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Computer Aided Calculation of Characteristics Points of Some Envelope Helical Surfaces

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ABSTRACT

Presented calculations and the choice of variables $u,v \in [0,1]$ of the given basic surface φ enable the creation of a versatile programme for the graphical processing of the characteristics not only of the helical but also of the rotational envelope surface Φ defined by the conical or cylindrical surface. Basic (conical or cylindrical) surface j can be generated from the basic curve defined by the vector function $\mathbf{r}(u)$, for $u \in [0,1]$ (1) applying a class of transformations defined by the matrix $\mathbf{T}(v)$, for $v \in [0,1]$ (2). The combined analytical and synthetical method resultes in the parametric equations of the characteristics (3) and in the coordinates of the meridian section points.

Keywords:

envelope surface, characteristics, meridian section, creative space, creative representation

Izračunavanje karakterističnih točaka ovojnice helikoidne plohe pomoću računala

SAŽETAK

Prikazana izračunavanja i izbor varijabli $u,v \in [0,1]$ zadane osnove plohe φ omogućavaju kreiranje programa za grafičku obradu osobitosti ne samo helikoidne, već i rotacijske ovojne plohe Φ definirane stožastom ili valjkastom plohom. Osnovna (stožasta ili valjkasta) ploha j može biti generirana iz osnovne krivulje definirane vektorskom funkcijom $\mathbf{r}(u)$, za $u \in [0,1]$ (1) primjenjujući klasu transformacija definiranu matricom $\mathbf{T}(v)$, za $v \in [0,1]$ (2). Kombinirana analitička i sintetička metoda rezultira parametarskim jednadžbama karakteristika (3) i koordinatama točaka meridijanskih presjeka.

Ključne riječi:

ovojna ploha, karakteristike, meridijanski presjek, kreativni prostor, kreativni prikaz

An envelope surface Φ is created by a continuous movement of a basic surface φ. Characteristics is a curve segment along which the envelope surface Φ touches the basic surface φ. At any point of the characteristics there exists a common tangent plane τ and a normal $\mathbf{c}$ to the basic surface φ and the envelope surface Φ.

The same envelope surface Φ can be created by the continuous movement of either characteristics, or the basic surface φ.

Let us deal with an envelope helical surface Φ created by a helical movement (this movement is a geometric transformation concatenated from a revolution about the axis o and a translation in the direction of the vector collinear to the axis o of revolution) of a conical or cylindrical surface φ. An envelope rotational surface is a special type of the envelope helical surface with the helical movement pitch $|\mathbf{z}_v|$ ($\mathbf{z}_v$ is the translation vector corresponding to the angle of revolution equal to 2π) equal to zero.

Study and realization of the construction of the envelope helical and rotational surface characteristics points are very important in the mechanical engineering practise. The classical construction of the characteristics points (mentioned in Kopincová [1]) can be substituted by computer processing and following graphical output.

In the Creative space (defined in Velichová [6] and described in Velichová [5]), in which we work with homogeneous coordinates (in correspondence with Qiulin [2]), let us create the basic surface φ. Let us define the basic curve segment of the conical or the cylindrical surface by a vector function

$$\mathbf{r}(u) = (\, x(u), y(u), z(u), 1) \qquad (1)$$

such, that it is defined and at least $C^{(1)}$ for $u \in [0,1]$ and its first derivative $(\, x'(u), y'(u), z'(u), 0)$ is a non-zero vector for $u \in [0,1]$.

Let the generating principle be the class of transformations represented by a regular square matrix of rank 4 in a form

$$T(v) = \begin{pmatrix} q(v) & 0 & 0 & 0 \\ 0 & q(v) & 0 & 0 \\ 0 & 0 & q(v) & 0 \\ x_1 v & y_1 v & z_1 v & 1 \end{pmatrix} \qquad v \in [0,1] \qquad (2)$$

where function $q(v) = 1 - v$ is pertinent to the conical surface and function $q(v) = 1$ to the cylindrical surface. Constants x_1, y_1, z_1 are coordinates of the conical surface vertex or they are coordinates of the cylindrical surface direction vector.

The analytic representation of the basic conical or the cylindrical surface φ can be then expressed as follows

$\mathbf{r}(u,v) = \mathbf{r}(u) \,.\, \mathbf{T}(v) =$

$= (x(u)\, q(v) + x_1\, v,\ \ y(u)\, q(v) + y_1\, v,\ \ z(u)\, q(v) + z_1\, v,\ 1)$

where $(u,v) \in [0,1] \times [0,1]$.

Let the clockwise helical movement with the axis o in the coordinate axis z be defined by the reduced pitch $v_0 = |\mathbf{z}_v|/2\pi$ (with respect to Velichová [6]) (see Fig.1).

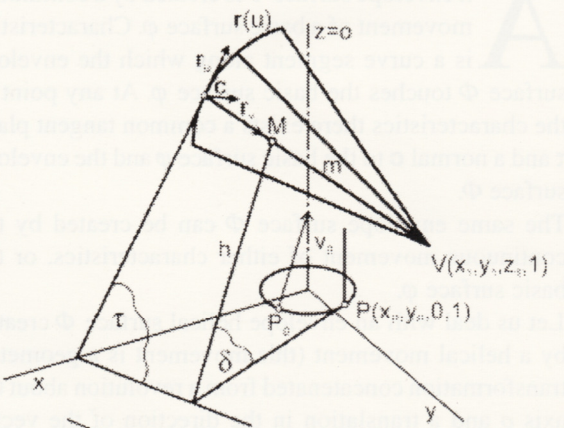

Fig.1

The point M of the characteristics can be determined as the intersection point of the line m on the surface φ and the auxiliary characteristics h of the tangent plane τ to the surface φ incident with the line m.. Characteristics h is a line on the surface of tangents to the helics (i.e. the envelope surface created from the tangent plane τ by the given helical movement). Characteristics $h = \tau \cap \delta$, where δ is a plane determined by the point P, the direction vector $\mathbf{c}$ and the direction vector $(0,0,1,0)$ of the axis o.

Parametric equations of the envelope helical surface characteristics (if it exists) will be

$$
\begin{aligned}
x &= x(u)\, q(\,\xi(u)\,) + x_1\, \xi(u) \\
y &= y(u)\, q(\,\xi(u)\,) + y_1\, \xi(u) \qquad (3) \\
z &= z(u)\, q(\,\xi(u)\,) + z_1\, \xi(u)
\end{aligned}
$$

for those values $u \in [0,1]$, for which $\xi(u) \in [0,1]$, while

$$\xi(u) = \frac{c_2(x(u) - x_P) + c_1(-y(u) + y_P)}{-c_2(x(u)a + x_1) + c_1(y(u)a + y_1)} \qquad (4)$$

and

$$
\begin{aligned}
c_1 &= a(y'(u)z(u) - z'(u)y(u)) + y'(u)z_1 - z'(u)y_1 \\
c_2 &= a(-x'(u)z(u) + z'(u)x(u)) - x'(u)z_1 + z'(u)x_1 \\
c_3 &= a(x'(u)y(u) - y'(u)x(u)) + x'(u)y_1 - y'(u)x_1
\end{aligned}
$$

$$x_P = \frac{v_0\, c_2\, c_3}{c_1^2 + c_2^2}(-1)^i$$

$$x_P = \frac{-v_0\, c_1\, c_3}{c_1^2 + c_2^2}(-1)^i$$

$$a = q'(v) = \text{const}$$

where vector $\mathbf{c} = (\,c_1\,, c_2\,, c_3\,, 0\,)$ is the direction vector of the basic surface φ normal and therefore it is also the direction vector of the envelope Φ normal in the point M of the characteristics.

$x_P\,, y_P$ - are coordinates of the auxiliary point $P(x_P\,, y_P, 0,1)$.

Constant value $i = 1$ is valid for clockwise and $i = 2$ for anticlockwise helical movement.

A special attention must be payed to the situations, in which the value c_1 and the value of the denominator of the relation (4) are equal to zero.

The shape of the envelope surface can be better comprehended by its meridian section than by the characteristics, which is usually a space curve segment. Coordinates of the point $M^* = (x^*, y^*, z^*, 1)$ located on the meridian section in the xz-plane can be obtained from the coordinates of the point $M = (x_M\,, y_M\,, z_M\,, 1)$ on the characteristics as the solutions of the following equations

$$x^* = \pm\sqrt{x_M^2 + y_M^2}$$

$$y^* = 0$$

$$z^* = z_M + \omega\, v_0$$

where ω is the directed angle of the revolution about the axis o to the xz-plane oriented in the helical movement direction.

Presented calculations and the choice of variables $(u,v) \in [0,1] \times [0,1]$ of the given basic surface enable the creation of a versatile programme for the graphical processing of the characteristics not only of the helical but also of the rotational (if $v_0 = 0$) envelope surface defined by the conical or cylindrical surface - see figures.

In the Fig.2 there is presented the characteristics and the principal meridian of the envelope rotational surface created by the revolution of a conical surface with the basic curve in a spatial Viviani curve.

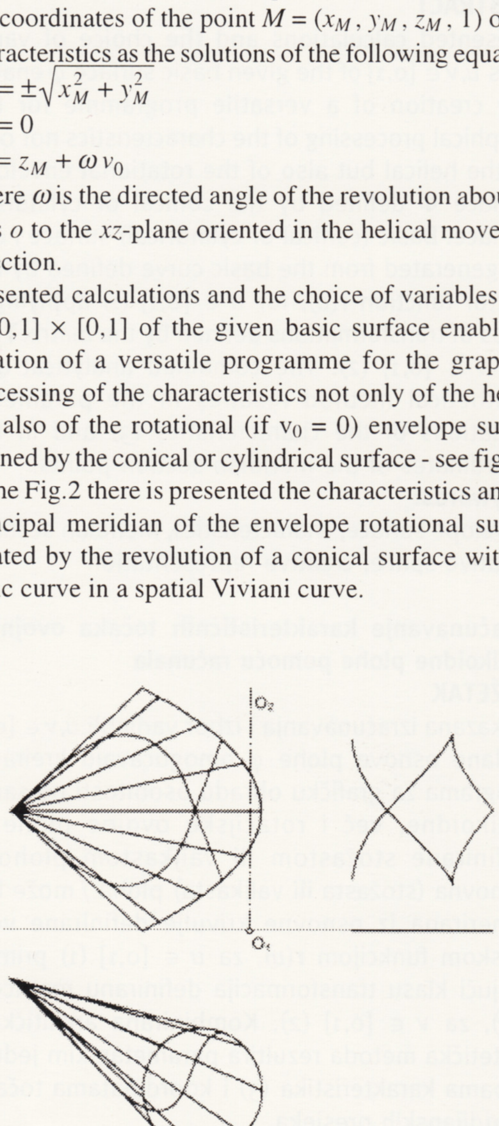

Fig.2

The illustration of the envelope surface generated by the helical movement of the basic conical surface characteristics, the basic curve of which is the circle, is in the Fig.3. The same movement of the envelope helical surface meridian produces the same envelope surface (see Fig.4).

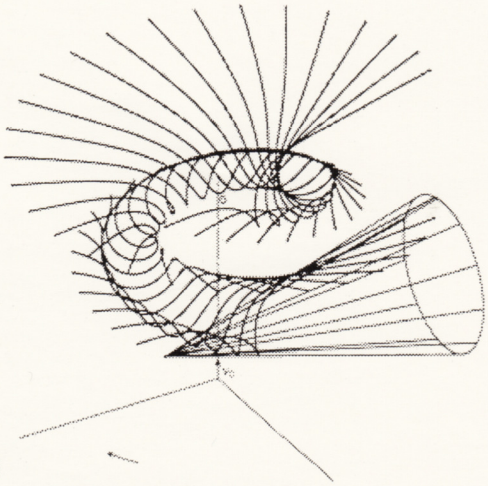

Fig.3

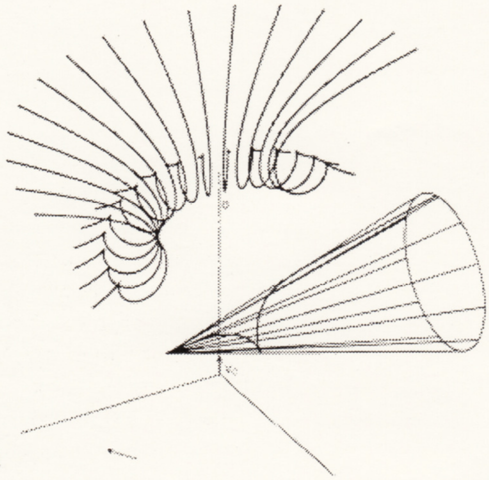

Fig.4

Fig.5 ilustrates a helical surface created by the characteristics of a basic cylindrical surface which basic curve segment is a circle. Other examples of the envelope surfaces can be found in the papers Szarková [3] and Szarková [4].

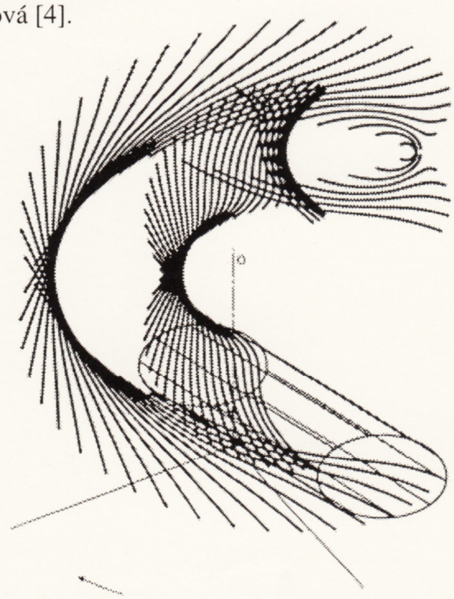

Fig.5

Graphical access to the processing of the given problematic enables a more effective function in the sphere of the machining tool design, because the basic surface of the desirable envelope surface can be fixed interactively and also all irregular cases, when the envelope surface cannot be defined or it is of an unsuitable shape, can be omitted.

Presented figures are examples of the characteristics, the meridian section and the basic surface φ of the envelope surface Φ in Monge projection method and Axonometry projections are outputs of the programme (written by the autor) on the screen and digital plotter.

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