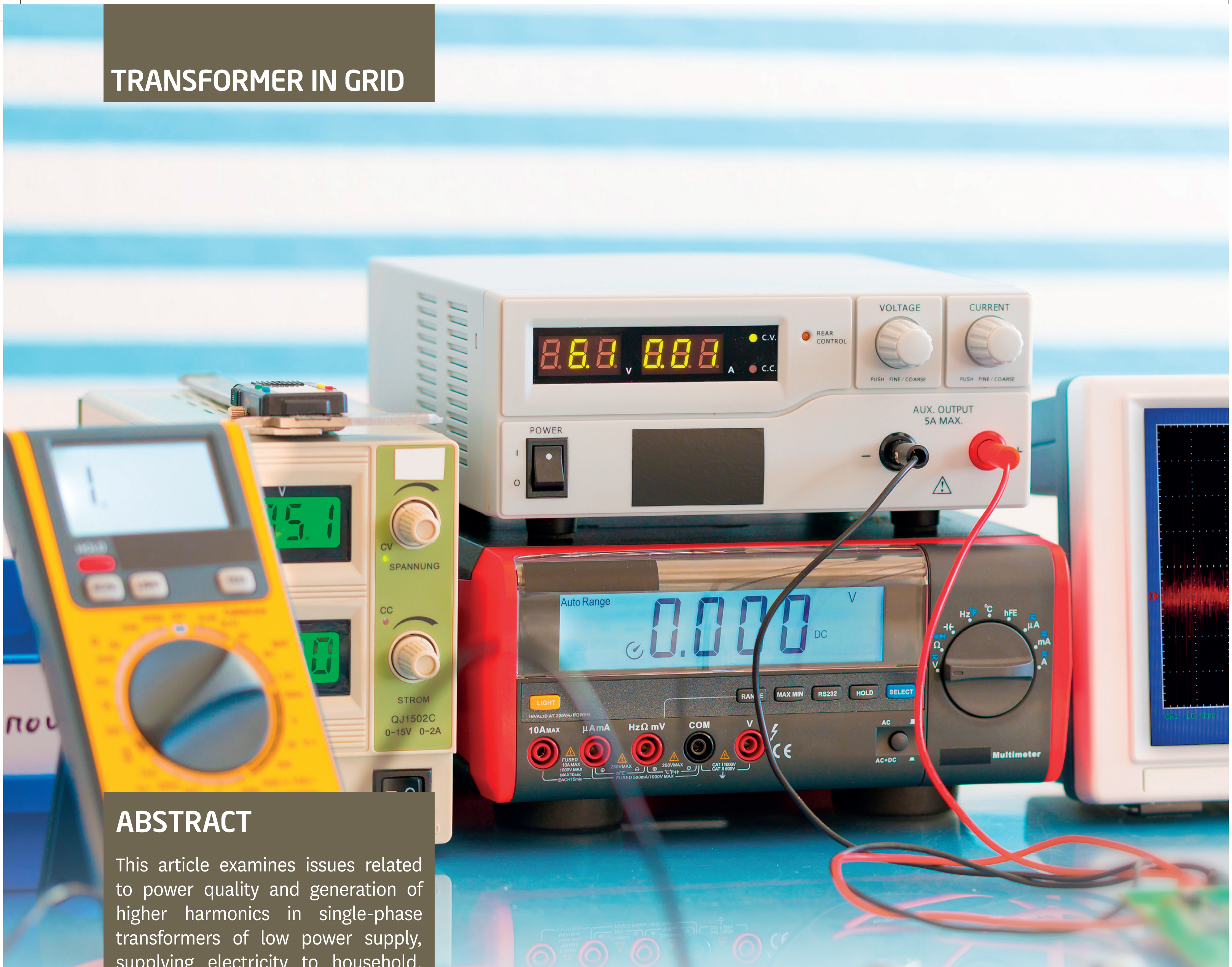


ABSTRACT

This article examines issues related to power quality and generation of higher harmonics in single-phase transformers of low power supply, supplying electricity to household, office and industrial equipment. Generation of higher harmonics of these transformers was simulated on basis of the results of the Finite Element Method (FEM) analysis of the magnetic field. Harmonic composition of the current in the primary side was determined using a Fourier transform applied to the curve of the current obtained through the FEM analysis. The results were fully confirmed by the experimental study of the harmonic composition of the current in the primary side of the investigated one-phase transformer.

The used approach is unique, and it enables manufacturers to explore the harmonic composition during the design stage, and take appropriate measures to reduce their magnitude.

KEYWORDS

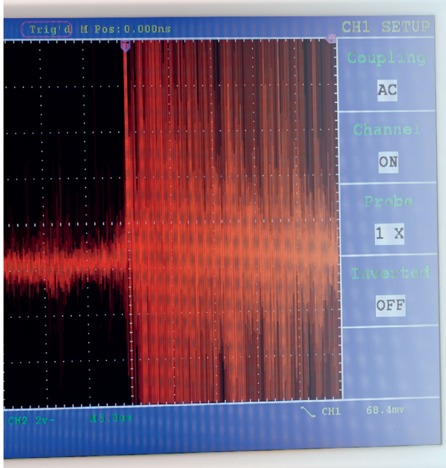
FEM, small transformers, harmonics, power quality, electrical power systems

Transformers and power quality – Part I

Modelling and researching generation of higher harmonics in small single-phase transformers used by domestic and industrial consumers

Petar Milkov UZUNOV

Due to the large number of small transformers, they pollute the electrical network and are a factor in lowering the quality of electrical power



1. Introduction

Electricity is one of the most widely used forms of energy in our time, which can be attributed to the easy reception, transmission and conversion of this type of energy to consumers. In terms of electricity consumption, this is a type of product which is characterized by a number of indicators that determine its quality. Users requiring high quality of supplied electricity will pay the right price for it.

In the transmission of electricity from the source to the consumer, there are various undesirable factors that can be observed: lightning strikes, frequency interference, high harmonics and shock loads. These fluctuations in power consumption and switching to different loads affect others in the network, thus reducing its basic parameters and quality, and adversely affecting the end users, i.e. consumers.

There is a lot of scientific research and literature discussing the effects of poor power supply quality on consumers [1-3]. Consumers themselves can also aggravate the quality of electricity [3]. However, the issue of which consumers exactly affect these quality indicators and how this works (i.e. the very mechanism of the process) has been less studied and requires more theoretical and experimental research.

Transformers are static electromagnetic devices which occupy an important place in the process of transmission and consumption of electricity. When an AC power source is connected to a transformer, current flows in its primary circuit, even when secondary circuit is open-circuited. This current is the current required to produce flux in the real ferromagnetic core, and it consists of two components:

- the magnetization current, which is the current required to produce the flux in the transformer core, and
- the core-loss current, which is the current required to make up for hysteresis and eddy current losses in the core.

From the graphical representation of the magnetization current in the classical theory of electrical machines [4], one can see that the magnetization current in the transformer is not sinusoidal. The higher-frequency components in the magnetization current are due to magnetic saturation in the transformer core, and they can be quite large compared to the fundamental component.

It should be noted here that contemporary designers design transformers so that they operate with a magnetic flux density close to saturation point for the maximum use of the ferromagnetic core material. In general, the more the transformer core is saturated, the larger the harmonic components will become.

Consequently, even when supplied with sinusoidal voltage, their no-load current will have a pure non-sinusoidal shape, i.e. transformers will generate high harmonics which penetrate into the grid and reduce the quality of electricity.

The objective of this article is to study the mechanisms of generating higher harmonics and the influence of small single-phase transformers used in household, office and industrial equipment, which are the external loads, on the quality of the power system. Specific studies were conducted on a single-phase transformer, shown in Figure 1a. Its technical data is outlined in Table 1 and its electrical scheme presented in Figure 1b.

Usually, electrical machine textbooks graphically present and explain a distortion of the shape of the current in the primary side [4], while other references [5, 6] describe various analytical models of the generation of higher harmonics in transformers.

The finite element method (FEM) offers the most accurate way to determine mechanisms of generating higher harmonics in transformers based on modelling and analysis of their magnetic field. FEM enables the analysis of the magnetic fields recording the nonlinearity in the used magnetic materials, which is difficult to obtain by the relevant analytical models that are based on the theory of electric and magnetic circuits.

The study of the generation of harmonics makes it possible to take appropriate actions to measure, control and improve the performance of transformers in order to prevent

Even a transformer supplied with sinusoidal voltage has non-sinusoidal no-load current, i.e. it generates high harmonics which penetrate into the grid and reduce the quality of electricity

them from polluting electricity supply and impairing its quality. Also, it enables proposition and implementation of relevant technical solutions to neutralize this harmful effect and improve power quality.

2. Modelling of the electromagnetic field of the transformer based on Maxwell's equations in differential form

2.1. Equations of the magnetic field

The quasi-stationary magnetic field of the transformer was modelled based on the system of differential equations of the electromagnetic field, recorded on the magnetic vector potential $\vec{A}$ and the potential of the electric field V.
The first Maxwell equation reads as follows:

$$\vec{\nabla} \times \vec{H} = \vec{J}. \qquad (1)$$

Here, on the right hand side the component $\frac{\partial \vec{D}}{\partial t}$ is ignored due to the electric flux density, and only the conductivity current density $\vec{J}$ is taken into consideration:

$$\vec{J} = \gamma \vec{E}. \qquad (2)$$

The equation for the continuity magnetic field principle is:

$$\vec{\nabla} \times \vec{B} = 0, \qquad (3)$$

where $\vec{B}$ is the magnetic flux density.
Equations (1) to (3), together with the relationship between the magnetic field intensity and the magnetic flux density, which corresponds to

$$\vec{B} = \mu \vec{H}, \qquad (4)$$

allow us to define the magnetic vector potential $\vec{A}$ as

$$\vec{B} = rot\,\vec{A}. \qquad (5)$$

The study of the generation of harmonics makes it possible to improve the performance of transformers and prevent pollution of electricity supply

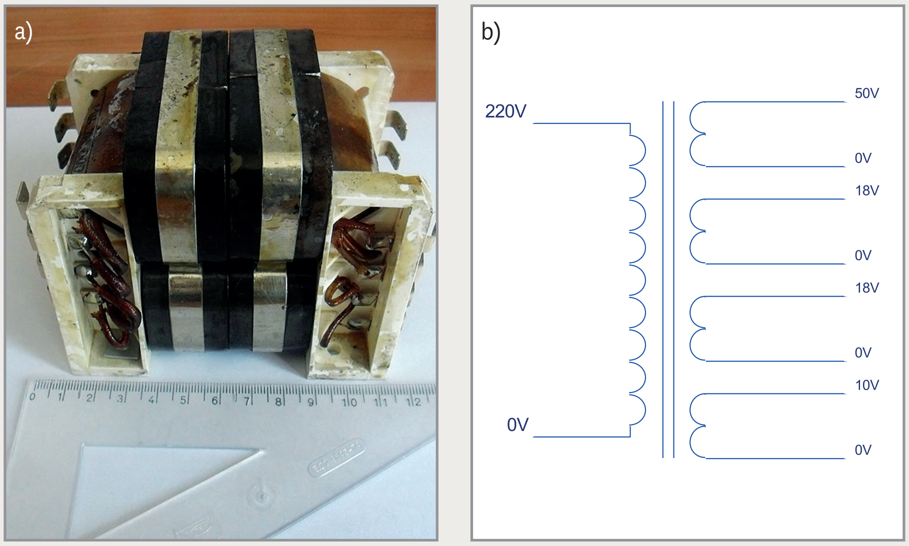


Figure 1. Investigated transformer: a) main view; b) electrical scheme of the transformer

From the equations (4) and (5) one can derive the following:

$$\vec{H} = \frac{\vec{B}}{\mu} = \frac{rot\,\vec{A}}{\mu}. \qquad (6)$$

From the second Maxwell's equation,

$$rot\,\vec{E} = -\frac{\partial \vec{B}}{\partial t}, \qquad (7)$$

and the equation (6), the following is derived:

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} + \vec{\nabla} \times (\nu \times \vec{B}). \qquad (8)$$

From (8) follows

$$\vec{E} = -\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t} + \nu \times \vec{\nabla} \times \vec{A}, \qquad (9)$$

where $\nu \times \vec{\nabla} \times \vec{A}$ represents moving of the field.

Applying (9), (2) and (8) to (1), the following result is obtained:

$$\vec{\nabla} \times \left(\frac{\vec{\nabla} \times \vec{A}}{\mu}\right) + \gamma \left(\frac{\partial \vec{A}}{\partial t} - \nu \times \vec{\nabla} \times \vec{A}\right) = -\gamma \vec{\nabla} V. \qquad (10)$$

The magnetic vector potential $\vec{A}$ is calculated by the use of (10). Then, using (5), (6) and (9), $\vec{B}$ and $\vec{H}$ are obtained.

Table 1. Technical data of the investigated transformer

Quantity	Value
Rated power [VA]	130
Primary voltage [V]	220
Rated frequency [Hz]	50
Secondary no-load voltage ±5 % [V] U_{21} / U_{22} / U_{23} / U_{24}	50 / 18 / 18 / 10
Rated secondary current [A] I_{21} / I_{22} / I_{23} / I_{24}	0.5 / 0.5 / 0.5 / 4.5
Number of turns of the primary winding	760
Number of turns of the secondary windings w_{21} / w_{22} / w_{23} / w_{24}	172 / 62 / 62 / 35
Diameter of the wire [mm] • primary winding • secondary winding	0.6 0.6 / 0.6 / 0.6 / 1.4
Material of the transformer core	M530
Class of insulation	F

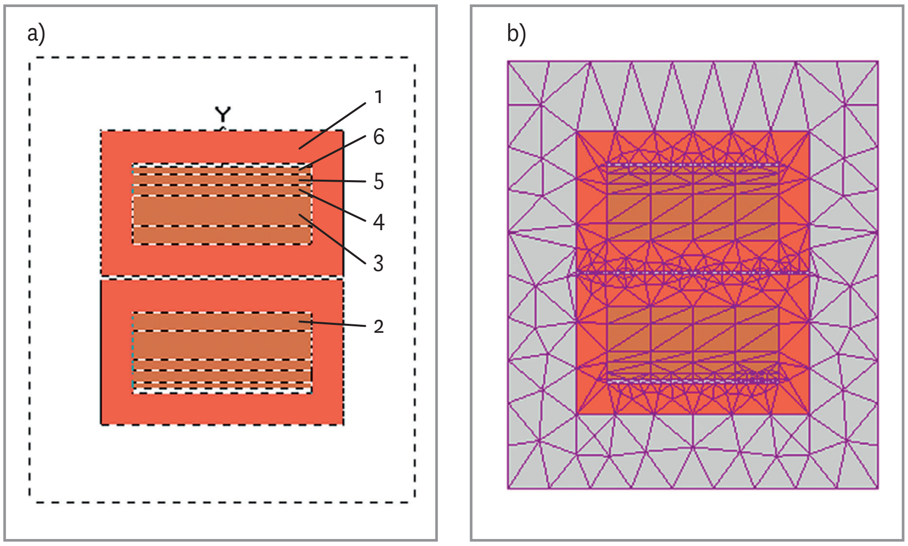


Figure 2. FEM model: a) region in which equations of the magnetic field are solved: 1 - magnetic core of the transformer; 2 - primary coil; 3,4,5,6 - secondary coils; b) discretized area

Part ∇V in (9) is taken from the definition of the source of electromotive force (e.m.f.).

2.2 Numerical modelling of the single-phase transformer

The analysis of the transformer's magnetic field was conducted in two dimensions in the region shown in Fig. 2a, divided into a network of 4,014 nodes and 7,960 triangular finite elements of the first order (Fig. 2b). The non-linear modelling of the material core was performed using the basic curve of magnetization shown in Fig. 3a.

The transformer was modelled by a combination of the circuit theory and the FEM, which is the best way to simulate the operation of an electromagnetic device and to test the electromagnetic processes therein.

The primary winding of the transformer was powered by a voltage source V_1 (Fig. 3b). Therefore, the source supplying the primary winding with a sinusoidal voltage had a set voltage amplitude, frequency and initial phase to model the network, supplying transformer with a sinusoidal voltage.

Series resistor R_1 was connected to the circuit of the primary winding. Its value can be calculated from no-load losses, as shown in the following equation:

$$R_1 = \frac{U_1^2}{P_{hyst} + P_{eddy}}, \qquad (11)$$

where

P_{hyst} is hysteresis losses in the transformer core,
P_{eddy} is eddy current losses in the transformer core,
U_1 is the effective value (RMS) of the primary voltage.

The non-linear modelling of the material core was performed using the basic curve of magnetization

The distribution of the magnetic field (Figure 4a) was obtained after solving the equation (10) for the time t = 0 to 120 ms with the increment of 1 ms – these were the six periods of the supply voltage. This number of periods was assigned in order to attenuate the transient process from the inclusion of the transformer and obtain the actual shape of the established magnetizing current of the transformer. Harmonic analysis of this current waveform was performed. Otherwise, if there is a non-steady state process, this will cause incorrect results of the Fast Fourier Transform (FFT) analysis.

To prove that the distortion of the current shape is due to the non-linearity of the magnetic characteristics of the ferromagnetic core material and its saturation, another analysis of the transformer magnetic field was performed, substituting the ferromagnetic core material with a material with a linear relation between the magnetic flux density and the magnetic field intensity, that is, a material with constant relative magnetic permeability $\mu \approx 1000$. As a result, sinusoidal shape of the current through the primary winding in no-load mode was obtained – i.e. the magnetizing current had sinusoidal shape in this particular case (Fig. 5).

3. Experimental study of the harmonic composition of the transformer magnetizing current

In order to verify the results of the harmonic analysis of the current obtained by FEM data post processing, an experiment was performed in which the waveform of the magnetizing current of the modelled transformer in no-load mode was measured using a digital oscilloscope. The picture of the experimental set-up is shown in Fig. 6.

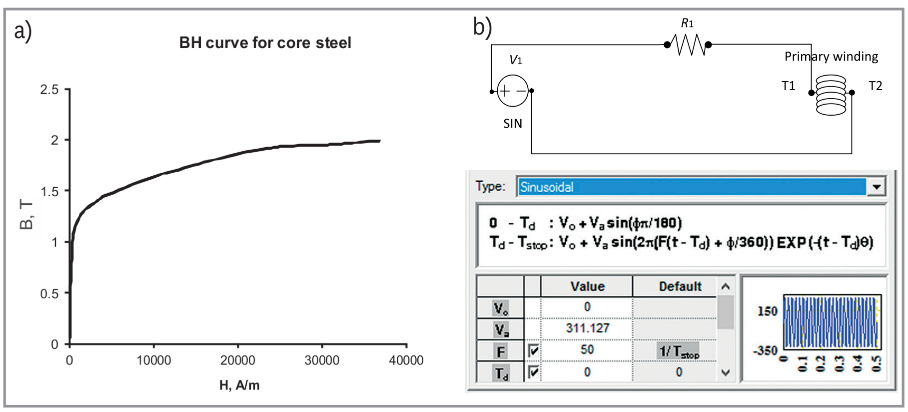


Figure 3. Modelling of the transformer: a) basic curve of magnetization of the magnetic steel M530; b) circuit and parameters of the power supply, source of e.m.f.

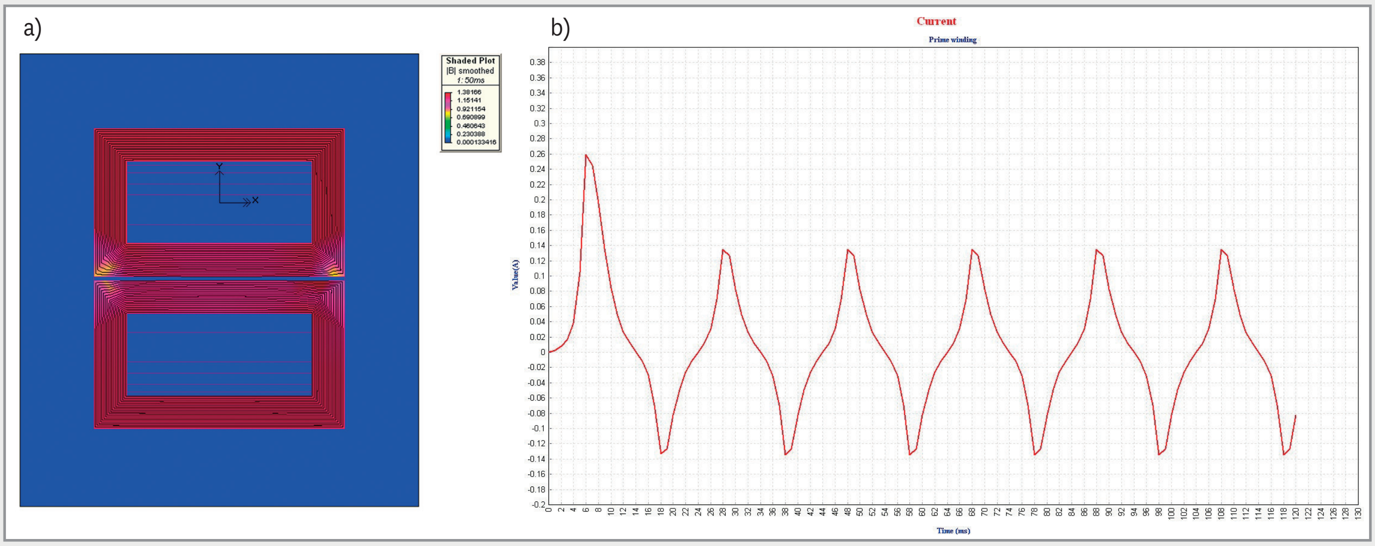


Figure 4. Results from the analysis of the transformer magnetic field: a) distribution of magnetic flux density for the time t = 2 ms; b) the current through the primary winding of the transformer

The tested single-phase transformer was powered by a laboratory autotransformer supplied with single-phase voltage of 220 V RMS with a pure sinusoidal shape. Its secondary windings were not connected to any load. The current waveform of the transformer was taken from the voltage drop on the series resistor R.

The current waveform displayed on the digital oscilloscope is shown in Fig. 7.

From Fig. 7 it is clearly visible that the current contains high harmonics generated only by non-linearity in the magnetic characteristics of the steel core and its saturation. The RMS value of the current was measured with the multimeter, which is a "true RMS" device which measured the real RMS value of the current, although it is non-sinusoidal. The measurement results are displayed in Table 2.

The digital oscilloscope has a built-in feature to help realize FFT. RMS values of harmonics are also provided in Table 2. In Fig. 8 the harmonic spectrum of the transformer current is shown. It was found that, in practice, apart from the fundamental harmonic with RMS value I_1, the current contains pronounced third and fifth harmonics, with RMS values I_3 and I_5 respectively.

4. Fourier's analysis of the transformer current

Fig. 7 shows the distortion of the shape of the no-load current of transformer, which is caused by higher harmonics generated due to non-linearity of the

To prove that the distortion of the current shape was due to non-linearity of the core, another analysis was performed substituting the ferromagnetic with linear core material

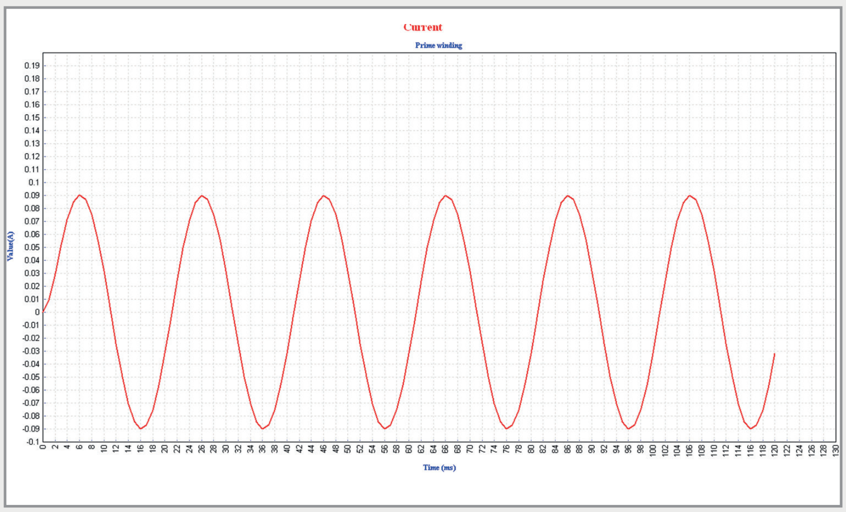


Figure 5. Waveform of the transformer magnetizing current when the core is made from magnetic material with linear properties

Table 2. Results of the experimental study of the current harmonic composition

Magnitude	Harmonic amplitudes				RMS value
	I_0	I_1	I_3	I_5	I
Unit	mA	mA	mA	mA	mA
Experience	18	64	18	6	70

To verify the results of the current harmonics obtained by FEM data post processing, the magnetizing current of the transformer was measured using a digital oscilloscope with a built-in FFT

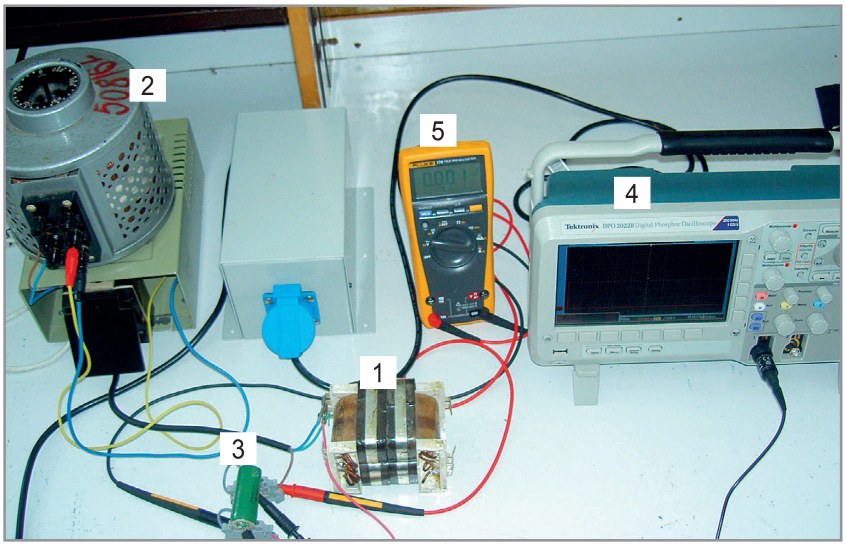


Figure 6. Experimental set-up: 1 – investigated transformer; 2 – laboratory autotransformer; 3 – resistance; 4 – digital oscilloscope; 5 – multimeter

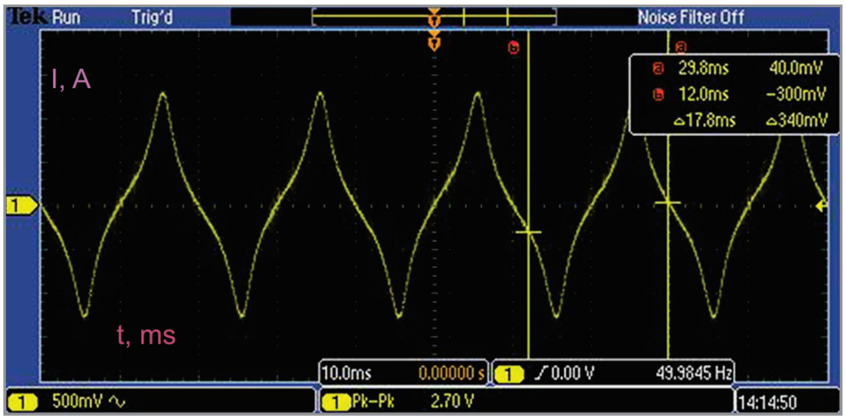


Figure 7. Waveform of the transformer current

magnetic characteristics of the steel core, and saturation effects.

In order to obtain a harmonic composition of the current through the primary winding, FFT was used. A mathematical programme provided harmonic analysis of the current waveform, as illustrated in Fig. 9. As a result, the harmonic spectrum of the current was obtained, shown in Fig. 9.

Fig. 9 shows a screen of a specially developed application displaying the shape of the curve of the current, and the results of the harmonic analysis of the transformer current.

It is obvious from the figure that the current of the transformer presents two more pronounced higher harmonics – the third and fifth, which are caused by non-linearity in the magnetic characteristics of the transformer core steel and which distort the shape of the current waveform.

From the theory of electrical engineering it is known that the RMS value of a non-sinusoidal magnitude, as transformer current, can be calculated by the following equation:

$$I = \sqrt{I_0^2 + \frac{I_{1m}^2}{2} + \ldots + \frac{I_{km}^2}{2}}, \qquad (12)$$

where I_{km} is the amplitude of the higher harmonics with number $k = 2, n$.

Using this equation and the results from the harmonic analysis, the RMS value of the current can be calculated. Table 3 presents a comparison of this RMS value, together with amplitudes of the harmonics, with the amplitudes of the experimentally measured higher harmonics. It was found that they differ little within the margin of the FEM calculation error, which indicates the correctness of the constructed FEM model of the transformer, especially the approach to modelling and the calculation of the harmonics generated by the transformer due to strong non-linearity in the magnetic properties of the core steel.

The 3rd and 5th current harmonics, which are caused by non-linearity in the magnetic characteristics of the transformer core steel, are more pronounced

Table 3. Comparison of the results of the harmonic analysis of the transformer current with FEM and experience

Magnitude	Harmonic amplitudes				RMS value
	I_0	I_1	I_3	I_5	I
Unit	mA	mA	mA	mA	mA
Experience	18	64	18	6	70
FEM	8	6.43	19	11	68

Conclusion

This article investigates the generation of higher harmonics in small single-phase transformers supplying power to household, office and industrial equipment. The results of the finite element analysis of the magnetic field have been used to obtain the waveform of the current in the primary side of the transformer operating at no load. Its shape is different from sinusoidal wave, i.e. the current contains higher harmonics, which occur due to non-linearity in the magnetic characteristics of the magnetic materials used in transformers, and due to saturation effect. To obtain magnitudes of these harmonics, harmonic analysis was performed by the FFT.

The results of the simulations with FEM were fully confirmed by measurements of the harmonic composition of the magnetizing current of the transformer with a digital oscilloscope. Despite their low power, the fact that there is a large number of such transformers in the electrical grid means that this is a serious pollutant and is therefore a factor in the reduction of the quality of electricity.

To remove this harmful effect and improve power quality, it is recommended to connect passive and active filters to the network, which are designed to filter out the 3rd, 5th and 7th harmonics.

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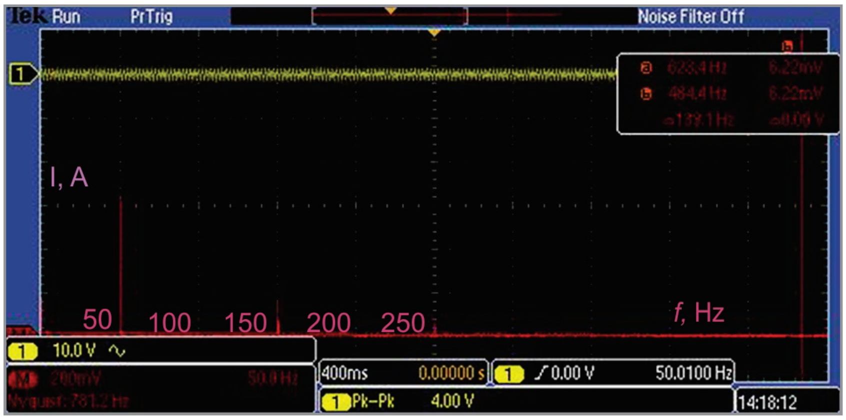

Figure 8. Harmonic analysis of the transformer current obtained by a digital oscilloscope

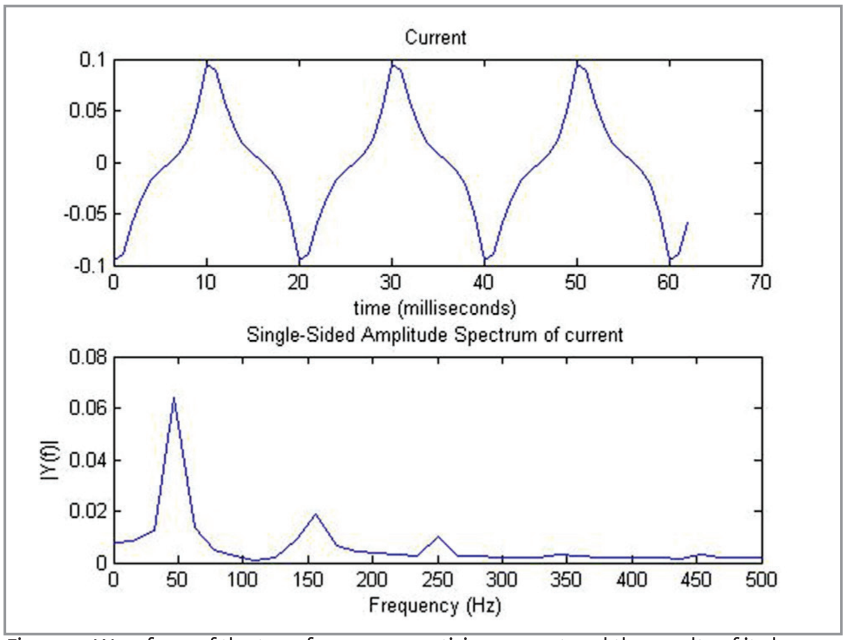

Figure 9. Waveform of the transformer magnetizing current and the results of its harmonic analysis (basic harmonic frequency 50 Hz)

Author

Petar Uzunov, Electricity System Operator, received his BSc and MSc degrees in electrical engineering from the University of Gabrovo, Bulgaria in 1988, and his PhD from the same university in 1998. From 1988 to 2007 he was an Assistant Professor, Chief Assistant Professor and Associate Professor at the Basic Principles of Electrical Engineering and Power Energetics Department at the Technical University of Gabrovo. From 2008 to 2012 he headed the department. From 2012 to 2015 he worked at Mechatronica SC, Gabrovo, Bulgaria as the Head of the R&D Department. He is the author of 10 textbooks, more than 60 articles, and two inventions. His research interests include optimal design of electrical machinery, simulations and research of electromagnetic processes in electrical machines and apparatus based on the analysis of electromagnetic field through Finite Element Method and hysteresis modelling.