RELIABILITY OF GLULAM BEAMS SUBJECTED TO BENDING

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Abstract: This paper presents probabilistic reliability analyses of glulam beams by using level I semiprobabilistic procedure according to Eurocode standards and level II first-order reliability method. The analyses were performed for normal design situations on two groups of girders: roof and ceiling girders. The girders were first dimensioned for the ultimate limit state of bending according to Eurocode 5, after which the limit state equations were set with stochastic values of basic variables. A sensitivity analysis showed that the effect of strength variation is not significant when the coefficient of strength variation is approximately 15%: the recommended value from previous literature. The reliability indices obtained in this study are compared with the corresponding standard values for structural categories RC2 and RC3 according to Eurocode 0.

Keywords: glulam beam; bending; reliability index; probability of failure; limit state equation; Eurocode 5

POUZDANOST LIJEPLJENIH LAMELIRANIH NOSAČA NA SAVIJANJE

Sažetak: U radu je prikazana analiza pouzdanosti lameliranih drvenih nosača na razini I., primjenom semiprobabilističkog postupka danog u Eurokodu i razini II., primjenom metode pouzdanosti prvog reda – FORM metoda. Analiza je provedena za normalnu proračunsku situaciju na dvjema različitim skupinama nosača: krovnim i stropnim nosačima. Nosači su najprije dimenzionirani prema graničnom stanju nosivosti na savijanje u skladu s Eurokodom 5, nakon čega su postavljene jednadžbe graničnog stanja sa stohastičkim vrijednostima osnovnih varijabli. Analizom osjetljivosti je pokazano da utjecaj varijacije čvrstoće na savijanje nije značajan ako se koeficijent varijacije čvrstoće na savijanje uzme oko 15 %, kolika je preporučena vrijednost iz literature. Rješenjem jednadžbi dobiveni su indeksi pouzdanosti koji su uspoređeni s normiranima za razred konstrukcije RC2 i RC3, prema Eurokodu 0.

Ključne riječi: drveni lamelirani nosač; savijanje; indeks pouzdanosti; vjerojatnost otkazivanja nosivosti; jednadžba graničnog stanja; Eurokod 5

1 INTRODUCTION

From an engineering standpoint, reliability represents the probability that a structure or structural element will fulfill its required function without failure under defined conditions in a defined period. The required function of each bearing structure is to carry the load acting on the object, or to have sufficient safety against failure. Failure may not only be related to the collapse of a particular element but may represent any deformation or damage threatening the structure functionality. Although reliability is expressed through a particular failure probability, it represents safety, serviceability, and durability.

The basic concept of structural reliability is given by the European standard EN 1990 [1] and the international standard ISO 2394 [2]. Additional explanations are given in a document issued by the Joint Committee of Structural Safety [3]. Reliability analysis, according to EN 1990 [1], is performed using a semiprobabilistic procedure (level I) according to the concept of limit states by utilizing appropriate partial factors. The procedure is in principle deterministic, in which the predetermined parameters in the limit states equation are defined using probabilistic and statistical methods. These are the so-called characteristic values of resistance and action on the structure. A partial safety factor is assigned to each of these characteristic values in the limit state equation, covering all uncertainties of that basic variable.

In this paper, the aim is to analyze and compare the reliability of glued-laminated timber beams by using the level I semiprobabilistic procedure (Eurocode standards) and level II first order reliability method (FORM) for a normal design situation. Reliability is thereby expressed using reliability index β, which is adopted as a measure of the safety level and is an inverse function of the failure probability p_f.

2 LOAD ANALYSIS AND BEAM DIMENSIONING

The reliability of the girders static system of a simple-supported glued-laminated timber beam was analyzed. The chosen material is a laminated wood of class GL 32h according to the EN 338 [4]. The analysis is performed for two groups of girders: roof and ceiling girders. Depending on their location, the roof girders were analyzed for two snowy regions according to [5]: zone I (100 m altitude), with a characteristic snow load s_k = 1.09 kN/m^2 on the ground; and zone II (600 m altitude), with a characteristic snow load s_k = 3.13 kN/m^2 on the ground. These zones are selected because the most populated areas of Croatia are located at these altitudes. The snow load on the roof is calculated by using the coefficient of roof shape μ = 0.80, which is the function roof slope. A permanent load of 1.0 kN/m^2 including the weight of roof girders is selected. The wind load is not taken into account because of the assumption that the snow is the dominant variable of action and the general lack of reliable statistical parameters of wind action, which are necessary in probabilistic analysis. The ceiling girders of two categories of the object are analyzed. The first imposed load is related to category C with a characteristic load q_k = 4.0 kN/m^2, and the second is related to category D with a characteristic load q_k = 5.0 kN/m^2 [6]. A permanent load of 3.0 kN/m^2, including the weight of the ceiling girder, is selected.

Girders are designed for bending according to the ultimate limit state by EN 1995-1-1 [7]. It was assumed that girders are laterally supported so that lateral torsional buckling is prevented. With the strength modification factor k_{mod}, the reduction of mechanical properties of material due to the influence of load duration and moisture content is considered. For roof girders, k_{mod} = 0.90 (for object service class 1 and the short-term action of snow) and for ceiling girders, k_{mod} = 0.80 (for object service class 1 and the medium-term action of imposed load) [7].

A cross-section subjected to bending is designed according to the following expressions [7]:

$$\sigma_{m,y,d} \le f_{m,y,d} \tag{1}$$

$$\frac{M_{y,d}}{W_y} \le k_{mod} \cdot \frac{f_{m,y,k}}{\gamma_M} \tag{2}$$

By developing expression (2), the following equation in which it is possible to determine the required beam height is obtained:

$$\frac{(1/8)\cdot(\gamma_G \cdot g_k + \gamma_Q \cdot p_k)\cdot L^2 \cdot e}{(1/6)\cdot b \cdot h^2} \le k_{mod} \cdot \frac{f_{m,y,k}}{\gamma_M} \tag{3}$$

where

$f_{m,y,d}$ design bending strength
$M_{y,d}$ design bending moment
W_y section modulus
k_{mod} strength modification factor
$f_{m,y,k}$ characteristic bending strength
γ_M partial factor for material properties
γ_G partial factor for permanent action
γ_Q partial factor for variable action
g_k characteristic value of permanent action
p_k characteristic value of variable action
L beam span
e beam distance
b cross-section width
h cross-section height

The girders were designed considering the entire capacity of the cross-section by using equation (3). A constant value of b = 18 cm was adopted for the beam cross-section. The girder span was varied from 6.00 to 12.00 m in 0.50 m steps, where the distance between girders is held constant at 5.0 m. Table 1 shows the calculation of the required height as a function of the location and purpose of the object.

Table 1 Required cross-section height as a function of beam span

Beam span *L* [m]	Roof beam *h* [cm]		Ceiling beam *h* [cm]	
	Zone I.	Zone II.	Category C	Category D
6.00	29	41	61	65
6.50	32	44	66	70
7.00	34	48	71	76
7.50	37	51	76	81
8.00	39	54	81	87
8.50	42	58	86	92
9.00	44	61	91	98
9.50	47	65	96	103
10.00	49	68	101	108
10.50	51	71	106	114
11.00	54	75	111	119
11.50	56	78	116	125
12.00	59	82	121	130

3 DEFINING THE LIMIT STATE EQUATION

For analyzing probabilistic reliability, it is necessary to first set the corresponding limit state equation. According to [8], the limit state equation for a cross-section subjected to stress in one particular direction in given as

$$g = z_d \cdot R \cdot X_M - \sum_i S_i \tag{4}$$

where z_d is a design variable of geometrical characteristics, R is the resistance, $\sum S_i$ is the sum of all possible load effects, and X_M is the model uncertainty.

3.1 Ultimate limit state equation

The ultimate limit state equation for maximum bending stress is given as

$$g(X) = M_R - M_E = 0 \tag{5}$$

$$g(X) = \left(X_M \cdot \frac{b \cdot h^2}{6} \cdot k_{mod} \cdot f_{m,k} \right) - \left(X_E \cdot \frac{(g+q) \cdot L^2 \cdot e}{8} \right) \tag{6}$$

where X_M and X_E are the model uncertainties of resistance and load. If all influential parameters are shown as random variables, X_i limit state equation takes the form

$$g(X) = X_1 \cdot \left[\frac{X_2 \cdot X_3^2}{6} \cdot X_4 \cdot X_5 \right] - X_6 \left[\frac{(X_7 + X_8) \cdot X_9^2 \cdot X_{10}}{8} \right] = 0 \tag{7}$$

where

- X_1 model uncertainty of resistance
- X_2 cross-section width
- X_3 cross-section height
- X_4 strength modification factor
- X_5 bending strength
- X_6 model uncertainty of load model
- X_7 permanent action
- X_8 variable action
- X_9 beam span
- X_{10} beam distance

4 STATISTICAL PARAMETERS OF BASIC VARIABLES

According to the probabilistic calculation method, that is, level II FORM, to determine the reliability index β as an inverse function of the failure probability p_f from all the statistical data of random variables that enter into the limit state equation, a mean value and standard deviation are needed. However, defining these variables is not straightforward because in literature it is difficult to determine suitable distribution functions and coefficients of variation for certain variables. This is partly because of the lack of test records or the general lack of conducted experiments. In this study, the statistical data given in [8–13] is selected, and each particular basic variable is described in the next paragraph.

1) The model uncertainty of resistance represents the ratio of the actual resistance of the girder and that obtained through experiments, and the lognormal distribution is assumed with mean value $\overline{X_1}$ = 1.0 and coefficient of variation V_1 = 0.10.

2) For cross-section width, a normal distribution is assumed with a mean value equal to the nominal $\overline{X_2}$ = 18 cm and the coefficient of variation V_2 = 0.01.

3) For cross-section height, a normal distribution is assumed with a mean value equal to the nominal $\overline{X_3}$ = h according to Table 1 and the coefficient of variation V_3 = 0.01.

4) The strength modification factor X_4 considers the humidity of space and long-term effects of action and is adopted as a deterministic parameter with values 0.90 and 0.80 for the roof and ceiling girders, respectively.

5) For bending strength X_5, the lognormal distribution was assumed with coefficient of variation V_5 = 0.15. However, because of insufficient number of experimental tests, the coefficient of variation in the parametric study was varied from 0.05 to 0.40 to determine its effect on the failure probability. The chosen material is wood of class GI 32h with 5% fractile value f_k = 3.2 kN/cm² [4]. The parameters of the lognormal distribution λ and ζ are determined by the following expression:

$$\zeta = \sqrt{ln\left(V^2+1\right)} \tag{8}$$

$$F\left(X_{0,05}\right) = F\left(32\right) = F_U\left(\frac{lnX_{0.05} - \lambda}{\zeta}\right) \tag{9}$$

where F_u represents cumulative function of a standard normal distribution. From these parameters, the mean value and standard deviation are obtained as follows:

$$\overline{X_5} = exp\left(\lambda + \frac{\zeta^2}{2}\right) \tag{10}$$

$$\sigma_5 = exp\left(\lambda + \frac{\zeta^2}{2}\right) \cdot \sqrt{exp\left(\zeta^2\right) - 1} \tag{11}$$

The calculated mean value and standard deviation as functions of the coefficients of variation are given in Table 2.

Table 2 Statistical parameters of bending strength variable

V_5	ζ	λ	$\overline{X}_5$	σ_5	$\overline{X}_5 / X_{0.05}$
0.050	0.050	1.245	3.478	0.174	1.087
0.075	0.075	1.286	3.630	0.272	1.134
0.100	0.100	1.327	3.789	0.379	1.184
0.125	0.125	1.368	3.958	0.495	1.237
0.150	**0.149**	**1.409**	**4.136**	**0.620**	**1.292**
0.175	0.174	1.449	4.323	0.757	1.351
0.200	0.198	1.489	4.520	0.904	1.413
0.225	0.222	1.529	4.728	1.064	1.477
0.250	0.246	1.568	4.946	1.236	1.545
0.275	0.270	1.607	5.175	1.423	1.617
0.300	0.294	1.646	5.415	1.624	1.692
0.325	0.317	1.684	5.667	1.842	1.771
0.350	0.340	1.722	5.931	2.076	1.853
0.375	0.363	1.760	6.207	2.328	1.940
0.400	0.385	1.797	6.495	2.598	2.030

6) The model uncertainty for load represents the results of negligence, for example, of 3D effects, inhomogeneities, interactions, boundary effects, and simplifications. Therefore, the lognormal distribution with mean value $\overline{X_6}$ = 1.0 and coefficient of variation V_6 = 0.10 is assumed.

7) For permanent load, a normal distribution with the mean value equal to the nominal value is assumed. For roof girders, $\overline{X_7}$ = 1.0 kN/m^2 and for ceiling girders, $\overline{X_7}$ = 3.0 kN/m^2. The coefficient of variation V_7 = 0.1.

8) For variable load X_8, the Gumbel distribution is selected for both groups of girders. Data of the snow load were obtained from [5], in which statistical parameters of snow regime were studied. Table 3 lists the data of mean values and standard deviations.

Table 3 Statistical parameters for variable load

Variable load		$X_{0,05}$ [kN/m^2]	$\overline{X}_8$ [kN/m^2]	V_8
Snow load [5]	Zone 1.	1,09	0,36	0,688
	Zone 2.	3,13	1,232	0,461
Imposed load [13]	Category C	4,0	0,940	0,74
	Category D	5,0	0,965	1,06

9) For girder span, a normal distribution with the mean value equal to the nominal value, that is, $\overline{X_9}$ = L, according to Table 1 is assumed. The coefficient of variation V_9 = 0.01.

10) For distance between girders, a normal distribution with the mean value equal to the nominal value, that is, $\overline{X_{10}}$ = 5.0 m is assumed. The coefficient of variation V_{10} = 0.01.

5 RESULTS OF PARAMETRIC ANALYSIS

According to the limit state equation (7) and the earlier defined basic variables, the parametric reliability was analyzed using the software package component reliability–time invariant (COMREL-TI), which is an integral part of the software package STRUREL [14]. The calculated reliability indices are compared with the standardized indices according to EN 1990 [1], and shown in Table 4. The relationship between reliability index and failure probability is shown in Table 5 [1].

Table 4 Recommended minimum values for reliability index β (ultimate limit states) [1]

Reliability Class (RC)	Minimum values for β	
	1 year reference period	50 years reference period
RC 3	5,2	4,3
RC 2	4,7	3,8
RC 1	4,2	3,3

Table 5 Relation between β and p_f [1]

p_f	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}
β	1,28	2,32	3,09	3,72	4,27	4,75	5,20

Figure 1 shows the calculated reliability indices for the roof and ceiling girders. The roof girders are observed to satisfy reliability class RC2 but not RC3. Furthermore, roof girders in snow zone I have slightly higher reliability indices compared with the same girders in snow zone II. Figure 1 shows the analysis of the ceiling girders, which satisfies the reliability class RC3. Thereby, girders from category C have higher reliability indices than girders from category D. From these results, conclusions can be drawn similar to those in [10], that is, with the increase in demands on the structure, the level of reliability of the structure designed according to Eurocode is changing and can be insufficient for reliability class RC3, such as in the example of snow load. The Eurocode for all types of construction predicts a reliability of class RC2 and reference period of 50 years. For instance, for reliability class RC2, the normed reliability index is β = 3.8, for which partial coefficients of resistance and action are calculated. Therefore, for exceptional structures, class RC3 is necessary to determine the value of partial coefficient to be used in the calculation because the values are not defined by standards [15], [16].

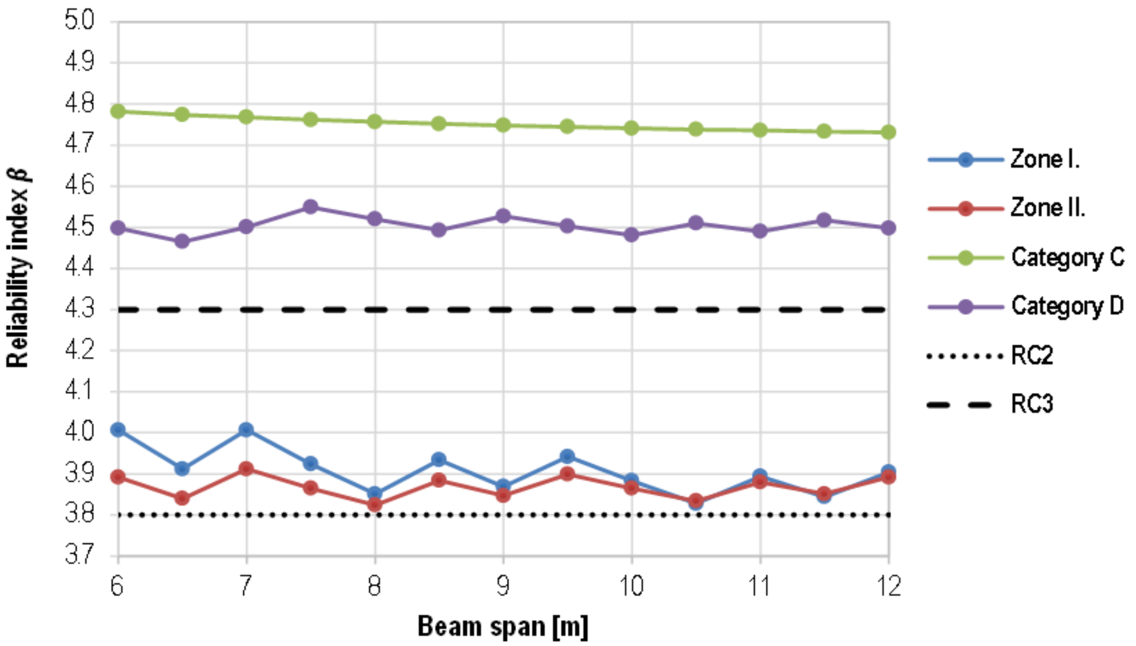

Figure 1 Reliability indices for girders of different purposes and spans

Figure 2 shows the sensitivity factor α_i for each variable that enters the limit state equation. It shows the impact of individual variable on the probabilistic analysis. Evidently, some random variables such as cross-sectional dimension, span, or distance between girders have almost no influence. Furthermore, permanent load has a small effect on both the analyzed groups of girders. Such variables may therefore be treated as deterministic. It is important to emphasize that the statistical parameters adopted from literature have an extremely low coefficient of variation, and therefore have low effect on the analyzed group of girders. Their exact contribution depends on the quality control of factory production and performance, and could be determined only on the basis of the analysis of existing built-in girders. Evidently, the greatest impact is caused by the variable of imposed load, followed by the material strength and model uncertainty of resistance and action. The modification factor is used as a deterministic parameter through analysis, and thus a sensitivity factor is equal to zero in all cases.

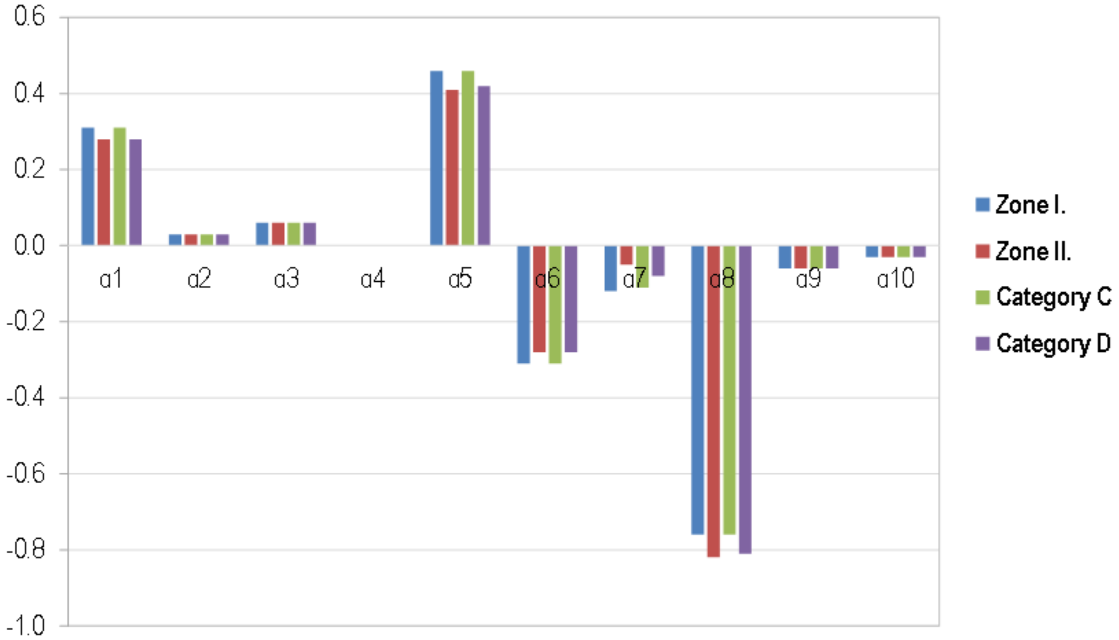

Figure 2 Sensitivity coefficient α_i for girders span L = 10 m

As the variable of bending strength has a great sensitivity coefficient α_5 and thus a great impact in the probabilistic analysis, parametric analysis is conducted according to the data given in Table 2 for both groups of girders with a span of 10 m. Figure 3 shows the change in the reliability index β according to the coefficient of strength variation, whereas Figure 4 shows the change in probability of failure p_f as a function of the coefficient of strength variation. Figure 3 shows that the relationship is not monotonic; instead, it has a maximum value between 0.125 and 0.15. Outside this range, the reliability indices decrease. A similar dependence can be observed in Figure 4, in which the failure probability is minimum inside the interval of the coefficient of strength variation from 0.125 to 0.150. From this figure, the probability of failure is observed to increase over 10-fold as the coefficient of variation is increased from 0.15 to 0.40.

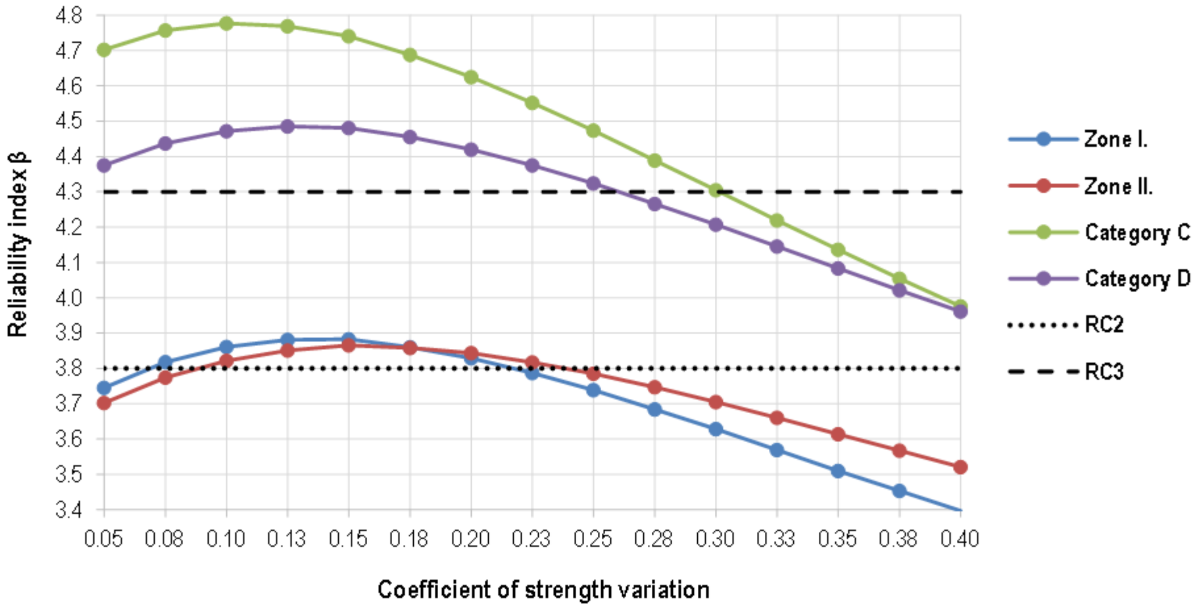

Figure 3 Reliability index β as a function of coefficient of strength variation

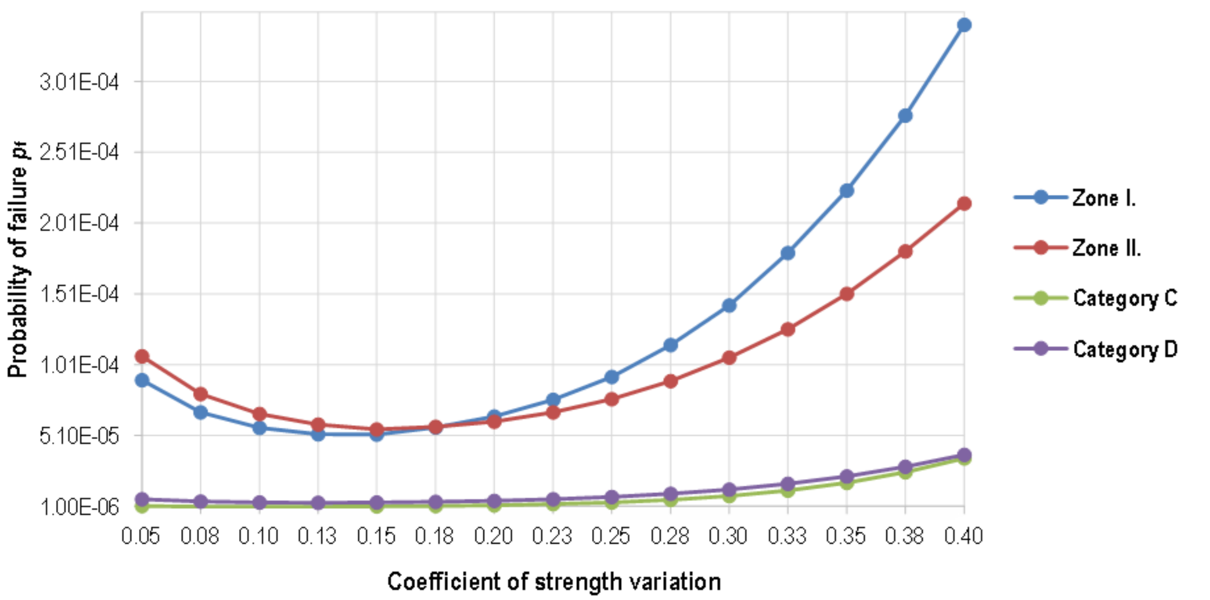

Figure 4 Probability of failure p_f as a function of coefficient of strength variation

As the change in the failure probability, as a function of the coefficient of strength variation, in the range from 0.125 to 0.150 is not very sensitive, the recommended value for the coefficient of strength variation for glued-laminated timber of 15% [12] is considered to be justified. Considering the failure probability, there seems to be no reason for attempting to decrease the strength variation, unless the characteristic value of material strength is affected. This low sensitivity of the strength variation is an advantage, considering the accuracy of reliability analysis of a glulam structure. This is because the lack of experimental studies prevents the knowledge of a precise value for strength variation.

6 CONCLUSION

This paper presented a comparative reliability analysis of glued laminated timber beams by using level I semiprobabilistic design method (according to Eurocodes) and level II probabilistic analysis using FORM. A bending limit state equation with a total of 10 independent random stochastic variables is defined.

The sensitivity analysis showed that the variable of bending strength greatly affects the reliability index. Owing to the insufficient number of experimental tests conducted so far, the coefficient of strength variation is not known precisely, and the recommended value according to the available test results is 15%. According to parametric analysis, the reliability is not very sensitive for this parameter, and thus it does not matter if it is not known precisely.

The reliability indices obtained for roof girders are higher than the minimum required for the reliability class RC2 (normal structure) according to EN 1990 [1], whereas the indices are lower than those required for reliability class RC3 (exceptional structure) for the same norm. In the case of ceiling girders, all the obtained reliability indices satisfy the minimum required for reliability class RC3. Therefore, with the increasing demands on the structure, the level of reliability of structures designed according to EN 1995-1-1 [7] for class RC3 may be insufficient. This is because standard EN 1990 [1] provides a partial safety factor for resistance and action of the common construction class RC2. As reliability class RC3 requires lower probability of failure, which is reflected in higher normed reliability index, higher values of partial factors should be determined. For timber structures of reliability class RC3, level II probabilistic analysis should be conducted because the analyzed example in this paper shows that reliability according to EN 1995-1-1 [7] may be insufficient.

References

[1] European Committee for Standardization (CEN): *EN 1990:2002, Eurocode 0: Basis of structural design*, Brussels, 2002.
[2] International Organization for Standardization (ISO): *General principles on reliability for structures*; ISO 2394, Third edition.
[3] Joint Committee of Structural Safety (JCSS): *Probabilistic Model Code, Part 1 – Basis of Design*, 2001.
[4] EN 338: Structural Timber – Strength classes, 2003.
[5] Zaninović, K.; Gajić-Čapka, M.; Androić, B.; Džeba, I.; Dujmović, D. 2001: Determination of a typical snow load, Građevinar, 53 (10), pp. 619-629.
[6] European Committee for Standardization (CEN): *EN 1991:2002*, *Eurocode 1: Actions on structures*, *Part 1-1: General actions – Densities, self-weight, imposed loads for buildings*, Brussels, 2002.
[7] European Committee for Standardization (CEN): *EN 1995-1-1:2004*, *Eurocode 5: Design of timber structures*, *Part 1-1: General – Common rules and rules for buildings*, Brussels, 2004.
[8] Köhler, J.; Sørensen, J. D.; Faber, H. V. 2007: Probabilistic modeling of timber structure, Structural Safety, 29(4), pp. 255-267, 2007, http://dx.doi.org/10.1016/j.strusafe.2006.07.007
[9] Torrati, T.; Schnabl, S.; Turk, G. 2007: Reliability analysis of a glulam beam, Structural Safety, 29(4), pp. 279-293, http://dx.doi.org/10.1016/j.strusafe.2006.07.011
[10] Androić, B.; Čizmar, D.; Rajčić, V. 2008: Reliability analysis for laminated timber girders, Građevinar, 60(6), pp. 513-518.
[11] Joint Committee of Structural Safety (JCSS): *Probabilistic Model Code, Part 2 – Load Models*, 2001.
[12] Joint Committee of Structural Safety (JCSS): *Probabilistic Model Code, Part 3 – Resistance models – Properties of Timber*, 2006.
[13] Tkalčević, V.; Džeba, I.; Androić, B. 2007: Reliability of castellated beams subjected to lateral-torsional buckling, Građevinar, 59(4), pp. 311-318.
[14] Strurel, *A structural Reliability Analysis Program System*, COMREL SYSREL Users Manual, RCP GmbH. 1999.
[15] Androić, B.; Dujmović, D.; Džeba, I. 2006: Inženjerstvo pouzdanosti 1, I.A. Projektiranje, Zagreb
[16] Milčić, V.; Peroš, B. 2003: Uvod u teoriju sigurnosti nosivih konstrukcija, Građevinski fakultet Sveučilišta u Splitu