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EXPRESSION OF UNCERTAINTY IN MEASUREMENTS OF ACTIVE POWER BY MEANS OF ELECTRODYNAMIC WATTMETER

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Original scientific paper
Application of the measurement uncertainty approach in expression of the measurement result of active power single-phased consumer is presented in this paper. The measurement of active power is carried out by means of electrodynamic wattmeter, which is in direct connection with the consumer. In this paper it is assumed that the results of the active power measurement by means of electrodynamic wattmeter are quite evenly spread between the highest and the lowest values of active power, i.e. rectangular or uniform distribution of measurement results is assumed.

Keywords: *direct connection, electrodynamic wattmeter, measurement uncertainty, rectangular distribution, true value*

Iskazivanje nesigurnosti kod mjerenja djelatne snage pomoću elektrodinamskog vatmetra

Izvorni znanstveni članak
U ovom radu prikazana je primjena koncepta mjerne nesigurnosti kod iskazivanja mjernog rezultata djelatne snage jednofaznog potrošača. Mjerenje djelatne snage izvodi se pomoću elektrodinamskog vatmetra, koji je u izravnom spoju s potrošačem. U ovom radu je pretpostavljeno da su rezultati mjerenja djelatne snage pomoću elektrodinamskog vatmetra jednoliko raspoređeni između najniže i najviše vrijednosti djelatne snage, odnosno pretpostavljena je pravokutna ili jednolika razdioba rezultata mjerenja djelatne snage.

Ključne riječi: *elektrodinamski vatmetar, izravni spoj, mjerna nesigurnost, prava vrijednost, pravokutna raspodjela*

1 Introduction

The measurement of the active power P (also called real power) is of great importance in power engineering, especially in area of the designing, testing, monitoring, and maintenance of the energy supply networks and of electrical equipment. For the measurement of the active power in AC and DC circuits there is a wide variety of instruments and transducers with recent trends from analogue toward digital wattmeters. Despite these trends, the indicating (deflecting) electrodynamic wattmeter is probably the most common used instrument in the measurement of the active power at mains frequency. Thus, this instrument will be used in this analysis of the results measurements of the active power.

Up until recently, the error approach (sometimes called *traditional approach* or *true value approach*) has been used in presenting results of active power measurements by means of electrodynamic wattmeter in almost all scientific and professional papers or text books, as for example [1÷7]. The error approach has long been a part of the practice of metrology, and thus it is understandable why it has been used in almost all scientific papers dealing with analysis of the results of the active power measurement. The error approach implies the belief that the measurand has a unique (true) value, which is by nature indeterminate and thus an error of measurement, which can be considered as deviation of the result of measurement from the true value of the measurable quantity (expressed in absolute or relative form), cannot be determined exactly, either. Since the true value is unknown, total error cannot be known and therefore only its expected value can be estimated. Both the true value and error (random and systematic) are abstract concepts. Their exact values cannot be determined. However, these concepts are nevertheless useful, because their estimates can be determined. In fact, as said above, measured value is an estimate of the true value. Thus, the error approach is valid as an idealized concept.

However, it is nowadays widely recognized, that when all of the known or suspected components of error have been evaluated and the appropriate corrections applied, there still remains uncertainty about the correctness of the stated result, i.e., a doubt about the quality of the result of the measurement. Thus, there is nowadays common practice in metrological publications, research, laboratories and industry, that results of measurements are expressed by means of the measurement uncertainty approach, which was introduced in the late 80's as a quantifiable attribute of a measurement, able to assess the quality of measurement process and result. Uncertainty of measurement is defined in *Guide to the Expression of Uncertainty in Measurement*- known by the acronym GUM, as a parameter associated with the result of a measurement that characterizes the dispersion of the values that could reasonably be attributed to the measurand, [8]. The parameter may be for example, a standard deviation called standard measurement uncertainty (or a specified multiple of it), or the half-with of an interval, having a stated coverage probability. In [8] are given general rules for evaluating and expressing uncertainty in measurement that can be followed at various levels of accuracy and in many fields. According to [8], the main requirement on the method for evaluating and expressing the uncertainty of a measurement is that it should be capable to provide an interval (a confidence interval) about the best estimated value of the measurement, within which may be expected to lie the values (that could reasonably be attributed to the measurand) with a given level of confidence. Measurement uncertainty is always associated with some probability. In Fig. 1 are illustrated interrelations between terms for true value, measured value P_M, absolute error ΔP and uncertainty range for a measurement of active power P.

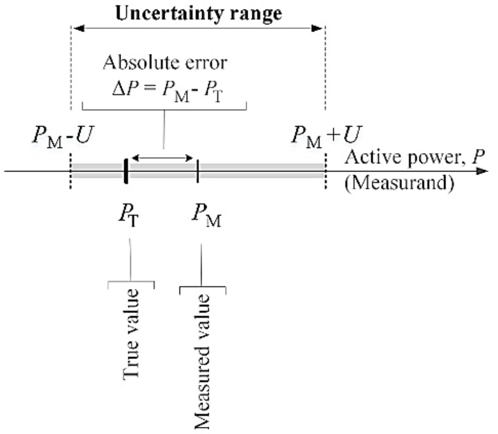


Figure 1 Interrelations between the terms: true value, measured value, absolute error and uncertainty range

According to abovementioned, intention in this paper is to present the concept of measurement uncertainty in expressing of the results of the active power measurements of the single-phase load by means of electrodynamic wattmeter, which does not have a compensating winding. In other words, the focus in this paper is on the active power measurement result in mains frequency circuit and its evaluated uncertainty rather than on the unknowable quantities "true" value of active power and measurement error. The electrodynamic wattmeter is used in this analysis, because it is the most often used electromechanical instrument for the measurement of DC and AC power. Intention in this paper is to illustrate that the measurement uncertainty approach, being based on practical concepts, may be more rational and modern than the error approach.

The evaluation of uncertainty in measurement of active power of the single-phase load using electrodynamic wattmeter is made in accordance with the GUM (*Guide to the Expression of Uncertainty in Measurement*), [8].The measurement of active power by means of electrodynamic wattmeter is a direct one. An electrodynamic wattmeter properly connected in the circuit gives direct indication of the active power being measured.

This paper gives a full description of the measurand P and of the systematic effects which affect wattmeter reading, as well as their corrections and then description of estimation of the active power measurement uncertainty using electrodynamic wattmeter.

Some more important system effects which can influence the accuracy of measurement result are recognised and quantified in this paper, too. Thus, corrections and correction factors have been applied to compensate for the following systematic effects:

- due to inductance of the wattmeter voltage coil, which causes that the current through it is not in phase with its voltage,
- due to power losses in the current or in the voltage coil of the wattmeter, depending on their method of connection.
- due to power losses in the connected ammeter and voltmeter.

After compensating for these systematic effects, the measurement uncertainty depends only on wattmeter accuracy class.

Generally, wattmeters can be connected directly, semi-directly, and indirectly. Wattmeters, connected directly, are used for the measurement of small power. Extension of the range of electrodynamic wattmeter can be carried out using current shunt and resistive voltage multipliers. The voltage measurement range of up to 750 V is extended by serially added resistors. For higher voltage and current ranges, instrument transformers must be used. Semi-direct connection of the wattmeter is used in low voltage applications. Indirect connection of the wattmeter is used in high voltage systems.

In the case of electrodynamic wattmeter utilisation, the measurement range is limited by its current and voltage range. Instrument current transformer (CT) with suitable transformation factor must be used for measurements where the current going through the load is higher than the current measurement range of the wattmeter.

In this paper will be analysed measurement uncertainty only for the case of direct connection of electrodynamic wattmeter in alternating electrical circuit with single-phase load. In the measurement practice, for the steady-state operation of single-phase electrical system with linear load supplied with sinusoidal voltage, the overwhelming majority of the active power measurements using electrodynamic wattmeter are single measurements, and only such kind of active power measurements will be analysed in this paper, too.

2 Theoretical background

The currently used definition of the active power is well established and undoubted for the steady-state operation of a single-phase electrical circuit with linear load supplied with sinusoidal voltage. According to the definition, the active power P is the average value (DC component) over one period T of the instantaneous power p, that is

$$P = \frac{1}{T}\int_0^T p(t)\,\mathrm{d}t = \frac{1}{T}\int_0^T u(t)\cdot i(t)\,\mathrm{d}t \tag{1}$$

where $u(t)$ and $i(t)$ are instantaneous voltage and current values, respectively.

If a linear load is supplied with a sinusoidal voltage in a single-phase circuit

$$u(t) = U\sqrt{2}\,\sin\left(\omega t + \varphi_\mathrm{u}\right) \tag{2}$$

and the source current value is

$$i(t) = I\sqrt{2}\,\sin\left(\omega t + \varphi_\mathrm{i}\right), \tag{3}$$

then inserting Eq. (2) and Eq. (3) in Eq. (1) gives well known expression for the active (real) power:

$$P = P_\mathrm{T} = U\cdot I\cdot\cos\varphi \tag{4}$$

where: U - effective (RMS) values of supply voltage (V), I - effective (RMS) values of load current (A), $\varphi = \varphi_u - \varphi_i$ - phase angle between supply voltage and load current, and cos φ - power factor.

Eq. (4) defines true value of the single-phase active power and its exact value cannot be determined by means of this active power measurement method, because the wattmeter cannot simultaneously measure the exact load voltage and load current.

3 Active power measurement using direct-indicating electrodynamic wattmeter

The measurement of the active power by means of electrodynamic wattmeter can be regarded as a direct one, because this power can be read directly from the indications of the wattmeter [2] and it can be calculated by means of the following equation:

$$P_{\mathrm{W}} = k_{\mathrm{W}} \cdot \alpha_{\mathrm{W}} \tag{5}$$

where: k_{W} – wattmeter constant (W/division), α_{W} - wattmeter deflection (division).

The operation of the electrodynamic wattmeter is based on the interaction of the magnetic fields of the movable coil (connected through a large auxiliary resistor parallel to the load) and fixed coil (connected in series to a load) as an electrical current passes through them. The electrodynamometer movement has a deflection proportional to the active (average) power dissipated in the load.

According to [7], it can easily be shown that the wattmeter reading (measured active power) P_{W} is given by:

$$P_{\mathrm{W}} = R_2 \cdot I_1 \cdot I_2 \cdot \cos\delta \tag{6}$$

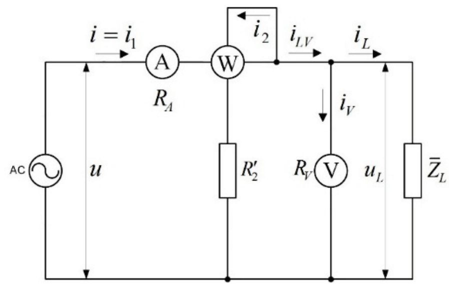
where: I_1 - the rms value of the current flowing through the current coil of the wattmeter (A), I_2 - the rms value of the current flowing through the voltage coil of the wattmeter (A), δ - the phase angle between $\underline{I}_1$ and $\underline{I}_2$ (°), R_2 - the sum of the resistance of the voltage coil of the wattmeter $R_{2\mathrm{S}}$ and the non-inductive high resistance R_2' connected in the series with the voltage coil (Ω).

This wattmeter reading P_{W} should be corrected by means of corrections factors, which are determined on the basis of the above mentioned systematic effects depending on wattmeter connections and on power factor cosφ. Wattmeter actual reading is slightly higher than the power in the load due to power losses in the current or in the voltage coils, depending on the method of their connection. Due to the inductance of the voltage coil, the current through it is not in phase with its voltage. This effect influences the wattmeter actual reading, too. These effects will be analysed in detail.

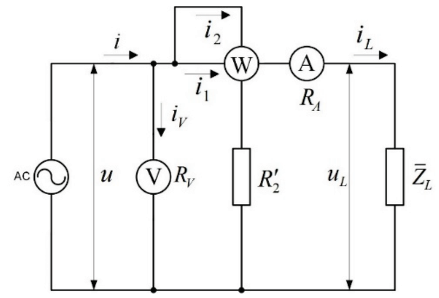
There are two alternative methods of direct connection of an electrodynamic wattmeter in a single-phase AC electrical circuit, depending on which the wattmeter coils are connected near the load:

1. Voltage (potential) coil on the load side, Fig. 2a,
2. Current coil on the load side, Fig. 2b.

In order to avoid situations which may lead to damage of electrodynamic wattmeter, it is recommended to control the voltage with a voltmeter and current with an ammeter, which are connected as shown in Fig. 2.



a) Voltage coil on the load side



b) Current coil on the load side

Figure 2 Connection diagrams of active power measurement using electrodynamic wattmeter

3.1 Method of connections 1: voltage coil on load side

Fig. 2a connection will be considered first. Phasor diagrams of this connection are shown in Fig. 3a and Fig. 3b for lagging and leading power factors, respectively. The following analysis will give necessary corrections for actual wattmeter reading, which should enable obtaining as much as possible better estimate of measured active power.

As seen from phasor diagrams in Fig. 3a and Fig. 3b,

$$\delta = (\varphi \pm \beta) - (\gamma + \varepsilon) \tag{7}$$

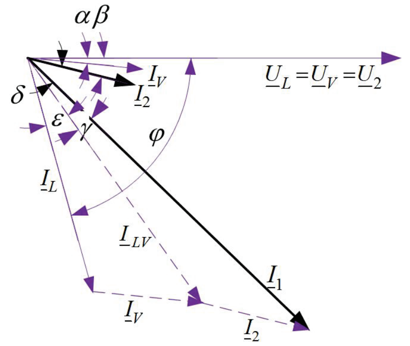
where: δ - phase angle between $\underline{I}_1$ and $\underline{I}_2$ (°), $\varphi = \arctan(X_{\mathrm{L}} / R_{\mathrm{L}})$ - phase angle of the load impedance, R_{L} - resistance of load in ohms, X_{L} - inductance of load in ohms,

$\beta = \tan^{-1}\left(\dfrac{X_2}{R_{2\mathrm{S}} + R_2'}\right) = \tan^{-1}\left(\dfrac{X_2}{R_2}\right)$ – phase angle of the voltage coil circuit impedance (°), X_2 - inductance of voltage coil in ohms, γ - phase angle between $\underline{I}_{\mathrm{LV}}$ i $\underline{I}_1$ (°), ε - phase angle between $\underline{I}_{\mathrm{L}}$ i $\underline{I}_{\mathrm{LV}}$ (°), (ε = 0° if the voltmeter is not connected).

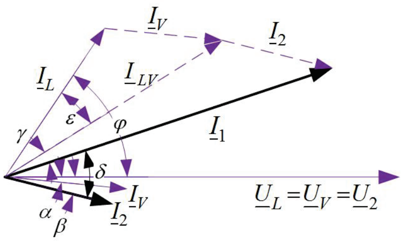
Note that Eq. (7) with β negative is for lagging power factor (inductive load) and with β positive is for leading power factor (capacitive load).

In Fig. 3, angle α denotes phase angle between $\underline{I}_{\mathrm{V}}$ and $\underline{U}_{\mathrm{V}}$. For better understanding of the following transformations, the next auxiliary phase angles will be defined:

$$\theta=\varphi-\beta;\quad \psi=\varphi+\beta;\quad \vartheta=\gamma+\varepsilon;\\ \tau=\varphi-\alpha;\quad \upsilon=\varphi+\alpha \tag{8}$$



a) phasor diagram for a lagging power factor of load



b) phasor diagram for a leading power factor of load

Figure 3 Phasor diagram during active power measurement by means of electrodynamic wattmeter in connection with voltage coil on load side

For lagging power factor and substituting Eq. (7) and Eq. (8) in Eq. (6) can be obtained:

$$\begin{aligned}P_W&=R_2I_1I_2\cos(\theta-\vartheta)=\\&=R_2I_1I_2(\cos\theta\cos\vartheta+\sin\theta\sin\vartheta)\end{aligned} \tag{9}$$

In an analogous way, for leading power factor and substituting Eq. (7) and Eq. (8) in Eq. (6) can be obtained:

$$P_{\mathrm{W}}=R_2I_1I_2(\cos\psi\cos\vartheta+\sin\psi\sin\vartheta) \tag{10}$$

Using phasor diagram shown in Fig. 3a) the following expressions can be obtained:

$$\begin{aligned}\cos\gamma&=\frac{I_{\mathrm{LV}}+I_2\cos(\theta-\varepsilon)}{I_1} & \sin\gamma&=\frac{I_2\sin(\theta-\varepsilon)}{I_1}\\ \cos\varepsilon&=\frac{I_{\mathrm{L}}+I_{\mathrm{V}}\cos\tau}{I_{\mathrm{LV}}} & \sin\varepsilon&=\frac{I_{\mathrm{V}}\sin\tau}{I_{\mathrm{LV}}}\end{aligned} \tag{11}$$

Substituting Eq. (11) in Eq. (9) and using appropriate trigonometric transformations, the following expression can be obtained:

$$P_{\mathrm{W}}=R_2\cdot I_2\cdot\left(I_{\mathrm{L}}\cdot\cos\theta+I_{\mathrm{V}}\cdot\cos(\alpha-\beta)+I_2\right) \tag{12}$$

Substituting for $I_2=\dfrac{U_{\mathrm{L}}\cos\beta}{R_2}$ and $I_{\mathrm{V}}=\dfrac{U_{\mathrm{L}}\cos\alpha}{R_{\mathrm{V}}}$ in Eq. (12) gives

$$\begin{aligned}P_{\mathrm{W}}&=U_{\mathrm{L}}I_{\mathrm{L}}\cos\theta\cdot\cos\beta+\frac{U_{\mathrm{L}}^2}{R_2}\cos^2\beta+\\&+\frac{U_{\mathrm{L}}^2}{R_{\mathrm{V}}}\cos\alpha\cdot\cos\beta\cdot\cos(\alpha-\beta)\end{aligned}. \tag{13}$$

Further transformation and adaptation of Eq. (13) gives expression of wattmeter reading for lagging power factor

$$\begin{aligned}P_{\mathrm{W}}&=\underbrace{U_{\mathrm{L}}I_{\mathrm{L}}\cos\varphi}_{\substack{\text{true value of}\\\text{active power}}}\cdot\frac{\cos\theta\cos\beta}{\cos\varphi}+\underbrace{U_{\mathrm{L}}^2\cos^2\beta/R_2}_{\substack{\text{active power loss}\\\text{in voltage coil}}}+\\&+\underbrace{U_{\mathrm{L}}^2\cos^2\alpha/R_{\mathrm{V}}}_{\substack{\text{active power loss}\\\text{in voltmeter}}}\cdot\underbrace{\cos\beta\cos(\alpha-\beta)/\cos\alpha}_{\cong 1}\end{aligned}. \tag{14}$$

For better estimation of the true value of load active power in the case of lagging power factor $\cos\varphi$ and according to Eq. (14), it can be concluded that the actual wattmeter must be reduced for power losses in the voltmeter and the voltage coil of the electrodynamic wattmeter. After that this difference must be multiplied with corresponding correction factor f_1, i.e.:

$$\left(P_{\mathrm{W}}-U_{\mathrm{L}}^2\cos^2\alpha/R_{\mathrm{V}}-U_{\mathrm{L}}^2\cos^2\beta/R_2\right)\cdot f_1=P_{\mathrm{K}} \tag{15}$$

where: P_{K} - corrected value of the actual wattmeter reading (W), and f_1 - correction factor which has the following value

$$f_1=\frac{\cos\varphi}{\cos\theta\cdot\cos\beta}=\frac{\cos\varphi}{\cos(\varphi-\beta)\cdot\cos\beta}. \tag{16}$$

According to Eq. (16), the curves of the correction factors f_1 are calculated for β = 0°10'; β = 0°30'; β = 1°; β = 0°30' and for various load phase angle values φ (lagging power factor), Fig. 4.

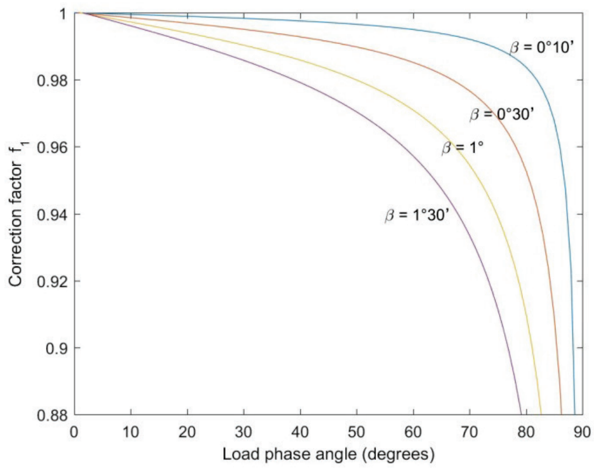


Figure 4 Curves of the correction factors f_1 for β = 0°10'; β = 0°30'; β = 1°; β = 0°30' and for various load phase angle values φ (lagging power factor).

In the case of leading power factor, the following expressions can be obtained using phasor diagram shown in Fig. 3b:

$$\begin{aligned}\cos\gamma&=\frac{I_{\mathrm{LV}}+I_2\cos(\psi-\varepsilon)}{I_1} & \sin\gamma&=\frac{I_2\sin(\psi-\varepsilon)}{I_1}\\ \cos\varepsilon&=\frac{I_{\mathrm{L}}+I_{\mathrm{V}}\cos\upsilon}{I_{\mathrm{LV}}} & \sin\varepsilon&=\frac{I_{\mathrm{V}}\sin\upsilon}{I_{\mathrm{LV}}}\end{aligned} \tag{17}$$

By analogy, substituting Eq. (17) in Eq. (10) and using appropriate trigonometric transformations, the following expression can be obtained in the case of leading power factor:

$$P_{\mathrm{W}} = R_2 \cdot I_2 \cdot \left(I_{\mathrm{L}} \cdot \cos\psi + I_{\mathrm{V}} \cdot \cos(\beta - \alpha) + I_2\right) \tag{18}$$

Also, in the case of leading power factor, shall be valid Eq. (15), with those differences that the factor f_1 has the value:

$$f_1 = \frac{\cos\varphi}{\cos\psi \cdot \cos\beta} = \frac{\cos\varphi}{\cos(\varphi + \beta) \cdot \cos\beta} \tag{19}$$

According to Eq. (19), the curves of the correction factors f_1 are calculated for β = 0°10'; β = 0°30'; β = 1°; β = 0°30' and for various load phase angle values φ (leading power factor), Fig. 5.

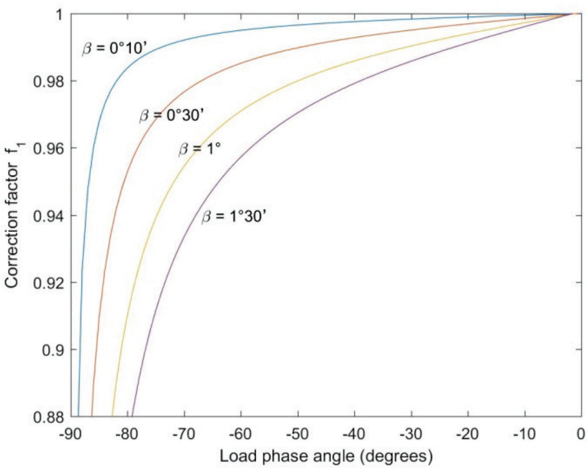


Figure 5 Curves of the correction factors f_1 for β = 0°10'; β = 0°30'; β = 1°; β = 0°30' and for various load phase angle values φ (leading power factor).

3.2 Method of connections 2: Current coil on load side

Connection from Fig. 2b will be considered now. Phasor diagrams of this connection are shown in Fig. 6a and Fig. 6b for lagging and leading power factors, respectively. The following analysis will give necessary corrections for actual wattmeter reading, which should make to obtain as much as possible better estimate of measured active power for this method of connection. As seen from phasor diagram in Fig. 6a and for lagging power factor, angle δ is defined by:

$$\delta = (\varphi - \beta) + \eta \tag{20}$$

where η is phase difference between $\underline{U}_{\mathrm{L}}$ and $\underline{U}_2$.

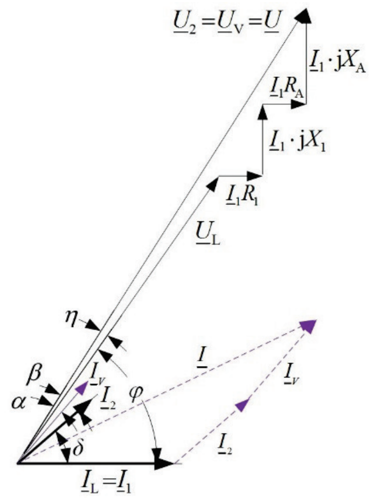
Substituting Eq. (20) in Eq. (6) can be obtained:

$$\begin{aligned} P_{\mathrm{W}} &= R_2 I_1 I_2 \cos(\theta + \eta) = \\ &= R_2 I_1 I_2 (\cos\theta \cos\eta - \sin\theta \sin\eta) \end{aligned} \tag{21}$$

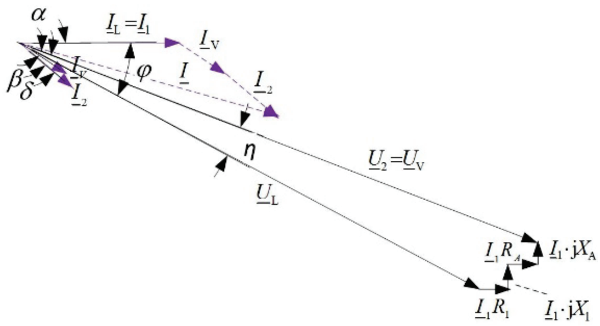
As seen from phasor diagram in Fig. 6a and for lagging power factor, it can be defined:

$$\begin{aligned} \cos\eta &= \frac{U_{\mathrm{L}} + I_{\mathrm{L}}(R_1 + R_{\mathrm{A}})\cos\varphi + I_{\mathrm{L}}(X_1 + X_{\mathrm{A}})\sin\varphi}{U_2} \\ \sin\eta &= \frac{I_{\mathrm{L}}(X_1 + X_{\mathrm{A}})\cos\varphi - I_{\mathrm{L}}(R_1 + R_{\mathrm{A}})\sin\varphi}{U_2} \end{aligned} \tag{22}$$

where: R_1, X_1 - resistance and inductance of the wattmeter current coil in ohms, R_{A}, X_{A} - resistance and inductance of ammeter in ohms.



a) phasor diagram for a lagging power factor of load



b) phasor diagram for a leading power factor of load

Figure 6 Phasor diagram during active power measurement by means of electrodynamic wattmeter in connection with current coil on load side

Substituting Eq. (22) in Eq. (21) and recalling that in this case $I_{\mathrm{L}} = I_1$, can be obtained:

$$\begin{aligned} P_W = \frac{R_2 I_2 I_{\mathrm{L}}}{U_2} &\left[U_{\mathrm{L}} \cos\theta + I_{\mathrm{L}}(R_1 + R_{\mathrm{A}})\cos(-\beta) + \right. \\ &\left. + I_{\mathrm{L}}(X_1 + X_{\mathrm{A}})\sin\beta\right] \end{aligned} \tag{23}$$

Substituting for $I_2 R_2 / U_2 = \cos\beta$ in Eq. (23) gives

$$\begin{aligned} P_{\mathrm{W}} = I_{\mathrm{L}} U_{\mathrm{L}} \cos\theta \cos\beta + I_L^2 R_1 \left(\cos^2\beta + \frac{X_1}{2R_1}\sin 2\beta\right) + \\ + I_L^2 R_{\mathrm{A}} \left(\cos^2\beta + \frac{X_{\mathrm{A}}}{2R_{\mathrm{A}}}\sin 2\beta\right) \end{aligned} \tag{24}$$

For better estimation of the true value of load active power in the case of lagging power factor cosφ and according to Eq. (24), it can be concluded that the actual wattmeter must be reduced for power losses in the ammeter and in the current coil of the electrodynamic wattmeter. After that, this difference must be multiplied with corresponding correction factor f_1 from Eq. (16), that is:

$$P_{\mathrm{K}}=\left[P_{\mathrm{W}}-I_L^2R_1\left(\cos^2\beta+\frac{X_1}{2R_1}\sin 2\beta\right)-\right.$$
$$\left.-I_L^2R_{\mathrm{A}}\left(\cos^2\beta+\frac{X_{\mathrm{A}}}{2R_{\mathrm{A}}}\sin 2\beta\right)\right]\cdot f_1 \tag{25}$$

As seen from phasor diagram in Fig. 6b and for leading power factor, angle δ is defined by:

$$\delta=(\varphi+\beta)-\eta=\psi-\eta \tag{26}$$

Substituting Eq. (26) in Eq. (6) can be obtained:

$$\begin{aligned}P_{\mathrm{W}}&=R_2\cdot I_1\cdot I_2\cdot\cos(\psi-\eta)=\\&=R_2\cdot I_1\cdot I_2\cdot(\cos\psi\cos\eta+\sin\psi\sin\eta)\end{aligned} \tag{27}$$

In an analogous way, substituting Eq. (22) in Eq. (27) and using appropriate trigonometric transformations, the following expression can be obtained in the case of leading power factor:

$$P_{\mathrm{K}}=\left[P_{\mathrm{W}}-I_L^2R_1\left(\cos^2\beta-\frac{X_1}{2R_1}\sin 2\beta\right)-\right.$$
$$\left.-I_L^2R_{\mathrm{A}}\left(\cos^2\beta-\frac{X_{\mathrm{A}}}{2R_{\mathrm{A}}}\sin 2\beta\right)\right]\cdot f_1 \tag{28}$$

where f_1 is given by Eq. (19).

The value of the quantity obtained after proper measurement and applying the above mentioned corrections due to the instrument and due to recognized systematic effects, represents now the estimated measured value.

4 Evaluation of uncertainty in the measurement of active power

The results of the active power, obtained according to one of the expressions: Eq. (15); Eq. (18); Eq. (25) or Eq. (28), can be only estimations of the measurand. According to GUM, such results should be accompanied with statements of the measurement uncertainty.

The uncertainty in the result of measurement generally consists of several components, which may be classified as either Type A or Type B evaluation. Uncertainties calculated by statistical analysis from repeated measurements are all Type A evaluation. Uncertainties obtained by estimation, worst case calculations or taken from reports and references or found by any other means are Type B evaluation. Both types of evaluation are based on probability distributions, and the uncertainty components resulting from either type are quantified by variances or standard deviations. Also, uncertainty of measurement can be expressed in two ways, namely in terms of absolute uncertainty or relative uncertainty.

Performing single measurement of active power, in this paper will be analysed only uncertainty of the type B, which is obtained from assumed uniform or rectangular distribution. Rectangular distribution is used when an equal probability of a measurement occurs within the bound limits. This type of distribution is normally associated with manufacturer specifications. Also, rectangular distribution is the default distribution which has to be used whenever the actual distribution is not known. Use of the rectangular distribution can always be justified as it represents the worst case scenario.

The uncertainty of the correction, applied to compensate the measurement result for a systematic effect, is a measure of the uncertainty of the measurement result due to incomplete knowledge of the required value of the correction. This uncertainty of the correction for a systematic effect will not be included in the evaluation of the uncertainty of the measurement result because its contribution to the uncertainty of the type B of the measurement result is insignificant in the measurement practice and may be ignored.

Thus, the measurement uncertainty will depend only on electrodynamic wattmeter, which gives direct indication of the active power being measured.

It has been shown [9] and quoted [10] that the standard deviation σ of a standard (partial) uncertainty having a rectangular distribution with limits $\pm G$ is given by

$$\sigma=\frac{G}{\sqrt{3}} \tag{29}$$

Maximum possible deviation $\pm G$ of the measured value from the conventional true value of measured quantity is defined by

$$G=\frac{\mathrm{AC}\cdot P_{\mathrm{R}}}{100}\ (\mathrm{W}) \tag{30}$$

where: AC - accuracy class of the wattmeter and P_{R} - is maximum value of the wattmeter measuring range.

For rectangular distribution, the probability that the measurand lies in the interval defined by standard uncertainty is comparatively low, i. e. 58 %.

Expecting rectangular distribution and using Eq. (29) and Eq. (30), the standard uncertainty of measured active power can be expressed in the form:

$$u_P=\sigma=\frac{\mathrm{AC}\cdot P_{\mathrm{R}}}{100\sqrt{3}}\ (\mathrm{W}) \tag{31}$$

Now, the value of the measured active power can be expressed as

$$P_{\mathrm{m}}=P_{\mathrm{K}}\pm u_{\mathrm{P}}\ (\mathrm{W}) \tag{32}$$

where P_{K} is determined according to one of the expressions: Eq. (15), Eq. (18), Eq (25) or Eq. (28).

In this paper intention is to provide an interval about the result of the measurement that may be expected to encompass a large fraction of the distribution of values that could reasonably be attributed to the measurand. Thus, expanded uncertainty is introduced in this analysis and its value is given by

$$U_{\mathrm{p}} = k \cdot u_{\mathrm{p}} \tag{33}$$

where k is a coverage factor. For rectangular distribution, the value of the factor k is usually in the range 1,5 to 1,73 and it is based on the coverage probability or level of confidence required of the interval, Tab. 1. Relation between coverage factor k and coverage probability p for rectangular distribution is given by

$$k = p\sqrt{3} \tag{34}$$

Table 1 The values of coverage factor k and of coverage probability p in the case of rectangular distribution

Coverage probability p	0,90	0,95	0,99	1,00
Coverage factor k	1,559	1,645	1,715	1,732

Finally, the value of the measured active power can be expressed by

$$P_{\mathrm{m}} = P_{\mathrm{K}} \pm U_{\mathrm{P}} \ (\mathrm{W}) \tag{35}$$

5 Illustrative example

In this practical example has been analysed the uncertainty in the measurement of active power of an unloaded AC single-phase squirrel cage electromotor using electrodynamic wattmeter and according to connection diagram from Fig. 2b. Measuring instruments and devices with the following data have been used in the measurements:

a) Analogous ammeter with R_{A}=0,7 (Ω), X_{A}= 0,000345 (Ω) and AC measurement range 0-2,5 A~.
b) Analogous voltmeter with R_{V}=1,006 (MΩ) and AC measurement range 0-250 V~,
c) Analogous electrodynamic wattmeter with R_1=0,432 (Ω), X_1= 0,232 (Ω), R_2=20,1 (kΩ), X_2= 43 (Ω), and AC measurement ranges: 0-2,5 A~; 0-250 V~ (K=10 -constant of the wattmeter in this range). Accuracy class of this wattmeter has been 1,5.
d) AC single-phase squirrel cage electromotor with nominal power 0,37 kW; nominal current 1,52 A; nominal voltage 220 V and nominal rotation speed 1350 rev^{-1}.

During this active power measurement, the following measured values have been obtained: I_{L} = 1,2 A, U_{V} = 220 V and α_{W} = 19,2.

From these values and from above mentioned data of the wattmeter can be calculated the following values:

$$P_{\mathrm{W}} = K \cdot \alpha_{\mathrm{W}} = 10 \cdot 19{,}2 = 192 \ (\mathrm{W}) \tag{36}$$

$$\cos\varphi \cong \frac{P_{\mathrm{W}}}{U_{\mathrm{V}} \cdot I_{\mathrm{L}}} = \frac{192}{220 \cdot 1{,}2} = 0{,}727 \tag{37}$$

$$\beta = \arctan\frac{X_2}{R_2} = \arctan\frac{43}{20100} = 7{,}4' \tag{38}$$

$$f_1 = \frac{\cos\varphi}{\cos(\varphi - \beta) \cdot \cos\beta} = \frac{0{,}727}{0{,}728 \cdot 0{,}999} = 0{,}9986 \tag{39}$$

Substituting values from Eq. (36), Eq. (38), Eq. (39), as well as the above mentioned values of I_{L}, R_{A}, X_{A}, R_1 and X_1 into Eq. (25) is obtained:

$$P_{\mathrm{K}} = 190{,}1 \ (\mathrm{W}) \tag{40}$$

Maximum possible deviation ±G is determined using Eq. (30) and it has the following value:

$$G = \frac{1{,}5 \cdot 625}{100} = 9{,}375 \ (\mathrm{W}) \tag{41}$$

The standard uncertainty u_{p} of active power is determined using Eq. (31) and it has the following value:

$$u_p = \frac{1{,}5 \cdot 625}{100 \cdot \sqrt{3}} = 5{,}413 \ (\mathrm{W}) \tag{42}$$

The value of the measured active power can be expressed according to Eq. (32), that is

$$P_m = 190 \pm 5{,}41 \ (\mathrm{W}) \ \text{(58 \% probability).} \tag{43}$$

For the rectangular distribution and most commonly used the overall uncertainty has to be scaled by using the coverage factor k = 1,645, to give a level of confidence of approximately 95 %. In this case, the value of the measured active power can be written:

$$P_m = 190 \pm 8{,}9 \ (\mathrm{W}) \ \text{(95 \% probability).} \tag{44}$$

The reported expanded uncertainty is based on a standard uncertainty multiplied by a coverage factor k = 1,645 providing a level of confidence of approximately 95 %.

Now, it can be concluded that true value of the active power measurement in this case lies in the range 190 ±8,9 (W) with the level of confidence of 95 %.

Additionally, measurement of active power has been performed under the same conditions by means of the new digital wattmeter, too. Accuracy class of this wattmeter has been 1,5, too. Result of active power measurement in this case was

$$P_{\mathrm{W}} = 196{,}7 \ (\mathrm{W}) \tag{45}$$

Digital wattmeter reading in Eq. (45) is assumed to be exact.

According to Eq. (44) and Eq. (45), it can be concluded that the result of the measurement obtained by means of the new digital wattmeter lies in the range of measurement uncertainty given in Eq. (44). Thus, the active power measurements carried out by means of the very old electrodynamic wattmeter can be considered as correct and the estimated uncertainty as relevant.

6 Conclusion

In this paper an approach is introduced with the aim of providing guidance for the estimation of measurement uncertainty of active power measurement of the single-phase load using electrodynamic wattmeter.

The uncertainty in a measurement result depends upon the three following elements:
1. The certain range in which the measured value is expected to lie. This range depends upon instruments,
2. Quality of taken measurements and
3. The confidence level at which the final result is to be stated.

A comprehensive analysis for the common systematic effects associated with active power measurements of single-phase load using electrodynamic wattmeter has been carried out in this paper. The expressions for the correction of systematic effects due to both types of the wattmeter connection for the lagging and leading power factor, and due to the reactance of the wattmeter voltage coil are given in this paper. The active power measurement using electrodynamic wattmeter is considered in this paper as a single measurement, because indications of such wattmeter do not change by repeated measurements for the steady-state operation of single-phase electrical system with linear load supplied with sinusoidal voltage. Thus, in this paper is considered only uncertainty of the type B, which is derived from the assumed rectangular probability density function. Only direct connection of the wattmeter is analysed in this paper for the active power measurement of the single-phase load, too. The authors' intention was to analyse estimation of the uncertainty of active power measurement for the semi-direct and indirect connection of both types wattmeter, electrodynamic and digital.

The analysis in this paper shows that estimation of the measurement uncertainty using electrodynamic wattmeter is not complex or difficult in principle. Illustrative example assists in understanding this task.

This paper has practical application and can be used for educational purposes.

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