

**APPLICATION OF ASYMETRIC S-FUNCTION FOR ANALYSIS  
AND VALUATION OF THE GROWTH OF BOARS****G. Kralik, R. Scitovski and Đ. Senčić****Summary**

Growth capacity of the pig is a subject of permanent research. The growth rate of pigs has different quantitative and qualitative course. Profitability of pig rearing is in use of progressive growth stage, i.e. phase. This study deals with the curves of growth of the Large White breed, based on the gain intensity of the animals tested with the body weights of 30 to 100 kg. According to the average daily gains the boars were deviced into three groups (0.81 - 0.90 kg, 0.91 - 1.0 kg and 1.01 to 1.10 kg). For description of the growth rate the asymmetric S- function was used modelled as:

$$f(t) = \frac{A}{(1 + be^{-cgt})^{1/g}}$$

Regarding the necessary time for achievement of certain mass, the stages are separated as: preparations ( $t=t_B$ ), growth-both progressive and degressive ( $t=t_B=t_C$ ) and phase of saturation ( $t=t_C$ ). The analysis showed that boars, entering the test in younger age, showed higher growth intensity than those beginning the test in older age (77.4; 83.4 and 82.6 days). In these boars the stage of progressive and degressive growth occurred earlier and lasted shorter period of time. The asymmetric S-function could be therefore used for valuation of the boars growth rate in older age based on the starting control measurements of animals.

**Introduction**

The phenomenon of growth as a material base of swine production constantly occupies the attention of researches. All occurrence forms, the structure of centres and the growth of swines during the life cyclus is differently qualitatively and quantitatively realized.

A numerous literature on models of biological growth and life cyclus of living organismus exists (1), (5), (7), (10), which are already researched for about hundred years<sup>1</sup>.

<sup>1</sup>It is thought that the first on this study on this field is Verhulst analysis of population in 1838 - issued 1845 in "Academie Royale des Sciences et des Lettres de Bruxelles".

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Analysing the weight gain an organism it is proceeded from the assumption that the weight increases in an exact time and it is proportional to the instantaneous weight of organism  $y(t)$  and to the instantaneous biological potential  $(A-y(t))$ , whereby  $A$  is the biological maximum of weight of average organism of the observed breed. Thereby, these two factors have different acting intensities on the weight gain in different time moments. With different combinations of these two factors, different growth models have been obtained (1), (6), (7), (9), (10), (11).

With the help of so-called generalized logistic function (6), (7), (11), (12) we shall try to describe the growth of young boars, define the growth characteristics and on that basis estimate the commotion of living mass for some samples regarding the age.

The economical utility in the production of pork declares in an optimal usage of progressive growth stage to obtain it, continuously are studies the specifics of growth of some breeds and swine respectively.

#### *Material and methods*

In the centre for biological test, on a sample ( $n=30$ ) examined were the growth characteristics of young boars of 30-100 kgs of live mass. The boars were kept in individual boxes, fed ad libitum, with a mixture containing 15% crude proteins. After including into test, the swines were weighted every 21 days till reaching the mass of 100 kg. At the same time controlled were: the total feed consumption in the test, average daily feed consumption and the feed conversion into the weight gain of live mass. The thickness of dorsal bacon was measured at the end of the test. According to the averagely obtained daily weight gain in the test, the boars were assigned into three groups:

1. Group - daily average gain: 0.81 - 0.90 kg
2. Group - daily average gain: 0.91 - 1.00 kg
3. Group - daily average gain: 1.01 - 1.10 kg.

By correlation analysis was examined the connection of the average weight gain during the fattening, connected to the swine age, test continuance, totally consumed feed, consumption of feed per feeding day, feed conversion and average thickness of dorsal bacon<sup>2</sup>.

To describe the growth of swines we shall apply the rule which was set up by Nelder (1961) and Lewandowsky (1974):

$$\frac{dy}{dt} = cy(t) \left[ 1 - \left( \frac{y(t)}{A} \right)^g \right] \dot{c}, \quad c > 0, g > 0 \quad (1)$$

according to which the weight gain in some particular time is proportional to instantaneous weight of organism  $y(t)$  and to instantaneous biological potential  $(A - y(t))$ ,

<sup>2</sup>The statistical processing of production swine characteristics and their mutual connection was performed on PC Computer with SPSS Package.

whereby A is the biological weight maximum of average organism of the examined breed. Thereby, these two factors, generally have different intensity activities at the weight gain in different time moments. Here is c the constant of proportionality and  $\gamma$  so-called coefficient of asymmetry by which the influence of mentioned factors at the weight gain is regulated.

The differential equation (1) is easily solvable. Its solving is so-called generalized logistic function:

$$y(t) = \frac{A}{(1 + b e^{-cgt})^{1/\gamma}}, \quad (b, c) \in R^2 \quad (2)$$

The inflection point, till which the progressive growth lasts, and from which starts the degressive growth of organism, for this function-model is assigned with:

$$I \approx \frac{1}{c\gamma} \ln \frac{b}{y}; \quad \frac{A}{(1 + \gamma^{1/\gamma})} \quad (3)$$

It is easily obvious (12) that the ordinate of inflection point  $y_1$  can accept the values from interval  $(A/e, A)$ . If  $\gamma = 1$ , this function becomes a logistic function for which is  $y_1 = A/2$ . If  $\gamma < 1$ , valid is:  $y_1 < A/2$  (it is said that the function is negatively asymmetric) and if  $\gamma > 1$ , valid is:  $y_1 > A/2$  (it is said that the function is positively asymmetric). Therefore  $\gamma$  we call as coefficient of asymmetry.

According to data  $(t_i, y_i)$ ,  $i = 1, \dots, m$  on instantaneous weight of organism  $y_i$ , in the moment  $t_i$  through application of method of smallest squares - optional parameters  $b$ ,  $c$ ,  $\gamma$  of generalized logistic function should be determined. As we want that during the evaluation of optimal parameters /during the avaluation of theoretical values  $y(t_i)$  respectively/ all data should be relatively equally represented, then it should be demanded that the squares sum of relative aberrations of actual from theoretical values should be minimal. That means that we shall look for minimum of function:

$$G(b, c, \gamma) = \sum_{i=1}^m \frac{(y_i - y(t_i; b, c, \gamma))^2}{y_i^2} \quad (4)$$

whereby  $y(t; b, c, \gamma)$  is generalized logistic function (2). If we introduce the indication:  $P_i = Y_i^{-2}$ , then the sum (4) we can write down as

$$G(b, c, \gamma) = \sum_{i=1}^m P_i \sum_{i=1}^m (y_i - y(t_i; b, c, \gamma))^2 \quad (5)$$

The numbers  $P_i$  we call ponders (or weights) of data, and the problem of minimization of function (5) is the standard problem of smallest squares for data  $(P_i, t_i, y_i)$ ,  $i=1, m$  and function- model  $y$ . Generally, the weights (ponders)  $P_i$  data we can also

define differently. Always that are positive numbers, and mostly they are taken in the interval (0,1). With their help we can influence on the contribution ("weight") of particular data.

This nonlinear problem of the smallest squares we shall solve through application of iterative modified Marquardt's method (see (2), (3)), whereby an appropriate linear problem of the smallest squares is solved by each step (see (4)).

When the values of optional parameters of generalized logistic functions are known - it is possible to evaluate the growth stages of the organism. For  $t=t_1$  this function is convex - it is said that in this interval the growth of organism is progressive. For  $t=t_1$  the function is concave - it is said that in this interval the growth of organism is degressive. Therefore it is interesting to find the maximum point  $t_B$  in the interval of progressive growth ( $t=t_1$ ), and in the interval of degressive growth ( $t=t_1$ ) the minimum point  $t_C$  (see (12)):

$$t_B = \frac{1}{cg} \ln \frac{2b}{g(g+3) + g(g+1)(g+5)}$$

$$t_C = \frac{1}{cg} \ln \frac{2b}{g(g+3) + g(g+1)(g+5)} \quad (6)$$

The stages of growth of observed organism can be now defined:

- Stage of preparing growth ( $t < t_B$ ),
- Stage of progressive growth ( $t_B < t < t_C$ ),
- Stage of growth retardation ( $t > t_C$ ).

Continuance of the test (number of days  $T$  in which the animal reaches a certain weight  $W$ ) can also be easily evaluated:

$$T = \frac{1}{cg} \ln b \xi^{(A_{\pi W})^g} - 1 \zeta^{-1} \quad (7)$$

#### Results of researches

The table 1 shows the production characteristics of boars and their mutual connection in the test of 30-100 kgs of live mass. The correlation coefficients show that between average daily weight gain and feed conversion and fattening continuance respectively, exists a negative, middle-strong and very strong connection ( $r=0.53^{II}$  and  $r = -0.99^{II}$ ). It is also stated that the connection between the average daily gain and the total feed consumption in the test, as well as the feed conversion into the gain, is positive and strong ( $r=0.60^{II}$  and  $0.59^{II}$ ). Because the average daily gain is in a significantly positive or negative correlation with other fattening characteristics, we were interested in the boars growth characteristics, according to the formerly defined groups.

The research results show that the young boars with highest growth intensity (Group 3) attained the live mass of 30 kgs earlier compared to other groups. The intensity of gain conditioned significant differences in the total feed consumption, as

well as highly significant differences in the fattening continuance among all swine groups. The differences in the daily feed consumption are highly significant between 1. and 3. and among 2. and 3. group of animals ( $P < 0,01$ ).

Table 1 - PRODUCTION CHARACTERISTICS OF BOARS IN TEST OF 30 - 100 KGS  
PROIZVODNI POKAZATELJI NERASTOVA U TESTU OD 30 - 100 KG

Characteristics Pokazatelji	$\bar{x} \pm S_x$	Correlation coefficients (r) - Koeficijenti korelacije (r)							
		2	3	4	5	6	7	8	
1. Age with attained 30 kgs (days) Dob s postignutih 30 kg (dani)	81.13 ± 1.44	0.83**	0.22	0.22	0.19	0.20	0.05	0.18	
2. Age with attained 100 kgs (days) Dob s postignutih 100 kg (dani)	156.06 ± 2.00	- 0.70**	0.70**	0.43*	0.43*	0.37	0.22		
3. Test continuance (days) Trajanje testa (dani)	74.60 ± 1.15		- 0.99**	0.60**	0.59**	0.53*	0.22		
4. Average daily gain (kg) Prosječni dnevni prirast (kg)	0,94 ± 0,14			- 0.63**	0.63**	0.49*	0.23		
5. Total feed consumption (kg) Ukupni utrošak hrane (kg)	182.07 ± 2,60				- 0.99**	0.36	0.51*		
6. Feed quantity per kg of gain (kg) Količina hrane po kg prirasta (kg)	2.60 ± 0.04					- 0.37	0.50*		
7. Feed quantity per test day (kg) Količina hrane po danu testa (kg)	2.45 ± 0.03						- 0.28		
8. Bacon thickness (mm) Debljina ledne slanine (mm)	15.32 ± 0.42							-	

\*  $P < 0.05$

\*\*  $P < 0.01$

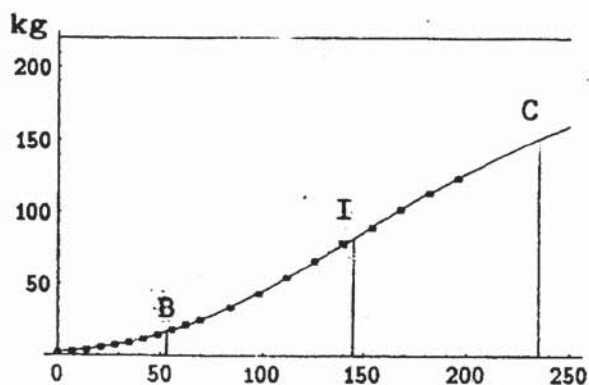


Figure 1 - GROWING CURVE OF FIRST GROUP

Table 2 - GROWTH CHARACTERISTICS OF BOARS REGARDING TO DAILY AVERAGE WEIGHT GAIN (KG)  
POKAZATELJI RASTA NERASTOVA OBZIROM NA PROSJEČNI DNEVNI PRIRAST (KG)

Characteristics Pokazatelji	1. Group Skupina 0.81-0,90	2. Group Skupina 0.91-1.00	3. Group Skupina 1.01-1.10	Signif. differ. among groups Signifikantnost razlika između skupina
1. No of boars Broj nerastova	10	10	10	
2. Age with 30 kgs (days) Dob s 30 kg (dani)	82.60 ± 2.70	83.40 ± 2.76	77.40 ± 1.67	N.S.
3. Age with 100 kgs (days) Dob sa 100 kg (dani)	164.30 ± 2.81	158.20 ± 2.87	145.70 ± 1.69	1:3**, 2:3**
4. Test continuance (days) Trajanje testa (dani)	81.70 ± 0.76	74.80 ± 1.87	67.30 ± 0.68	All groups** Sve skupine**
5. Total feed consumption (kg) Ukupni utrošak hrane (kg)	190.20 ± 4.47	185.80 ± 3.34	170.20 ± 3.06	All groups* Sve skupine *
6. Feed per kg of weight gain (kg) Hrana po kg prirasta (kg)	2.72 ± 0.06	2.65 ± 0.05	2.43 ± 0.04	1:3**, 2:3**
7. Feed per test day (kg) Hrana po danu testa (kg)	2.32 ± 0.06	2.48 ± 0.05	2.53 ± 0.03	N.S.
8. Bacon thickness (mm) Debljina ledne slanine (mm)	15.92 ± 0.72	15.48 ± 0.63	14.56 ± 0.81	N.S.

\* P < 0.05 \*\* P < 0.01 N.S. P > 0.05

Curves-models of growth as generalized logistic shape functions (2) are obtained according to the measurement data and the assumption on biological weight maximum of Large White  $A = 220$  kgs. In the Table 3 are shown the parameters of these functions-models for the reported groups, their inflection points and points which limit the growth stages.

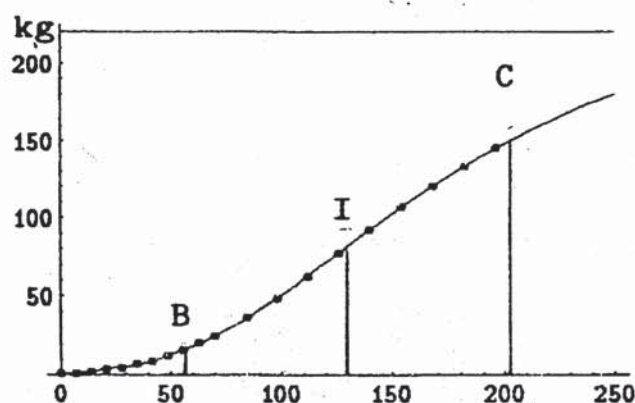


Figure 2 - GROWING CURVE OF THIRD GROUP

Table 3 - PARAMETERS OF FUNCTIONS-MODELS OF BOARS GROWTH  
PARAMETRI FUNKCIJA-MODELA RASTA NERASTOVA

	1. Group 1. Skupina	2. Group 2. Skupina	3. Group 3. Skupina
b	0.046368	0.048497	0.055287
c	1.067189	1.124951	1.321355
$t_i$	144	140	129
$Y_i$	81.3	81.3	81.3
$t_b$	53.14	54.41	56.24
$t_c$	234.34	226.30	202.58

Figure 1 i 2 show the appropriate generalized logistic function and the growth stage for the first and third observed group.

According to this analysis it could be registered that, for example for the first observed group, the stage of progressive growth continues from the end of the second till the end of eight fattening month, with the highest growth intensity toward the end of fifth fattening month.

Knowing the parameter value  $c, y$  for some particular swine group, we can directly solve the differential equation (1) along with some beginning condition. Concretely that means that knowing the weight of a particular animal at some definite time, we can forecast its weight in the future.

The differential equation (1) along with the beginning condition:  $y(t_0)=y_0$  we can solve by the method of discretization:

$$y_{n+1} - y_n = hcy_n \frac{1 - (y_n/A)^c}{1 + (y_n/A)^c}, \quad h = 1, \quad (8)$$

whereby  $Y_0 = 30$  kgs.

As an example we shall observe per three choosed animals from the 1. and 3. group. In the Table 4 we compared the results obtained through prognosis and the actual results.

Table 4 - COMPARISON OF PROGNOSIS AND ACTUAL GROWTH  
USPOREDBA PROGNOZE I STVARNOG RASTA

Tested animals Testirane životinje	Necessary time for 100 kgs Potrebno vrijeme za 100 kg	
	Actual stvarno	Prognosis prognoza
1. Group (1) Skupina (1)	159	162
1. Group (2) Skupina (2)	163	166
1. Group (3) Skupina (3)	172	174
3. Group (1) Skupina (1)	140	145
3. Group (2) Skupina (2)	145	147
3. Group (3) Skupina (3)	146	148

### Conclusion

Research results of growth characteristics of Large White boars in the test of 30-100 kgs show the following:

- The daily weight gain is in a significant connection with other fattening features. Accordingly, through a selection of animals for a higher intensity of weight gain it is influenced on the test continuation to be shorter and on conversion improvement of nutritive matters into weight gain.

- Generalized logistic function is suitable for the description of characteristics of boars growth during the fattening period. Swines with a higher average of daily weight gain attain the maximum speed of weight gain (point of inflection) in earlier age, and the phase of progressive growth lasts relatively shorter time compared to the animals with a lower average of daily weight gain.

- By knowing the breed parameters ( $A, c, y$ ) and data on the live mass in the first part of the test, it is possible through application of generalized logistic function-model to estimate reliably the time in which the animals will attain some programmed live mass (for example 100 kgs).

- The described model enables well-timed selection of animals according to the growth capacity of each individual.



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#### PRIMJENA ASIMETRIČNE S-FUNKCIJE U ANALIZI I VREDNOVANJU RASTA NERASTOVA

##### Sažetak

Kapacitet rasta svinja predmet je stalnih istraživanja. Rast svinja različito se kvantitativno i kvalitativno ostvaruje. Gospodarska korist od svinjogojstva je u korištenju progresivne faze rasta. U radu se istražuju krivulje rasta nerastova velikog jorkšira na osnovu intenziteta prirasta u testu od 30-100 kg. Prema postignutim prosječnim dnevnim prirastima nerastovi su podjeljeni u tri skupine (0,81 - 0,90 kg, 0,91 - 1,0 kg i 1,01 - 1,10 kg). Za opisivanje rasta upotrebljena je asimetrična S-funkcija, model:

$$f(t) = \frac{A}{(1 + be^{-cgt})^{1/\alpha}}$$

Obzirom na potrebno vrijeme za postizanje određene mase, razgraničene su faze: pripreme ( $t=t_b$ ), rasta - progresivnog i regresivnog ( $t=t_b=t_c$ ) i zasićenja ( $t=t_c$ ). Analiza je pokazala da nerastovi koji su ušli u test u mlađoj dobi imaju veći intenzitet prirasta od onih koji su započeli test u starijoj dobi (77,4; 83,4 i 82,6 dana). Kod tih nerastova faze progresivnog i regresivnog rasta nastupaju ranije i traju kraće vrijeme. Primjenjena asimetrična S-funkcija može se koristiti i za procjenu rasta nerastova u kasnijoj dobi, na osnovi početnih kontrolnih mjerenja životinja

Primljeno: 22. 10. 1993.