score of 5. The ratings form the entries of a real valued matrix, R and are bidirectional. These entries of matrix R are assumed to be the 'raw' trust values of the agents as [1] notes that trust and reputation are closely linked.

Let T be the matrix of "raw" trust values of the agents. As trust is the future expectation of the current reputation ratings, then $E(T) = \alpha R = R_K$. We estimate the "raw" trust values of the matrix T by discounting the singular values obtained from the SVD with a factor α. This eliminates the noise which represents the future expectations based on current observations in trust estimation. The noise is eliminated by adopting different accuracy threshold from 10 % to 90 %. The error is defined as [20]:

$$\alpha = \sqrt{\frac{\sum_{i=1}^{k} \sigma_i^2}{\sum_{i=1}^{N} \sigma_i^2}}. \tag{2}$$

This is the relative error for a sum of thefirst k terms of the SVD outer product expansion [7]. SVD is used for optimal low rank approximation and a partial SVD can be used to construct a rank k approximation. Given a matrix R, we can represent it as the product of two orthonormal matrices U and V and a diagonal matrix S as $R = USV$. The low rank approximation is to make $\| \tilde{T} - VV^T \tilde{T}^T \|$ as small as possible for the estimation of the trust levels from the reputation ratings [9]. The matrix $\tilde{T} = R_k$ can be decomposed with SVD and thus the discounted matrices. We use simulation with $U \sim (0,1)$, for $1 \le R \le 5$ based on Matlab version 7.0.1.

We compare trust and reputation levels based on the simulated peer to peer reputation ratings which are a real-valued matrix. SVD extracts the trust matrix by discounting the reputation matrix using SVD.

Table 1: Glossary of all matrix names used in the study.

Name	Meaning
T	"Raw" trust levels matrix
R	Reputation ratings matrix
$\widetilde{T} = R_k$	Estimated trust levels matrix (discounted with α)

RESULTS AND DISCUSSIONS

In Table 2, the Friedman test is for the first seven values of the error term $\alpha \in [0,1;0,7]$. We have truncated the last two values in the Table 2 as they basically do not add more information in our analysis. At α = 0,1 the test is statistically significant indicating that in estimating trust from reputation, the noise component is evident. At α = 0,2 and above, the test is not statistically significant. This shows that we can only discount reputation ratings with an error term of 20 % to achieve our desired results in estimating trust from the reputation ratings. Increasing the error term above 30 % does not improve on the model performance. Thus, 20 % is the optimal level of discounting of reputation ratings to estimate trust values of agents in a social network.

Table 2. Friedman tests comparison between Trust, Reputation and Noise.

Error term, in %	10	20	30	40	50	60	70
Reputation & Trust	0,0184	0,1573	0,2513	0,1573	0,1083	0,1573	0,1573
Trust & Noise	0,0073	0,1083	0,2513	0,1573	0,1083	0,1573	0,2513
Reputation & Noise	0,0073	0,1083	0,1573	0,1573	0,1573	0,1573	0,2513

In Figure 1, when the three columns are compared, the first is different from the second and the third. These results are similar to those in Table 2 which shows that after the discounting of 20 %, there is no added value with continued noise elimination. This shows that rank approximation in this scenario is ideal when the error term is set at 20 %, any discounting above this level does not improve the model.

In Figure 2, as evident in Table 2, the second and third columns are similar while the first column is different. We observed that the discounting is optimal when $\alpha = 0{,}2$ which is depicted in the second column of Figure 2. Generally, there is a high similarity between trust and reputation as noted in [1]. Reputation rating is thus an important factor in estimating trust and this rating is an accurate expected value of trust.

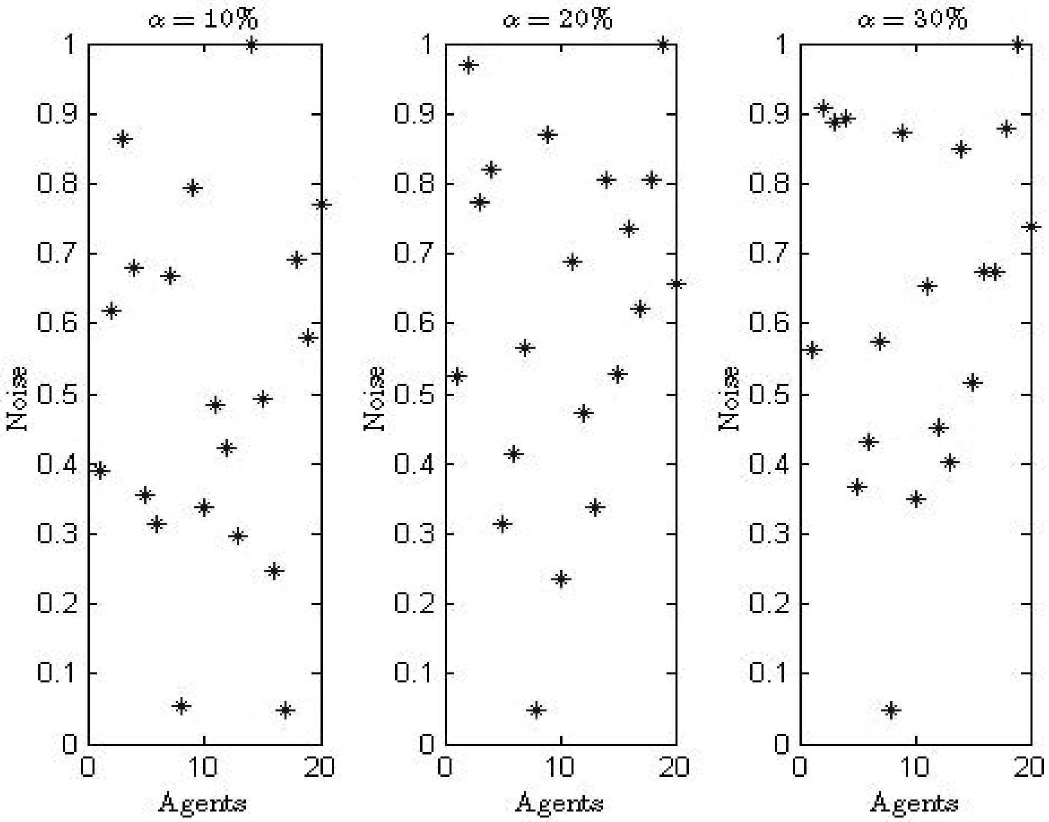

Figure 1. Noise levels for the first three error terms.

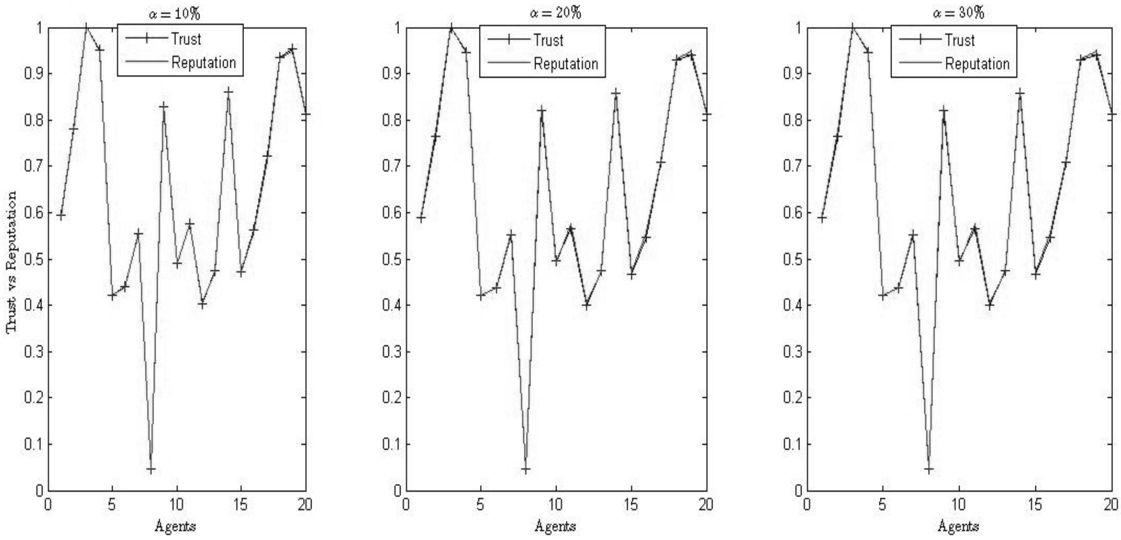

Figure 2. Relationship between trust and reputation levels.

CONCLUSIONS

Trust values in a social network are estimated based on a real-valued matrix of the agents' reputation ratings. The ratings are discounted using SVD as trust is the future expectations based on the current reputation ratings. An optimal discounting of 20 % is optimal with the noise levels remaining constant after this value. Trust and reputation are observed to be closely related. In general, SVD is a versatile technique that is ideal for estimating trust using reputation ratings. Further research in using it to estimate trust and distrust levels in a social network can be applied.

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