Using Simple Mathematics in the Sailing Problem

Korištenje jednostavne matematike u plovidbenom problemu

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Summary

In this article, we will discuss a problem of open sea sailing on a yacht. The yacht will be heading to a set point that is positioned upwind from the yacht. We will create a mathematical model and find the optimal starting direction for the yacht, so that it reaches the set point in shortest time possible.

KEY WORDS

yacht, restricted area, angle of turning, velocity made good

Sažetak

U ovom članku diskutirat ćemo problem plovidbe jahte otvorenim morem. Jahta će putovati prema određenoj točci, koja je smještena niz vjetar jahte. Stvorit ćemo matematički model i utvrditi optimalnu početnu poziciju jahte, tako da dođe do zacrtane točke u najkraćem mogućem vremenu.

KLJUČNE RIJEČI

jahta, ograničen prostor, kut okreta, brzina kretanja

1. INTRODUCTION

We introduce this problem as a possible task for high school students. As we mentioned earlier, the task is to find the optimal (fastest) route for the yacht that is trying to reach an upwind point from a given position. The yacht is on an open sea and we do not consider any other influences such as streams or wind blind spots behind islands. We will also consider that the yacht is not losing any extra time by making turns when changing direction. The wind will not be changing its strength and direction and the task will be spatially limited to a half plane determined by the starting and the finishing point.

2. WHICH DIRECTION IS THE BEST ONE?

One of the commonly known facts about sailing is that the yacht cannot travel in the angle sector approximately 70° wide (in the direction right upwind) as illustrated in the Figure 1 [1-4].

It must become quite clear that as the direction of the yacht gets more and more against the wind direction, the yacht will be moving slower, but it would also be moving closer to the upwind point. Every yacht has its own manufacturing parameters which might be different from model to model, so two yachts of the same type might differ in their performances (for example the coefficient of ascendance or the speed in certain wind angles).

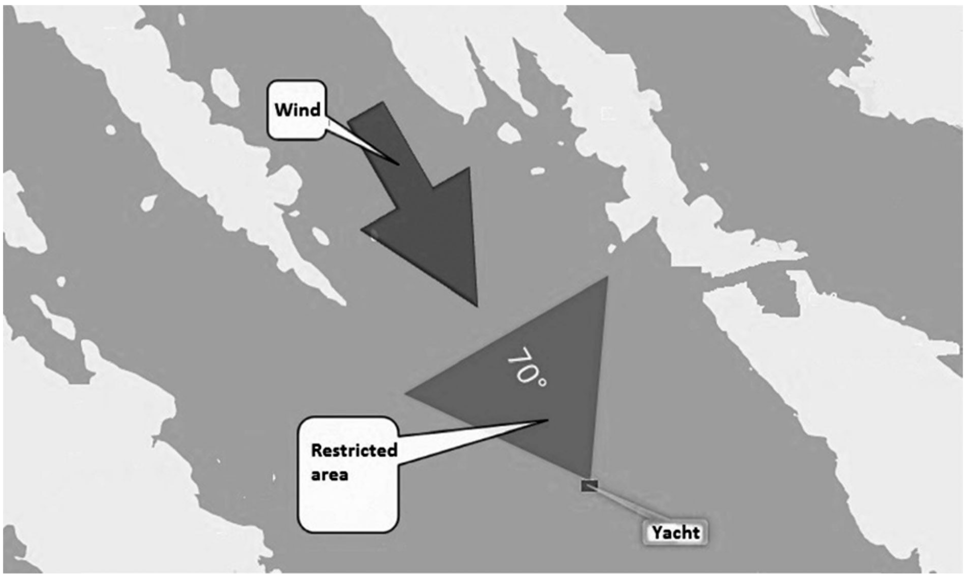

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Figure 1 Yacht cannot travel in the angle sector approximately 70° wide

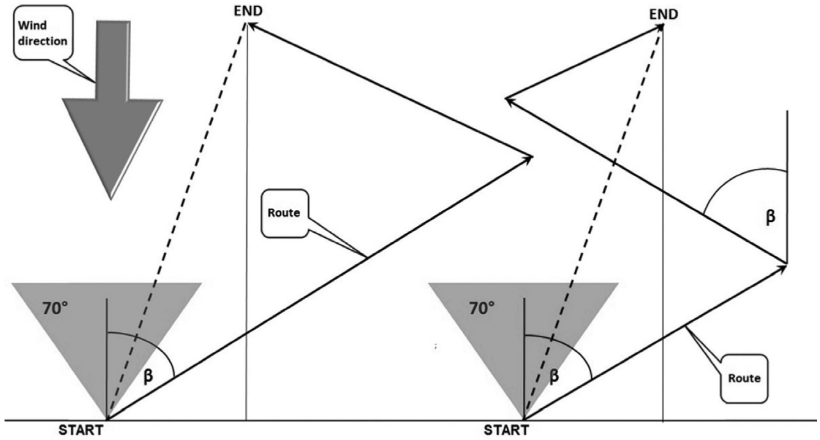

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Figure 2 Two possible scenarios of fixing the starting angle

Another important factor which can influence the resulting speed of the yacht might be the angle between lengthwise axis of the yacht and its sail [5-7].

Assuming that we ignore these facts because they would make the task unnecessarily harder, the speed of the yacht will be with any given constant angle also unchanging. The problem still stays the same: How do we plan the route, so that the time needed for reaching the destination is minimal?

3. PLAN OF THE ROUTE

Let's assume that the yacht won't be able to reach the finishing point going only one direction but it will need to make at least one turn. We can also presume that the space the yacht will be travelling is of a constant width in our scenario we measured 32 Nm (Nautical miles).

We can fix the starting angle and denote it by β. Now let's think about our options. There are two possible scenarios as in Figure 2.

When we explore the illustration more thoroughly, we can make an interesting observation. By summing all of the partial routes in both mentioned scenarios, we will get to a surprising result - they are equal. This renders the total amount of turns irrelevant (considering the yacht will set off in the same angle after it makes each turn). But why is this happening? We can find the explanation using characteristics of the right triangle – see Formula (1) [7], [8]:

$$s_1 + s_2 + \cdots s_n = \frac{y_1 + y_2 + \cdots y_n}{\cos\beta} = \frac{32}{\cos\beta} \quad (1)$$

We can illustrate this relation in Figure 3.

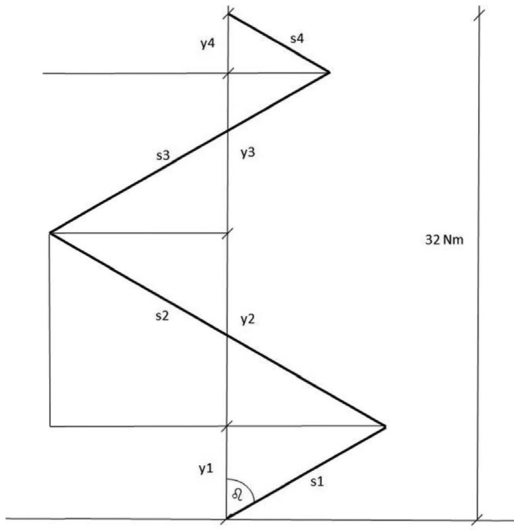

Figure 3 The illustration of the formula (1)

4. HOW TO DETERMINE THE IDEAL ANGLE?

As we mentioned earlier, the speed of the yacht will change with the angle it is going. The real world example will tell us that in the range of a marginal angle and the angle $\beta = 90°$, the speed will at first rise and then start to drop. This relation can be enumerated using a reasonably simple function that would approximate yacht's speed based on the starting direction. For our example we went with the quadratic function in the form

$$v(\beta) = \frac{-2}{225}\beta^2 + \frac{52}{45}\beta - \frac{194}{9} \quad (2)$$

We have guessed the coefficients by approximation initial conditions [8]. When exploring a real world scenario, we would collect these data empirically. As we are looking for the optimal choice of the starting direction, a small error in the exact form of the function isn't that important. The graph in the Figure 4 shows the dependency of the length of the route measured in Nautical miles and the starting direction in the form

$$s(\beta) = \frac{32}{\cos\beta} \quad (3)$$

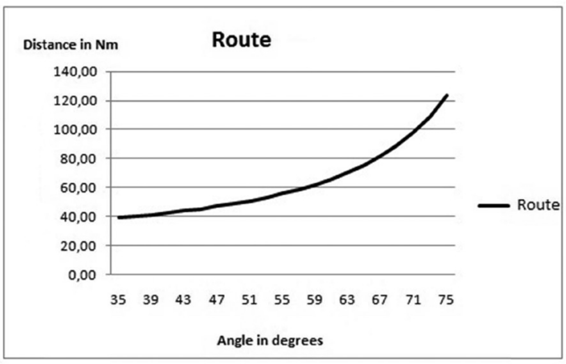

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Figure 4 The dependency of the length of the route measured in Nautical miles and the starting direction

In Figure 5, we can see the dependency of the speed on the starting direction and Figure 6 shows the dependency of the total time on the starting angle, (1 Nm=1852 m) [7].

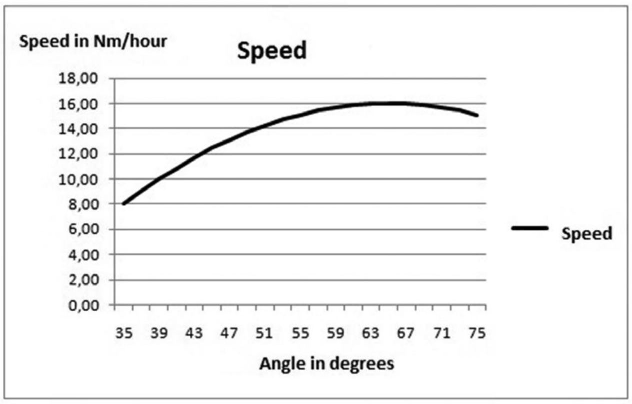

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Figure 5 The dependency of the speed on the starting direction

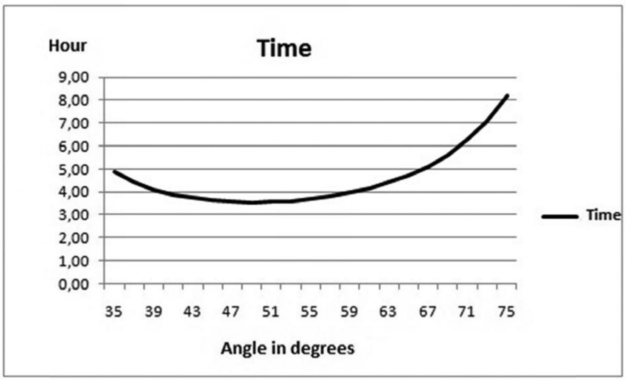

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Figure 6 The dependency of the total time on the starting angle

Table 1 The complete data for determining the optimum angle

Angle [°]	Route [Nm]	Speed [Nm/hour]	Time [hour]
35	39.06	8.00	4.88
37	40.07	9.03	4.44
39	41.18	9.99	4.12
41	42.40	10.88	3.90
43	43.75	11.70	3.74
45	45.25	12.44	3.64
47	46.92	13.12	3.58
49	48.78	13.72	3.55
51	50.85	14.26	3.57
53	53.17	14.72	3.61
55	55.79	15.11	3.69
57	58.75	15.43	3.81
59	62.13	15.68	3.96
61	66.01	15.86	4.16
63	70.49	15.96	4.42
65	75.02	16.00	4.73
67	81.90	15.96	5.13
69	89.29	15.86	5.63
71	98.29	15.68	6.27
73	109.45	15.43	7.09
75	123.64	15.11	8.18

Source: authors

The graph in Figure 6 and the data in Table 1 clearly state that under given circumstances the optimal starting direction is β =49° (the value is mere approximation of the minimum) and the yacht will reach its destination in 3 hours and 55 minutes. Figure 7 shows the resulting route [6].

5. VMG AGAINST THE WIND

After you input the target location, that is positioned upwind from your current location, into the GPS, the device will display the VMG (Velocity Made Good). VMG leads towards crisscross

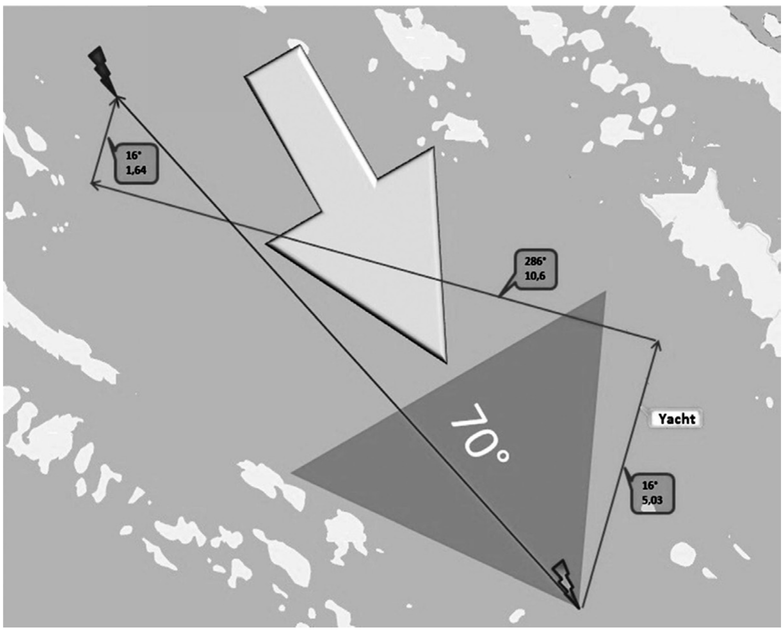

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Figure 7 The resulting route

cruising inside a cone (triangle) with its vertex in the target location see Figure 8.

However, a trajectory like this will not be efficient, since the second half of it would be full of turns that will very slowly converge to the goal. This problem can be solved by adjusting your target location and moving it approximately 10 miles upwind from (basically behind) the desired location. This way you will receive a real VMG for the entirety of the cruise, see Figure 9 [2], [8-10].

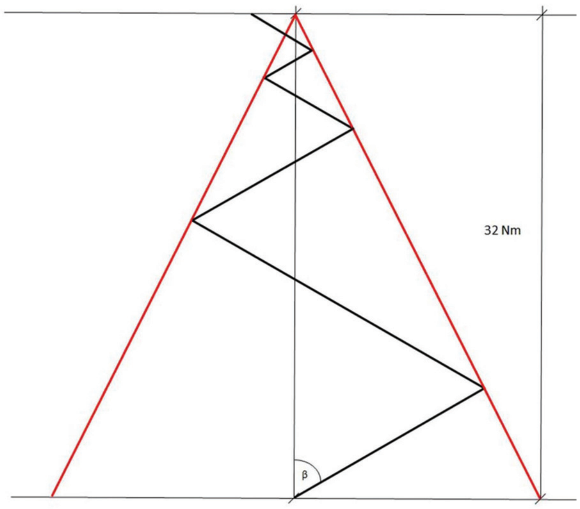

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Figure 8 VMG leads towards crisscross cruising inside a cone (triangle) with its vertex in the target location

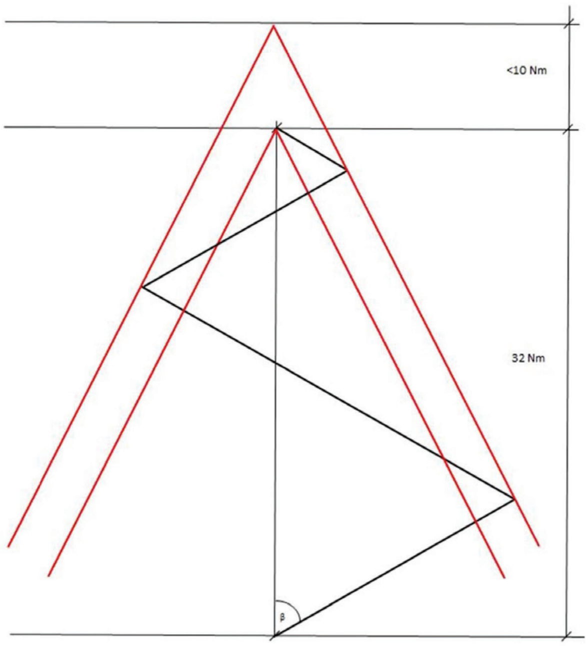

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Figure 9 Real VMG for the entirety of the cruise

6. CONCLUSION

Students can refresh their knowledge of mathematics and physics using this motivational task. Even the process of finding the fact that the length of the route is not dependent on the number of turns may be interesting. For reaching the correct result, a differential calculus might be used and this task may serve as a limit theory exercise. We avoided this so that it is suitable for younger students of lower grades. Our specific case could be solved by minimization of the function describing the dependency of time on the angle β in the form [2], [4], [11-14]

$$t(\beta) = \frac{s(\beta)}{v(\beta)} = \frac{32}{\cos\beta\left(\frac{-2}{225}\beta^2 + \frac{52}{45}\beta - \frac{194}{9}\right)} \tag{4}$$

where β and $s(\beta)$ are given in (1) and (2).

The minimum value might be also estimated in Figure 6. This topic can be expanded into more dimensions by including changing conditions, obstacles in the route like islands or more yachts. Then we would have to create a model which can be used to calculating the fastest route.

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