Croatian Journal of Philosophy
Vol. XV, No. 43, 2015

Chrysippus' Indemonstrables *and Mental Logic*

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Stoic logic assumes five inference schemata attributed to Chrysippus of Soli. Those schemata are the well-known indemonstrables. A problem related to them can be that, according to standard propositional calculus, only one of them, modus ponens, is clearly indemonstrable. Nevertheless, I try to show in this paper that the mental logic theory enables to understand why the Stoics considered such schemata to be basic kinds of arguments. Following that theory, four of them can be linked to 'Core Schemata' of mental logic and the only one that is more controversial is modus tollens. However, as I also comment, some assumptions of Stoic philosophy, which can be interpreted from the mental logic theory, can explain why this last argument was included into the set of the indemonstrables as well.

Keywords: Classical logic, *indemonstrable*, mental logic, reasoning schemata, Stoic.

Introduction

The basic arguments that, according to Stoic logic lead inferences are the five *indemonstrables*: modus ponens, modus tollendo ponens, modus ponendo tollens (1), modus ponendo tollens (2), and modus tollens. Chrysippus of Soli is said to be the philosopher that identified them. For example, Sextus Empiricus, in *Adversus Mathematicos* 8.223, states this fact. It is true that, as indicated by O'Toole and Jennings (2004), there is a certain discussion regarding this point. Nonetheless, what is important for this paper is that the Stoic idea seems to be that those five schemata are rules that cannot be proved and that, however, serve to demonstrate all the other inferences.

Given that, according to standard propositional calculus, it is obvious that only one of them, modus ponens, is really indemonstrable, and that the other four arguments can be derived by means of other rules of that calculus, one might ask why the Stoics considered the *indemonstrables* to be so basic schemata. In my view, a contemporary theory on human reasoning can respond to that question. The theory is the mental logic theory (e.g., Braine & O'Brien 1998a; O'Brien 2009; O'Brien & Manfrinati 2010). Following it, people reason by using a mental logic that is different to classical logic. Mental logic is not really in contradiction with classical logic. In fact, all the valid inferences in mental logic are also valid in classical logic. The difference is that mental logic does not admit some formal rules of classical logic, and that, therefore, classical logic enables inferences that are not accepted in principle by mental logic. In this way, mental logic only considers the rules that, according to empirical research, individuals truly apply. Thus, it distinguishes different kinds of rules and describes the order and the circumstances in which such rules are used. However, what is more important here is that the mental logic theory claims that there are 'Core Schemata' in human mind that people always use when they reason about inferences with certain formal structures. Those Core Schemata are basic, since they only involve one step for finding a conclusion, and I think that the correspondences that can be found between Chrysippus' *indemonstrables* and the Core Schemata of mental logic can explain why the Stoics attributed a status so essential to the *indemonstrables*.

Thus, in this paper, I will try to show that four of the *indemonstrables*, and not only modus ponens, can be considered to be really basic in the system proposed by the mental logic theory. The difficult points are only, as I will also indicate, that, while disjunctions are exclusive in Stoic logic, that is not necessarily the case in mental logic, and that, due to this fact, modus ponendo tollens (2) must be interpreted as a derived version of modus ponendo tollens (1).

As it will be also shown, the only problematic inference is modus tollens. As it is known, this rule is not a basic rule in standard propositional calculus or in systems such as that of Gentzen (1935). Nonetheless, this schema is problematic for the aims of this paper because, in the same way, cannot be linked to any Core Schema in mental logic. Besides, modus tollens causes many difficulties in human reasoning research (see, e.g., Johnson-Laird & Byrne 2002, and Espino & Byrne 2013) and, as reported by cognitive science literature, individuals do not always apply it (see, e.g., Byrne and Johnson-Laird 2009, and López-Astorga 2013). In any case, I think that there are reasons that explain why the Stoics included it into the set of the *indemonstrables*. Such reasons are compatible with the theses of the mental logic theory and I will account for this idea below.

Nevertheless, before doing it, I will argue in favor of the thesis that mental logic allows considering four *indemonstrables* (all of them except modus tollens) to be basic schemata. Each of the five sections of this paper hence addresses one *indemonstrable*. I will begin by the simplest one, i.e., modus ponens, and finish with the most complex one, i.e., modus tollens.

Modus ponens

Modus ponens is an argument in which the first premise is a conditional *ἀξίωμα*. This Greek word is often translated as 'proposition'. Although I am aware that the exact meaning of the word is discussed (see, e.g., O'Toole & Jennings 2004), for simplicity I will adopt that translation in the following pages. The Stoics usually expressed modus ponens in this way:

"If the first, the second;
but the first;
therefore, the second" (O'Toole & Jennings 2004: 476).

Its formal structure hence is as follows:

x → y, x // Ergo y

Where '→' stands for conditional relationship.

In my view, it is obvious that modus ponens is a basic and essential reasoning rule. That is evident in Gentzen's system and in standard propositional calculus. In addition, it is a Core Schema, schema 7, in the description of mental logic proposed by Braine and O'Brien (1998b). So, it can be said that it is a schema that people use where possible. Likewise, its importance is also clear for axiomatic systems, both those based on classical logic and those based on non-classical logics. Furthermore, its structure is quite simple. Given a conditional proposition, if the *ἡγούμενον*, that is, the antecedent (or, in the previous quote, 'the first') happens, then the *λῆγον*, that is, the consequent (or, in the previous quote, 'the second') must happen as well. Because these facts, it is not surprising that the Stoics thought that modus ponens is an *indemonstrable*. Indeed, it seems that they really were right.

Modus tollendo ponens

In this case, the first premise is a disjunctive proposition. The argument was often expressed in the following way:

"Either the first or the second;
but not the first;
therefore, the second" (O'Toole & Jennings, 2004: 476).

Thus, its formal structure is:

x ∨ y, ¬x // Ergo y

Where '∨' means disjunction and '¬' represents denial.

As it can be noted, the rule is that, if one of the disjuncts of a particular disjunction is denied, then the other disjunct must be correct. The problem of this inference is that it can be demonstrated in standard propositional calculus. The derivation could be this one:

[1] $x \vee y$ (premise)
[2] $\neg x$ (premise)
[3] x (assumption)
[4] $\neg y$ (assumption)
[5] $x \cdot \neg x$ ($\cdot$I 2, 3)
[6] $\neg\neg y$ (RA 4–5)
[7] y ($\neg$E 6)
[8] y (assumption)
[9] y (reiteration 8)
[10] y ($\vee$E 1, 3–7, 8–9)

Where '$\cdot$' is conjunction, '$\cdot$I' refers to the conjunction introduction rule (x, y // Ergo $x \cdot y$), 'RA' represents the Reductio ad Absurdum strategy (if x is supposed and a contradiction such as $y \cdot \neg y$ is found, then $\neg x$ must be drawn), '$\neg$E' denotes the denial elimination rule ($\neg\neg x$ // Ergo x), and 'vE' stands for the disjunction elimination rule ($x \vee y$, $x \rightarrow z$, $y \rightarrow z$ // Ergo z).

Ten steps are many steps and one might think that they do not describe the real process that human mind makes in arguments in which modus tollendo ponens is involved. However, although this deduction is truly complex in mental logic, it is not absolutely impossible in it. The derivation includes rules that are schemata in mental logic. $\cdot$I is a Feeder Schema, in particular, schema 8 in Braine and O'Brien's (1998b) system. A Feeder Schema is not a Core Schema. Nevertheless, Feeder Schemata play an important role in mental logic, since they are used when they can offer relevant information that enables to use other rule. On the other hand, $\neg$E is a Core Schema in that same system, in particular, schema 1 in Braine and O'Brien's (1998b) description.

The difficulties are linked to Reductio ad Absurdum and vE. Reductio ad Absurdum is, certainly, a strategy enabled by mental logic. Nonetheless, it does not take part in the 'Direct Reasoning Routine'. It is an 'Indirect Reasoning Strategy' and, for this reason, it is hard to use and it is not always applied. This is a real problem because in the previous derivation Reductio ad Absurdum is used two times. Besides, it can be thought that other controversial point related to Reductio ad Absurdum is that, from other perspectives, it is argued that the logical systems allowing resorting to Reductio ad Absurdum do not really describe human reasoning, since contradictions enable to conclude any proposition in formal logic (e.g., Johnson-Laird 2010). The idea seems to be that contradictions or incompatibilities, i.e., cases of $x \cdot \neg x$, are not only linked in logic to Reductio ad Absurdum, but also to the Ex Contradictione Quodlibet principle. However, I think that this criticism is only opportune for a theory claiming that human beings reason following classical logic, standard propositional calculus or systems such as that of Gentzen (1935). Indeed, in those cases, any formula can be supposed and, if it causes a contradiction, its negation can be drawn. Nevertheless, in mental logic incompatibilities only refer to Reductio ad Absurdum, and not to Ex Contradictione Quodlibet. In this last logic, any proposition cannot be assumed. A proposition can only be supposed if it can be true, and "Nothing follows from a contradiction except that some assumption is wrong" (Braine & O'Brien 1998c: 206). Therefore, based on mental logic, criticisms such as this one are not true problems. The difficulties are facts such as those indicated, i.e., the fact that Reductio ad Absurdum is hard to apply, the fact that it is not often used, and the fact that the previous deduction requires it to be applied two times.

As far as vE is concerned, it can be said that there is a Core Schema in mental logic that can correspond to it. That schema is schema 5 in Braine and O'Brien (1998b) and can be expressed, with other symbols, as follows:

$x_1 \vee \ldots \vee x_n, x_1 \rightarrow y, \ldots, x_n \rightarrow y$ // Ergo y

There is no doubt that this schema is very akin to vE. The problem is that, in modus tollendo ponens, the premises $x_1 \rightarrow y, \ldots, x_n \rightarrow y$ do not appear. They need to be made in some way (steps 3–7 and 8–9 in the previous deduction), and, undoubtedly, this means an additional effort.

It is hence evident that, although the system presented by Braine and O'Brien (1998b) allows proving modus tollendo ponens, that deduction is very hard to do in their system, and, according to the general theses and predictions of the mental logic theory, such a demonstration is very unlikely to be done. Regardless the fact that human reasoning does not seem to make inferences with so many steps in an automatic way, the empirical results reported in Braine and O'Brien (1998b) and in Braine, Reiser, and Rumain (1998) suggest that modus tollendo ponens is a simple and basic rule that only requires one step to be applied (that is, that people tend to infer y directly from x v y and ¬x). Individuals appear to solve reasoning problems involving modus tollendo ponens very quickly and, in addition, the percentage of errors in this kind of problems is very low. In this way, it can be thought that, for these reasons, mental logic assumes that the following argument is a Core Schema:

$x_1 \vee \ldots \vee x_n, \neg x_i$ // Ergo $x_1 \vee \ldots \vee x_{i-1} \vee x_{i+1} \vee \ldots \vee x_n$

Indeed, Braine and O'Brien (1998b) state that a schema similar to this one (with other symbols) is clearly a Core Schema of mental logic (in particular, it is their Core Schema 3) whose percentage of errors is only 2,5%. It is obvious that this schema corresponds to modus tollendo ponens and the fact that it can be considered to be a basic Core Schema enables to understand why the Stoics thought that it is an *indemonstrable*. As said, maybe modus tollendo ponens could be demonstrated in mental logic system. However, empirical evidence and experimental results indicate that it is usually applied in a rapid way, and that it is a schema naturally used by human beings.

Modus ponendo tollens (1)

In the version 1 of modus ponendo tollens, the first premise is a denied proposition. In particular, it is a denied conjunction. It was often expressed as follows:

"Not both the first and the second;
but the first;
therefore, not the second" (O'Toole & Jennings 2004: 476).

So, the logical form of this inference could be:

$\neg(p \cdot q)$, p // Ergo $\neg q$

Modus ponendo tollens (1) is an inference that can be proved in standard propositional calculus as well. The derivation could be the following:

[1] $\neg(x \cdot y)$ (premise)
[2] x (premise)
[3] y (assumption)
[4] $x \cdot y$ ($\cdot$I 2, 3)
[5] $\neg(x \cdot y) \cdot (x \cdot y)$ ($\cdot$I 1, 4)
[6] $\neg y$ (RA 3–5)

Again, in principle, it could be thought that the system proposed by Braine and O'Brien (1998b) allows demonstrating modus ponendo tollens (1) and that, therefore, it is not a basic rule. $\cdot$I is a Feeder Schema in mental logic and Reductio ad Absurdum is a possible strategy in that same logic. Nevertheless, as said, mental logic considers Reductio ad Absurdum to be a complex strategy that is not always used and that is not applied by every individual. In this way, it seems that, when reasoners face to premises such as $\neg(x \cdot y)$ and x, they resort to a simple schema that allows them to derive $\neg y$. The empirical results reported by Braine and O'Brien (1998b) and Braine et al. (1998) suggest that this is the case and that people do not really follow the previous six steps. Because of such results, other mental logic Core Schema—schema 4 in Braine and O'Brien (1998b)—has a form similar to this one:

$\neg(x_1 \cdot \ldots \cdot x_n)$, x_i // Ergo $\neg(x_1 \cdot \ldots \cdot x_{i-1} \cdot x_{i+1} \cdot \ldots \cdot x_n)$

It is not hard to note that modus ponendo tollens (1) can be directly related to this Core Schema, which, according to Braine and O'Brien (1998b) has an error rate of 4%. It hence is also clear why modus ponendo tollens (1) is an *indemonstrable* in Stoic Logic.

Modus ponendo tollens (2)

Again the first premise is a disjunction. The problem now is that the disjunction is exclusive. The usual wording of modus ponendo tollens

(2) is this one:

"Either the first or the second;
but the first;
therefore, not the second" (O'Toole & Jennings 2004: 476).

Obviously, this argument is only valid if its disjunction is exclusive. Thus, its formal structure could be as follows:

$x \underline{\vee} y, x$ // Ergo $\neg y$

Where '$\underline{\vee}$' stands for exclusive disjunction.

But it can be stated that disjunction is exclusive in modus ponendo tollens (2) not only because, according classical logic, its formal structure requires it, but also because it seems that all disjunctions are exclusive in Stoic logic. We have some testimonies in this regard (most of them mentioned by O'Toole & Jennings 2004). For example, Gellius, speaking about disjunctions, in *Noctes Atticae* 16.8, states that "*Ex omnibus, quae disiunguntur, unum esse verum debet, falsa cetera*". It is absolutely clear that what Gellius means is that, in a particular disjunction, only one disjunct can be true. All the other disjuncts must be false. Other example can be taken from Galen, who, in *Institutio Logica* 5.1, says the same idea again, i.e., that "… *τῶν διεζευγμένων εν μόνον ἔξόντων ἄληθές*,…", that is, that, in disjunctions, only one disjunct is true. Of course, more examples can be offered, but I think that these two examples are representative enough to understand the Stoic view about disjunctions.

In any case, the fact that the Stoics consider all disjunctions to be exclusive can be, in principle, problematic because disjunctions are inclusive in systems such as standard propositional calculus. Nevertheless, as it is known, the problem disappears if we take the following equivalence into account:

$(x \underline{\vee} y) = (x \vee y) \cdot \neg(x \cdot y)$

Certainly, standard propositional calculus can work with exclusive disjunctions by virtue of this equivalence. Thus, it can be said that modus ponendo tollens (2) is not also indemonstrable in classical logic. The derivation can be this one:

[1] $(x \vee y) \cdot \neg(x \cdot y)$ (premise)
[2] x (premise)
[3] $\neg(x \cdot y)$ ($\cdot$E 1)
[4] …

Where '$\cdot$E' is the conjunction elimination rule ($x \cdot y$ // Ergo x).

From step 4 on, the deduction is the same as that of the previous section, i.e., as that of the modus ponendo tollens (1). So, the same arguments and criticisms could be repeated here. Nonetheless, I think that the points that are important to comment in this case are the following:

On the one hand, if disjunctions are always exclusive in Stoic logic, it could be thought that this fact affects modus tollendo ponens too. However, this is not a problem, since, as it can be noted, it does not matter whether the disjunction in modus tollendo ponens is inclusive or exclusive. The arguments and comments indicated in the corresponding section continue to be valid even if the disjunction of the first premise of modus tollendo ponens is exclusive.

On the other hand, the equivalence $(x \veebar y) = (x \vee y) \cdot \neg(x \cdot y)$ can be assumed in mental logic as well. This assumption would not cause difficulties to the mental logic theory. In this theory, $\cdot$E is other Feeder Schema and, therefore, there need be no additional problems in this way. The only aspect that would have to be highlighted is that the mental logic theory would not accept deductions as large as that corresponding to the derivation in classical logic of $\neg y$ from $[(x \vee y) \cdot \neg(x \cdot y)]$ and x. According to mental logic, in the previous deduction, reasoners would apply schema 4 and, in step 4, would draw $\neg y$. So, mental logic schema 4 and the previous equivalence not only allow understanding why the Stoics thought that modus ponendo tollens (2) was an *indemonstrable* too, but also why modi ponendo tollens (1) and (2) are so linked. Both of them refer to the logical form $\neg(x \cdot y)$ and hence to a Core Schema, schema 4, in the system proposed by Braine and O'Brien (1998b).

Modus tollens

This is the *indemonstrable* that is more difficult to explain because, as mentioned, cognitive science literature shows that people do not often make this inference correctly. Its usual expression is this one:

> "If the first, the second;
> but not the second;
> therefore, not the first."
> (O'Toole & Jennings 2004: 476).

Its logical form hence is as follows:

$x \rightarrow y, \neg y$ // Ergo $\neg x$

Given that modus tollens can be proved in standard propositional calculus and does not correspond to any Core Schema or to any schema of other type in mental logic, it can be argued that, by considering it to be one of the five *indemonstrables*, the Stoics made a mistake. However, it can also be thought that the Stoics had any reason to adopt modus tollens, and the aim of this section is to check whether or not that reason can be found.

As it is well known, modus tollens is not a basic rule in standard propositional calculus and its conclusion must be derived by means of several steps. The usual inferential process attributed to it is akin to this one (see, for example, Byrne & Johnson-Laird, 2009, or López-Astorga, 2013):

[1] $x \rightarrow y$ (premise)
[2] $\neg y$ (premise)
[3] x (assumption)
[4] y (MP 1, 3)
[5] $y \cdot \neg y$ ($\cdot$I 2, 4)
[6] $\neg x$ (RA 3–5)

Where 'MP' means 'modus ponens'.

Thus, it is clear that, following classical logic, modus tollens is a derived rule and is not as basic as, for example, modus ponens (which, as it can be noted, needs to be used in the deduction corresponding to modus tollens). The problem in this case is, as said, that is not even a schema in mental logic. The inference is, of course, possible in mental logic, but, as in the case of standard propositional calculus, it depends on Reductio ad Absurdum and modus ponens, which means that it is an inference that is hard to apply and less frequent than others. One might ask why the Stoics thought that it is one of the *indemonstrables* and so basic.

It appears that the Stoics analyzed problems such as this one, since they were very concerned with the criteria that conditional propositions had to fulfill. Perhaps, they already noted that people do not always use modus tollens and offered a solution. Nevertheless, if we pay attention to ancient sources, it seems that their solution was related to the characteristics that a proposition needed to have to be considered as a conditional, and not to the *indemonstrables* themselves. In this way, one might suppose that the idea was that modus tollens was only applied when the first premise of the argument was really a conditional. If that was not the case, modus tollens was not used. This is my hypothesis and it is based on criteria such as that mentioned by Diogenes Laërtius at 7.73: "*συνημμένον οὖν ἀληθές ἐστιν οὗ τὸ ἀντικείμενον τοῦ λήγοντος μάχεται τῷ ἡγουμένῳ, οἷον 'εἰ ἡμέρα ἐστί, φῶς ἐστι.*'" What Diogenes Laërtius indicates is that an actual relation between the antecedent (*ἡγούμενον*) and the consequent (*λῆγον*) is needed. As I interpret this quote, without that relation, it is not possible to state that the proposition is a real conditional. Thus, it can be understood that the Stoic criterion is that, if the consequent is denied, the antecedent must denied too. O'Toole and Jennings' (2004) thesis on this point is very illustrative. According to them, the key seems to be the translation of the word '*μάχεται*', which is interpreted as 'conflicts' and refers to "some degree of common content" (O'Toole & Jennings 2004: 492) between the two clauses of conditional.

Undoubtedly, Diogenes Laërtius' example at 7.73, i.e., 'If it is day, it is light' (O'Toole & Jennings' 2004, translation) is very clear. If it is not light, then necessarily it is not day. Therefore, it can be said that, according to the Stoics, a conditional such as $x \rightarrow y$ is a real conditional only when it is also true that $\neg y \rightarrow \neg x$. So, it appears that modus tollens is applied only when this last requirement is fulfilled. Equally, from this perspective, it is obvious the reason why modus tollens is not used in some cases. When the relation between the antecedent and the consequent is random, the use of modus tollens is not secured. For example, in the proposition 'If I wear white trousers, then I wear red shoes', there is no an evident relation between the then-clause and the if-clause. The fact that I do not wear red shoes does not necessarily involve that I do not wear white trousers. However, in Diogenes Laërtius' example, the situation is different. If it is not light, as said, then necessarily it is not day. In this last case, modus tollens can be applied in a rapid and automatic way and without effort. Nevertheless, in the case of the white trousers and the red shoes, it is obviously harder to use.

It can be thought that the Stoics were considering cases similar to those in which an invited inference can be found (Geis & Zwicky 1971) or in which the phenomenon of conditional perfection happens (e.g., Auwera 1997a, 1997b; Horn 2000; López-Astorga 2014; Moldovan 2009), that is, cases in which the conditional leads to propositions such as $\neg x \rightarrow \neg y$ or it is transformed into a biconditional. As it is well known, when a conditional such as x –> y is perfected, it is transformed into $(x \rightarrow y) \cdot (y \rightarrow x)$, or, if preferred, into $x \leftrightarrow y$. Of course, this could be an interesting idea, since, for example, a perfected conditional leads one to think that only two scenarios are possible: a scenario in which both x and y are true, and a scenario in which both x and y are false. Thus, if the first premise is $x \leftrightarrow y$ and the second one is $\neg y$, it is clearer that only one option is possible: $\neg x$.

However, I think that the explanation that can be offered from the mental logic theory is simpler and has a more evident link to Diogenes Laërtius' previous quote. In mental logic, modus tollens is not, as indicated, an accepted schema. Nonetheless, the mental logic theory can explain why this rule is applied without difficulties in certain cases. The theory admits that pragmatics plays an important role in human inferential processes (Braine & O'Brien, 1998d) and, therefore, that pragmatics can provide information, i.e., some premise, which is not explicitly mentioned in the inference. In this way, it can be stated that modus tollens is only easily used when pragmatics refers to a premise such as $\neg y \rightarrow \neg x$. Thus, if Diogenes Laërtius' example were the first premise in a modus tollens inference, the true deduction could be as follows:

[1] $x \rightarrow y$ (premise)
[2] $\neg y$ (premise)
[3] $\neg y \rightarrow \neg x$ (pragmatic premise)
[4] $\neg x$ (MP 2, 3)

Step 3 indicates that reasoners, by virtue of their general knowledge, know that if it is not light, then it cannot be day. And this last pragmatic premise allows deriving, by means of a simple application of modus ponens, $\neg x$ in step 4.

So, it can be said that the Stoics considered modus tollens to be an *indemonstrable* because they thought that it could only be used with real conditionals, i.e., with relations between x and y involving both $x \rightarrow y$ and $\neg y \rightarrow \neg x$. Based on the mental logic theory, however, it is not necessary to distinguish between real and false conditionals. When a conditional refers to a pragmatic premise such as $\neg y \rightarrow \neg x$, modus tollens appears to be used without effort. Nevertheless, what really happens is that the pragmatic premise allows applying modus ponens. It is evident that, if this last argument is accepted, it is easy to understand why the Stoics assumed that modus tollens was an *indemonstrable*. Although both accounts—that of Stoic logic and that of mental logic—are different, both of them share an important idea: the reference to $\neg y \rightarrow \neg x$ is needed to directly derive $\neg x$ from $x \rightarrow y$ and $\neg y$. Without that reference, modus tollens is problematic because it can be thought that the conditional is not a true conditional (Stoic logic) or that $\neg x$ cannot be concluded by means of just one or two simple steps (mental logic). The proponents of the mental logic theory could hence state that the propositions that the Stoics took as real conditionals are actually conditionals that refer to a pragmatic premise such as $\neg y \rightarrow \neg x$.

But it is also interesting that, from frameworks such as Stoic logic and mental logic, other problems related to modus tollens commented in cognitive science literature can be solved as well. For example, Johnson-Laird and Byrne (2002) and Espino and Byrne (2013) mention inferences with the formal structure of modus tollens in which $\neg x$ is never concluded. Such inferences often have a first premise, i.e., the premise corresponding to the conditional proposition, which can be considered to be difficult or controversial. A clear case of premise of this kind is this one:

"If Rachel is in Brazil she is not in Rio" (Espino & Byrne 2013: 102).

If an argument of modus tollens with this proposition as its first premise is thought, the conclusion that is derived is hard to accept. In particular, the inference would be as follows:

[1] If Rachel is in Brazil, then she is not in Rio (premise)
[2] Rachel is in Rio (premise)
[3] Rachel is not in Brazil (MT 1, 2)

Obviously, 'MT' means 'modus tollens'.

As it can be noted, steps 2 and 3 are absolutely incompatible, since it is not possible to be in Rio and not to be in Brazil. According to Stoic logic, the solution of this problem is evident: the denial of the consequent (she is in Rio) does not involve the denial of the antecedent (Rachel is not in Brazil). So, the first premise is not an actual conditional and modus tollens cannot be applied.

Nevertheless, following mental logic, the solution is also obvious. There is a pragmatic premise, but that premise is not $y \rightarrow \neg x$ (the antecedent of this last formula is y, and not $\neg y$, because the consequent of the first premise states that she is not in Rio and hence is denied), which explains why modus tollens is not immediately applied. The true pragmatic premise is in this case $y \rightarrow x$ (i.e., 'If Rachel is in Rio, then she is in Brazil'), since, as said, a situation in which Rachel is in Rio and is not in Brazil cannot be thought. Therefore, based on the mental logic theory, it can be argued that the actual inferential process would be this one:

[1] $x \rightarrow \neg y$ (premise)
[2] y (premise)
[3] $y \rightarrow x$ (pragmatic premise)
[4] x (MP 2, 3)

As shown in step 4, a simple application of modus ponens leads reasoners to conclude that Rachel is in Brazil. And this shows why it is so unusual that $\neg x$ (Rachel is not in Brazil) is drawn in this type of inferences. Obviously, the process could continue and, in step 5, $\neg y$ could be inferred from steps 1 and 4 by means of modus ponens. Nonetheless, in that case, a contradiction (steps 2 and 5) would be found, which would indicate that a premise is false (for example, that of step 1).

In any case, what is important is that, as explained in the previous pages, the mental logic theory enables to understand the reasons that leaded the Stoics to state that the five arguments reviewed—modus ponens, modus tollendo ponens, modus ponendo tollens (1), modus ponendo tollens (2), and modus tollens—were indemonstrable. Four of them are demonstrable in standard propositional calculus, but, as claimed by the mental logic theory, people do not reason paying attention to the principles and rules of classical logic.

Conclusions

As commented above, only modus ponens seems to be an indisputable basic schema, since it is so in standard propositional calculus. Modus tollendo ponens, the two versions of modus ponendo tollens, and modus tollens can be proved in that calculus. However, given that modus tollendo ponens, modus ponendo tollens (1), and modus ponendo tollens (2) are Core Schemata in mental logic, the only problem appears to be modus tollens.

Indeed, if we assume that human mind does not follow classical logic, but mental logic, it is not difficult to understand why the Stoics considers the first four arguments (all but modus tollens) to be so basic. Nevertheless, this last idea requires two points to be taken into account. Firstly, it is true that disjunctions are exclusive in Stoic logic. Nonetheless, exclusive disjunctions are possible in mental logic. It is only necessary to attribute to them the logical form $(x \vee y) \cdot \neg(x \cdot y)$. This logical form makes the two versions of modus ponendo tollens very similar and allows one to note that they really refer to the same schema in mental logic (schema 4 in Braine & O'Brien 1998b). Secondly, as far as modus tollendo ponens is concerned, it does not matter whether disjunction is exclusive or inclusive. The corresponding schema (schema 3 in Braine & O'Brien, 1998b) can be used without difficulties both when it is exclusive and when it is inclusive.

Therefore, as mentioned, the only controversial *indemonstrable* is modus tollens. However, as also commented, it is not hard to understand why it is an important argument for the Stoics. According to them, an actual conditional is that in which there is a clear link between the antecedent and the consequent, and in which $\neg y$ is obviously incompatible with x. In this way, from the perspective of mental logic, what Stoic logic claims is that a conditional is only true if it is linked to a pragmatic premise with the logical form $\neg y \rightarrow \neg x$. Thus, it seems that modus tollens is applied, but the schema that is really used is modus ponens. So, because it is very easy to deduce $\neg x$ from $x \rightarrow y$, $\neg y$, and $\neg y \rightarrow \neg x$, the reasons why the Stoics assumed that modus tollens was an *indemonstrable* are evident.

In this way, we can think about an extension of mental logic including modus tollens. The idea would be to consider modus tollens to be a valid schema provided that $\neg y$ is incompatible or inconsistent with x, i.e., provided that the Stoics' requirement is fulfilled. Nevertheless, the mental logic system described in Braine and O'Brien (1998a) does not need to assume this new rule. That system admits that pragmatic premises play a role in human reasoning, and that, as indicated, if $\neg y \rightarrow \neg x$ is accepted as a pragmatic premise in an inference with the logical structure of modus tollens, just modus ponens (which is schema 7 in Braine & O'Brien 1998b) must be applied for drawing $\neg x$. From this point of view, to add this new rule would only make the mental logic system unnecessarily more complex, without giving it more predictive or explicative scope.

In any case, paying attention to the Stoics again, it can be said that they were aware that the material interpretation of conditional is problematic. According to that interpretation, if the antecedent of a conditional is false, it is absolutely guaranteed that the conditional in entirety is true. Therefore, a proposition such as 'if elephants can fly, then human beings are oviparous' is necessarily true, since it is false that elephants can fly. Maybe cases such as this one leaded the Stoics to assume their criterion indicated by Diogenes Laërtius at 7.73. Following that criterion, the previous conditional would not be a real conditional. The reason is that the fact that human beings are not oviparous does not have any link or relation to the possibility that elephants can fly. The proponents of mental logic also noted these difficulties and rejected the material interpretation of conditional as well. Based on the mental logic theory, it cannot be stated that human beings reason considering the traditional truth tables (which, as it is well known, are consistent with the material interpretation). Human reasoning follows syntactic rules, and, in particular, the syntactic schemata that experimental results reveal (not all the rules of calculi such as standard propositional calculus). Undoubtedly, this is an important point, and, given that both an ancient theory (Stoic logic) and a current theory (mental logic) agree on it, it can be worth continuing to take this thesis into account.

Acknowledgments

This paper is a result of the Project N. I003011, "Algoritmos adaptativos e inferencias lógicas con enunciados condicionales", supported by the Directorate for Research of Talca University (Dirección de Investigación de la Universidad de Talca), Chile. The author, who is also the main researcher of that Project, would like to thank the mentioned institutions for their help in funding this paper.

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