A NEW PATH PLANNING APPROACH BASED ON ARTIFICIAL ELECTRIC POTENTIAL ENERGY

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Abstract:

Path planning is one of the most fundamental desired autonomous navigation capabilities for aircrafts. A sensible environment modeling method plays a significant role in improving the path planning algorithm, and the electric potential principle has a unique advantage in this regard. Due to the random node generation of the sampling-based algorithm, it is difficult to generate an optimum path. In the integration of electric potential cost function and probability function, the calculation has approved that there is a negative correlation between the path cost value and probability value, that is, the lower the cost value, the higher the probability that the path nodes is to be selected. Meanwhile, the electric potential value of the entire path is also used to evaluate the safety of an entire route. The simulation results depict that, compared with other traditional methods, the algorithm proposed in this article has distinctive superiority in raising and enhancing computational efficiency and path safety.

1 Introduction

Most of missions require aircrafts to follow predefined paths autonomously, so path planning is fundamental for all of them. Currently, a gradually increasing number of aircraft in the airspace, limited paths resources, rapid changes of weather conditions, and other factors extremely affect flight safety so that flight environment has become seriously complex. Meanwhile, the requirements of emergency tasks which must apply aircraft are gradually increasing and therefore the high computational efficiency and path security level of the path planning algorithm are extremely important.

The purpose of path planning is to find a suitable path for aircraft considering certain constraints (such as threats, path length, fuel consumption, arrival time, etc.). Due to an increasing number of further findings in path planning methods, researchers constantly seek to make modifications for many of these methods, such as topology method [1], artificial potential field [2], grid method [3], gravitational search algorithm [4], intelligent optimization algorithms [5,6] and so on. The topology method generates the path through analyzing the geometrical relationship between the vertices of the obstacle and the boundary. The method is actually a geometric modeling method for the environment, and the calculation process is fairly simple. The computational complexity is affected by the number of obstacles, if

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the location of the obstacle is changed, then it is necessary to reconstruct the map. Since the time when the artificial potential field method was proposed, it has been used widely in path planning problem, although it has some defects, such as: the "trap" area of the potential field, the search blind spot among adjacent barriers. According to the grid method, environment is represented by grids, and the heuristics information is stored in each grid. The classical grid-based methods, such as A*, D* algorithm, can get the feasible path by the calculation of the cost map, although the computation of these algorithms is inefficient and can only be used for the low-dimension and discrete space. The intelligent optimization algorithm such as genetic algorithm (GA), ant colony algorithm, particle swarm optimization (PSO), artificial fish swarm algorithm, artificial bee colony algorithm and so on, can well solve the path planning problems. GSA is similar to PSO algorithm, and the convergence is superior to the PSO and GA. Iterations in such algorithms have a magnificent relevance to the accuracy of the results, if times of iterations are short, the accuracy cannot sometimes be satisfactory. In comparison with other methods mentioned above, these algorithms are easy to fall into local minima, and the calculation process can be more complicated in dealing with the problem of path planning in complex environment and high dimensional space. In recent years, researchers proposed some optimization algorithms based on the laws of physics, such as: gravitational search algorithm, physicomimetics method [7], electromagnetism-like algorithm [8], central force optimization [9] and so on. These algorithms use the physical laws to describe the relationship among the various factors, and have some advantages in solving the optimization problems including path planning.

Sampling-based methods can solve the problem of high-dimensional path planning. The rapidly exploring random tree (RRT) [10] algorithm is one of them, and this method has been widely used for the complex constrained path planning problem in two-dimensional even high-dimensional space [11]. In [12], the algorithm improves the convergence ability of RRT algorithm by using A* algorithm, but it cannot avoid the local minimum problem. In [13, 14], the RRT algorithm is optimized by the terrain cost function and the geometric method. The method depicted in [15] can reduce the computational cost of the algorithm, but the influence of the environment complexity is not taken into account. Karaman and Frazzoli [16] prove that the probability of the optimal solution which is obtained from the RRT algorithm is 0, and the improved RRT* algorithm can get the optimal result. However, the improved methods mentioned above do not consider the distribution of the environmental threats intensity, the safety level of path and computational efficiency are not considered comprehensively in the algorithm.

Electric potential field theory is applied to environment modeling in this paper. As we all know, electric potential energy will change when a charge moves in the potential field, therefore, we have combined probability knowledge with this principle, established probability selection mechanism of path node and safety evaluation method.

This paper is organized as follows. In Section II, we mainly analyze the potential field distribution characteristics are analyzed, and the environmental model established. Section III introduces the basic principle of the electric potential energy, path node probability selection mechanism, and then Section IV completely describes the proposed algorithm. Section V is a simulation experiment and analysis. Section VI shows the conclusion.

2 Path planning problem and environment model

The main objective of path planning is to find the appropriate path from the start to the end point; the main issues to be considered are the safety of path and computational efficiency of algorithms. Establishing an appropriate model to describe the threatening conditions is the basis for efficient path planning. Thus, flight environment threats and obstacles are: anti-aircraft artillery, missiles, mountains, high buildings, and no-fly zone. For non-aggressive obstacles, boundaries can be directly set. For offensive threats, traditional methods commonly use the round or spherical scope to represent the threat model. These methods can not accurately describe the intensity of the threat. Electric potential field has some advantages in environment modeling. Typically, the closer the plane to threat, the more dangerous it is. That is, threat intensity is in positive correlation with relative distance. This is right in accordance with electric potential field theory, the closer to the charge, the stronger the electric potential. Inspired by this characteristic, we use the electric potential field to describe the threat intensity of the environment.

2.1 Electric potential field distribution principle

In the two-dimensional Cartesian coordinate system, the coordinate of P is (x,y), the coordinate of q_i is (x_i, y_i), the electric potential of the charge q_i at the point P is:

$$V_p = \sum_{i=1}^{n} \frac{1}{4\pi\varepsilon_0} \frac{q_i}{\sqrt{(x-x_i)^2+(y-y_i)^2}} \tag{1}$$

Suppose there are n charges $q_1, q_2, q_3, \cdots\cdots, q_n$, according to the superposition principle of electric potential field, we can work out the electric potential of a certain point P, the formula is defined as follows:

$$V_p = \sum_{i=1}^{n} V_{p_i} = \sum_{i=1}^{n} \frac{q_i}{4\pi\varepsilon_0 r_i} \tag{2}$$

r_i is the distance between P and q_i.

The electric field intensity is:

$$\mathbf{E} = -\nabla\mathbf{V} = -\mathbf{a}_x \frac{\partial V}{\partial x} - \mathbf{a}_y \frac{\partial V}{\partial y} \tag{3}$$

According to the above conclusion, we can get electric dipole potential distribution.

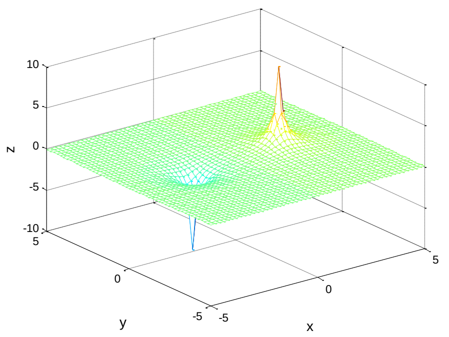

Figure 1. Electric potential of electric dipole.

As shown in Fig. 1, (x,y) is the coordinate, z is the electric potential.

2.2 Environment modeling

Electric field intensity is the physical quantity describing the electric field strength. The electric field intensity depends on the quantity of charge. We assume that the threat is positive charge, the coordinate is (x_i, y_i), the electric potential at the point P is:

$$V_o = \sum_{i=1}^{n} V_{o_i} = \sum_{i=1}^{n} \frac{q_{o_i}}{4\pi\varepsilon_0 r_i}, \qquad r_i \in (0, r_1) \tag{4}$$

r_i is the distance between ith threat and a certain point. Since kill probability of weapons is zero outside the effective distance, fire coverage range is $0 < r_i < r_1$. Assuming that the aircraft is a point charge, it does not affect the electric potential field distribution. Here there is a target point, and since the target point is negatively charged, then the potential becomes:

$$V_{all} = \sum_{i=1}^{n} \frac{q_{o_i}}{4\pi\varepsilon_0 r_i} - \frac{q_t}{4\pi\varepsilon_0 r_t}, \ r_t \in (0, \infty) \tag{5}$$

3 Randomized strategy

This chapter mainly introduces the electric potential energy principle, and studies the relationship between the electric potential energy and path. In order to choose appropriate path nodes, we used the Gaussian kernel function to establish the probability selection mechanism. Finally, we proposed the complete path planning method.

3.1 Electric potential energy

We usually define the electric field intensity as $E=F/q$, q is a charge, F is an electric field force. If q is a unit of electricity, the electric field intensity is equal to the electric field force.

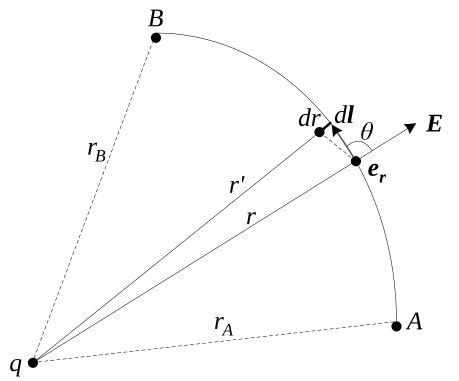

Figure 2. A charge moves in the electric potential from A to B.

As shown in Fig. 2 the point charge e_r moves in the electric field generated by charge q, their distance is r, $\boldsymbol{dl}$ is a small path along AB. Electric potential energy is:

$$\mathrm{dW} = \mathrm{q_0}\mathbf{E}\cdot \mathrm{d}\boldsymbol{l} \tag{6}$$

The electric field intensity is

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{\mathrm{q}}{\mathrm{r}^2}\boldsymbol{e_r} \tag{7}$$

Using (6), we obtain

$$\mathrm{dW} = \frac{1}{4\pi\varepsilon_0}\frac{\mathrm{qq_0}}{\mathrm{r}^2}\mathrm{dr}$$

And $\boldsymbol{e_r}\bullet\mathrm{d}\boldsymbol{l} = \mathrm{d}l\cos\theta = \mathrm{dr}$, so the whole electric potential energy along the path is:

$$\mathrm{W} = \int \mathrm{dW} = \frac{\mathrm{qq_0}}{4\pi\varepsilon_0}\int_{\mathrm{r_A}}^{\mathrm{r_B}}\frac{\mathrm{dr}}{\mathrm{r}^2} = \frac{\mathrm{qq_0}}{4\pi\varepsilon_0}\left(\frac{1}{\mathrm{r_A}} - \frac{1}{\mathrm{r_B}}\right) \tag{8}$$

And because

$$\mathrm{V} = \int_{\mathrm{r}}^{\infty}\mathbf{E}\bullet\mathrm{d}\boldsymbol{l} = \frac{\mathrm{q}}{4\pi\varepsilon_0}\frac{1}{\mathrm{r}} \tag{9}$$

Using (8), we obtain

$$\mathrm{W} = \mathrm{q_0}(\mathrm{V_A} - \mathrm{V_B}) \tag{10}$$

In a certain range, the aircraft will be affected by the superimposed electric potential generated by the threat and target point at the same time. That is to say, the charge is moving in an electric field, energy of the charge changes. For superimposed electric field, at two points i,j, the energy is:

$$\Delta \mathrm{W}_{i,j} = q_0(V_{all}(i) - V_{all}(j)) \tag{11}$$

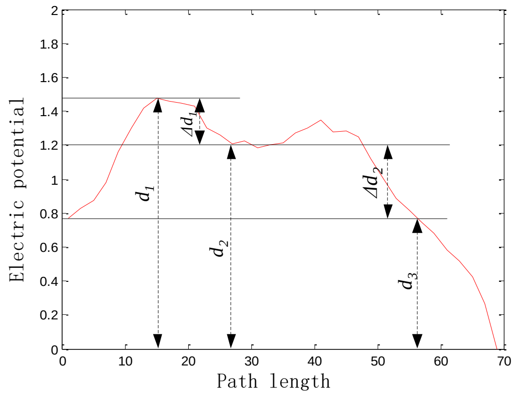

Figure 3. Electric potential and path length.

In Fig. 3 the electric potential energy is determined by the relative electric potential value of the start point x_{init} and the end point x_{goal}. The reduction of potential is U, and $\sum_i \Delta \mathrm{U}_{i+} + \sum_j \Delta \mathrm{U}_{j-} = 0$. Namely, the increase in electric potential $\Delta\mathrm{U}_+$ is equal to the decrease $\Delta\mathrm{U}_-$, and

$$\mathrm{W}_{ij} = q\mathrm{U}_{ij} \tag{12}$$

So, we only consider the increased electric potential energy.
Path length is l, electric potential energy is $\mathrm{w(p)}$, for one path, potential energy can be expressed as:

$$\mathrm{W(p)} = q_0\sum(V_{all}(i) - V_{all}(j)) + \delta l \tag{13}$$

Path coefficient δ is a very small number (e.g., $\delta = 0.01$), it can guarantee the shortest path to be selected if the paths have the same electric potential energy $q_0\sum(V_{all}(i) - V_{all}(j))$. That is, the shorter the path is, the smaller the $\mathrm{W(p)}$.

3.2 Probability selection mechanism

In the potential field, we combined the Gauss kernel function with the electric potential energy, thus establishing the probability selection mechanism, which can effectively promote the path node to bias toward the target point. To prevent local minimum, we retained some nodes which are away from the target point if they are in accordance with the probability condition. Calculation process is as follows: Gaussian kernel function is

$$f(\mathrm{x})=\exp\left(-\frac{\mathrm{x}(i)-\mathrm{x}(j)}{2k^2}\right) \qquad (14)$$

k is a given parameter. $\mathrm{x}(i)-\mathrm{x}(j)$ represents $\mathrm{V}_{all}(i)-\mathrm{V}_{all}(j)$, if $\mathrm{x}(i)-\mathrm{x}(j)>0$, probability that the note j will be selected can be calculated according to the above-mentioned formula; if $\mathrm{x}(i)-\mathrm{x}(j)\le 0$, it indicates the node biased toward the target point, probability is 1.

Assume that there are n paths, each path has m nodes. The probability of given ith path is:

$$\mathrm{p}_i=\prod_{j\in[1,\mathrm{m}]}(\mathrm{p}^i{}_{j\_\mathrm{rand}})(\mathrm{p}^i{}_{j\_\mathrm{select}})$$

$\mathrm{p}^i{}_{j\_\mathrm{rand}}$ is the probability of an randomly given node, $\mathrm{p}^i{}_{j\_\mathrm{select}}$ is its selection probability. For ith path, the probability that each parent node randomly generated the new node is the same that is $1/n$, so

$$\mathrm{p}_i=\frac{1}{\mathrm{n}^\mathrm{m}}\prod_{j\in[1,\mathrm{m}]}(\mathrm{p}^i{}_{j\_\mathrm{select}})$$

Using (14), we have $\mathrm{p}_i=\frac{1}{\mathrm{n}^\mathrm{m}}\prod_j f(x_j)$, that is

$$\mathrm{p}_i=\frac{1}{(\mathrm{n})^\mathrm{m}}\exp\left(-\frac{\sum\Delta\mathrm{V}_j}{(2k^2)^\mathrm{m}}\right)$$

Using (13), and ignored the path coefficient δ, we get

$$\mathrm{p}_i=\frac{1}{(\mathrm{n})^\mathrm{m}}\exp\left(-\frac{\mathrm{W}(\mathrm{p}_i)}{\mathrm{q}_0(2k^2)^\mathrm{m}}\right) \qquad (15)$$

Then we can get the following conclusion: the smaller the $\mathrm{w}(\mathrm{p}_i)$, the greater the probability p_i. To reach the target point first, the probability of path with the least potential energy must reach its maximum.

4 Path planning method

Here, node grows around the parent node with a certain step according to the model of RRT algorithm. When the electric potential energy decreases, a new node is added to the extended tree. When it is faced with threat, electric potential energy rapidly becomes large enough so that the tree expansion fails, selection probability mechanism is initiated to determine whether to accept the new node or not, and also to calculate the probability. Repeat the above process to make sure that the path is a bias toward the target point, and finally a small proportion of the path selects the shortest branch. The complete method is as follows:

First, we define the environment C. Assuming that there are several threats in the environment, the aircraft is a positive charge, the target point and threats are positive charges. We set the influence scope of the potential field generated by each charge and target point. Assuming that the target point is a negative charge, the quantity of this electric charge is large, and its potential field covers the whole environment. C_{free} is non-threatening area, T is an extended tree, x is the point of T, and $\mathrm{T}\in\mathrm{C}_{free}$. x_{init} is the start point, x_{goal} is the target point, x_{rand} is the random point, and $x_{rand}\in\mathrm{C}_{free}$.

Then we start to select the new node. Randomly generate a node x_{rand}, we calculate the distance between x_{rand} and all nodes in the tree, assuming that x_{near} is the closest node in the tree to x_{rand}. $\mathrm{Dis}(x_i,x_j)$ is geometric distance of two nodes, here we use Euclidean distance. We have $\mathrm{Dis}(x_{near},x_{rand})\le\mathrm{Dis}(x,x_{rand})$. On the straight line between the point x_{rand} and x_{near}, a node point x_{new} is defined and the condition met that $\mathrm{Dis}(x_{near},x_{rand})=\rho$, and step $\rho>0$. Then parent node of x_{near}, x_{near} and x_{new}, the three

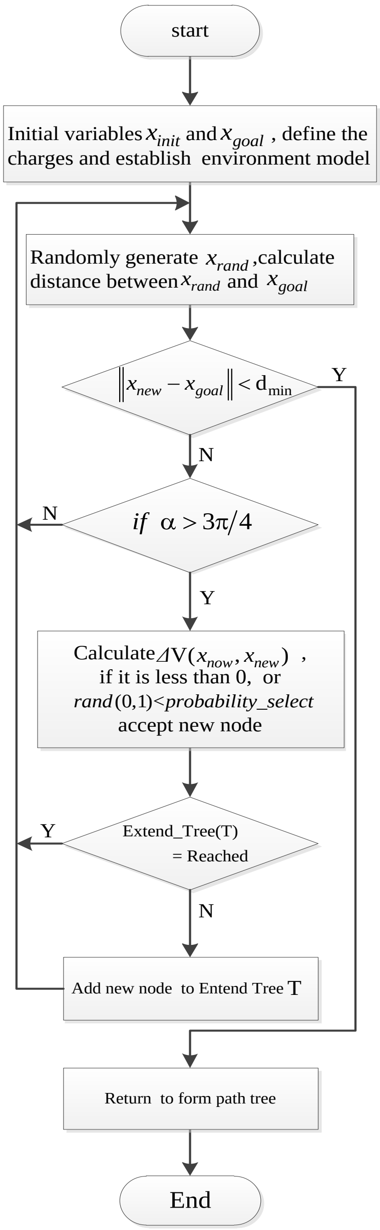

Figure 4. Algorithm flow chart.

nodes form a corner, in order to ensure that the path is smooth, we define that angle of the corner must be greater than 135° , if not, reselect a new node. Final step is probability selection. Calculate the potential change $\Delta V(x_{near}, x_{new})$ between x_{near} and x_{new}; if $\Delta V(x_{near}, x_{new}) \le 0$, x_{new} is added to the tree, otherwise the probability of being selected is $P = \exp\left(-\frac{x_{near} - x_{new}}{2k^2}\right)$. If there is no new node, then reselect x_{rand} . Repeat the above steps. The algorithm flow chart is as follows, $\mathrm{Extend\_Tree(T) = Reached}$ indicates that the new node of the tree has reached the target point.

5 Simulation

In this section, we conducted simulation experiments to compare the efficiency of the different algorithms, security level and length of the paths. Simulation software: Matlab 7.0; computer operating system: Windows XP, CPU is Inter Core i3, frequency is 3.3GHz. Assuming that the size of environment is 50×50, $\delta = 0.01$, $k = 1/\sqrt{2}$.
Firstly, under the same conditions we compare RRT algorithm, A* algorithm and proposed algorithm to test the computation time, path length and other factors. There are 13 threats, the threat intensity is determined by the quantity of electric charge, which are +1C and +2C, respectively, the target point is -10Cand the aircraft is +1C.

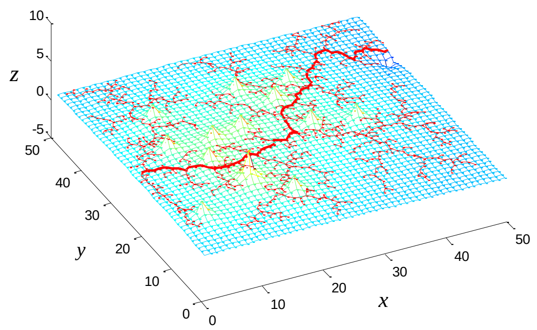

Figure 5. RRT algorithm.

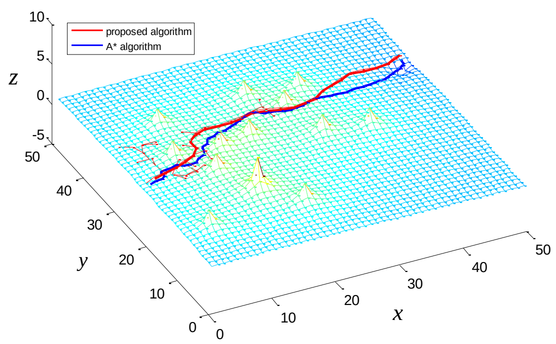

Figure 6. Proposed algorithm and A algorithm.*

Table 1. Algorithm performance comparison

Algorithm	Time	length	Electric potential
	s	km	J
Proposed	0.9	78	-27.0476
A*	9.4	76	-26.3611
RRT	4.3	92	-31.8905

As shown above, nodes are given randomly in RRT algorithm. The biggest advantage of this way is its strong ability to search in the unknown space, but as most of the search is invalid, computational costs have also increased dramatically, and the generated path is not optimal. In Fig. 6, the blue line represents the path generated by the A* algorithm, and the red line is generated by the proposed algorithm. Since the electric potential value reflects the attraction of the target point, the lower the absolute value of the electric potential value is, the shorter the length of the path. It is evident that, A* algorithm (here the Manhattan distance as the heuristic function) can find the shortest path to the target point, but in the complex environment, calculation time is the longest, and the algorithm does not consider the aircraft's motion constraints. The path generated by proposed algorithm is quite short and smooth, whereas the calculation time is the shortest.

Secondly, we compare the security of the paths generated by the three methods according to the above simulation results. Taking into account the aircraft's dynamic performance and the unknown factors in the environment, the closer proximity of threat, the more dangerous it is. Therefore, it is necessary to evaluate the influence of all the threats on each node of the path. We proposed evaluation method, evaluation function is:

$$V_{threat} = \sum_{i=1}^{n} V_{o_i} = \sum_{i=1}^{n} \frac{q_{o_i}}{4\pi\varepsilon_0 r_i},\ r_i \in (0,\infty) \quad (16)$$

Assuming that the center of threat is at the origin of the coordinate frame, the electric potential along the gradient direction is shown in Fig. 7. The closer proximity to threat, the higher the intensity of threat is. When close to the center of threat, the threat intensity is increased rapidly, so, when the distance is greater than 5, the threat gradually becomes 0.

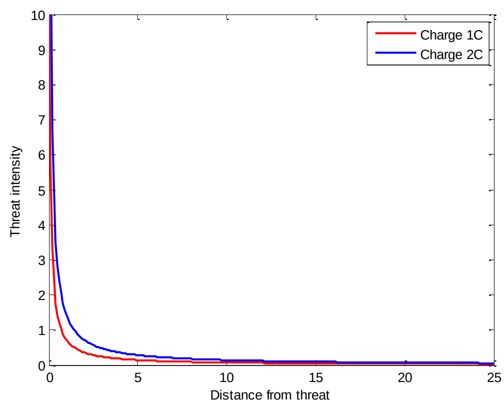

Figure 7. Threat intensity of different electric charges.

In order to compare the safety level of the paths, threat intensity is calculated as follows: the coordinate of x_{goal} as the center of the circle, draw circles with different radii respectively; as the radius length is an arithmetic sequence, there are a series of intersections between each circle and the path, so take it as a data point. We can get thus a number of points from each path. According to the above formula, we calculate each data point and add them together as the intensity of threat.

As shown in Fig. 8, the number of data points and the threat intensity are linearly related. In this paper, the threat intensity of the path generated by the algorithm is approximately equal to that of the RRT algorithm, and it is less than the A* algorithm, which indicates that the paths are far from threat, and that the security of paths is better.

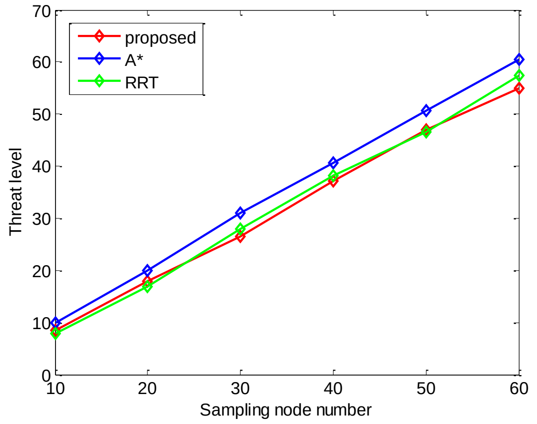

Figure 8. Comparison of three algorithm threat intensity.

Thirdly, we test the algorithms in "chasm" area. Chasms area is narrow low-cost region surrounded by the increasing cost. Because RRT makes sampling randomly, it is hard for the algorithm to exactly select the point in a narrow region.

As we can see, the circular region shown in Fig. 9 represents the chasm area. RRT cannot make sampling in such narrow regions. The path generated by the proposed algorithm can pass through the area although it failed several times. Among collected failure times in 50 tests, there are 36% of failure times in the proposed algorithm while in the RRT there is 96%. Proposed algorithm performance in the test is not very good.

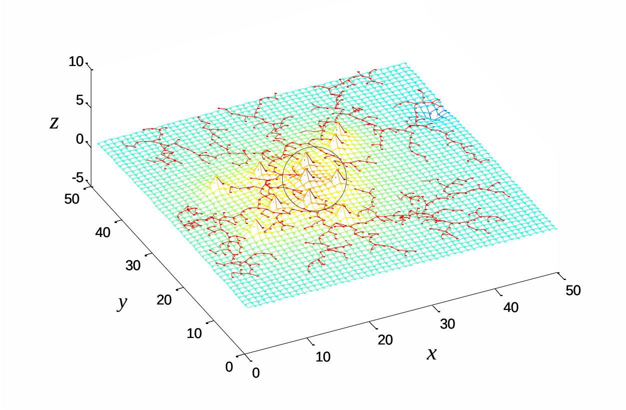

Figure 9. RRT in chasm area.

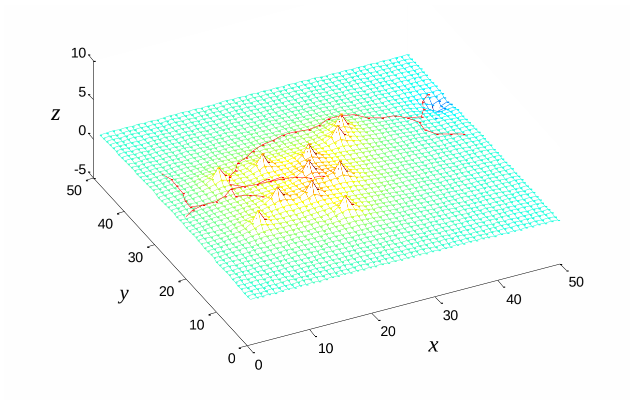

Figure 10. Proposed algorithm is tested in chasm area for the first time.

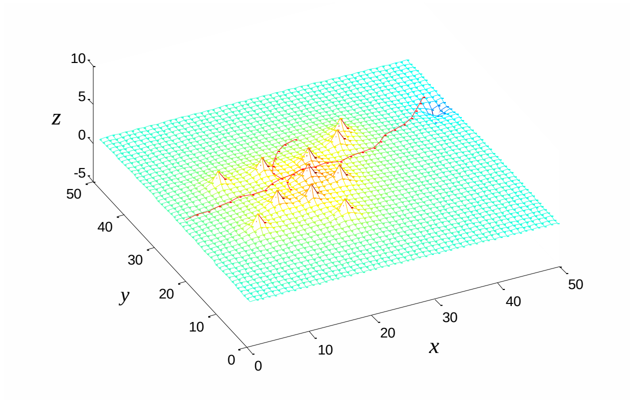

Figure 11. Proposed algorithm is tested in chasm area for the third time.

Finally, we conducted the local minimum test, compared the proposed algorithm with the RRT-A* algorithm. There are 6 threats arranged in the form of anti "C". The start point coordinate is $(1,22)$, and the target point is $(30,20)$. The threats and target points are 1C and -10C, respectively.

The constraint condition is $\alpha > 3/4\pi$. In Fig. 12, the algorithm does not consider the motion constraints, and the generated path is much worse than that of Fig 13. The local minimum test shows that the RRT-A* algorithm improves the RRT algorithm by heuristic function, but local minimum problem has not been well solved. Proposed algorithm can effectively solve the problem by the probability selection mechanism.

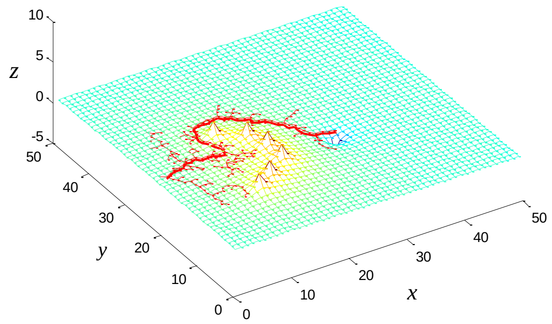

Figure 12. Proposed algorithm local minimum test without constraint condition.

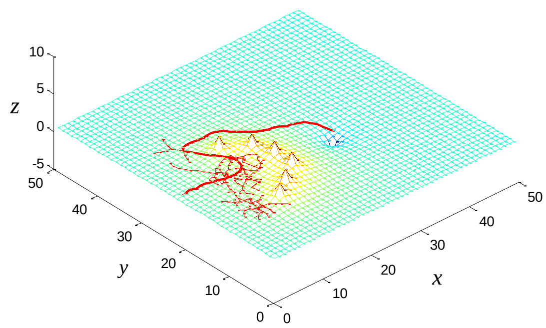

Figure 13. Proposed algorithm local minimum test under constraint condition.

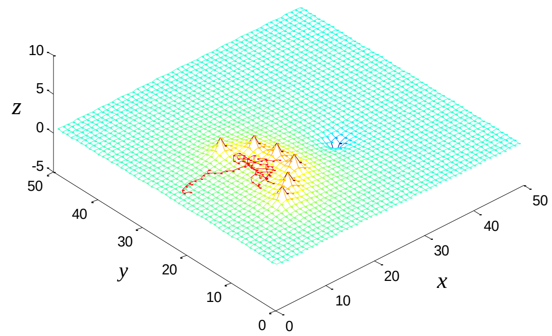

Figure 14. RRT-A algorithm local minimum test.*

6 Conclusion

The traditional method is not very suitable for the path planning problem in complex environment and high-dimensional space. Inspired by the principle of the electric potential energy and distribution principle of the electric potential field, the environmental model and probability selection mechanism were established, and finally the new path planning method was also proposed. It is worth mentioning that this method has high computational efficiency, but also applicability of the algorithm in complex environment and high-dimensional space is verified. Besides, the study of this paper shows that the principle of the electric potential energy can provide a new mechanism for path planning. However, for real-time planning problems, further studies are needed.

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