Optimal Sensor Spacing in Experimental Determination of Heat Transfer Coefficient by Inverse Analysis

F. Balkan* and M. H. Sezar
Department of Chemical Engineering, Ege University,
35100 Bornova, Izmir, Turkey
Tel: +90 232 388 76 00, Fax: +90 232 388 77 76;
*E-mail: firuz.balkan@ege.edu.tr

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It is shown that there exists an optimal spacing of thermo-sensors in the determination of the experimental heat transfer coefficient of a fluid flowing over a plate. The problem is considered as an inverse heat transfer problem with long thin fin model. The heat transfer coefficient of the fluid is estimated from simulated steady-state temperature measurements along the plate. It is shown theoretically that the inner product of the sensitivity vector, $\boldsymbol{J}^T\boldsymbol{J}$, should be maximum and the group $m\ n\ d$ should be equal to 1.692 to obtain the most accurate coefficients, where m is a system parameter containing heat transfer coefficient h, n is the number of thermo-sensors and d is the sensor spacing. These results are also verified by simulated experiments.

Key words:
Inverse heat transfer, parameter estimation, convective heat transfer coefficient

Introduction

Inverse heat transfer problems have become popular over the last three decades, especially through the development of computer hardware.[1–3] These problems, having been used in a broad area, can be classified as inverse heat conduction, inverse convection problems, and parameter estimation, function estimation, estimation of boundary conditions, and estimation of geometry of boundary of thermal systems. In direct heat transfer problems, the temperature field of a given thermal system with all known physical and thermal properties is determined by solving the mathematical model that describes the process using all the initial and/or boundary conditions of any kind. Whereas, in inverse heat transfer problems (IHTP), some unknown physical and thermal properties and/or some unknown boundary conditions of the system are estimated by processing the space-wise or transient temperature measurements taken in the system. In other words, a direct problem determines the "effect" from the "cause(s)" while an inverse problem estimates the "cause(s)" from the "effect". The "causes" to be estimated from temperature measurements are sometimes physical parameters such as heat transfer coefficient, thermal conductivity, constant wall heat flux, etc.; sometimes the parameters of the functions for the temperature-dependent physical properties, the space-wise or transient variations of wall heat flux or inlet fluid temperature, etc. Hence, an inverse problem may often be considered as a "parameter estimation problem". Due to experimental error that is inherent, the parameters can be determined only with some uncertainties; hence, the term "estimation" is used. Moreover, the parameters to be estimated may often be very sensitive to experimental errors. For this reason, inverse problems belong to a class called "ill-posed", while direct heat transfer problems are "well-posed". For a problem to be well-posed, its solution should satisfy the following three conditions introduced by *Hadamard*:[3] existence, uniqueness, and stability with respect to input data. Despite the ill-posed character, the solution of an inverse problem can be obtained through its reformulation in terms of a well-posed problem, such as a minimization problem associated with some kind of regularization (stabilization) technique. Different methods based on such an approach have been successfully used in the past for the estimation of parameter and functions. They include the Levenberg-Marquardt method of parameter estimation and the conjugate gradient method of parameter and function estimation.[4]

Sawaf and *Ozisik*[5] estimated temperature dependent thermal conductivity and heat capacity of an orthotropic medium using Levenberg-Marquardt method. *Huang* and *Yan*[6] used conjugate gradient method to estimate thermal conductivity and heat capacity simultaneously. *Huang*, *Ozisik* and *Sawaf*[7] estimated contact conductance during metal casting using conjugate gradient method. *Bokar* et al.[8] presented an inverse analysis for estimating the time-varying inlet temperature in laminar flow inside a parallel plate duct. In these and most of other works of inverse analysis, the parameter or parameters were estimated successfully using different optimization algorithms and the effect of experimental conditions -such as the number, standard deviation and the location of the thermo-sensors(s), spacing and/or time interval for temperature measurements- on the uncertainty of the estimated parameters were discussed.[1–8]

In this work, an inverse analysis is performed in the determination of convective heat transfer coefficient of a fluid in cross-flow over a thin long plate as shown in Fig. 1. The heat transfer coefficient, h, is considered as an unknown system parameter and it is determined from the simulated steady-state temperature measurements along the plate. This classical problem was discussed by *Beck*[2] using temperature responses of the long thin fin model. Actually, in the experiments the wall temperatures are measured, but average plate temperatures are used in the calculations. Thin fin approximation assumes that surface and averaged integrated temperatures are equal at all the points along the plate. Thus, plate temperatures do not vary across the plate wall. This model or the isothermal thin wall approach assumes zero Biot number and leads us to the ideal surface with zero thickness but finite thermal conductivity. Accuracy of thin fin approximation has been studied and it has been shown that the error of approximation is decreased as the Biot number decreases.[9] This model considerably simplifies the procedure of parameter estimation.

The objective of the work is obviously not to report the heat transfer coefficient estimations for this relatively simple system. Instead, it is to show the existence of a criterion for the best parameter estimation. It is clear that every system will have its own criterion. It is hoped that this paper will be a challenge in obtaining such criteria for more complex systems.

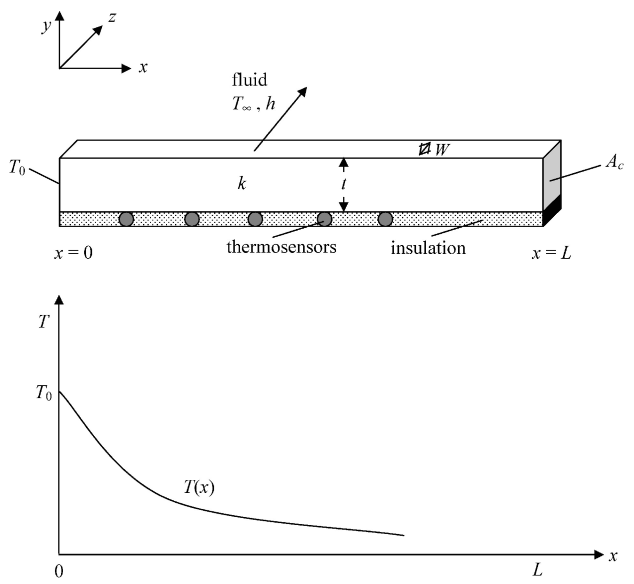

Fig. 1 – *Simulated experimental system and steady-state temperature profile along thin long plate (Bi << 1)*

In the classical experimental determination of heat transfer coefficients, the surface heat flux is measured, as well as the surface and bulk fluid temperatures. The heat transfer coefficient is then calculated from the classical Newton's law of cooling. The most important disadvantage of the classical technique is the difficulty in measuring the heat flux, due to inevitable heat losses. Whereas, the inverse technique used in this work does not need the measurement of heat flux. The heat transfer coefficient is determined by steady-state temperature measurements only. Since a system parameter is to be determined, the actual IHTP can be considered as a "parameter estimation" problem. The two important questions arising immediately are associated with the optimal spacing between thermo-sensors and with the uncertainties in the parameters to be determined. In this work, as a new topic of practical interest, a criterion for optimal spacing between sensors is developed to obtain the parameter with minimum uncertainty, and the "expected" parameter uncertainties are calculated for different experimental conditions. The theoretical results are then tested with simulated experiments.

The direct problem

The thermal system is shown in Fig. 1. A fluid at temperature T_∞ flows cross-wise in z-direction over the top surface of a long narrow thin plate of length L. All surfaces are insulated except the top surface which is subjected to convective cooling and the left surface is maintained at known constant temperature T_0 (say, by steam condensation). With the thin fin model, conduction is assured to be in x-direction only. Convection loss with a known uniform heat transfer coefficient h occurs in y-direction. Uniform h along x-direction is assured by choosing cross-flow rather than parallel-flow over the plate. The objective of the direct problem is to determine one-dimensional steady-state temperature distribution, $T = T(x)$ from the known coefficient h.

The steady-state energy balance for this system is given by the following equation:

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}x^2} - m^2\theta = 0 \tag{1.a}$$

where,

$$\theta = T - T_\infty \tag{1.b}$$

$$m = \sqrt{hP/kA_\mathrm{c}} \tag{1.c}$$

P is perimeter, k thermal conductivity, A_c cross-sectional area of the plate.

Boundary conditions are as follows:

$$1)\ x = 0;\quad \theta = \theta_0 = T_0 - T_\infty \tag{1.d}$$

$$2)\ x = L;\ \mathrm{d}\theta/\mathrm{d}x \approx 0 \text{ (long plate)} \tag{1.e}$$

The general solution of the above differential equation is

$$\theta = A\,\mathrm{e}^{-mx} + B\,\mathrm{e}^{mx} \tag{1.f}$$

The constants A and B are determined from the boundary conditions:

$$A = \theta_0/[1 + \exp(-2\,m\,L)] \tag{1.g}$$

$$B = \theta_0 \exp(-2\,m\,L)/[1 + \exp(-2\,m\,L)] \tag{1.h}$$

The inverse problem

For the inverse problem considered here, the parameter m of eq. (1.a) is regarded as an only unknown, when the root (left face) temperature of the plate, T_0 (or θ_0) is assumed to be known. The additional information obtained from steady-state temperature measurements (T_i) taken at n discrete points (x_i) along the plate is used to estimate the parameter m, from which the heat transfer coefficient h can be calculated later using eq. (1.c).

Simulated experimental temperatures

Obviously, it is inevitable to conduct real experiments to determine the values of parameter m. However, the temperature profile can be obtained by simulated temperature measurements as explained below, since the aim of this study is to find out a criterion for the optimal sensor spacing in determining the parameter with minimum uncertainty for fixed number of sensors, rather than determining the value of the parameter. Simulated temperatures at discrete points x_i (i = 1, 2, … n) are obtained by adding random errors, generated from a "random number generator" code, to the temperatures that are the solutions of the direct problem, as follows,[3]

$$Y_i = T_i \text{ (direct)} + \omega\,\sigma \tag{2}$$

where

Y_i – simulated temperatures containing random errors

T_i (direct) – exact temperatures of the direct problem

σ – standard deviation of the measurement errors

ω – random variable with normal distribution. For 99 % confidence level $-2.576 < \omega < 2.576$.

So, we have a vector of simulated temperatures,

$$Y = (Y_1,\ Y_2,\ \dots\ ,\ Y_n) \tag{3}$$

as well as a vector of temperatures calculated from the mathematical model for any m:

$$T(m) = (T_1,\ T_2,\ \dots\ ,\ T_n) \tag{4}$$

The estimation of the parameter m is based on the fitting of the measured temperature distribution along the plate to the theoretical model given by eq. (1.f). The elements of two vectors above are the temperatures at discrete coordinates $x_i = i\ d$, where d is equal spacing between temperature sensors. Estimation of the parameter is based on the minimization of the ordinary least square norm given by the functional,

$$S(m) = \sum_{i=1}^{n} [Y_i - T_i(m)]^2 \tag{5}$$

where

$S(m)$ – sum of squares of errors (or the objective function)

$Y_i = Y(x_i)$ – measured temperature at x_i

$T_i(m)$ – calculated temperature at x_i

Eq. (5) can be written in matrix form as

$$S(m) = [\boldsymbol{Y} - \boldsymbol{T}(m)]^T [\boldsymbol{Y} - \boldsymbol{T}(m)] \tag{6}$$

The parameter m, is evaluated as

$$\frac{\mathrm{d}S(m)}{\mathrm{d}m} = -2\left[\frac{\mathrm{d}T^T(m)}{\mathrm{d}m}\right][\boldsymbol{Y} - \boldsymbol{T}(m)] = 0 \tag{7}$$

Let m^* be the exact parameter value that could have been obtained if the measurements were errorless. Since the measurements are subject to the experimental errors, the parameters are inevitably determined with any uncertainties. Neglecting higher order terms, $\boldsymbol{T}(m)$ may be linearized in the vicinity of m^* as,

$$\boldsymbol{T}(m) = \boldsymbol{T}(m^*) + \boldsymbol{J}(m - m^*) \tag{8}$$

where $\boldsymbol{J} = \mathrm{d}\boldsymbol{T}(m^*)/\mathrm{d}m$ is the sensitivity matrix[3] (or, sensitivity vector for one parameter problems). Due to inevitable random errors in the measurements, an error vector $\boldsymbol{E}$ may be defined as follows,

$$\boldsymbol{E} = \boldsymbol{Y} - \boldsymbol{T}(m^*) = (e_1,\ e_2,\ e_3,\ \dots,\ e_n) \tag{9}$$

The components of the vector $\boldsymbol{E}$ are assumed to be described by Gaussian distribution and different components to be non-correlated:[3]

$$\langle e_i \rangle = 0 \quad \text{and} \quad \langle e_i\, e_j \rangle = \sigma^2\,\delta_{ij} \tag{10}$$

where δ_{ij} is Kronecker delta. From eq. (9),

$$\boldsymbol{Y} = \boldsymbol{E} + \boldsymbol{T}(m^*) \tag{11.a}$$

and using eq.(8),

$$\boldsymbol{Y} = \boldsymbol{E} + \boldsymbol{T}(m) - \boldsymbol{J}(m - m^*) \tag{11.b}$$

$$\boldsymbol{Y} - \boldsymbol{T}(m) = \boldsymbol{E} - \boldsymbol{J}(m - m^*) \tag{11.c}$$

Therefore, the objective function $S(m)$ in eq. (6) may be written as

$$S(m) = \boldsymbol{E}^T\boldsymbol{E} - 2\boldsymbol{J}^T\boldsymbol{E}(m - m^*) + \boldsymbol{J}^T\boldsymbol{J}(m - m^*)^2 \tag{12}$$

The derivative of $S(m)$ is set to zero to minimize it:

$$\frac{\mathrm{d}S(m)}{\mathrm{d}m} = -2\boldsymbol{J}^T\boldsymbol{E} + 2\boldsymbol{J}^T\boldsymbol{J}(m - m^*) = 0 \tag{13}$$

From the above equation, the uncertainty of the parameter m is obtained as

$$m - m^* = \frac{\boldsymbol{J}^T\boldsymbol{E}}{\boldsymbol{J}^T\boldsymbol{J}} \tag{14.a}$$

Now,

$$(m - m^*)^2 = \frac{(\boldsymbol{J}^T\boldsymbol{E})^2}{(\boldsymbol{J}^T\boldsymbol{J})^2} \tag{14.b}$$

$$(\boldsymbol{J}^T\boldsymbol{E})^2 = (\boldsymbol{J}_1 e_1 + \boldsymbol{J}_2 e_2 + \ldots + \boldsymbol{J}_n e_n)^2 \tag{14.c}$$

$$= \boldsymbol{J}_1^2 e_1^2 + \boldsymbol{J}_2^2 e_2^2 + \ldots + \boldsymbol{J}_n^2 e_n^2 + 2\sum_{i \neq j} \boldsymbol{J}_i e_i \boldsymbol{J}_j e_j \tag{14.d}$$

For single parameter problem, since the sensitivity matrix $\boldsymbol{J}$ is a vector, $\boldsymbol{J}^T\boldsymbol{J}$ becomes the inner product. Additionally, since the errors are assumed to be non-correlated ($\langle e_i\, e_j \rangle_{i \neq j} = 0$), eq. (14.b) reduces to

$$(m - m^*)^2 = \frac{\boldsymbol{J}_1^2 e_1^2 + \boldsymbol{J}_2^2 e_2^2 + \ldots + \boldsymbol{J}_n^2 e_n^2}{(\boldsymbol{J}_1^2 + \boldsymbol{J}_2^2 + \ldots + \boldsymbol{J}_n^2)^2} \tag{15.a}$$

From eq. (10), $e_1^2 = e_2^2 = \ldots = e_n^2 = \sigma^2$. Thus, we find

$$(m - m^*) = \frac{\sigma^2}{\boldsymbol{J}_1^2 + \boldsymbol{J}_2^2 + \ldots + \boldsymbol{J}_n^2} = \frac{\sigma^2}{\boldsymbol{J}^T\boldsymbol{J}} \tag{15.b}$$

and

$$\delta m = |m - m^*| = \frac{\sigma}{\sqrt{\boldsymbol{J}^T\boldsymbol{J}}} \tag{16}$$

It follows from eq. (16) that δm, the uncertainty of m, will be minimum in the experiment for which the inner product ($\boldsymbol{J}^T\boldsymbol{J}$) is maximum. The relative uncertainty of m for any experiment can now be estimated as follows,

$$\frac{\delta m}{m^*} = \frac{\sigma}{m^*\sqrt{\boldsymbol{J}^T\boldsymbol{J}}} = \frac{\sigma}{(m \pm \delta m)\sqrt{\boldsymbol{J}^T\boldsymbol{J}}} \approx \frac{\sigma}{m\sqrt{\boldsymbol{J}^T\boldsymbol{J}}} \tag{17}$$

As seen, to minimize the relative uncertainty in the experiments with n thermo-sensors, ($m^2\boldsymbol{J}^T\boldsymbol{J}$) should be maximized.

A criterion for optimal sensor spacing, d_{opt}

By definition, i-th component of the sensitivity vector is

$$\boldsymbol{J}_i = -Ax_i e^{-mx_i} + Bx_i e^{mx_i} \tag{18}$$

and

$$\boldsymbol{J}^T\boldsymbol{J} = \sum_{i=1}^{n} (-Ax_i e^{-mx_i} + Bx_i e^{mx_i})^2 \tag{19}$$

Using a discrete variable $\eta_i = mx_i$ defined by *Ilyinsky* et al.[10] and substituting the expressions for A and B into the above equation, we have

$$\begin{aligned} \boldsymbol{J}^T\boldsymbol{J} = {} & \frac{\theta_0^2 e^{-4mL}}{m^2(1 + e^{-2mL})^2} \sum_{i=1}^{n} \eta_i^2 e^{2\eta_i} - \\ & - \frac{2\theta_0^2 e^{-2mL}}{m^2(1 + e^{-2mL})^2} \sum_{i=1}^{n} \eta_i^2 + \\ & + \frac{\theta_0^2}{m^2(1 + e^{-mL})^2} \sum_{i=1}^{n} \eta_i^2 e^{-2\eta_i} \end{aligned} \tag{20}$$

$\boldsymbol{J}^T\boldsymbol{J}$ can be normalized as follows,

$$(\boldsymbol{J}^T\boldsymbol{J})^* = m^2\boldsymbol{J}^T\boldsymbol{J} \big/ (\theta_0 / (1 + e^{-2mL}))^2 \tag{21}$$

$$\begin{aligned} (\boldsymbol{J}^T\boldsymbol{J})^* = {} & \sum_{i=1}^{n} \eta_i^2 e^{-2\eta_i} - 2e^{-2mL} \sum_{i=1}^{n} \eta_i^2 + \\ & + e^{-4mL} \sum_{i=1}^{n} \eta_i^2 e^{2\eta_i} \end{aligned} \tag{22}$$

For large number of points the summations may be replaced by integrals, and defining a combined variable $z = m\,n\,d$, the summations in eq. (22) may be written in integral form as follows,

$$\begin{aligned} \sum_{i=1}^{n} \eta_i^2 e^{-2\eta_i} & = \frac{1}{md} \int_0^{mnd} \xi^2 e^{-2\xi} \mathrm{d}\xi = \\ & = \frac{n}{4z} [1 - (2z^2 + 2z + 1)e^{-2z}] \end{aligned} \tag{23.a}$$

$$\sum_{i=1}^{n}\eta_i^2 = \frac{1}{md}\int_0^{mnd}\xi^2 \mathrm{d}\xi = nz^2/3 \qquad (23.b)$$

$$\sum_{i=1}^{n}\eta_i^2 e^{2\eta_i} = \frac{1}{md}\int_0^{mnd}\xi^2 e^{2\xi}\mathrm{d}\xi = \qquad (23.c)$$

$$= \frac{n}{4z}[(2z^2-2z+1)e^{2z}-1]$$

Using eq. (21), the relative uncertainty of parameter, eq. (17), can be rewritten as

$$\frac{\delta m}{m^*} \approx \frac{\sigma}{m\sqrt{\boldsymbol{J}^T\boldsymbol{J}}} = \frac{\sigma}{\sqrt{(\boldsymbol{J}^T\boldsymbol{J})^*}}\frac{1+e^{-2mL}}{\theta_0} \qquad (24.a)$$

For $m\ L$ values not being so small (around 10 in this work), exponential term can be ignored and eq. (24.*a*) simplifies to

$$\frac{\delta m}{m^*} \approx \frac{\sigma}{\theta_0\sqrt{(\boldsymbol{J}^T\boldsymbol{J})^*}} \qquad (24.b)$$

or,

$$\delta m \approx \frac{\sigma\, m^*}{\theta_0\sqrt{(\boldsymbol{J}^T\boldsymbol{J})^*}} \qquad (24.c)$$

It follows from eqs. (24.b-c) that, for a given fluid flow system (namely m^* is fixed), for fixed base temperature difference, θ_0, and standard deviation of thermo-sensor, σ, the parameter uncertainties become minimum when $(\boldsymbol{J}^T\boldsymbol{J})^*$ is maximum. The variation of $(\boldsymbol{J}^T\boldsymbol{J})^*$ with z for various values of $m\ L$ are shown in Fig. 2. It follows from the figure that $(\boldsymbol{J}^T\boldsymbol{J})^*$ does not have a global maximum but local maxima for $m\ L < 5$. For $m\ L \geq 5$ however, the graphs overlap and posses unique global maximum at a unique z value. This value is found analytically to be $z = m\ n\ d_{\mathrm{opt}} = 1.692$. This value is independent of standard deviation of temperature measurements and it can be used as a criterion for optimum spacing for the best estimation of the parameter.

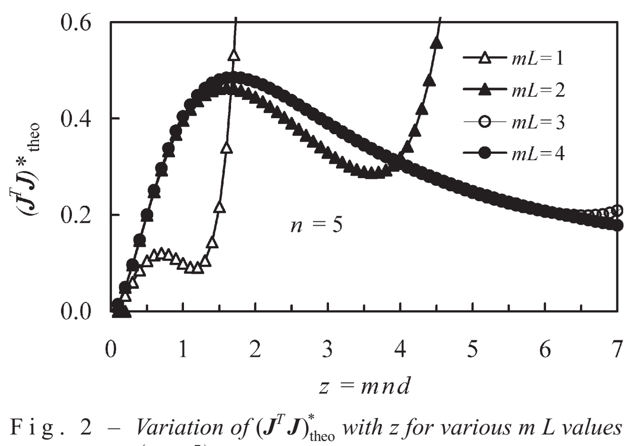

Fig. 2 – *Variation of $(\boldsymbol{J}^T\boldsymbol{J})^*_{\mathrm{theo}}$ with z for various m L values (n = 5)*

Simulated experiments and verification of the criterion

A number of simulated experiments were conducted to verify the theoretical result for the optimum spacing. In the experiments two different numbers of sensors (n = 5 and 10) and various sensor spacings were used to show the variation of $(\boldsymbol{J}^T\boldsymbol{J})^*$ with z for two standard deviation values (σ = 0.01 and 0.10). The value of parameter m in each experiment was determined by Levenberg-Marquardt method of minimization. Any experiment was repeated 50 times under the same conditions and the calculated values were averaged. The averaged results are given in Figs. 3 and 4. In all the experiments the exact value of m and L were taken as 10 m^{-1} and 1 m, respectively, so that the condition $m\ L \geq 5$ is justified.

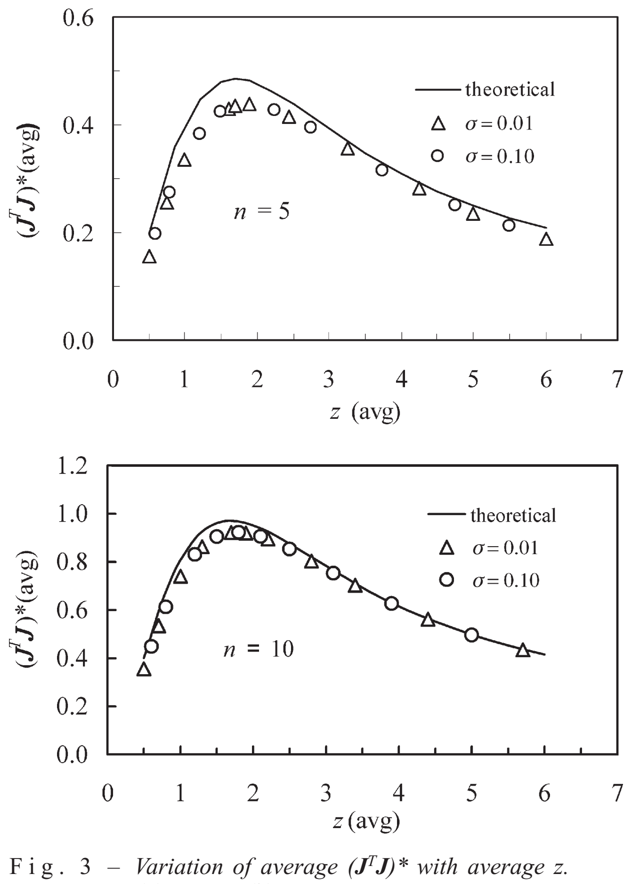

Fig. 3 – *Variation of average $(\boldsymbol{J}^T\boldsymbol{J})^*$ with average z. (a) n = 5, (b) n = 10*

The validity of 1-D approximation in the simulated experiments

The Biot number for a long plate of thickness t insulated at one-side is defined as:

$$Bi = \frac{h\,t}{k} \qquad (25.a)$$

From eq. (1.c), h can be written as,

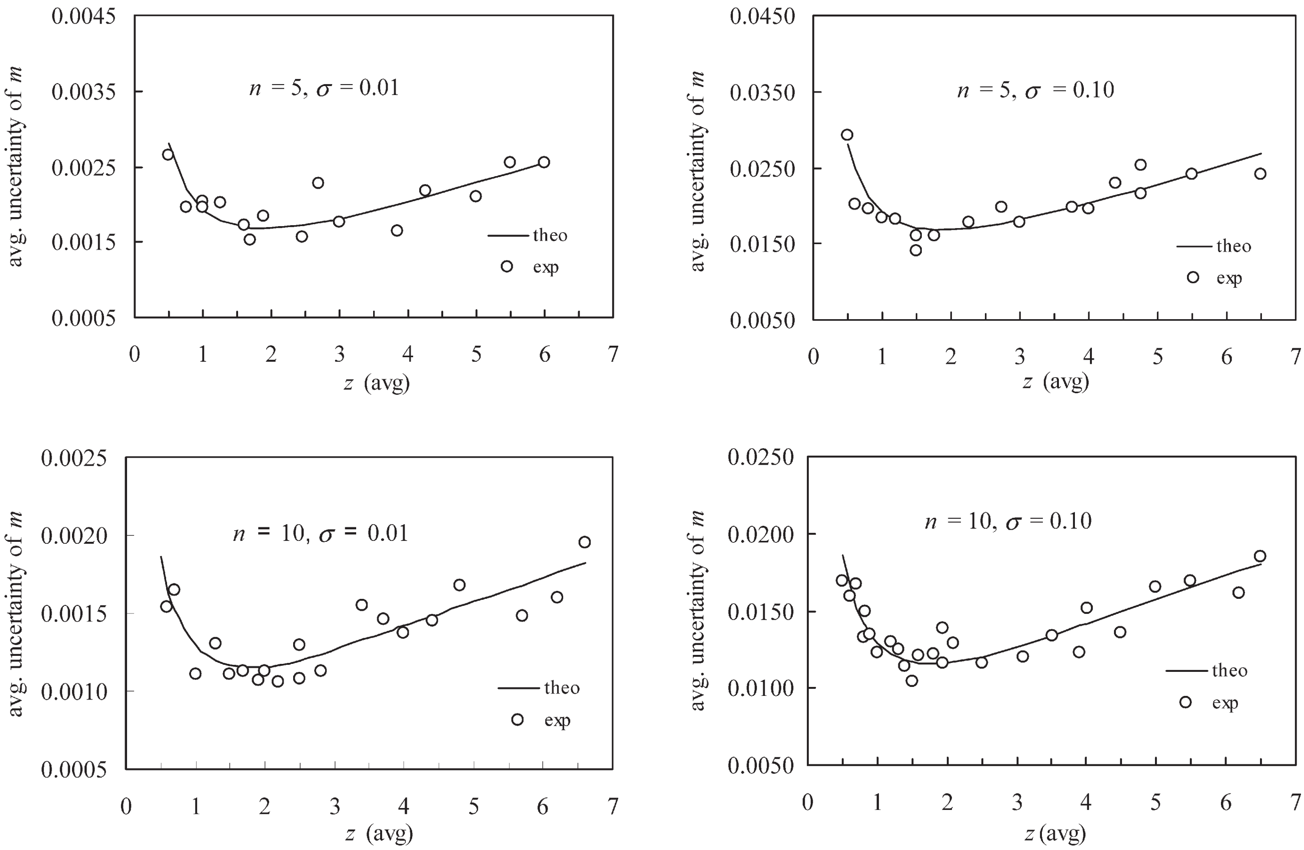

Fig. 4 – *Variation of average uncertainty of m with average z. (a) n = 5, σ = 0.01, (b) n = 5, σ = 0.10, (c) n = 10, σ = 0.01, (d) n = 10, σ = 0.10*

$$h = m^2 \frac{kA_c}{P}$$

The cross-sectional area and the perimeter of the plate (see Fig. 1) are $A_c = W\,t$, $P = W + 2\,t$.

So,

$$h = m^2 \frac{kWt}{W + 2t} \approx m^2 kt \quad (\text{Since } t << W) \quad (25.\text{b})$$

Biot number can then be simplified as follows,

$$Bi = \frac{ht}{k} \approx \frac{(m^2 kt)t}{k} = (mt)^2 \qquad (25.\text{c})$$

m was taken as 10 during the simulated experiments; taking, for example, 2 mm for plate thickness, t, then Biot is calculated as

$$Bi \approx (mt)^2 = (10 \cdot 2 \cdot 10^{-3})^2 = 0.0004 < 0.1.$$

So, 1-D approximation is also justified.

Results and discussion

First of all, as seen in Figs. 3–4, the standard deviation of temperature measurements, σ, has no effect on experimental $(\boldsymbol{J}^T\boldsymbol{J})^*$ values, as expected from the theory. Although experimental $(\boldsymbol{J}^T\boldsymbol{J})^*$ values are lower than theoretical ones, as n increases from 5 to 10, experimental values approach theoretical ones. For example, the ratio $(\boldsymbol{J}^T\boldsymbol{J})^*_{\text{exp}} / (\boldsymbol{J}^T\boldsymbol{J})^*_{\text{theo}}$ is 0.779 for $z = 0.5$ when $n = 5$; this ratio increases to 0.887 for $z = 0.5$ when $n = 10$. It is no surprise since we assumed large number of sensors in replacing the summations by the integrals in the theoretical part of the study.

The maximum $(\boldsymbol{J}^T\boldsymbol{J})^*_{\text{exp}}$ occurs at $z = 1.900$ for $\sigma = 0.01$ and at $z = 1.899$ for $\sigma = 0.1$ when 5 sensors are used; while z value is 1.700 for both standard deviations when 10 sensors are used. These z values, especially 1.700, are very close to 1.692 found theoretically for infinite number of sensors.

According to eq. (24.c), the uncertainty of the parameter, δm, will be minimum when $(\boldsymbol{J}^T\boldsymbol{J})^*$ is maximum. Thus, δm is expected to be minimum at $z = m\ n\ d = 1.692$ for any σ, theoretically. This is verified by the simulated experiments as shown in Figs. 4(a)-(d). These figures also show that the uncertainty is almost linearly proportional to σ, as in eq. (16).

The result that $z_{\text{opt}} = m\ n\ d_{\text{opt}} = 1.692$, may be used as a criterion in determining heat transfer coefficients from the real temperature measurements. In the lack of a prior information on h (namely, m), a few number of experiments are conducted with some trial sensor spacings. Calculated z values are then compared with z_{opt}. Comparisons lead us to the optimum spacing. Obviously, less effort is required when one has prior information on h. For example, if m is expected to be in the range of 10–20, and if the maximum number of sensors which can be placed practically is 10; optimum d will be in the range of 16.92–8.46 mm.

Due to the exponential characteristic of the temperature distribution, the "weak information" (nearly parallel temperature profile) near the end of the plate will lead us to the diminishing in accuracy of the parameter due to the increasing negative effect of the experimental errors. So, there must be an optimum fraction of the plate length to take all the measurements. The above criterion may also be used to determine this fraction as follows:

Interpreting ($n\ d_{opt}$) as the measured length, L_{meas}, the criterion can be written as, $m\ L_{meas} = 1.692$. We know also that $m\ L \geq 5$. Dividing both expressions side by side, we obtain $L_{meas}/L \leq 0.338$. This means that n sensors with equal spacing should be placed from the left side up to at most 33.8 % of the plate length.

Conclusions

In this study, it is shown that a criterion for optimum sensor spacing exists in the experimental determination of heat transfer coefficient of a fluid in cross-flow over a thin long plate by an inverse analysis. The steady-state temperatures along the plate are simulated by adding random experimental noises to the temperatures of the direct problem. The inverse problem is solved and the system parameter m that contains the coefficient h is determined using Levenberg-Marquardt method of minimization. The simulated experiments are repeated with two different numbers and standard deviations of temperature sensors. After theoretical work, the condition z = $m\ n\ d = 1.692$ is found as a criterion, and it is verified by the simulated experiments. The criterion assures the best coefficient value that can be determined experimentally for any number and quality of temperature sensors, although the accuracy in the coefficient value will increase obviously with increasing number and quality of sensors used. It is planned to extend the present study to more complicated systems with more than one parameter.

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* Finance Academy, Math Modeling Department, Leningradski prospect, 49, Moscow 125468, Russian Federation, Russia.

List of Symbols

A_c – cross-section area of plate, m^2
d – spacing between temperature sensors, m
e_i – random measurement error
$\boldsymbol{E}$ – error vector
h – convective heat transfer coefficient, W m^{-2} K^{-1}
$\boldsymbol{J}$ – sensitivity matrix (or vector)
k – thermal conductivity of plate material, W m^{-1} K^{-1}
L – length of plate, m
m – system parameter, $\sqrt{hP/kA_c}$, m^{-1}
m^* – exact parameter value, m^{-1}
n – number of sensors
P – plate perimeter, m
S – sum of squares of errors
T_i – calculated temperature, °C
x, x_i – conductive heat transfer direction, discrete coordinate for temperature measurements, m
y – convective heat transfer direction
Y_i – measured temperature (by simulation), °C
z – flow direction, combined variable, $m\ n\ d$, –

Greek symbols

δm – uncertainty of parameter m
δ_{ij} – Kronecker delta
η_i – discrete variable, mx_i
ω – random variable with normal distribution. For 99 % confidence level $-2.576 < \omega < 2.576$.
σ – standard deviation of the measurement errors
θ – excess temperature, $T - T_\infty$, °C
ξ – dummy variable

Subscripts

∞ – bulk fluid conditions
0 – root (left side) of the plate
i – discrete value for x, T and Y
meas – measured
opt – optimum

Superscripts

T – Transpose

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