Mercator's Projection – a Breakthrough in Maritime Navigation

Mercator projekcija – napredak u pomorskoj navigaciji

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Summary

This paper is focused on Mercator's projection as a breakthrough in maritime navigation. In the paper, the principle and properties of Mercator's projection are described. The advantages, disadvantages and current utilization are mentioned.

KEY WORDS

Mercator's projection, navigation, mapping, azimuth

Sažetak

Ovaj članak se usredotočuje na Mercator projekciju kao napredak u pomorskoj navigaciji. U članku su opisani principi i osobine Mercator projekcije. Prikazani su prednosti, nedostaci i sadašnje korištenje.

KLJUČNE RIJEČI

Mercator projekcija, navigacija, kartografija, azimut

1. INTRODUCTION

One of the oldest logistic problems is finding a correct route from one place to another. The increasing sea travel has made the problem topical on a global scale. To solve this issue, more or less precise maps have been created. The invention of Mercator's projection was a breakthrough not only in cartography, but also in sea travel.

This discovery is listed as one of the greatest milestones in mathematics [1]. The main benefit of Mercator's projection is that it is conformal (i.e., it preserves angles). Using this fact in navigation with a constant azimuth is very simple. Mercator's mapping is used for creating maps also in present days.

Note that current logistic problems are being studied very intensively (see for example [2-5]).

2. MERCATOR'S LIFE

One of the most important cartographers in history, Gerardus Mercator (1512 - 1594), was born as Gerard de Creme (Gerard De Kremer) in the Flemish town of Rupelmonde (nowadays a part of Belgium). At that time, Latin was the language of Europe's educated elite and young scholars latinized their names routinely. Therefore, Gerard De Kremer decided to latinize his name to "Gerardus Mercator".

He studied in Hertogenbosch and Louvain. In 1532 Mercator received a master's degree at the Louvain University. He studied humanities and philosophy, attended lectures given by an excellent mathematician and astronomer Gemma Frisius (1508-1555). Later, he focused on geography and its related disciplines. Gerardus Mercator became a respected specialist in cartography. He profited from creating high-quality maps and globes. He established his own workshop in the city of Duisburg. A very interesting fact is that he worked as a volunteer teacher of mathematics at local higher school and, moreover, he prepared syllabus of mathematical education.

Mercator also devised a new method of constructing globes (terrestrial and celestial), that enabled extensive production. His innovation completely transformed the process of making globes. At his time a globe was created by engraving wooden or brass spheres, Mercator's technique consisted of laminating paper and plaster on a wooden construction. To these days, several of his globes have been preserved. Mercator is often referred to as "a father of atlas". He was the first who used the word "atlas" to denote a collection of maps.

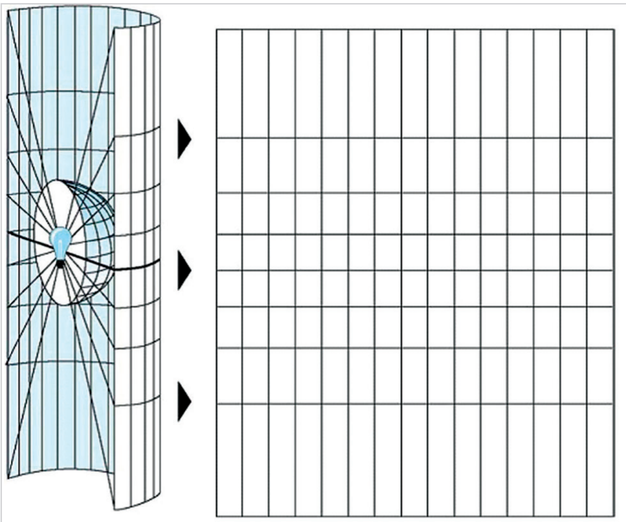
In 1569 he created a wall map, which made him famous as a brilliant cartographer. This is one of the most famous maps in history. The map was created by using cylindrical projection upon a plane with a correction to conformality. In other words, this projection preserves angles. The Mercator's projection is the most common projection used in maritime navigation. This enables great simplification of maritime navigation (planning sea roads), because it was possible to use only a compass and the Mercator's map.

3. MERCATOR'S PROJECTION AND MATHEMATICAL NOTES

Mercator's projection is a cylindrical projection adjusted to be conformal (it means that this projection preserves angles). Moreover, compared to other conformal mappings, Mercator's projection possesses a unique feature: the loxodrome (curve that crosses meridians in a constant angle (Figure 2)) is projected to a straight line.

Because of this feature, maps utilizing Mercator's projection were an invaluable tool used in maritime navigation (almost necessary for maintaining a constant azimuth of sail before the existence of satellite navigation). For the purpose of sailing it is sufficient to connect starting and target points of the planned way, find the angle between this line and any meridian in the map (Figure 3). Therefore, it was possible for a 16th century sailor to determine the correct bearing to follow in order to arrive at the intended destination. In the 18th century the utilization of Mercator's maps expanded even more with the invention of the marine chronometer, which enables to determine longitude using stellar navigation.

The principle of Mercator mapping is a projection of the sphere (resp. spheroid) to the cylinder, the equator is usually its line of tangency (Figure 1). There are several modifications of this mapping, which differ in line of tangency (e.g. a meridian or a great circle).



Source: [6]

Figure 1 Principle of normal Mercator's projection

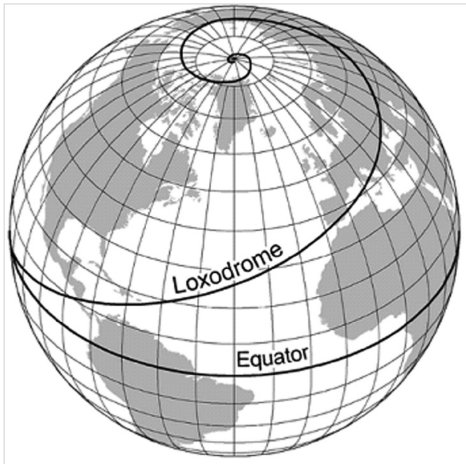
The surface of the cylinder is a map (in the meaning of the surface unrolled to plane). Note that the meridians and the parallels create a rectangular grid on the map. The distance between the meridians is constant, while the distance between the parallels increases with a coefficient , where is the latitude.

An important issue in creating maps is that the whole surface of the Earth cannot be projected onto a single map. For mapping of the Earth we need more than one map, because bijection from the whole surface of the globe onto the plane does not exist. To create an ideal map, one needs a projection which preserves angles, distances and areas. Unfortunately, there is no projection which preserves all these measurements. These issues follow from topological properties of space and properties of projection (this fact is known as Gauss egregium theorem, [7]). The properties of map projection are studied by methods of differential geometry [7-9]. For purposes of navigation preserving angles is the most important property.

The exact method that Mercator used to create his map is not known. He probably used graphical means using a compass and a straight edge. Required mathematical apparatus (logarithm, calculus, differential geometry) was not invented in his time. Edward Wright (1558? –1615) was the first mathematician who described the mathematical properties of Mercator mapping in his work *Certaine Errors in Navigation* (in 1599). The mathematical properties were studied in a lot of scientific papers. We recommend an article [10] in which the author derived equations of Mercator's projection using only elementary calculus.

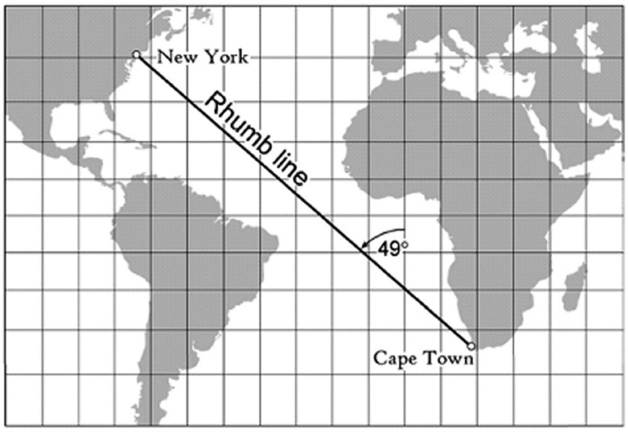
The equations of Mercator's projection contain a natural logarithm. However, logarithm function was not known until 1614 (twenty years after Mercator's death). Several years after, the mathematician Henry Bond discovered similarity between Wright's tables and tables of function for certain values of .

In the 20th century, an unpublished paper by an English mathematician Thomas Harriot (1560 – 1621) was found. This paper describes a method similar to the one devised by Wright in 1599 [11].



Source: [12]

Figure 2 The loxodrome



Source: [12]

Figure 3 The Rhumb line

Equations of the mapping [11] for a sphere are in the form

$$x = R\ \ln\tan\left(\frac{\varphi}{2}+\frac{\pi}{4}\right), y = R\,\lambda \qquad (1)$$

where *R* is the radius of the Earth, is the latitude and is the longitude.

In the Mercator's projection, equations of loxodrome are as follows [13], [14]:

$$\lambda = \pm \ln\alpha\ \ln\tan\left(\frac{\varphi}{2}+\frac{\pi}{4}\right) \qquad (2)$$

where is a constant azimuth for navigation.

4. CONCLUSION

In the article we comment some advantages, disadvantages and current utilization.

Advantages of Mercator's projection:

- preserves angles and therefore also shapes of small objects
- close to the equator, the distortion of lengths and areas is insignificant
- a straight line on the map corresponds with a constant compass direction, it is possible to sail and fly using a constant azimuth
- simple navigation
- meridians and parallels make a rectangular grid on the map

Aside from nautical and aerial use, Mercator's projection is practical for visual mapping of the Earth's globe (without polar areas).

Disadvantages of Mercator's projection:

- Mercator's projection does not preserve lengths or areas further from the equator
- the smallest distance on the globe is not realized by a straight line on the map
- not suitable for teaching geography and in political maps (this projection distorts areas and sizes of states and continents, for example Africa and Greenland seem to be the same size)
- line with the shortest distance is not obvious from the map

Nowadays, Mercator projection is still used in mapping for some nautical, aerial and military maps. For example, NATO utilizes a special type of this projection (UTM). The most frequent utilization is in maps of the oceans (sea and air navigational charts), GoogleMaps and for GPS (see [15-17]). It is also one of useable projections in digital panoramic photography.

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