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LUMPED PARAMETER MODEL WITH INNER STATE VARIABLES FOR MODELING HOT DEFORMABILITY OF STEELS

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A novel method for description of stress-strain relationship for hot deformability of steels is presented. Laplacian transformation of stress-strain data obtained on cylindrical hot compression tests offer simple description of dynamical input/output relationship between strain (input) and stress (output). In this paper, strain/stress relationship is described using transfer function of the third order. Parameters of transfer function are determined by numerical optimization for temperatures 1 000, 1 100 and 1 200 °C and logarithmic strain rates 10, 5 and 1 s^{-1}. Obtained relative model error is around 1 % for logarithmic deformations in the range $0{,}1 < \varphi < 0{,}8$.

Key words: Alloyed steel, Hot deformability, stress-strain curves, mathematical model, Laplacian transformation

INTRODUCTION

Experimentally obtained stress-strain curves on cylindrical hot compression tests describe hot deformability of test material. Dense data obtained by hot compression test enable a wide range of possibilities for mathematical model determination: identification of known model's structure parameters using least-squares, numerical optimization of parameters using criterion function, etc.. Mathematical models for prediction of stress-strain curves are of great interest for industry and have been studied for decades, since a lot of hot working process data can be derived from tests: peak or steady state stress [1, 2], whole flow curves or processing maps [3-8], the initiation of Dynamic Recrystallization (DRX) [3-4, 9-12], grain size prediction [13] etc.. Many fragmented models for different purposes are reported in literature.

To overcome the partial and fragmented models for the description or prediction of only some variables of the same dynamical system, a wider context should be taken into account describing whole hot deformation curves (φ, σ) at various strain rates and temperatures ($\dot{\varphi}$, ϑ). One of the first and most coarse model describing the whole curve is Hajduk's equation [8]. Johnsons-Cook model is used by He et al. [5] and models accuracies seems slightly better compared to Hajduk. Later on, whole curve models are described using Zener-Hollomon parameter with hyperbolic sine stress dependence [3,4,6]. Modified approach using dislocation density as single inner state variable yields better model accuracy compared to hyperbolic sine stress dependence [1]. Two reviews on models describing hot deformability are suggested [2,13].

Approach of Alberto et al. [1] and Chavy et al. [6] demonstrated that modelling based on solving differential equations results in significantly more accurate flow curves description compared to static dependencies without inner state variables (Zener-Hollomon parameter, Hajduk's equation). Although Alberto et al. [1] proposed simple and analytically solvable first order system, where inner variable is dislocation density, the model yields better accuracy compared to stateless models. For model's parameter fitting, Alberto et al. [1] used numerical optimization.

An additional reflection on currently used models describing flow curves should be done. Sub-process dynamics during hot deformability, such as dislocation density, static and dynamic recrystallization, grain size, etc., are dynamic processes interacting among each other and their dynamics varies with temperature and strain rate. In other words, hot deformability is a dynamic process and stress-strain relationship should therefore be considered as input/output relationship of the dynamical system with several internal states, where most probable system parameters for given steel composition are function of strain rate and temperature ($\dot{\varphi}$, ϑ).

The most frequently used and useful tool for mathematical descriptions of dynamic systems are Differential Equations (DE). Numerical methods for solving DE are either time based (state space) or frequency based. The latter is a Laplacian transformation, which is an integral transformation and is used for Linear Time Invariable (LTI) systems. One benefit of using Laplacian transformation is simple description of input/output relationship of dynamic systems using transfer function. Besides this, Laplacian transformation has been used for analysis of analog circuit electronics and control systems for decades, leaving behind large amounts of methods and software for analysis, engineering, stability testing etc..

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In this paper, whole hot deformation flow curves (φ,σ) obtained by model based on Hajduk's equation [8] and newly proposed model using transfer function are compared. The newly proposed model is of lumped parameter type and uses inner state variables based on realization of transfer function. Parameters of both models are determined by numerical optimization.

MATERIALS AND METHOD

Hot compression tests at temperatures 1 000, 1 100 and 1 200 °C and logarithmic strain rates 10, 5 and 1 s^{-1}. The specimens were held at the deformation temperature for 10 minutes and then deformed. Were carried out in TA Instruments 805 D/A deformation dilatometer; the samples were cylindrical with a 5 mm diameter and 10 mm long, made from 50CrV4 steel.

One of the simplest models is Hajduk's equation, which enables direct calculation of stress σ for given strain, strain rate and temperature in a range of deformation ε between 0,1 < φ < 0,8. The equation is.

$$\sigma_p(X) = k_{f0} * A_\varepsilon \varphi^n * e^{-m_\varphi \varphi} * A_{\dot{\varphi}} \varphi^{m_{\dot{\varphi}}} * A_\vartheta \varphi^{-m_\vartheta \vartheta}$$

Model parameters X are $(k_{f0}; A_\varphi; n; m_\varphi; A_{\dot{\varphi}}; m_{\dot{\varphi}}; A_\vartheta; m_\vartheta)$ are for experimental steel found at (30,3; 0,338; 25,4; 1,14; 5,33; 0,129; 5,30; 0,0041). Comparison of measured and calculated stress and relative error between them for 1 000 °C is shown in Figure 1. For temperatures 1 100 and 1 200 °C relative error is shown in Figure 2. The main reason for poor prediction accuracy of Hajduk's equation is, that inner states (grain size, static and dynamic recrystallization, etc.) are not considered.

To apply Laplacian transformation on stress-strain curve, variable *t* of Laplacian transform (which is usually time) represents logarithmic deformation φ. Since Laplacian transformation is used for time *t* (in our case φ) invariable system, partial models are determined for each experimental strain rate and temperature pair (φ̇, ϑ). Note that, hot pressure test experiments as a result offer a continuous curve in space (φ,σ). Transfer function *H(s)* of third order is employed in this case:

$$H(s) = \frac{as^3 + bs^2 + cs + d}{es^3 + fs^2 + gs + h} \qquad (1)$$

Model parameters are coefficients of transfer function polynomials *(a,..,h)* + *initial states X* = (a, b, c, d, e, f, g, h, x_0, x_1, x_2) and are again determined by numerical optimization technique as described and used by Alberto et al. [1].

RESULTS AND DISCUSSION

Values of the obtained parameters for *H(s)* are shown in Table 1. Agreement of measured and calculated stress-strain curves using (1) for ϑ = 1 000 °C is shown in Figures 3 - upper, and relative error Figure 3 - lower. In Figure 4, relative error for ϑ = 1 100 °C – upper and ϑ = 1 200 °C lower.

Table 1 **Values of obtained H(s) parameters for strain rates and temperature pairs ($\dot{\varphi}$, ϑ)**

ϑ	1 000	1 000	1 000
$\dot{\varphi}$	10	5	1
a	3,82	3,51	532
b	433	451	4 105
c	3 288	3 399	39 306
d	0,453	0,497	6,64
e	4,02	4,47	27,9
f	16,6	17,9	282
g	0,0730	0,0064	0,0524
h	1,55·10-5	0,0110	4,42
x_0	0,124	0,1256	0,0917
x_1	0	0	0
x_2	0	0	0
ϑ	1 100	1 100	1 100
$\dot{\varphi}$	10	5	1
a	1,07	1,01	212
b	30,6	39,2	2 867
c	174	280	2 002
d	0,044	0,064	5,35
e	0,347	0,568	34,4
f	1,19	2,28	188
g	0,000947	0,000878	0,0542
h	0,017	0,012	2,20
x_0	0,0153	0,128	0,119
x_1	0	0	0
x_2	0	0	0
ϑ	1 200	1 200	1 200
$\dot{\varphi}$	10	5	1
a	41,3	37,7	641
b	278	234	9 769
c	2 096	2 489	48 869
d	0,667	0,597	18.5
e	3,44	3,23	143
f	26,5	32,5	835
g	0,0152	0,0136	0,18
h	1,087	1,025	$2{,}2 \cdot 10^{-14}$
x_0	0,0961	0,0864	$2{,}3 \cdot 10^{-14}$
x_1	0	0	0
x_2	0	0	0

Selection of the third order Transfer Function (TF) is a result of several trial and error experiments by changing the order of *H(s)* polynomials and determination of *H(s)* parameters using numerical optimization. The first and second order TF yields in model accuracies around 5 % and 2 % using the same methodology as described for used third order *H(s)* where accuracies are around 1 %. Additional reason for using third order TF is fact, that *n*-th order TF has *n* internal states. At least stress σ, dislocation density ρ and grain size are three internal states of hot deformability process and therefore 3rd order transfer function is used.

Applying the Laplacian transformation on (φ̇, σ) curve proved as a successful tool for an accurate description of the stress-strain curves of hot compression tests in wide range of strains. A detailed inspection of the obtained deviations between the experimental results and model obtained stresses shows that higher deviations occur for slower and colder tests (φ̇, ϑ). This fact points to some structurally unsuitable inter-state

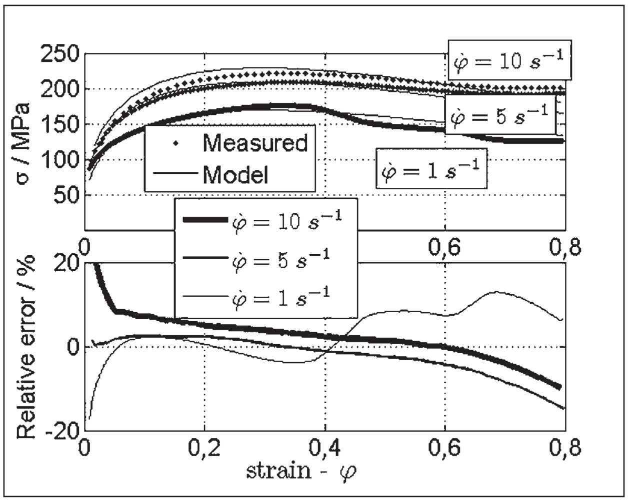

Figure 1 Comparison of experimentally obtained and calculated stress-strain curves using Hajduk's equations for 1 000 °C (upper) and relative error (lower)

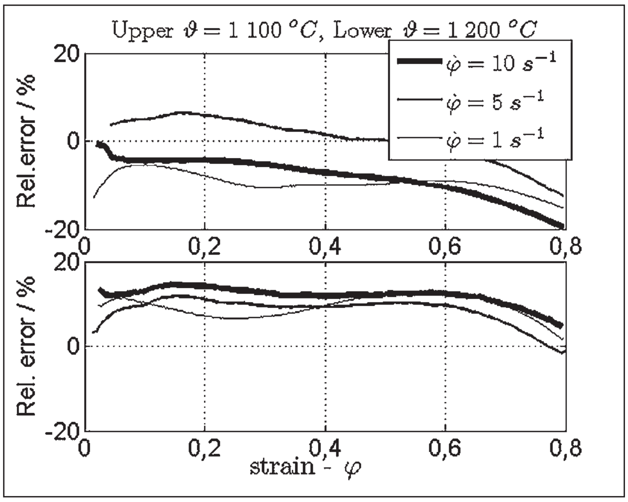

Figure 2 Relative error of Hajduk's equation for 1 100 °C and 1 200 °C for strain rates $\dot{\varphi}$ = 10, 5 and 1 s-1

connections of the model. Unsuitable model part is related to dynamic recrystallization phenomena description, which arise and dominate at slower and colder tests.

One benefit of the proposed method is that initial states of transfer function can be used, although it is not fully clear what they represent. Disadvantage of the proposed method is that model's parameters are to be determined for each experimental strain rate and temperature pair $(\dot{\varphi}, \vartheta)$. Since the method is of lumped parameter type, fast enough computations are possible for real-time applications.

CONCLUSIONS

Stress-strain curves prediction using Laplacian transformation is a novel approach. The same model structure covers the whole tested range of process variables $(\varphi, \dot{\varphi}, \vartheta)$, but model parameters change by process variables $(\dot{\varphi}, \vartheta)$. Obtained model accuracy in the whole range is around 1 %. Process variables at the greatest

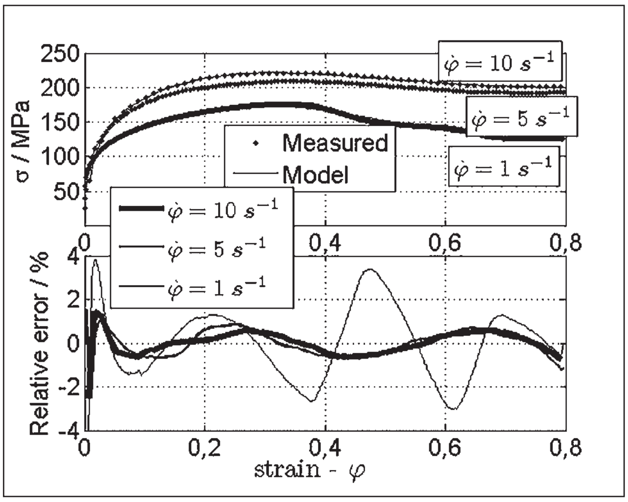

Figure 3 Comparison of experimentally obtained and calculated stress-strain curves at 1 000 °C for strain rates $\dot{\varphi}$ = 10, 5 and 1 s-1 and relative error (lower).

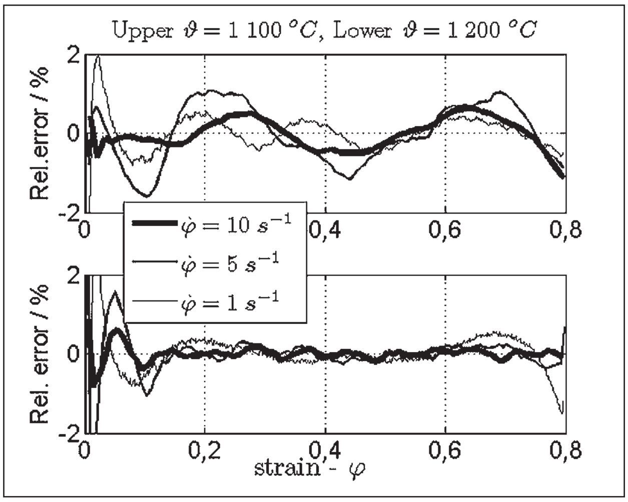

Figure 4 Relative error of H(s) transfer function for 1 100 °C and 1 200 °C for strain rates $\dot{\varphi}$ = 10, 5 and 1 s-1

deviations between measured and modeled strains suggest that model has still some structural unsuitable parts, which are probably related to dynamic recrystallization. Obtained models accuracy shows, that the technique is promising. However, selection of plain transfer is not suitable for description of internal states. Future effort should therefore be directed into defining multiple input – multiple output dynamical system, where inputs, outputs and internal states will represent known physical phenomena (dislocation density, grain size, stress,…) and solving it using tools for description of dynamical systems (time or frequency based). Numerical optimization technique has however proved as effective and accurate to fit model's parameters to experimentally obtained data.

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