Lot Sizing Heuristics Performance

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Abstract Each productive system manager knows that finding the optimal trade-off between reducing inventory and decreasing the frequency of production/replenishment orders allows a great cut-back in operations costs. Several authors have focused their contributions, trying to demonstrate that among the various dynamic lot sizing rules there are big differences in terms of performance, and that these differences are not negligible. In this work, eight of the best known lot sizing algorithms have been described with a unique modelling approach and have then been exhaustively tested on several different scenarios, benchmarking versus Wagner and Whitin's optimal solution. As distinct from the contributions in the literature, the operational behaviour has been evaluated in order to determine which one is more suitable to the characteristics of each scenario.

Keywords Lot Sizing, Heuristics Performances, Unconstrained Single Item Lot Sizing Problem

1. Introduction

The lot sizing problem represents a traditional issue in operations management and operations research since 1913, when Harris [1] first published the economic order quantity model in his famous article "how many parts to make at once." Moreover, those software houses which develop enterprise resource planning (ERP) and materials requirement planning (MPR) applications continuously confirm their appreciation for every lot sizing rule, which is able to approach those critical problems providing fairly good sub–optimal results in a few seconds. Indeed, lot sizing heuristics are embedded in MRP software [2,3]; thus, their efficiency and effectiveness reflects on the company's flexibility and key performance indicators. The lot size problem was modelled in various ways and several solutions were proposed. The incapacitated single-item lot size problem (USILP) represents the starting point for each research investigation on lot sizing problems: indeed, the "single item" or "incapacitated" simplifications do not always prevent the model from being applied in real cases to solve industrial problems. Its most general model - the USILP - takes as an input a vector D[$d_1...d_T$] of T values representing the production requirements in each period t :1…T and provides a solution in a similar vector L[$l_1...l_T$] in which, for each period, the lot size to be ordered is determined taking into account only the inventory carrying cost (h) and the order launch cost (S). This problem was solved to the optimum in 1958 by Wagner and Whitin [4] and several improvements [5, 6, 7, 8, 9, 10] have enabled the reduction of its complexity to O(TlogT).

Historically, research has seen an effort in trying to modify the USILP in order to contemplate all those specific issues which could have brought back the scenario model to the real industrial problem. The Harris model was first modified by Rogers [11] and then by Hanssmann [12] in order to insert limited capacity [13, 14, 15] and multiple items, thus introducing the so-called *economic lot scheduling problem* and bringing it to the *NP*–hard level [16, 17, 18, 19]. In 1969, Zangwill [20] introduced *backlogging* into the USILP; afterwards, a new formulation was given by Pochet and Wolsey in 1988 [21]. Zangwill even treated the case of *multiple facilities*, introducing the use of W_{jkt} "transfer variables" which represented the quantities that must be transferred from plane j to plane k at period t. A similar approach was used in 2000 by Sambasivam and Schmidt [22]. Perishable goods complications were introduced in the general problem in 1963 by Ghare and Shrader [23], and afterwards analysed in the case of continuous-time [24, 25, 26, 27, 28, 29]. Concerning the latter category, in 2000 Hsu [30] proposed a dynamic programming algorithm which found the optimal solution in $O(T^4)$. Other extensions of the lot–sizing problem include the analysis of reworking issues – a first complete analysis can be found in Simpson [31] and later in Richter and Sombrutzki [32] – or the case of bounded production [33]. In order to include priorities among lots, Eppen and Martin [34] introduced the distinction between *big time bucket* and *small time bucket* models, which are brought back respectively to the classes of *discreet lot sizing problems* (DLPs) and *discreet lot scheduling problems* (DLSPs). More recent studies concern the introduction of the concept of *time windows*: specifically, Lee et al. [35] recall that a real industrial supply agreement may provide that demand can be satisfied in a certain period of time instead of a at due date; thus, the supplier should rely on *grace periods*. They analyse this aspect either in the case in which backlogging is allowed or where it is not, and propose two algorithms: $O(T^3)$ and $O(T^2)$. On the other hand, for Dauzere-Peres et al. [36] the *time window* has another meaning, indicating indeed the period of time in which the lot may be processed. They approach this problem with a $O(T^4)$ pseudo–polynomial algorithm. As can be sees, lots of contributions are present in the literature on all the possible variants of the lot sizing problem. Certain authors have devoted entire publications to the classification of general lot sizing problems [37,38]. However, according to Haddock and Hubicky [39], the most commonly used lot sizing technique in real industrial contexts is the simple *lot-for-lot* rule, followed by *fixed order quantity* and *fixed period quantity*; this, despite the fact that, since the 1960s, research has concentrated on finding better performing techniques. In our opinion, this is due to the fact that the results from complete and fair comparisons have not been adequately disseminated.

2. USILP heuristics performances

Lot sizing heuristics analysis still represents a major field of research in operations research and management. Recently, several authors have focused on classifying different solution approaches for various classes of lot sizing problems (LP): dynamic capacitated LP [40], stochastic LP [41], rolling horizon LP [42] and online LP [43]. Regarding the unconstrained LP, some authors provided interesting insights on specific solution techniques: genetic algorithms [44], evolutionary algorithms [45] and neighbourhood searches [46].

As far as USILP algorithms' performance is concerned, various authors cite the work of Nydick and Weiss [47], in which it is shown how the traditional techniques *economic order quantity* (EOQ) and *lot-for-lot* (L4L) may perform poorly while other algorithms – *part period balancing* (PPB), *silver meal* (SM) and *Groff's method* (GM) – achieve much better results. Jeunet and Jonard [48] measured the performance of some algorithms in uncertain environments - i.e., demand variability and a certain grade of flexibility of the production system - a modified heuristic which copes with this matter is shown even in Kropp et al. [49]. In this sense, they found that certain algorithms seem to be more suitable for stable environments, while others for unstable environments; *silver meal* (SM) and the *part period simplified* (PPS) algorithms represent the best trade-off between *cost-effectiveness* and *robustness*. One of the most complete analysis is that of Simpsons [50]: he analyses the following nine heuristics: least unit cost (LUC), least total cost (LTC), silver meal (SM), part period balancing (PPB), Groff's method (GM), McLaren's order moment (MOM), economic order interval (EOI), maximum part period gain algorithm (MPG), considering even Wagner and Whitin's (WW). The reason for including WW in a benchmark test among other heuristics resides in the fact that Simpson tested these algorithms on a *rolling refresh* scenario – which is the standard way of managing production planning in the great part of manufacturing industries – into which the WW algorithm is not optimal anymore; actually, rolling refresh may even be considered as an *extension* of the case of USILP [38]. Simpsons' test is performed through simulation and analysing the algorithms' behaviour on a planning horizon which varies from 4 to 20 periods. The demand distribution variability (Gaussian distribution) is set to three different levels. The inventory holding cost is constant while the order launch cost is a parameter of the simulation. In his article, Simpsons shows that the MPG algorithm performs better than the others, as well as WW on a rolling refresh scenario, while LUC achieves the worst results. However, the classification resulting from Simpsons' analysis does not seem to be so useful in depicting application scenarios for each algorithm; moreover, simulating algorithms' behaviour on a rolling

refresh scenario implies assuming certain hypotheses during the system's *frozen period* – moreover, such results may significantly change depending upon whether the frozen period is *order-based* or *period-based* [51] and involves *safety stock usage* [52].

For this reason, in this work, the most known USILP algorithms have been exhaustively tested across several different scenarios; the *operational* behaviour has been evaluated, while the pure mathematical point of view – which is what industry still refuses to acknowledge – has momentarily been set aside.

The algorithms that have been tested are: least unit cost (LUC), silver meal (SM), Groff's method (GM), Freeland and Colley (FC), part period simplified (PPS), part period balancing (PPB), McLauren's order moment (MOM), maximum part period gain (MPG); as in Simpsons' work, the Wagner Whitn algorithm (WW) has been used as a benchmark, provided that in this specific case (the fixed planning horizon) WW returns the optimal solution.

The algorithms target is to find the lot sizing solution, which corresponds to the minimum cost – inventory holding cost plus order launch cost – in different scenarios with uncertain demand on a fixed planning horizon. The performance of the algorithms, indeed, is measured with two parameters that are recorded in each run:

- *Optimality Distance* (OD): this value represents the cost-delta between the solution given by the algorithm and the minimum cost of the optimal solution, obtained from WW.
- *Optimality Percentage* (OP): this value represents the percentage of the optimal solution found by the algorithm - i.e., the number of times out of 2000 - that the tested algorithm reached the performance of the WW algorithm.

The inventory holding cost and order launch cost have been kept fixed throughout the simulation, so that the economic part period (EPP) was constant, thus obtaining a fair comparison among the performances of each algorithm in all the different scenarios.

3. A review of the main USILP heuristics

In order to compare the algorithms, they need to be brought back to a common formulation framework. Let R be a vector $[R_1, R_2,… R_x]$ where R_x is the number of periods of which the requirements are grouped in the xth-order launch. Clearly, from R, the lot size vector L[$l_1…l_T$] can be easily derived.

In almost all the following heuristics, R can be found through a simple procedure that can be formalized in pseudo C++ code as follows:

```
Begin
    n = T;    'T is the length of the planning period'
    i = 1;    'i is the last period'
    j = 1;    'j is the first period'
    x = 1;    'x is the order number'
    R = {0};  'number of periods grouped in an order
              launch'

        While j ≤ n
        {
        if  (condition) then j = j+1;
        else
        {
        Rx = j +1–i ;
        j = j +1;
        i = j ;
        x = x +1;
            }
        }
End
```

This algorithm follows a *forward* approach evaluating - period by period - the *condition* in order to decide whether to launch the order or not. Besides the *condition*, the complexity of this approach is O(T). The differences among the first 7 heuristics out of the 8 presented here are, then, only in terms of the *condition*.

1. Least Unit Cost [53]: through this simple rule, the requirements in the various periods are aggregated in a single order launch unless the unit cost for period $k + 1$ is greater than that of period k. Recalling that:

d_k is the requirement in period k ;
S is the order launch cost;
h is the inventory holding cost per period,

the order launched in period i will group the requirements of all the periods from i to j until:

$$\frac{S + h\sum_{k=i}^{j+1}(k-i)d_k}{\sum_{k=i}^{j+1}d_k} > \frac{S + h\sum_{k=i}^{j}(k-i)d_k}{\sum_{k=i}^{j}d_k}$$

that is, given i, find the greater value of j subject to the *condition*:

$$F = \sum_{k=i}^{j+1}(j-k+1)d_k \leq \frac{S}{h} = EPP \qquad (1)$$

where EPP is the economic part period, defined as S/h.

2. Silver–Meal Algorithm (also called *Minimum Cost per Period*) [54]: the criterion is similar to that used in the least unit cost heuristic but, instead of considering the cost per unit, the cost per period is evaluated. Thus, the order launched in period i will group the requirements of all the periods from i to j until:

$$\frac{S+h\sum_{k=i}^{j+1}\left(k-i\right)d_k}{j+1} > \frac{S+h\sum_{k=i}^{j}\left(k-i\right)d_k}{j}$$

that is, given *i*, find the greater value of *j* subject to the *condition*:

$$F = \sum_{k=i}^{j}\left(i-k\right)d_k + j(j-i+1)d_{j+1} \le \frac{S}{h} \qquad (2)$$

4. Part-period Simplified (often referred as *Least Total Cost*) [55]: PPM simply operates with the *part–period* criteria: recalling that, for each order, its part–period is defined as the sum of the amounts stored multiplied for their storage periods; for each alternative lot–sizing, it chooses the solution in which the part-period is much nearer to but not bigger than the economic part period. Thus, the *condition* becomes:

$$F = \sum_{k=i}^{j}\left(k-i\right)d_k \le \frac{S}{h} \qquad (3)$$

5. Part-period Balancing [56]: this technique is very similar to the previous one except for the fact that *j* is chosen so that the part-period is the nearest to the EPP; thus, the *condition* is:

$$F = \sum_{k=i}^{j}\left(k-i\right)d_k + \frac{(j-i+1)}{2}d_{j+1} \le \frac{S}{h} \qquad (4)$$

3. Groff's Method [57]: with this heuristic, *k* periods are grouped in the order launch from period *i* = *j* +1–*k* to period *j*, and the *condition* is:

$$F = \frac{(j-i)(j-i+1)}{2}d_j \le \frac{S}{h} \qquad (5)$$

Despite the apparently nonlinear formulation of the constraint, the GM heuristic is based on a part-period comparison criteria as well. It has been shown in (3) that the computation of the part-period in period *i* up to period *j* results as follows:

$$d_{i+1} + 2d_{i+2} + 3d_{i+3} + ... + (j-i-1)d_{j-1} + (j-i)d_j$$

GM assumes the hypothesis of constant demand (i.e., $d_k = d_j$, $\forall k \in [1...n]$) with the value of the last period requirement. Thus, the series becomes:

$$d_j \cdot \sum_{k=i}^{j}(k-i) =$$
$$= d_j \cdot \sum_{n=0}^{j-i}(n) = d_j \cdot \frac{(j-i)(j-i+1)}{2}$$

6. Freeland and Colley [58]: in this heuristic, only the demand in the analysed period, multiplied by the number of periods of storage, is compared to the EPP. The *condition* becomes:

$$F = \left(j-i+1\right)d_j \le \frac{S}{h} \qquad (6)$$

7. McLaren's Order Moment [59]: this heuristics tries to combine the advantages of economic order quantity with the part-period criteria. The number of grouped periods *k* is determined with two checks: the part–period must not exceed the order moment target value (OMT), but the Freeland and Colley rule is applied anyway to the last analysed period. In other words, at first *k* is found searching the greatest value of *j* so that the *condition* is:

$$F_1 = \sum_{k=i}^{j}\left(k-i\right)d_k \le OMT \qquad (7)$$

Subsequently, a second *condition* is checked:

$$F_2 = \left(j-i-1\right)d_j \le \frac{S}{h} \qquad (8)$$

so that if $F_2 > EPP$, then the number of grouped periods will be *k*–1 instead of *k*. The order moment target is defined as follows:

$$OMT = \mu\left(\sum_{t=1}^{T^*-1}t + \left(TBO - T^*\right)T^*\right) \qquad (9)$$

where:

μ is the average value of the demand, in all periods;
TBO is the Time Between Orders, equal to EOQ/μ;
T^* is equal to TBO truncated in two decimals.

8. The Maximum Part-period Gain [60]: as distinct from the previous heuristics, MPG does not follow a *forward* procedure. Briefly, in step 0, all of the part–periods PP_k in each period *k* are set equal to the requirements d_k, thus starting from a lot-for-lot condition. Then, the algorithm searches for the smallest PP_k on the entire planning horizon; the requirement d_k is satisfied with storage from the previous *k*–1 period. Thus, the part–periods valued are updated and period *k* is deleted. The algorithm iterates until all the part–periods in each period are greater than EPP. Note the fact that MPG does not suffer the disadvantages of the algorithms based on forward procedures and so renders as unfair the performance comparison. This has been taken in into account in the rest of the analysis.

4. Performances analysis results

For each algorithm, 152 different scenarios were tested. The stochastic variable which describes the demand follows a normal distribution; thus, each scenario is characterized by an average demand and a different demand standard deviation. Each scenario was tested 2,000 times with randomly generated samples. The average demand value (mean, μ) varies from $2,000 \le \mu \le 40,000$ units per period with an interval of 250 units,

while the demand standard deviation value (σ) varies from $1,000 \leq \sigma \leq 7000$ with an interval of 500 units. The length of the planning horizon is set to $T = 12$ periods. EPP is set equal to 20,000 units.

The following graphs show the OD (*optimality distance*) and OP (*optimality percentage*) of each heuristic in accordance with the average value and to the standard deviation in order to determine the performances of the algorithms in various different scenarios. Increasing the standard deviation, the colour of the lines varies from black (lower σ) to white (higher σ). In the general case, we would reasonably expect poorer performances (higher OD and lower OP) in more uncertain scenarios (higher σ).

4.1 Least Unit Cost

The performance graphs of LUC clearly show great sensitivity to the standard deviation: when the average demand is lower than EPP, the OP in higher σ scenarios falls below 10% (white lines), while in lower σ scenarios it can reach 100% (black lines). On top of this, LUC graphs show poor performance when the average demand gets nearer to the EPP. This is evident in the N-shaped OP pattern represented in Figure 2.
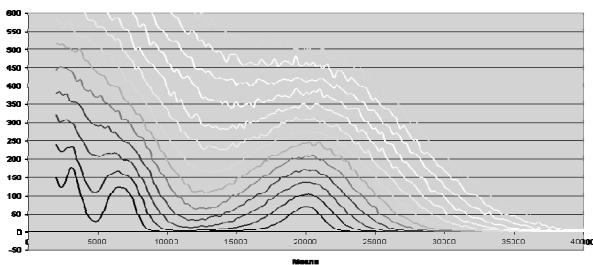
Figure 1. Optimality distance for the LUC heuristic
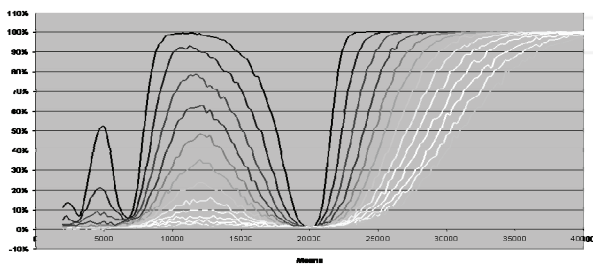
Figure 2. Optimality percentage for the LUC heuristic

This far from optimal behaviour of LUC has already been put forward as evidence in previous works [50, 61] and it is mainly due to the fact that LUC does not use the part–period ratio to determine the lot size but, as can easily be shown in the following example. Let us take ($i = 4$; $j = 7$); in accordance with (1), the LUC algorithm compares as:

$$F = \sum_{k=i}^{j+1} \left(j + 1 - k \right) d_k \leq \frac{S}{h}$$

therefore:

$$F = 4d_4 + 3d_5 + 2d_6 + d_7 \leq \frac{S}{h}$$

Meanwhile, a more appropriate comparison should have been made among the EPP and the part-period, resulting from the chosen solution - i.e., in accordance with the DeMatteis criteria (3):

$$F = \sum_{k=i}^{j} \left(k - i \right) d_k \leq \frac{S}{h}$$

therefore:

$$F = d_5 + 2d_6 + 3d_7 \leq \frac{S}{h}$$

The differences between using (1) or (3) clearly increases with the TBO and this explains why LUC achieves higher performances when TBO values are low (when μ=10,000, the average TBO is 2; when μ=15,000 the average TBO is 1,65).

4.2 Silver Meal

The SM algorithm is based on a criteria that is almost identical to LUC. Indeed, and similar to the LUC graph, it shows the "N-pattern" of OP in Figure 4, with poor performances when the average demand corresponds to EPP. However, a significant improvement comes from the introduction of the cost-per-period ratio: the differences in OD between higher and lower σ scenarios in Figure 3 are much smaller with respect to the LUC case shown in Figure 1. As far as OP is concerned, Figure 4 shows that the SM algorithm reached approximately 50% of optimal solutions when the average demand equals EPP, guaranteeing at least approximately 25% of optimal solutions, even in those scenarios with higher standard deviations.
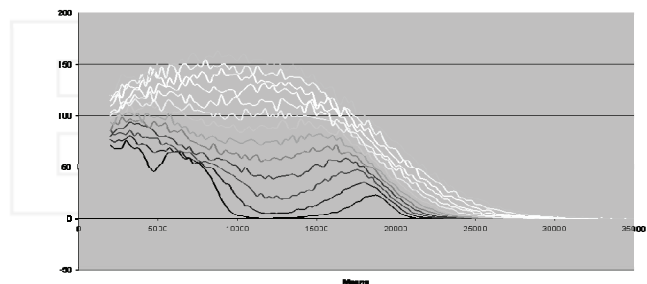

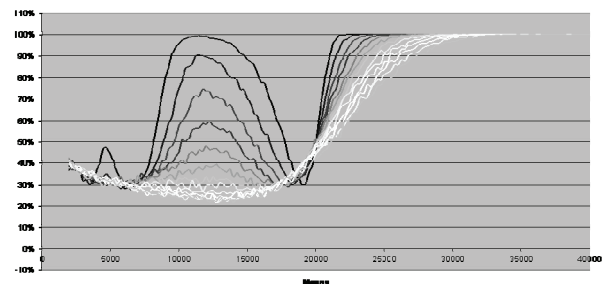**Figure 3.** Optimality distance for the SM heuristic

Figure 4. Optimality percentage for the SM heuristic

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4.3 Part-period Simplified

PPS performances are apparently slightly inferior to those of SM, being present in the OP N-pattern with a minimum percentage 20% of optimal solutions for low values of the average demand, even in low σ scenarios (black lines in Figure 6).
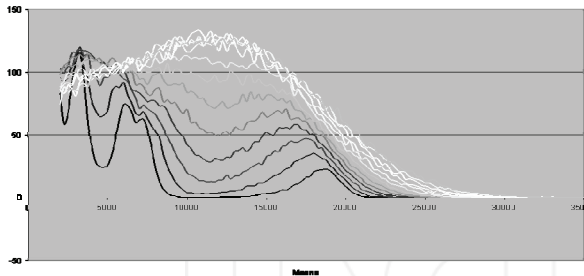

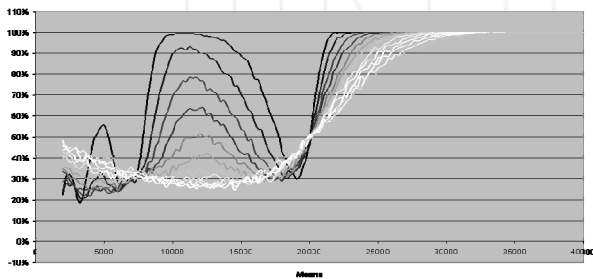**Figure 5.** Optimality distance for the PPS heuristic

Figure 6. Optimality percentage for the PPS heuristic

However, comparing the cost variance of the solutions found by both heuristics in the graphs shown below, PPS shows itself to be a more precise algorithm: the PPS cost variance (Figure 7) is reduced with respect to that of SM (Figure 8).
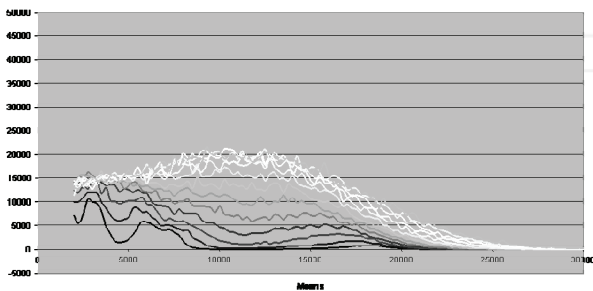
Figure 7. Cost variance of the PPS solutions
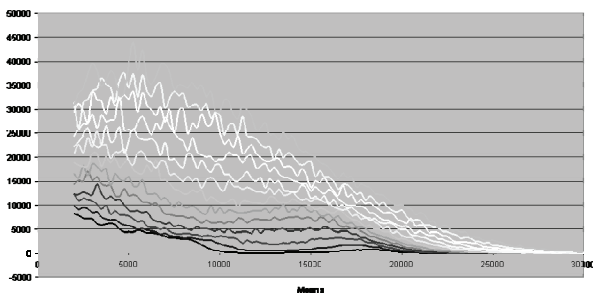
Figure 8. Cost variance of the SM solutions

4.4 Part-period Balancing

The PPB does not succeed in guaranteeing a minimum percentage of optimal solutions, and the performance is very poor when the average demand is lower than EPP/2, as is clearly shown either by the high values of OD or by the low values of OP in Figure 9 and 10 respectively. In the latter, the N-pattern is again evident.
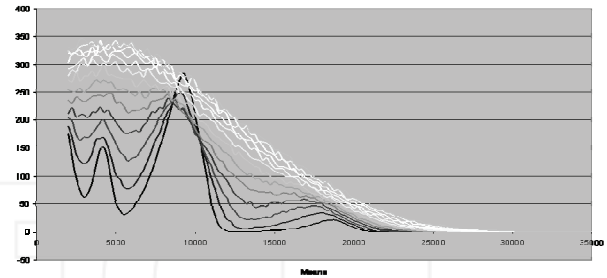

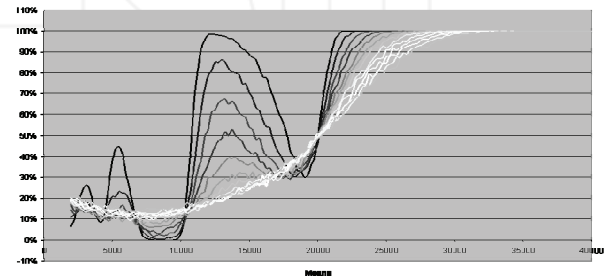**Figure 9.** Optimality distance for the PPB heuristic

Figure 10. Optimality percentage for the PPB heuristic
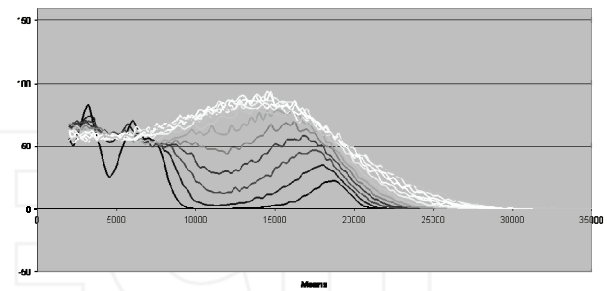
4.5 Groff's Method

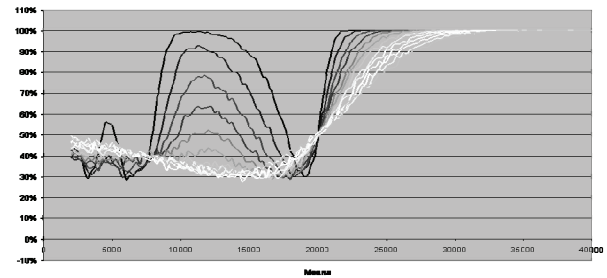**Figure 11.** Optimality distance for the GM heuristic

Figure 12. Optimality percentage for the GM heuristic

The GM heuristic shows a good performance; the peak OD values (white lines) in Figure 11 are the lowest so far. With respect to the results of SM and PPS, the OD graphs

clearly show that the distances among the solutions found by the algorithm and the optimal ones are minor; However, even its OP graph follows the previously shown "N-pattern", with a lower peak near EPP. The speed at which the optimal solutions are found - when the mean goes over EPP - is roughly the same while the lower bound of the percentage of optimal solutions is slightly higher at approximately 30%.
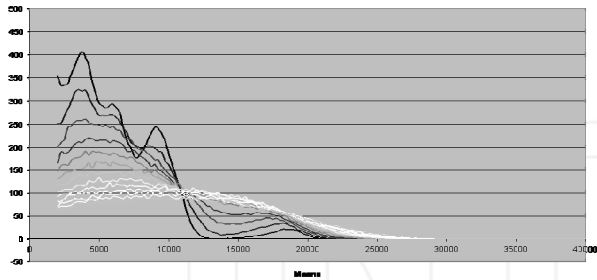
4.6 Freeland and Colley

Figure 13. Optimality distance for the FC heuristic
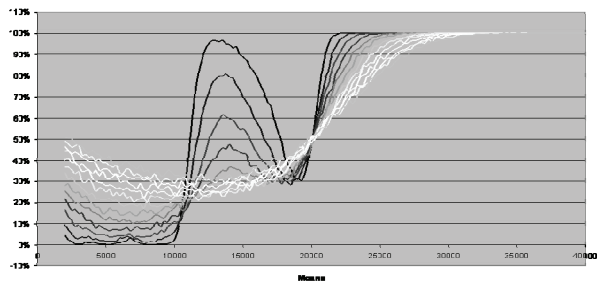

Figure 14. Optimality percentage for the FC heuristic

As is clear from the OD graph, the Freeland and Colley heuristic is quite precise when coping with average demand greater than EPP/2. On the other hand, when $\mu <$ EPP/2, and according to the OP graphs, increased percentages of optimal solutions are counterintuitive in the case of high variance. In this demand interval, the heuristic displays critical behaviour.

4.7 McLaren's Order Moment

The graphs displays behaviour similar to PPB, but the introduction of the second constraint (8) manages to lower the OD values, slightly improving the performances.
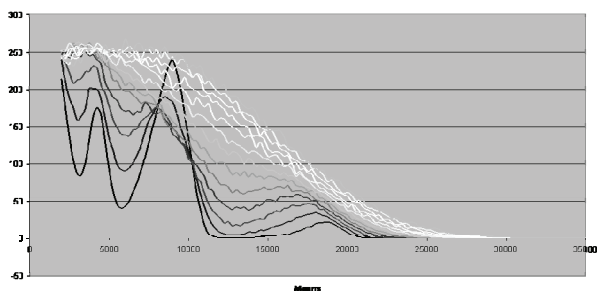

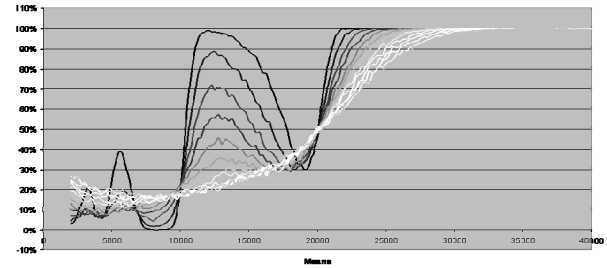**Figure 15.** Optimality distance for the MOM heuristic

Figure 16. Optimality percentage for the MOM heuristic

4.8 Maximum Part-period Gain

MPG is not a *forward algorithm;* indeed, as was expected, this heuristic does not perform like the previous ones: darker lines are at the bottom of the OP graph (Figure 18) and at the top of the OD graph (Figure 17). Thus, performance increases with the variance of the demand. On top of this, no N-pattern is evident. However, the optimality distance is very low with respect to the results of the other heuristics, while the percentage of optimal solutions found for $\mu <$ EPP is higher than 90%. In any case, it is important to recall that MPG may increase the nervousness of the heuristic when facing schedule updating [50]
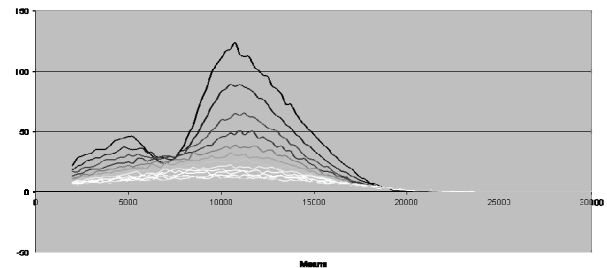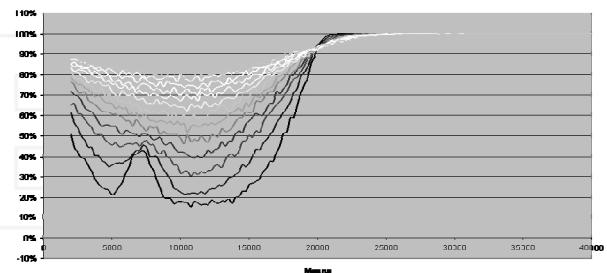

Figure 17. Optimality distance for the MPG heuristic

Figure 18. Optimality percentage for the MPG heuristic

	LUC	SM	GM	PPS	PPB	FC	MOM	MPG
UB OD	927,23	163,6	93,48	133,15	344,05	407,68	267,93	123,85
LB OP	0,05%	22,30%	27,70%	18,30%	0%	0%	0,05%	15,25%
Avg OD	208,2	42,79	28,98	40,19	81,11	52,52	70,07	9,81
Avg OP	44%	67%	70%	68%	61%	63%	62%	84%

Table 1. Numerical performance indicators

Table 1 shows the numerical results of some performance indicators: Upper Bound OD, Lower Bound OP (as a percentage), average OD, average OP (as a percentage). It is possible to see that MPG guarantees a superior average in terms of performance but GM follows immediately

afterwards, guaranteeing an average percentage of 70%. Figure 12 shows a minimum percentage of 27,70% optimal solutions for GM, in 26,000 experiments.

Identifying a scenario through its main characteristics (μ, σ), the heuristic that may offer the best performance can be located on the map, as shown in Figure 19.
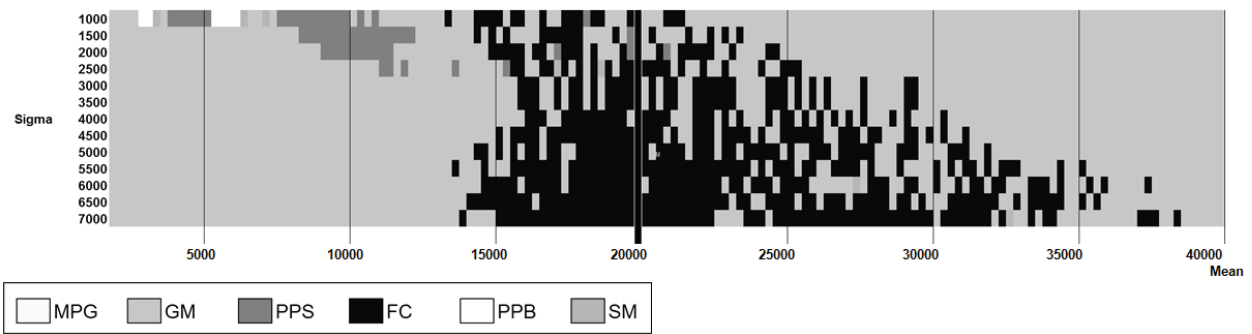

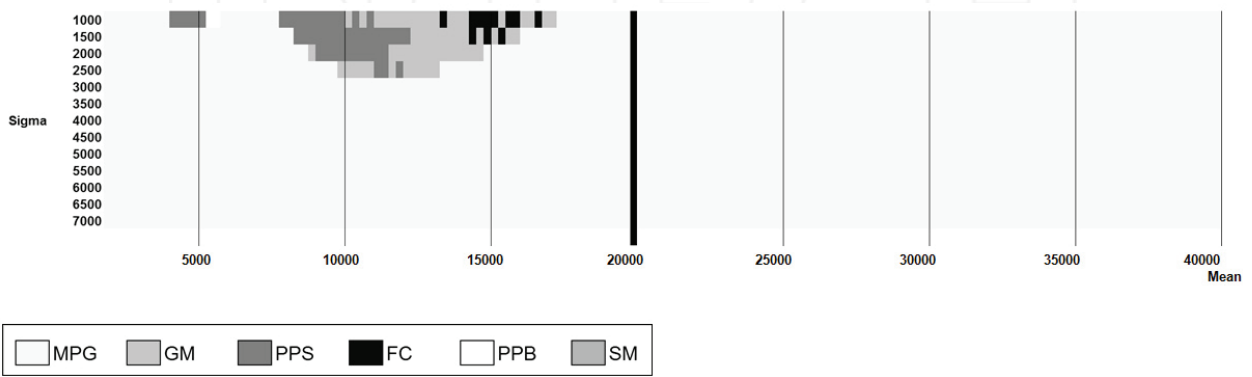**Figure 19.** Suitability of presented heuristics

Figure 20. Suitability of the GM, PPS, FC, PPB and SM heuristics

Clearly, MPG results to be the most effective heuristics in the bulk of all possible scenarios, while GM and PPS are suitable for a small number of cases of deterministic scenarios with quite small average demand. In order to perform a fairer comparison, in Figure 20, MPG has been excluded and only the *forward* algorithms have been located on the map.

Excluding MPG, as far as the experiment presented is concerned, Groff's Method and Freeland and Colley's heuristics seem to be the most adaptable lot sizing methods.

5. Conclusions

For each period inside a planning time horizon, lot sizing procedures determine the opportunity for launching an order or - by way of contrast - carrying the inventory from the previous phases. Input data is represented by (net) requirements. Thus, different scenarios can be characterized by two main elements: the mean value of the period's requirements and its standard deviation. All the presented heuristics show variable performance in different scenarios and so it is opportune to choose the most appropriate in relation to the case that needs to be approached. For this reason, this paper presented an extensive experiment for 8 of the main basic lot sizing

techniques, which have been translated into a comparable mathematical framework and then benchmarked against Wagner and Whitin's exact algorithm.

Two main indicators were considered: the percentage of the optimal solution (OP) and the average cost increase (OD) of the solutions found by each analysed heuristic. Once more, Orlicky's least unit cost algorithm – the only one that does not consider a part-period criterion – has shown the worst performance. It is noticeable that all the heuristics that followed a *forward* computation approach – i.e., that determine the lot sizing alternatives scanning the planning period from the beginning to the end – show a performance trend that depicts a peculiar "N-pattern", with local minimum in correspondence with the economical part-period value. The N-pattern is absent in the only algorithm that does not follow a forward computation approach - the maximum part-period gain.

Future works on the analysis of LP heuristics performance should concentrate on testing these approaches in real industrial cases. Thus, the authors are now working on the implementation of a subset of these heuristics in the MRP software of one of the largest Italian manufacturing companies of home appliances - which at present only works with the *fixed order quantity* and *fixed*

period quantity rules - in order to verify the theoretical results on a real data set.

6. References

[1] Harris FW (1913) How many parts to make at once. The Magazine of Management, 10: 135-136

[2] Bregni A, D'Avino M, Schiraldi MM (Working Paper) A revised and improved version of the MRP algorithm: RevMRP.

[3] D'Avino M, De Simone V, Schiraldi MM (2013) Revised MRP for reducing inventory level and smoothing order releases: a case in manufacturing industry. Production Planning & Control (forthcoming)

[4] Wagner HM, Whitin TM (1958) Dynamic version of the economic lot size model. Management Science, 5, 1: 89-96

[5] Evans JR (1985) An efficient implementation of the Wagner–Whitin algorithm for dynamic lot-sizing. Journal of Operations Management, 5, 2: 229–235

[6] Aggarwal A, Park JK (1986) Improved Algorithms for economic lot size problems. Operations Research, 34: 237-249

[7] Federgruen A, Tzur M (1991) A simple forward algorithm to solve general dynamic lot-sizing models with n periods in O(nlogn) or O(n) time. Management Science, 37: 909-925

[8] Wagelmans PM, Van Hoesel S, Kolen A (1992) Economic lot sizing: an O(n log n) that runs in linear time in the Wagner Whitin case. Operations Research, 40, 1: S145-S156

[9] Van Hoesel A, Ruik R, Salomon M, Van Wassenhove LN (1994) The single item discrete lot-sizing and scheduling problem: Optimization by linear and dynamic programming. Discrete Mathematics, 48: 289-303

[10] Ahuja RK, Hochbaum DS (2008) Solving linear cost dynamic lot sizing problems in O(n log n) time. Operations research, 56, 1: 255-261

[11] Rogers J (1958) A computational approach to the economic lot scheduling problem. Management Science, 4: 264-291

[12] Hanssmann F (1962) Operation Research in Production and Inventory Control. John Wiley, 1962

[13] Bitran GR, Yanasse HH (1982) Computational complexity of the capacited lot size problem. Management Science, 28, 10: 1174-1186

[14] Sandbothe RA, Thompson GL (1990) A forward algorithm for the capacited lot size model with stockouts. Operations Research, 38, 3: 474-486

[15] Du Merle O, Goffin JL, Trouiller C, Vial JP (1999) A Lagrangian relaxation of the capacitated multi-item lot sizing problem solved with an interior point cutting plane algorithm. Approximation and Complexity in Numerical Optimization: Continuous and discrete problems, Kluwer Academic Publishers, 1999

[16] Hsu WL (1983) On the general feasibility test of scheduling lot sizes for several products on one machine. Management Science, 29: 93-105

[17] Yao MJ, Elmaghraby SE (1964) The economic lot scheduling problem under power-of-two policy. Computers and Mathematics with Applications, 41: 465-471

[18] Elmaghraby SE (1978) The economic lot scheduling problem (ELSP): review and extensions. Management Science 24: 587-598

[19] Lopez MA, Kingsmann BG (1991) The economic lot scheduling problem: theory and practice. International journal of Productions Economics 23: 147-164

[20] Zangwill W (1969) A backlogging model and a multi-echelon model of a dynamic economic lot size production system: a network approach. Management Science, 15: 506-527

[21] Pochet Y, Wolsey LA (1998) Lot-size models with backlogging: Strong reformulation and cutting planes. Mathematical Programming, 40: 317-335

[22] Sambasivam M, Schmidt CP (2000) A solution procedure to solve incapacitated lot sizing for multi-plant, multi-period problems with inter-plant transfers. Technical report, University of Putra Malaysia, Faculty of Economics and Management, 2000

[23] Ghare PM, Shrader GF (1963) A model for exponentially decaying inventories. Journal of Industrial Engineering, 14: 238-243

[24] Hariga M (1996) Optimal EOQ models for deteriorating items with time-varying demand. Journal of Operation Research Society, 47: 1228-1246

[25] Papachristos S, Skouri K (2000) An optimal replenishment policy for deteriorating items with time-varying and partial exponential type backlogging. Operations Research Letters, 27: 175-184

[26] Friedman Y, Hoch Y (1978) A dynamic lot size model with inventory deterioration. INFOR, 16: 183-188

[27] Nahmias S (1982) Perishable inventory theory: A review. Operations Research 30: 680-708

[28] Rajagopalan S (1992) Deterministic capacity expansion under deterioration. Management Science, 38: 525-539

[29] Covert RP, Philip GC (1973) An EOQ model for items with Weibull distribution deterioration. AIIE Transactions 5: 323-326

[30] Hsu VN (2000) Dynamic economic lot size model with perishable inventory. Management Science, 46, 8: 1159-1169

[31] Simpson VP (1978) Optimum solution structure for a repairable inventory problem. Operations Research, 26: 270-281

[32] Richter K, Sombrutzki M (2000) Remanufacturing planning for the reverse Wagner/Whitin models. European Journal of Operational Research, 121: 304-315

[33] Love SF (1973) Bounded production and inventory models with piecewise concave costs. Management Science, 20, 3: 313-318

[34] Eppen GD, Martin RK (1987) Solving multi-item capacitated lot sizing problems using variable redefinitions. Operations Research, 35: 832-848

[35] Lee Y, Cetinkaya S, Wagelmans APM (2001) A Dynamic Lot-Sizing Model with Demand Time Windows. Management Science, 47, 10: 1384-1395

[36] Dauzere-Peres S, Brahimi N, Najid N, Nordli A (2002) The single item lot sizing problem with Time Windows. Technical Report, Ecole des Mines de Nantes, France, 2002

[37] Bahl HC, Ritzman LP, Gupta JND (1987) Determining lot sizes and resources requirements: a review. Operations Research, 35, 3: 329-345

[38] Brahimi N, Dauzere-Peres S, Najid NM, Nordli A (2006) Single item lot sizing problems. European Journal of Operations Research 168: 1–16

[39] Haddock J, Hubicky DE (1989) Which lot sizing techniques are used in Materials Requirements Planning? Production and Inventory Management,30, 3: 53-56

[40] Buschkühl L, Sahling F, Helber S, Tempelmeier H (2010) Dynamic capacitated lot-sizing problems: a classification and review of solution approaches. OR Spectrum, 32, 2: 231-261

[41] Winands EMM, Adan IJBF, van Houtum GJ (2011) The stochastic economic lot scheduling problem: A survey. European Journal of Operational Research, 210, 1: 1-9

[42] Tiacci L, Saetta S (2012) Demand forecasting, lot sizing and scheduling on a rolling horizon basis. International Journal of Production Economics, 140, 2: 803-814

[43] Van den Heuvel W, Wagelmans APM (2010) Worst-Case Analysis for a General Class of Online Lot-Sizing Heuristics. Operations Research, 58, 1: 59-67

[44] Goren HC, Tunali S, Jans R (2010) A review of applications of genetic algorithms in lot sizing. Journal of Intelligent Manufacturing, 21, 4: 575-590

[45] Han Y, Cai J, Kaku Y Li Y, Chen Y, Tang J (2012) Evolutionary algorithms for solving unconstrained multilevel lot-sizing problem with series structure. Journal of Shanghai Jiaotong University – Science, 17, 1: 39-44

[46] Xiao Y, Kaku Y, Zhao Q, Zhang R (2012) Neighborhood search techniques for solving uncapacitated multilevel lot-sizing problems. Computers & Operations Research, 39, 3: 647–658

[47] Nydick Jr RL, Weiss HJ (1989) An evaluation of variable-demand lot-sizing techniques. Production and Inventory Management, 30, 4: 41-48

[48] Jeunet J, Jonard N (2000) Measuring the performance of lot-sizing techniques in uncertain environments. International Journal of Production Economics, 64: 197-208

[49] Kropp DH, Carlson DC, Jucker JV (1983) Heuristic lot-sizing approach for dealing with MRP system nervousness. Decision Sciences, 14: 159-169

[50] Simpson NC (2001) Questioning the relative virtues of dynamic lot sizing rules. Computers & Operations Research, 28: 899-914

[51] Sridharan SV, Berry WL, Udayabhanu V (1988) Measuring master production schedule stability under rolling planning horizons. Decision Sciences, 19, 1: 147-166

[52] Sridharan SV, LaForge RL (1990) An analysis of alternative policies to achieve schedule stability. Journal of Manufacturing and Operation Management, 3, 20: 53-73

[53] Orlicky J (1974) Material Requirements Planning. McGraw Hill, New York, 1974

[54] Silver EA, Meal HC (1973) A heuristic selecting lot size requirements for the case of a deterministic time-varying demand rate and discrete opportunities for replenishment. Production and Inventory Management, 14, 2: 64-74

[55] DeMatteis JJ (1978) An economic lot sizing technique: the part period algorithm. IBM Systems Journal, 7, 1: 30-38

[56] Narasimhan S, McLeavy DW, Billigton P (1995) Production Planning and Inventory Control, 2nd edition. Prentice Hall, 1995

[57] Groff GK (1979) A lot sizing rule for time phased component demand. Production and Inventory Management, 20, 4: 66-74

[58] Freeland JR, Colley JL (1982) A simple heuristic method for lot sizing in a time-phased reorder system. Production and Inventory Management, 23, 1: 15-21

[59] McLaren BJ (1977) A Study of Multiple Level Lot Sizing Procedures for Material Requirements Planning Systems. PhD dissertation, Purdue University

[60] Karni R (1981) Maximum part-period gain (MPG): a lot sizing procedure for unconstrained and constrained requirements planning systems. Production and Inventory Management, 22: 91–98

[61] Axater S (1985) Performance bounds for lot sizing heuristics. Management Science, 31, 5: 634-640