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A novel approach to modeling price volatility of sovereign debt instruments – the example of the Croatian government's debt-based instruments

Novi pristup modeliranju cijene državnih dužničkih vrijednosnih papira – primjer dužničkih vrijednosnih papira Republike Hrvatske

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Abstract

Debt-based financial instruments are specific due to the maturity component and conventional approaches in estimating their volatility may not be applicable. This paper focuses on modeling and forecasting price volatility of sovereign debt instruments while taking into account their maturity. In doing so we propose a simple and useful technique for obtaining the desired confidence of volatility estimates. The proposed approach provides price volatility estimates for debt instruments issued by Croatian government denominated in HRK and in EUR.

Keywords: debt instruments, volatility, Croatia

JEL Classification: FC13, C41, G12, G17

Sažetak

Dužnički financijski instrumenti su specifični zbog komponente dospijeća koju sadržavaju te stoga konvencionalni pristupi u procjeni njihovih volatilnosti mogu biti neprimjereni. Rad je usmjeren na modeliranje i prognoziranje volatilnosti cijene dužničkih vrijednosnih papira uzimajući u obzir komponentu njihovog dospijeća. U radu se uzima u obzir ovisnost volatilnosti dužničkog vrijednosnog papira o njegovom dospijeću i predlaže jednostavna i primjenjiva tehnika procjene volatilnosti uz željeni interval pouzdanosti. Korištenjem predloženog pristupa u radu su provedene procjene volatilnosti dužničkih vrijednosnih papira denominiranih eurima kao i u kunama koje je izdala Republika Hrvatska.

Ključne riječi: dužnički instrumenti, volatilnost, Hrvatska

JEL classification: FC13, C13, C41, G12, G17

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1. Introduction

Comelli (2012) points out that sovereign debt instruments have become a key way of funding for emerging market economies and an increasingly important asset class for investors as well. Volatility estimates of financial instruments are often obtained using time-series approaches on high frequency data samples. One of the most famous and frequently used approaches is the autoregressive conditional heteroscedasticity (ARCH) approach, initially introduced by Engle (1982). The number of ARCH models is extremely large and such an approach is efficient in removing conditional heteroscedasticity from financial time series (Arabi, 2012; Çağlayan et al., 2013). The generalised ARCH model (GARCH) introduced by Bollerslev (1986) is the most frequently used model in describing volatility of financial series in literature as well as in market analyses (Berüment and Günay, 2003; Oduncu, 2011).

Debt-based financial instruments are specific due to their maturity components where debt instrument volatility may depend on the time to maturity. Given that each subsequent day a debt instrument draws closer to maturity, debts instruments are specific for their time-varying maturity property. Debt instruments with a longer time to maturity may experience higher volatility whereas those with shorter time to maturity a lower volatility. This phenomenon is defined as maturity-dependent volatility. The main aim of this paper is to test maturity-dependent volatility on samples of Croatian government debt instruments denominated in HRK and in EUR. Subsequently, we will propose a simple and applicable technique to estimate debt instrument volatilities and following the proposed technique will recommend volatility measures for Croatian government debt instruments. Furthermore, the proposed technique addresses the issue of measuring the risk of a newly issued bond that has no history in trading prices.

The paper consists of five parts. Following the introductory part, the second part provides a brief overview of the literature. The third part of the paper proposes an empirical strategy and methodology, with the results of the study presented in the fourth part. The final part provides a conclusion.

2. Brief related literature overview

Alfonso et al. (2014) used the Exponential Generalized Autoregressive Conditional Heteroskedasticity (EGARCH) model, developed by Nelson (1991) to model sovereign debt volatilities. The ARCH approach is often used for analysing determinants of sovereign bond yields on emerging market (EM) (Comelli, 2012; Csonto and Ivaschenko, 2013; Jaramillo and Weber, 2012). Besides the ARCH approach, Alfonsoet al. (2014) applied the value-at-risk (VaR) to mean-variance portfolios with and without taking into account the effect of credit rating information on stock and bond return volatilities.

Leavens (1945) offers a quantitative example, considered the first VaR measure. Markowitz (1952) and Roy (1952) use VaR to calculate the means of selected portfolios and to optimise the risk and returns. VaR is often used for the portfolio theory (Tobin, 1958; Treynor, 1961; Sharpe, 1964; Lintner, 1965; Mossin, 1966). VaR measures are best suited for equity portfolios. Applying VaR to either debt instruments entails modelling term structures. Dusak (1973) applies VaR measures but does not address the term structure issue. Garbade (1986) proposes VaR measures modelled in a way that each bond price depends on its sensitivity to yield changes. Garbade (1987) extends his previous work and introduced buckets that enabled substituting a large portfolio of bonds with a smaller portfolio of representative bonds. There are three basic approaches used to compute VaR, with numerous variations for each approach. The measure can be computed by making assumptions about return distributions for risks, and by using the variances and covariance across these risks. It can also be estimated by running hypothetical portfolios through historical data or using Monte Carlo simulations (see Jorion, 2001). Britten-Jones and Schaefer (1999) deal with non-linear instruments in portfolios and developed Quadratic Value at Risk measures. VaR estimates rely on assuming a normal distribution, however the corresponding empirical distribution has fatter tails than that of a normal distribution. Studies have applied the Extreme Value Theory to model tail behaviour based only on extreme values. Bali (2003) points out that standard VaR approaches can be significantly improved using the Extreme Value Theory. Marimoutou, Raggad, and Trabelsi (2009) compare the Extreme Value Theory approach to other approaches and found out that the Extreme Value Theory outperforms GARCH, historical simulation and filtered historical simulation. Litzenberger and Modest (2008) explain tail risk by utilising Markov regime switching processes to capture time varying risk exposures in different market conditions or different regimes. Even though the VaR measure has often criticised following the global financial crises of 2007, nowadays it has become widespread and is a frequently applied risk measure within financial institutions.

The overview of literature contains various techniques to estimate volatilities. Accordingly, we propose one such technique that takes into account maturity dependence and has been specifically designed for estimating volatility of debt-based financial instruments.

3. Empirical strategy and methodology

Instead of contractual maturity, we calculate and observe effective maturity using the equation (1):

$EM=\frac{1}{TIP}\sum_{i=1}^{n} IP_i \times DP_i$	1

where:

n - number of cash inflows;

DP_i - maturity of the cash inflow IP_i;

IP_i - cash inflow amount with maturity DP_i;

TIP - total amount of inflow or sum of all inflows and

EM - effective maturity in years.

Here we use a four-day liquidation period since we found that four days was a reasonable enough period to liquidate the position. To provide a unique measure for positive and negative change in price, we use a discrete return in its absolute amount. Hence, the four-day discrete return is calculated using the equation (2):

$DR_i=\left\|\frac{c_i-c_{i-4}}{c_{i-4}}\right\|;\ i=1,..,n$	2

where:

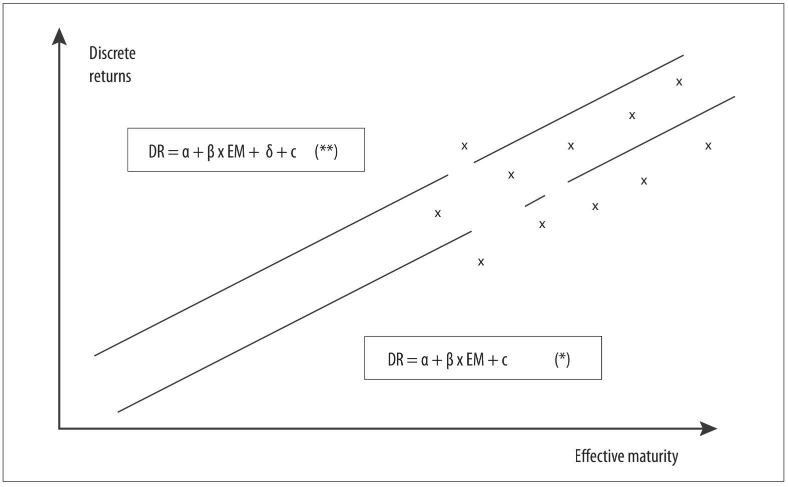
n - number of observations,

c_i - price of debt-based financial instrument at day i,

DP_i - discrete return of debt financial instrument at day i.

Figure 1 illustrates the proposed empirical strategy. Debt instruments with longer time to maturity may experience higher volatility or discrete returns. Accordingly, we firstly intend to explain the differences in discrete returns based on the differences in effective maturity. The ordinary least square (OLS) is used as an estimator to obtain the linear regression model. Equation (*) in Figure 1 represents the estimated linear regression model, but using the equation (*) in Figure 1 to estimate the discrete return may lead to underestimated discrete returns i.e. volatility. Regulatory requirements for financial institutions often prescribe at least a 99% confidence interval, hence using just the equation (*) fulfilling the requirements is not possible. Nonetheless, this upward shift of the estimated regression (*) while keeping the slope or β coefficient constant results in 99% of discrete returns being positioned below the regression line and

Figure 1 llustration of empirical strategy



Source: the authors.

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Table 1 Augmented Dickey-Fuller Test results for the observed variables

Variable		P - value
EME	around the constant	0.0000
DRE	around the constant	0.0000
EMH	around the constant	0.0000
DRH	around the constant	0.0000

Source: the authors.

may be a satisfactory solution.

Following the proposed methodology as illustrated in Figure 1, debt instrument volatility may be estimated knowing only its effective maturity as calculated by the equation (1). The data sample on Croatian government debt denominated in HRK counts 36.402 four-day discrete returns dating from 5 June 2013 to 10 May 2016 and the data sample for Croatian government debt denominated in EUR counts 17.590 four-day discrete returns dating from 13 November 2001 up to 10 May 2016. The data is available from the Bloomberg data service. We applied the proposed methodology and estimated the volatility of Croatian government debt instruments denominated in HRK and in EUR.

4. Results and discussions

The analysis was performed on a two pooled data sample, one containing discrete returns for debt instruments denominated in HRK and the other pooled data sample containing discrete returns for debt instruments denominated in EUR. Firstly, we tested the stationarity properties for the observed variables using the Augmented Dickey-Fuller Test. The Augmented Dickey–Fuller test (ADF) is a test for a unit root in a time series sample but also needs to be performed for a pooled and cross-sectional sample as well. The unit root test is carried out under the null hypothesis of the existence of a unit root against the alternative hypothesis that assumes no unit root is present. The test results are shown in Table 1.

As expected, Table 1 shows that for the usually accepted significance level of 1%, the observed variables are stationary at the levels.

Thereafter, we estimated the linear regression model, as (*) illustrated in Figure1, where DRH represents four-day discrete returns for a sample of instruments sample denominated in HRK and EMH represents the corresponding effective maturity. The variables are in log values. Results of the estimation are given in Table 2.

The estimated results in Table 2 show that approx. 21% of differences in discrete returns of debt instruments denominated in HRK can be explained by the differences in corresponding effective maturities.

The same procedure was used to estimate the linear regression model for debt instruments denominated in EUR where DRE represents four-day discrete returns for a sample of instruments denominated in EUR and EME represents the corresponding effective maturity.

As shown in Table 3, the results indicated that the estimated model has a 1% significance level and the determination coefficient at the level of 15%. Therefore, a 15% change in discrete returns for debt instruments denominated in EUR is attributed to differences in the corresponding effective maturities.

To obtain the desired confidence level, we perform

Table 2 Estimated model of maturity dependence for debt instruments denominated in HRK

Variable	Model description	Constant value (α)/ value of coefficient (β)	P-value
DRH	dependent variable		
α	constant	-1.181905	0.0000
EMH	independent variable	0.472134	0.0000
Significance of defined model (F-test)			0.0000
Determination coefficient (R^2)	0.208550		

Source: the authors.

Table 3 Estimated model of maturity dependence for debt instruments denominated in EUR

Variable	Model description	Constant value (α)/ value of coefficient (β)	P-value
DRE	dependent variable		
α	constant	-0.931287	0.0000
EME	independent variable	0.408666	0.0000
Significance of defined model (F-test)			0.0000
Determination coefficient (R^2)	0.150946		

Source: the authors.

a parallel upward shift of the estimated regression line as (**) illustrated in Figure 1. As a matter of fact, line (*) and line (**) share the same slope coefficient (*β*) but have different intercepts. Therefore, to obtain the regression line (**), the intercept of the regression line (*) needs to be adjusted. Adjusting the intercept depends on maintaining the desired confidence level in our estimates. The intercept adjustments (*δ*) for various confidence intervals are shown in Table 4.

Table 4 shows the confidence interval and corresponding adjustment for intercepts for debt instruments denominated in EUR as well as debt instruments denominated in EUR.

Subsequently, in line with the proposed empirical strategy, Table 5 shows the volatility estimates for debt instruments denominated in EUR and debt instruments denominated in HRK for various confidence intervals and effective maturities.

As is evident in Table 5, volatility estimates with a 99% confidence interval for Croatian government debt instruments denominated in HRK with an effective maturity of one year amounts 0.95%, while volatility estimates with a 99% confidence interval for Croatian government debt instruments denominated in EUR with an effective maturity of one year amounts 1.20%. Furthermore, using the 99% confidence interval, instruments denominated in EUR are more volatile than their counterparts denominated in HRK. A confidence interval of 100% represents the maximum absolute discrete return. Hence, we found higher maximum discrete returns in the data sample for debt instruments denominated in HRK.

5. Conclusions

Debt-based financial instruments are specific due to their maturity component and therefore conventional approaches in estimating volatility may not be applicable. In our study, we have taken into account the maturity dependence of debt instrument volatility and proposed a simple and applicable technique to obtain the desired confidence in the respective volatility estimates. This proposed approach based on a confidence interval of 99% resulted in price volatility estimates for debt instruments denominated in HRK and issued by Croatian government ranging from 0.95% to 3.43%, depending on their effective maturity. Price volatility estimates for debt instruments denominated in EUR and issued by the Croatian government using 99% confidence intervals range from 1.20% to 3.63%, depending on their effective maturity. These estimates are based on a large data set, and accordingly these calculated volatilities should be stable with

Table 4 Model parameters for volatility estimates of Croatian government debt instruments with various confidence intervals

Currency	Confidence interval	Intercept adjustment(δ)	Coefficient (β)	Initial intercept (α)
EUR	99%	1.0103	0.4087	-0.9313
EUR	99.6%	1.1698	0.4087	-0.9313
EUR	99.9%	1.3537	0.4087	-0.9313
EUR	100%	1.5392	0.4087	-0.9313
HRK	99%	1.1613	0.4721	-1.1819
HRK	99.6	1.3345	0.4721	-1.1819
HRK	99.9%	1.5441	0.4721	-1.1819
HRK	100%	2.0149	0.4721	-1.1819

Source: the authors.

Table 5 Volatility estimates of Croatian government debt instruments in % with various confidence intervals and effective maturities for HRK and EUR

	HRK				EUR			
Effective maturity\ Confidence interval	99%	99.60%	99.90%	100%	99%	99.60%	99.90%	100%
1	0.95	1.42	2.30	6.81	1.20	1.73	2.65	4.05
2	1.32	1.97	3.19	9.44	1.59	2.30	3.51	5.38
3	1.60	2.39	3.87	11.44	1.88	2.71	4.14	6.35
4	1.84	2.73	4.43	13.10	2.11	3.05	4.66	7.14
5	2.04	3.04	4.92	14.56	2.32	3.34	5.11	7.83
6	2.22	3.31	5.37	15.87	2.49	3.60	5.50	8.43
7	2.39	3.56	5.77	17.06	2.66	3.84	5.86	8.98
8	2.55	3.79	6.15	18.17	2.81	4.05	6.19	9.48
9	2.69	4.01	6.50	19.21	2.94	4.25	6.49	9.95
10	2.83	4.21	6.83	20.19	3.07	4.44	6.78	10.39
11	2.96	4.41	7.14	21.12	3.20	4.62	7.05	10.80
12	3.08	4.59	7.44	22.01	3.31	4.78	7.30	11.19
13	3.20	4.77	7.73	22.86	3.42	4.94	7.55	11.57
14	3.32	4.94	8.,01	23.67	3.53	5.09	7.78	11.92
15	3.43	5.10	8.27	24.45	3.63	5.24	8.00	12.26

Source: the authors.

no potential pro-cyclical effects. Finally, following the proposed approach, it is possible to measure the risk of newly issued debt instruments that have no history in trading prices.

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