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A MODEL WITH STORAGE LIMITATION AND SIMULATED DEMAND AS FRESH MEAT INVENTORY MANAGEMENT SUPPORT

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SUMMARY

An important aspect of retail outlets which offer different kinds of fresh meat is the careful planning of inventory. In this context, inventory management includes ensuring the required quantity of meat, storing it in an adequate way, and also minimizing the setup cost, holding cost, shortage cost and possible losses that might occur if the goods remain unsold. By devising and applying appropriate models companies can significantly improve their decision making connected to harmonizing the demand and different inventory costs. This paper presents an inventory model with limited storage space, characterized by the estimate of meat demand based on simulated values generated by computer from beta distribution. If the determined quantities exceed the available storage space, the Lagrange multiplier which allows for simultaneous decrease of the originally established values, could be used in the model. This eventually results in the optimum order quantity of different meat products, as well as the associated total storage costs.

Key-words: inventory model with storage limitation, computer simulated demand, fresh meat inventory, inventory costs, optimization

INTRODUCTION

Inventory planning is an important issue in the operation of retail outlets that trade in fresh meat products. The aim of inventory management in this case is to ensure the required quantities of different kinds of fresh meat, with minimum setup cost and holding cost. To achieve this, it is necessary to understand in detail all the costs related to inventory. Different costs incurred in the process of purchasing meat products all belong to setup cost. Holding cost includes not only the costs of adequate storage of fresh meat products, but also the expenditure arising from funds being tied up in inventory. In addition to the aspects stated above, the management needs to take into account the losses arising from insufficient inventory, i.e. shortage cost. In this context a special problem appears to be the loss of consumer confidence. In order to maximize their income, retail outlets need to make appropriate estimates of optimum order quantity for different kinds of fresh meat, paying particular attention to the shelf-life of such products and the costs arising from product deterioration.
In order to improve the decision-making in the domain of inventory management, several models have been developed. This paper will present the inventory model with storage limitation. In the case of fresh meat in retail outlets, storage limitation is determined by the capacity of their cold storage for such products. Taking into account the available storag1e, daily demand, setup cost, holding cost and the space required for their storage, the model yields the optimum quantity of different kinds of fresh meat to be held as inventory, and associated total inventory cost. If it turns out that the storage space required for keeping the determined meat quantities is larger than the available cold storage, we keep decreasing simultaneously the initially determined values by changes in Lagrange multiplier until storage limitation is satisfied.

MATERIAL AND METHODS

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The proposed model has its origins in the inventory model with storage limitation. As such, it is based on quantitative optimisation methods. Its specific feature is the estimate of demand for meat by means of computer simulation. Given that the demand for meat is simulated in the model, it belongs to the group of stochastic models. The simulated demand values are generated from beta distribution. Such a procedure is significantly accelerated by using adequate computer and software support. The paper will particularly emphasize this aspect of the model. In addition, after theoretical explication of the model we present a simple example in order to assess better practical possibilities of its application.

RESULTS AND DISCUSSION

Theoretical features of the model

The starting assumption of the inventory model with storage limitation should have at least two different kinds of products to be stored in limited storage space. It is furthermore assumed that each item is replenished instantaneously. For the sake of simplicity, it can be assumed that setup cost remains the same regardless of the quantity ordered. In this case, the inventory model can be presented as follows:

$$\text{minimize } C(q_1,\ldots,q_n) = \sum_{i=1}^{n}\left(\frac{K_i D_i}{q_i} + \frac{h_i q_i}{2}\right)$$

subject to

$$\sum_{i=1}^{n} s_i q_i \le S$$

$$q_i > 0 \text{ , for all } i$$

Where

- n = Number of items
- q_i = Optimum order quantity of the *i*th item
- K_i = Setup cost of the *i*th item
- D_i = Demand rate per unite time of the *i*th item
- h_i = Holding cost per unite time of the *i*th item
- s_i = Storage area requirements per unite of the *i*th item
- S = Maximum storage area

In the stated model optimum value of q_i is obtained by minimizing the objective function, whereas the constraints reflect the condition that total ordered quantity of all kinds of products should fit into available storage space. Here it is assumed that the optimum quantity of each product must be above zero.

To determine the optimum order quantity accompanied by minimum costs, the objective function needs to be derived by each of the variables q_i, and then the partial derivatives obtained in this way need to be equalized to zero:

$$\frac{\partial C}{\partial q_i} = -\frac{K_i D_i}{q_i^2} + \frac{h_i}{2} = 0$$

In this case we get the following formula for calculating the optimum order quantity of *i*th item:

$$q_i = \sqrt{\frac{2K_i D_i}{h_i}} \tag{1}$$

If the determined sum of optimum quantities of all items exceeds the available storage space, it is necessary to establish new optimum values. With this aim the Lagrange function is defined as follows:

$$L(q_1,...,q_n,\lambda)=\sum_{i=1}^{n}\left(\frac{K_iD_i}{q_i}+\frac{h_iq_i}{2}\right)-\lambda\left(\sum_{i=1}^{n}s_iq_i-S\right)$$

In the stated expression λ represents the Lagrange multiplier whose value is below zero. As in the preceding case, the optimum values of q_i and λ can be found by equating the obtained partial derivatives to zero:

$$\frac{\partial L}{dq_i}=-\frac{K_iD_i}{q_i^2}+\frac{h_i}{2}-\lambda s_i=0$$

$$\frac{\partial L}{\partial\lambda}=-\sum_{i=1}^{n}s_iq_i+S=0$$

The second equation implies that the sum of optimum order quantities must be equal with available storage space, whereas the first equation yields the formula for calculating the optimum order quantity of *i*th items:

$$q_i=\sqrt{\frac{2K_iD_i}{h_i-2\lambda s_i}} \tag{2}$$

In determining the optimum quantities q_i the Lagrange multiplier λ decreases until the available storage space is used to the full.

A specific feature of the described model is simulation of demand for meat on the basis of beta distribution whose parameters are α and β. The beta distribution has the probability density function:

$$f(x)=\frac{(x-a)^{\alpha-1}(b-x)^{\beta-1}}{B(\alpha,\beta)(b-a)^{\alpha+\beta-1}},\ a\le x\le b,\ \alpha,\beta>0\,.$$

In the stated formula *a* represents the minimum, and *b* the maximum range limits, whereas $B(\alpha,\beta)$ represents the beta function. Simulation of demand for meat products can be based on other theoretical distributions as well, but characteristics of beta distribution justify its selection. Among other things, adequate determination of range limits for this distribution prevents negative demand values in the simulation process. In addition, it is more likely that a higher number of consumers will want to buy smaller quantities of fresh meat, and a smaller number will tend to buy larger quantities. It is therefore realistic to assume that the demand for fresh meat does not follow the rules of one of symmetric distributions. Since parameters α and β allow the beta distribution to be defined as positively asymmetrical, it thus satisfies this condition of a simulation model as well.

In the simulation process, the first thing to be determined for each kind of meat is the smallest (*a*) and the largest (*b*) daily quantity demanded. Such an estimate can be based on the meat sales data recorded in a given time interval. Parameters α and β are also assessed on the basis of this data:

$$\alpha=\frac{\bar{x}-a}{b-a}\left(\frac{\frac{\bar{x}-a}{b-a}\left(1-\frac{\bar{x}-a}{b-a}\right)}{\frac{s^2}{(b-a)^2}}-1\right),\ \beta=\left(1-\frac{\bar{x}-a}{b-a}\right)\left(\frac{\frac{\bar{x}-a}{b-a}\left(1-\frac{\bar{x}-a}{b-a}\right)}{\frac{s^2}{(b-a)^2}}-1\right),$$

Where

$\bar{x}$ = Sample mean,

s^2 = Sample variance.

Simulated demand values are obtained by founding beta distribution values with accompanying probabilities, determined by means of random number generator. The values simulated in this way are then used to calculate the optimum order quantities of fresh meat which satisfy the condition of storage limitation. To achieve a more accurate assessment of optimum quantities and total costs, it is advisable to repeat the simulation procedure as many times as possible. Taking into account the capabilities of today's computers, this is no problem at all. What this paper is trying to emphasize is precisely the need to establish an adequate database on fresh meat sales, and to use state-of-the-art information and communication technologies in order to improve inventory management. Reiteration of the described procedure yields a larger number of solutions. Optimum quantities of meat products to be held as inventory can then be determined as a mean of all the solutions obtained in this manner.

A hypothetical example of devising and solving the model

Let it be assumed that we need to store of three kinds of fresh meat in the cold storage planned for storing S=40 containers with capacity of 20 kg each. This means that $s_1=s_2=s_3=1$ container. Let it further be assumed that the setup cost amounts to K_1=20 MU (monetary units), K_2=15 MU and K_3=30 MU, whereas the daily holding cost is h_1=2 MU, h_2=3 MU and h_3=1 MU. On the basis of sales data, the management has established the amounts of minimum and maximum daily demand for meat, as well as the values of mean daily demands with accompanying variances. The data are shown in kg, and are used to calculate the parameters α and β (table 1).

Table 1. Hypothetical demand data required for calculating the parameters α and β

Item	Minimum demand	Maximum demand	Sample mean	Sample variance	α	β
1	150	300	200	800	1.750	3.500
2	200	350	250	700	2.048	4.095
3	100	200	130	400	1.275	2.975

By means of a computer simulation which requires adequate software, the values of daily demand for all three kinds of meat have been generated from the beta distribution. For the first kind of meat we got the simulated demand of 194.241 kg, for the second 224.723 kg, and for the third 152.568 kg. Since meat is stored in containers with 20 kg capacity, daily demand for the first kind of meat is D_1=10, for the second D_2=12, and for the third daily demand is D_3=8 containers. Optimum order quantity for the first kind of meat, calculated by means of formula (1) is q_1=14.142, for the second q_2=10.954, and for the third q_3=21.909 containers. Since these optimum order quantities exceed the available space in cold storage by 7.005 containers, we need to apply formula (2). The quantities of inventory that satisfy the storage limitation are obtained for λ=-0.285. For this value of Lagrange multiplier the determined quantities are q_1=12.476, q_2=10.042, and q_3=17.485 containers. By inserting the necessary values into the objective function we can get total inventory cost, which in this case, amounts to 83.963 MU.

To improve the assessment of optimum order quantity, the simulation of daily demand values should be carried out as many times as possible. Repeating, the described procedure will produce distributions of optimum quantities for three kinds of meat products. Their means will in this case represent optimum order quantity. Naturally, any solutions representing the number of containers expressed as non-integers should be rounded and expressed as whole numbers.

CONCLUSION

Fresh meat inventory management in retail outlets is a very sensitive issue. Inadequate assessment of this inventory can result in significant financial losses. It is therefore very important to make a correct assessment of the inventory level that would satisfy the demand for fresh meat and minimize the setup cost and holding cost. This paper has presented the inventory model with storage limitation both theoretically and through an example. An important point of reference in this model is the estimate of demand for fresh meat based on computer simulation. The simulation is carried out in the framework of beta distribution whose characteristics justify its usage in the simulation process. Similarly to any abstract construction of reality, the presented model cannot fully eliminate the losses connected with inventory holding, but it can certainly improve the process of inventory management to a significant degree.

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