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Duopoly innovation under product externalities

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This study argues that product substitutability and complementary have major effects on the relationship between innovation and competition and some interesting conclusions are derived. First, innovative investment is reduced with market power. The total quantity of products and social welfare are increased with market power while decreased with increasing of substitutability or deceasing of complementary. Second, the equilibrium products and innovative investment are lower than those under social optimality. Finally, by comparison with Cournot quantity competition, Bertrand price competition is keener. But the main conclusions are the same under both kinds of competitions.

Keywords: innovation; duopoly; product complementary; product substitutability

JEL classification: D43, L13

1. Introduction

Since Schumpeter (1942) proposed innovation theory, many people fixed their attention on innovation behaviour and the relationship between innovation and competition became an important topic in economics and management. Schumpeter (1942) issued that monopoly stimulates innovation, but 20 years later Arrow (1962) declared that competition motivates innovation. Since them, the debate about the relationship between competition and innovation is continuous. Some people supported Schumpeter (e.g. Demsetz, 1969; Gilbert & Newbery, 1982; Yi, 1999), some people sustained Arrow (e.g. Recently Holmes, Levine & Schmitz, 2012; Vives, 2008), while some others held their own opinion (e.g. Aghion et al., 2005; Sacco & Schmutzer, 2011).

What is the relationship between innovation and competition? Different people have different conclusions, so we do not try to give the final answer to that question. The purpose of this study is to make efforts to reveal that relationship under some special conditions.This study addresses the innovation of the only firm with cost advantage,[1] while in some literature, all firms launch innovative investment.[2] For example, Sacco and Schmutzer (2011) discussed that all firms launch innovation. Taking spillover into account, D'aspremont and Jacquemin (1998) explored both cooperative and non-cooperative innovation theory. Wang and Yang (2002) further developed this cooperative innovation theory under a vertically related market structure. The main contributions of this study lie as follows. First, considering externalities[3] (including substitutability and

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complementary), this article examines Arrow's (1962) innovation theory. Second, taking externalities into account, this study expands Vives's (2008) research. Considering the effects of externalities, this study finds that innovative investment competition is fiercer than that under product quantity competition in Vives's (2008) study. Finally, this study expands the theory about substitutability. Substitutability not only effects product competition, but also effects firm's innovation investment.

Taking product externalities, including substitutability and complementary into account, both under Cournot quantity competition and Bertrand price competition, this study further addresses the relationship between innovation and competition. This study argues that complementary and substitutability have major effects on the relationship between innovation and competition. Market power has negative effects on innovative investment. Both total quantity of products and social welfare are reduced with substitutability. High market power improves the total quantity of products and social welfare. Moreover, higher substitutability reduces profits of firms with higher costs of production. Furthermore, we find that social welfare under equilibrium is lower than under optimality. Besides, by comparing Bertrand with Cournot, this study argues that Bertrand competition is more intense than that of Cournot competition.[4]

2. Literature review

Many people focus their attention on the relationship between innovation and competition. So before stating our own research, we will introduce some major prior studies.

Following Arrow's idea, there exist many important papers. Recently, Holmes, Levine and Schmitz (2008) developed a theory of switchover disruptions under the monopoly market structure, and their conclusions support Arrow's idea. Vives (2008) developed an innovation theory under competition and his study took market structure as a major fact. Chen and Sappington (2010) further discussed innovation under vertically related market structures. Vives (2008) and Chen and Sappington (2010) drew the same conclusions as Arrow (1962).

On the other hand, Arrow's proposal has been intensely debated for many years. Perhaps the most famous critique came from Demsetz (1969) and Yi (1999). Demsetz (1969) and Yi (1999) discussed that Arrow's idea manifests that increased competition yields less innovation. Another important critique is proposed by Gilbert and Newbery (1982), in which Arrow's assumptions were changed and it showed that a monopolist has a greater incentive to adopt new technologies. In other words, those earlier studies hold the similar viewpiont as Schumpeter (1942).

More significantly, some other people took the strategic effects of innovation into account. Maybe the most important research came form Brander and Spencer (1983) and Spence (1984). Strategic effects means cost reduction innovation reduces output of its competitors. Brander and Spencer (1983) declared that strategic effects increases total amount of R&D and total output. Spence's (1984) research considered spillover of R&D. Then Bester and Petrakis (1993) investigated how product substitutability incites cost reduction innovation both under Betrand competition and Cournot competition. In their model, two firms acted under a social planner. As a result, two firms should produce heterogeneity products, or the social planner would only operate the more efficient firm. And they issued that one cannot draw a general conclusion about the relationship between innovation and substitutability. That two firms should act under a social planner could also be seen as a drawback of Bester and Petrakis's (1993) study, because all conclusions in their study were constrained in planning economics. More interestingly, Nie and Chen (2012) investigated doupoly competition with input constraints.

In theoretical studies, there also exists plenty of important literature about the relationship between innovative investment and competitive pressure. Arrow (1962) initially advanced that monopolistic industries would be less innovative than competitive ones. Aghion et al. (2005) further examined an inverted-U relationship between competition and innovation. Schmutzler (2007) discussed the relationship between R&D and competition with a two-stage model. Under a linear demand function, Sacco and Schmutzer (2011) confirmed the U-shaped relationship between competition and innovation with numerical simulations. Narajabad and Watson (2011) studied dynamic innovation under production differentiation. Furthermore, Regarding empirical conclusions, using the Statistics Canada 1999 Survey of Innovation, Tang (2006) examined that this relationship is affected by many factors, such as substitutability, constant arrival of competing products and so on. Tang (2006) confirmed that market power has negative effects on the innovative investment under substitutability. Patel and Ward (2011) estimated competition in innovative market with patent citation patterns.

Summarising from those prior studies, we obtain the conclusions that there are three kinds of issues about the relationship between innovation and competition. First, competition stimulates innovation. Second, competition inhibits innovation. And the relationship between them is U or an inverted-U shape. But we think that the relationship between innovation and competition are depended. On the one hand, the relationship between innovation and competition depends on the descriptions of innovation, such as cost-reducing or quality-inproving innovation. On the other hand, which depends on the descriptions of competition, such as market concentration or substitutability. So we will investigate the relationship between innovation and competition under the special state that firms produce with different efficiency and only the low cost firm invests innovation, but not to join in the debate. Different from other studies, the value interval of γ in our study is from -1 to 1, which means γ can been regarded as substitutability as well as complementary.

The rest of this article is organised as follows. The model is established in Section 3. In this model, complementary and substitutability along with innovative investment are introduced. Then model analyses are present in Section 4, which contains two parts. The model of Cournot quantity competition is analysed in part A. By analysing the model, some useful conclusions are drawn. The model of Bertrand price competition is addressed in part B. The price and social welfare under Bertrand are compared with those under Cournot. Some remarks are presented in the final section.

3. Model

The model of duopoly innovation is established. Different from other studies (Aghion, et al., 2005; Sacco, Schmutzler, 2011; Schmutzler, 2007), this study applies a one-stage model.[5] In our model, product externality is fully addressed. Assume that there are two producers, and denoted by $i \in N = \{1, 2\}$. q_i represents the product quantity of firm i, $i = 1, 2$. Given the prices of the two firms, p_1 and p_2, the representative consumer's (net) utility function is outlined by the following function.

$$u(p_1, p_2, q_1, q_2) = A(q_1 + q_2) - p_1 q_1 - p_2 q_2 - \frac{1}{2}(q_1^2 + q_2^2) - \gamma q_1 q_2. \tag{1}$$

In the above, $A > 0$represents the maximum price that consumers are willing to pay for either good, and $\gamma \in [-1, 1]$ denotes the externality parameter. The inverse linear demand functions are outlined as follows, which are directly obtained from (1).

$$p_1 = A - q_1 - \gamma q_2, \; p_2 = A - q_2 - \gamma q_1. \tag{2}$$

$\gamma = 0$ means that two goods are independent; $\gamma = 1$ indicates perfect substitutes; $\gamma = -1$ signifies perfect complements of the firms' products. Furthermore, $\gamma \in [-1, 0)$ means that one firm has positive externality on the other firm, or both firms benefit from each other's production, while $\gamma \in [0, 1]$ represents that the products of the two firms are substitutes.

We note that (1) is employed in Liu and Wang (Lin and Henry, 2013), and Sacco and Schmutzer (2011) with $\gamma \in [0, 1]$. The parameter $\gamma \in [-1, 1)$ is also regarded as the externality of one firm exerted on the other. This study extends the models of Liu and Wang (2011) and Sacco and Schmutzer (2011).

Initially, the firms' marginal costs of production are c_1 and c_2, respectively. We assume that $c_2 = c_1 + \tau$, where $\tau > 0$ denotes the cost advantage or market power of the first firm. The first firm launches innovative investment I_1 at the same time when it make output decision,[6] while the second firm makes no innovative investment. This is the benchmark model of innovation of Arrow (1962) or Holmes, Levine and Schmitz (2008). The cost associated with innovative investment I_1 is $\frac{1}{2}I_1^2$. Profit functions of the two firms are given as follows.

$$\pi_1 = [p_1 - c_1(I_1)]q_1 - \frac{1}{2}I_1^2, \tag{3}$$

$$\pi_2 = (p_2 - c_2)q_2. \tag{4}$$

With innovative investment I_1 of the first firm, the marginal cost of the first firm becomes $c_1(I_1)$, where $c_1(0) = c_1$ and $c_1(I_1)$ is convex and continuously decreasing in I_1. We further assume that $\frac{\partial c_1(I_1)}{\partial I_1} > -1$. Since the reduction in marginal cost incurred by innovation is not too much, this hypothesis is reasonable. To simplify the problem, we further assume that $c_1(I_1) = c_1 \; g(I_1) = (c_2 - \tau)g(I_1)$, where $g(I_1) > 0$ is continuously decreasing for all I_1 and convex. Obviously, $g(0) = 1$.

4. Model analyses

To compare different models, we analyse the duopoly model based on quantity (or Cournot) competition and price (or Bertrand) competition. Most conclusions are similar in the two models, which mean the conclusions are robust. Furthermore, we find that Bertrand competition seems fiercer than Cournot competition.

A. Results under Cournot quantity competition.

The model is addressed in this section. The equilibrium is outlined and characterised under Cournot. Under Cournot competition, the two firms compete in quantity. From (2)–(4), profit functions are restated as follows:

$$\pi_1^C = [A - q_1 - \gamma q_2 - c_1(I_1)]q_1 - \frac{1}{2}I_1^2, \tag{5}$$

$$\pi_2^C = (A - q_2 - \gamma q_1 - c_2)q_2. \tag{6}$$

The two firms maximise (5) and (6), respectively by choosing q_1, I_1 and q_2. The equilibrium is discussed next. Since $c_1(I_1)$ is convex and continuously decreasing, both (5) and (6) are concave functions in the respective choice variables. Therefore, there exists a unique equilibrium to (5)–(6). The equilibrium is determined by the following first-order optimal conditions.

$$\frac{\partial \pi_1^C}{\partial q_1} = f_1 = [A - \gamma q_2 - c_1(I_1)] - 2q_1 = 0. \tag{7}$$

$$\frac{\partial \pi_1^C}{\partial I_1} = f_2 = -q_1 \frac{\partial c_1(I_1)}{\partial I_1} - I_1 = 0. \tag{8}$$

$$\frac{\partial \pi_2^C}{\partial q_2} = f_3 = (A - \gamma q_1 - c_2) - 2q_2 = 0. \tag{9}$$

Equation (8) manifests that the marginal cost incurred by innovative investment is exactly equal to the marginal benefit caused by innovative investment. Obviously, from (7) and (9) we have the relationship that $q_1 > q_2$.

By virtue of (7)–(9), we have the following conclusion

Proposition 1. *Given* c_2*, a larger cost advantage yields lower innovative investment and higher total quantity in the industry.*

Proof: See Appendix. ■

Remarks: We have concluded that market power has a negative relationship with innovative investment under substitutability, successfully explaining the empirical results of Tang (2006). This conclusion is consistent with those in Vives (2008) and Arrow's idea (1962). Conclusions of Proposition 1 are more extensive than those of Tang (2006). Apparently, q_1 increases with its cost advantage while q_2 decreases with first firm's cost advantage. Besides, $\frac{\partial q_1}{\partial \tau} + \frac{\partial q_2}{\partial \tau} > 0$, which means that the total products in this industry are increasing with cost advantage of the first firm. Larger market power of the first firm increases the quantity of products of the first firm and the total quantity of products in the industry. Moreover, cost advantage of the first firm has more effects on its own products than that of the second firm because of $|\frac{\partial q_1}{\partial \tau}| > |\frac{\partial q_2}{\partial \tau}|$.

The equilibrium quantity and price are further described next. (7) indicates that $q_1 = p_1 - c_1(I_1)$. (9) implies that $q_2 = p_2 - c_2$. From (7)–(9), we have

Proposition 2. *Under the equilibrium state, the total quantity of outputs $q = q_1 + q_2$, q_2 and p_2 all decrease with γ. Both I_1 and q_1 decrease with γ if $2q_2 > \gamma q_1$ but increase with γ if $2q_2 \leqslant \gamma q_1$.*

Proof: See Appendix. ■

Remarks: The above proposition illustrates that the total quantity of products, q_2 and p_2 decrease with the externality parameter. Obviously, according to Proposition 2, the first firm's innovative investment and its quantity are reduced with positive externality (product complementary or $\gamma < 0$).

The total quantity of production of the two firms is further discussed. According to the above analysis, we have the interesting relationship $\frac{\partial q_1}{\partial \gamma} + \frac{\partial q_2}{\partial \gamma} < 0$, which means that as γ increases, total outputs of the industry decrease. Because of larger substitutability parameter, competition in the industry becomes fiercer and the total demand for products in this industry is reduced. By the proof of Proposition 2, we also have the significant relationship that $|\frac{\partial q_1}{\partial \gamma}| < |\frac{\partial q_2}{\partial \gamma}|$, which indicates that γ has more effects on the quantity of the second firm than that of the first firm.

Here, the profits of two producers are remarked. By the envelope theorem, we have the following conclusion.

Proposition 3. *Under equilibrium state, the profits of the second firm satisfy:* $\frac{\partial \pi_2^C}{\partial \gamma} < 0$.

Proof. See Appendix. ■

Remarks: With a large γ, the two firms compete drastically and the second firm undertakes a loss. In other words, γ reduces the profit of the second firm. It is not sure about the effect of parameterγon the first firm's profits.

Actually, from (2), (7)–(9), we immediately have the following equations $\pi_1^C = q_1^2 - \frac{1}{2}I_1^2$ and $\pi_2^C = q_2^2$. Obviously, $\frac{\partial \pi_2^C}{\partial \gamma} < 0$. It is difficult to determine the effects of γ on $\pi_1^C = q_1^2 - \frac{1}{2}I_1^2$.

Social welfare (SW) is addressed next. Social welfare (SW) is the sum of consumer surplus (CS) and producer surplus (PS). CS is given by (1) and PS is given by (3) and (4).

$$SW = CS + PS = A(q_1 + q_2) - \frac{1}{2}(q_1^2 + q_2^2) - \gamma q_1 q_2 - c_1(I_1)q_1 - \frac{1}{2}I_1^2 - c_2 q_2. \quad (10)$$

The social optimal solution is determined by (10). The first order optimal conditions are given as follows:

$$\frac{\partial SW}{\partial q_1} = A - q_1 - \gamma q_2 - c_1(I_1) = 0 \quad (11)$$

$$\frac{\partial SW}{\partial I_1} = -q_1 \frac{\partial c_1(I_1)}{\partial I_1} - I_1 = 0, \quad (12)$$

$$\frac{\partial SW}{\partial q_2} = A - q_2 - \gamma q_1 - c_2 = 0. \quad (13)$$

Comparing (7)–(9) with (11)–(13), we draw the following conclusions:

Proposition 4. *The quantity of products under duopoly is lower than that of social optimality. Under equilibrium based on (7)–(9), social welfare in the industry is increased with market power while reduced with substitutability.*

Proof: Denote the solution of (7)–(9) by $(q_1^{C,*}, q_2^{C,*}, I_1^{C,*})$ and the corresponding social welfare by $SW^{C,*}$. Let the social optimal solution to (11)–(13) be $(\bar{q}_1, \bar{q}_2, \bar{I}_1)$ and the corresponding social welfare is $\bar{SW}$. Apparently, $A - q_1^{C,*} - \gamma q_2^{C,*} - c_1(I_1^{C,*}) = q_1^{C,*}$ and $A - q_2^{C,*} - \gamma q_1^{C,*} - c_2 = q_2^{C,*}$. Therefore, $\bar{q}_1 > q_1^{C,*}$ and $\bar{q}_2 > q_2^{C,*}$. From (12), we have $\bar{I}_1 > I_1^{C,*}$.

Given c_2, we have

$$\begin{aligned}\left.\frac{\partial SW}{\partial \tau}\right|_{(q_1^*, q_2^*, I_1^*)} &= \left.\frac{\partial SW}{\partial q_1}\frac{\partial q_1}{\partial \tau} + \frac{\partial SW}{\partial I_1}\frac{\partial I_1}{\partial \tau} + \frac{\partial SW}{\partial q_2}\frac{\partial q_2}{\partial \tau} - \frac{\partial c_1(I_1)}{\partial \tau} q_1\right|_{(q_1^*, q_2^*, I_1^*)} \\ &= \left.\frac{\partial (q_1 + q_2)}{\partial \tau} + g(I_1) q_1\right|_{(q_1^*, q_2^*, I_1^*)} > 0\end{aligned},$$

and

$$\left.\frac{\partial SW}{\partial \gamma}\right|_{(q_1^*, q_2^*, I_1^*)} = \left.\frac{\partial SW}{\partial q_1}\frac{\partial q_1}{\partial \gamma} + \frac{\partial SW}{\partial I_1}\frac{\partial I_1}{\partial \gamma} + \frac{\partial SW}{\partial q_2}\frac{\partial q_2}{\partial \gamma} - q_2 q_1\right|_{(q_1^*, q_2^*, I_1^*)}$$
$$= \left.\frac{\partial(q_1 + q_2)}{\partial \gamma} - q_2 q_1\right|_{(q_1^*, q_2^*, I_1^*)} < 0.$$

Therefore, the social welfare in this industry is increased with market power while reduced with higher substitutability or lower complementary.

Conclusions are therefore achieved and the proof is complete. ■

Remarks: Proposition 4 implies that the equilibrium products of the two firms are lower than those of the social optimum. There exists underinvestment in innovation. Under equilibrium, social welfare is increased with market power because of lower costs of the first firm. Social welfare is reduced with higher substitutability or lower complementary because larger parameter γ yields more fierce competition.

From Propositions 1–4, we learn that the firms benefit from innovation and social welfare is also promoted by innovation, which means government and firms should share the cost of innovation. That is the reason why the government spends a lot of their revenue in firms' innovations every year.

According to (7)–(9) and (11)—(13), governmental subsidies in innovation can efficiently improve quantity of products and social welfare.

B. Results Under Bertrand Price Competition.

Here the Bertrand price competition is discussed, in which the two firms compete in prices. If $\gamma^2 \neq 1$, (2) is restated as follows:

$$q_1 = \frac{A(1-\gamma) - p_1 + \gamma p_2}{1-\gamma^2}, \; q_2 = \frac{A(1-\gamma) - p_2 + \gamma p_1}{1-\gamma^2}. \tag{14}$$

If $\gamma^2 = 1$, $p_1 = p_2$ under Bertrand competition, which is an existing conclusion in the literature. So $\gamma^2 = 1$is not considered in the following. Hence, the substitutability is partial in this section:

By (3), (4) and (14), the two firms aim to solve the following problems:

$$\max_{p_1, I_1} \quad \pi_1^B = [p_1 - c_1(I_1)]\frac{A(1-\gamma) - p_1 + \gamma p_2}{1-\gamma^2} - \frac{1}{2}I_1^2, \tag{15}$$

$$\max_{p_2} \quad \pi_2^B = (p_2 - c_2)\frac{A(1-\gamma) - p_2 + \gamma p_1}{1-\gamma^2}. \tag{16}$$

Since both (15) and (16) are concave, there exists a unique solution which is determined by the first order conditions of (15)–(16). The corresponding first order optimal conditions are listed as follows:

$$\frac{\partial \pi_1^B}{\partial p_1} = g_1 = \frac{A(1-\gamma) + \gamma p_2 + c_1(I_1)}{1-\gamma^2} - \frac{2p_1}{1-\gamma^2} = 0, \tag{17}$$

$$\frac{\partial \pi_1^B}{\partial I_1} = g_2 = -\frac{A(1-\gamma) + \gamma p_2 - p_1}{1-\gamma^2}\frac{\partial c_1(I_1)}{\partial I_1} - I_1 = 0, \tag{18}$$

$$\frac{\partial \pi_2^B}{\partial p_2} = g_3 = \frac{A(1-\gamma) + \gamma p_1 + c_2}{1-\gamma^2} - \frac{2p_2}{1-\gamma^2} = 0. \tag{19}$$

Considering (17)–(19) and checking Propositions 1–4, we immediately have the following conclusions.

Proposition 5. *Propositions 1–4 all hold under Bertrand price competition.*

Proof: Similar to the proofs of Propositions 1–4. Based on (17)–(19), the corresponding conclusions are drawn. ■

Remarks: The above proposition manifests that the relationships between market power and price as well as quantity under Bertrand competition are similar to those under Cournot competition.

Denote the equilibrium under Bertrand competition by $(p_1^{B,*}, p_2^{B,*}, I_1^{B,*}, q_1^{B,*}, q_2^{B,*})$ and the corresponding social welfare by $SW^{B,*}$. We then have the following relationships.

Proposition 6. *Comparing the equilibrium under Bertrand with that under Cournot, we have* $p_1^{C,*} \geqslant p_1^{B,*}, p_2^{C,*} \geqslant p_2^{B,*}$, $q_1^{C,*} \leqslant q_1^{B,*}, q_2^{C,*} \leqslant q_2^{B,*}, I_1^{c,*} \leqslant I_1^{B,*}$ *and* $SW^{C,*} \leqslant SW^{B,*} \leqslant \bar{S}\bar{W}$.

Proof: See Appendix. ■

Remarks: The above conclusions illustrate that Bertrand competition seems to be much fiercer than Cournot competition both in prices and in innovative investment. Therefore, prices under Bertrand are lower than those under Cournot, while quantity of outputs, innovative investment and social welfare under Bertrand are all higher than those under Cournot.

Here an explanation is presented about the above conclusions. Under Bertrand competition, firms compete directly and this competition seems fiercer than Cournot competition. Therefore, prices under Bertrand are lower and quantities are higher than those under Cournot.

5. Concluding remarks

The conclusions in this article are consistent with some eminent prior research, including Arrow (1962), Qiu (1997), Tang (2006), and Vives (2008), although this research uses different models and different competition structures. That means the conclusions are quite robust. This article finds that innovation decreases with market power,[7] the same as Arrow (1962). It also finds higher substitutability yield lower total quantity of products in the industry. Using the Bowley linear demand system, Vives (2008) and Qiu (1997) reach similar conclusions. This research reaches the same conclusion as Qiu (1997) that price is lower and output is larger in Bertrand than in Cournot competition. And you can also find empirical support of the conclusions in Tang (2006). And different from Bester and Petrakis (1993), we need no social planner.

This study addresses the relationship between innovation and competition under product externality based on Arrow's (1962) innovation theory. For the different measures of market power, we measure it with cost advantage while Vives (2008) with Lerner index, this article contrasts with Vives's (2008) conclusion about the effect of market power on innovation. By a duopoly model both under quantity (Cournot) and price (Bertrand) competition, the conclusions, in which market power has negative effects on innovative investment, supports the empirical evidence in Tang (2006). Higher substitutability yields lower total quantity of products in the industry and lower profits of the firm with higher costs of production. The equilibrium products and innovative investment are all lower than social optimum. Social welfare is also reduced with higher substitutability because a larger externality parameter yields fiercer competition. These conclusions are robust under different models.

There are some further researching topics following this work. This study discusses the innovation of firms with cost advantage and it is interesting to extend to general situations. When two firms simultaneously launch innovative investment, it is interesting to capture. Besides, if positive externality (or product complementary) is strong enough, firms will be co-innovation or free-riding. These are our further research topics.

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Notes

1. Cost advantage can also be seen as market power. A firm with cost advantage has the power to change price first. The price is given in this study, but cost advantage can be use as competition threat to the competitor.
2. Only the cost advantage firm innovates is reasonable and the reason for the cost disadvantage one does not innovate is that it does not have enough money to innovate or the revenues are less than its costs when it innovates.
3. Noting that externalities here mean the interaction between products produced by different firms, including substitutability and complementary, which are different from the externality of innovation in of other studies such as Spence (1984) and Bester and Petrakis (1993).
4. Though Bertrand competition is fiercer than Cournot competition, the conclusions under both models are robust.
5. On the one hand, only one firm invests innovation in our model. On the other hand, we assume that firm makes innovation and output decision at the same time (or the firm makes output decision soon after innovation). So we set up a one-stage game model.
6. Which means we assumes simultaneous decision in the innovation stage between R&D and production (non-strategic R&D).
7. This study gets the same conclusion as Arrow (1962) but market power in this article is measured in different way from Arrow.

References

Armin, S. (2007). *The relation between competition and innovation – why is it such a mess*. Mimeo, Nov: University of Zurich. 2007.

Borghan, N., & Randal, W. (2011). The dynamics of innovation and horizontal differentiation. *Journal of Economic Dynamics & Control, 35*, 825–842.

Claude, D., & Alexis, J. (1998). Cooperative and noncooperative R&D in duopoly with spillovers. *American Economic Review, 78*, 1133–1137.

Dario, S., & Armin, S. (2011). Is there a U-shaped relation between competition and investment? *International Journal of Industrial Organization, 29*, 65–73.

Darshak, P., & Michael, W. R. (2011). Using patent citation patterns to infer innovation market competition. *Research Policy, 40*, 886–894.

Harold, D. (1969). Information and efficiency: Another viewpoint. *Journal of Law and Economics, 12*, 1–22.

Helmut, B., & Emmanuel, P. (1993). The incentives for cost reduction in a differentiated industry. *International Journal of Industrial Organization, 11*, 519–534.

Henry, W. X., & Bill, Y. Z. (2002). Cooperative and noncooperative R&D in vertically related markets. *Seoul Journal of Economics, 15*, 1–14.

Holmes, Thomas J., Levine, David K., & Schmitz, James A, Jr. (2012). Monopoly and the incentive to innovate when adoption involves switchover disruptions. *American Economic Journal-Microeconomics, 4*, 1–33.

James, B. A., & Barbara, S. J. (1983). Strategic commitment with R&D: The symmetric Case. *The Bell Journal of Economics, 14*, 225–235.

Jian-min, T. (2006). Competition and innovation behavior. *Research Policy., 35*, 68–82.

Kenneth, A. (1962). Economic welfare and the allocation of resources for inventions. In R. Nelson (Ed.), *The Rate and Direction of Inventive Activity* (pp. 609–626). Princeton, NJ: Princeton University Press.

Larry, Qiu D. (1997). On the dynamic efficiency of Bertrand and Cournot equilibria. *Journal of economic theory, 75*, 213–229.

Lin, L., & Henry, W. X. (2013). Entry bias and product substitutability, *Journal of Institutional and Theoretical Economics, 169*, 304–319.

Philippe, A., Nick, B., Richard, B., Rachel, G., & Peter, H. (2005). Competition and innovation: An inverted-U relationship. *Quarterly Journal of Economics, 120*, 701–728.

Pu-yan, N., & You-hua, C. (2012). Duopoly competitions with capacity constrained input. *Economic Modelling, 29*, 1715–1721.

Richard, G. J., & David, N. M. G. (1982). Preemptive patenting and the persistence of monopoly. *American Economic Review, 72*, 514–526.

Sang-Seung, Y. (1999). Market structure and incentives to innovate: The case of Cournot oligopoly. *Economic Letters, 65*, 379–388.

Schumpeter Joseph Alois. (1942). *Capitalism*. New York: Socialism and Democracy.

Spence, M. (1984). Cost Reduction, Competition, and Industry Performance. *Econometrica, 52*, 101–122.

Xavier, V. (2008). Innovation and competitive pressure. *The Journal of Industrial Economics, 56*, 419–469.

Yongmin, C., & David, S. E. M. (2010). Innovation in vertically related markets. *The Journal of Industrial Economics, 58*, 373–401.

Appendix

Proof of Propositions 1

Denote the Jacobian matrix of (7)–(9) to be:

$$D = \begin{bmatrix} \frac{\partial f_1}{\partial q_1} & \frac{\partial f_2}{\partial q_1} & \frac{\partial f_3}{\partial q_1} \\ \frac{\partial f_1}{\partial I_1} & \frac{\partial f_2}{\partial I_1} & \frac{\partial f_3}{\partial I_1} \\ \frac{\partial f_1}{\partial q_2} & \frac{\partial f_2}{\partial q_2} & \frac{\partial f_3}{\partial q_2} \end{bmatrix} = \begin{bmatrix} -2 & -\frac{\partial c_1(I_1)}{\partial I_1} & -\gamma \\ -\frac{\partial c_1(I_1)}{\partial I_1} & -q_1\frac{\partial^2 c_1(I_1)}{\partial (I_1)^2} - 1 & 0 \\ -\gamma & 0 & -2 \end{bmatrix}$$

Denote

$$D_\tau = \begin{bmatrix} \frac{\partial f_1}{\partial q_1} & \frac{\partial f_2}{\partial q_1} & \frac{\partial f_3}{\partial q_1} \\ \frac{\partial f_1}{\partial \tau} & \frac{\partial f_2}{\partial \tau} & \frac{\partial f_3}{\partial \tau} \\ \frac{\partial f_1}{\partial q_2} & \frac{\partial f_2}{\partial q_2} & \frac{\partial f_3}{\partial q_2} \end{bmatrix} = \begin{bmatrix} -2 & -\frac{\partial_1(I_1)}{\partial I_1} & -\gamma \\ -\frac{\partial c_1(I_1)}{\partial \tau} & -q_1\frac{\partial^2 c_1(I_1)}{\partial I_1 \partial \tau} & 0 \\ -\gamma & 0 & -2 \end{bmatrix}$$

Apparently, $\det D = \left[q_1 \frac{\partial^2 c_1(I_1)}{\partial (I_1)^2} + 1\right](-4+\gamma^2) + 2\left[\frac{\partial c_1(I_1)}{\partial I_1}\right]^2 < 0$. This inequality comes from the hypotheses that $0 > \frac{\partial c_1(I_1)}{\partial I_1} > -1$ and $\gamma \in [-1, 1]$. By the implicit function theorem, there exists a unique solution to (7)–(9), which is differentiable. Meanwhile,

$$\frac{\partial I_1}{\partial \tau} = -\frac{\det D_\tau}{\det D} < 0,$$

where

$$\begin{aligned}\det D_\tau &= -4q_1 \frac{\partial^2 c_1(I_1)}{\partial I_1 \partial \tau} + q_1 \frac{\partial^2 c_1(I_1)}{\partial I_1 \partial \tau}\gamma^2 + 2\frac{\partial c_1(I_1)}{\partial I_1}\frac{\partial c_1(I_1)}{\partial \tau} \\ &= 4q_1 \frac{dg(I_1)}{dI_1} - q_1 \frac{dg(I_1)}{\partial I_1}\gamma^2 - 2\frac{\partial c_1(I_1)}{\partial I_1} g(I_1) \\ &= \frac{dg(I_1)}{dI_1}[4q_1 - q_1\gamma^2 - 2c_1 g(I_1)] < 0.\end{aligned}$$

$$\frac{\partial q_1}{\partial \tau} = -\frac{\det\begin{bmatrix} -\frac{\partial c_1(I_1)}{\partial \tau} & -q_1 \frac{\partial^2 c_1(I_1)}{\partial I_1 \partial \tau} & 0 \\ -\frac{\partial c_1(I_1)}{\partial I_1} & -q_1 \frac{\partial^2 c_1(I_1)}{\partial (I_1)^2} - 1 & 0 \\ -\gamma & 0 & -2 \end{bmatrix}}{\det D} > 0,$$

$$\frac{\partial q_2}{\partial \tau} = -\frac{\det\begin{bmatrix} -2 & -\frac{\partial c_1(I_1)}{\partial I_1} & -\gamma \\ -\frac{\partial c_1(I_1)}{\partial I_1} & -q_1 \frac{\partial^2 c_1(I_1)}{\partial (I_1)^2} - 1 & 0 \\ -\frac{\partial c_1(I_1)}{\partial \tau} & -q_1 \frac{\partial^2 c_1(I_1)}{\partial I_1 \partial \tau} & 0 \end{bmatrix}}{\det D} < 0.$$

Obviously, we have $\frac{\partial (q_1+q_2)}{\partial \tau} > 0$.

Conclusions are therefore achieved and the proof is complete. ■

Proof of Propositions 2

According to (7)–(9) and the analysis in Proposition 1, we have

$$\begin{aligned}\frac{\partial q_1}{\partial \gamma} &= -\frac{\det\begin{bmatrix} -q_2 & 0 & -q_1 \\ -\frac{\partial c_1(I_1)}{\partial I_1} & -q_1 \frac{\partial^2 c_1(I_1)}{\partial (I_1)^2} - 1 & 0 \\ -\gamma & 0 & -2 \end{bmatrix}}{\det D} \\ &= -\frac{\left[-q_1 \frac{\partial^2 c_1(I_1)}{\partial (I_1)^2} - 1\right](2q_2 - \gamma q_1)}{\det D} \begin{cases} < 0 & 2q_2 > \gamma q_1 \\ \geqslant 0 & 2q_2 \leqslant \gamma q_1. \end{cases}\end{aligned}$$

Therefore, q_1 decreases with γ if $2q_2 > \gamma q_1$ while increases with γ if $2q_2 \leqslant \gamma q_1$.

$$\begin{aligned}\frac{\partial q_2}{\partial \gamma} &= -\frac{\det\begin{bmatrix} -2 & -\frac{\partial c_1(I_1)}{\partial I_1} & -\gamma \\ -\frac{\partial c_1(I_1)}{\partial I_1} & -q_1 \frac{\partial^2 c_1(I_1)}{\partial (I_1)^2} - 1 & 0 \\ -q_2 & 0 & -q_1 \end{bmatrix}}{\det D} \\ &= -\frac{\left[-q_1 \frac{\partial^2 c_1(I_1)}{\partial (I_1)^2} - 1\right](2q_1 - \gamma q_2) + q_1\left[\frac{\partial c_1(I_1)}{\partial I_1}\right]^2}{\det D} < 0.\end{aligned}$$

The above inequality follows from $q_1 > q_2$, $\gamma < 1$ and $|\frac{\partial c_1(I_1)}{\partial I_1}| < 1$ (or $0 > \frac{\partial c_1(I_1)}{\partial I_1} > -1$). Apparently, because $\gamma < 1$, we have $\frac{\partial q_1}{\partial \gamma} + \frac{\partial q_2}{\partial \gamma} < 0$. Combined with $q_2 = p_2 - c_2$, we therefore have that both $q = q_1 + q_2$ and price p_2 decrease with γ.

$$\frac{\partial I_1}{\partial \gamma} = -\frac{\det\begin{bmatrix} -2 & -\frac{\partial c_1(I_1)}{\partial I_1} & -\gamma \\ -q_2 & 0 & -q_1 \\ -\gamma & 0 & -2 \end{bmatrix}}{\det D} = -\frac{\frac{\partial c_1(I_1)}{\partial I_1}(2q_2 - \gamma q_1)}{\det D}\begin{cases} <0 & 2q_2 > \gamma q_1 \\ \geq 0 & 2q_2 \leq \gamma q_1 \end{cases}$$

I_1 decreases with γ if $2q_2 > \gamma q_1$ but increases with γ if $2q_2 \leqslant \gamma q_1$. Conclusions are therefore achieved and the proof is complete. ■

Proof of Propositions 3

We first show the relationship $\left|\frac{\partial q_1}{\partial \gamma}\right| < q_1$ based on the above analysis.

$$\left|\frac{\partial q_1}{\partial \gamma}\right| = \left|\frac{\left[-q_1\frac{\partial^2 c_1(I_1)}{\partial (I_1)^2} - 1\right](2q_2 - \gamma q_1)}{\left[-q_1\frac{\partial^2 c_1(I_1)}{\partial (I_1)^2} - 1\right](4-\gamma^2) + 2\left[\frac{\partial c_1(I_1)}{\partial I_1}\right]^2}\right| < \left|\frac{\left[-q_1\frac{\partial^2 c_1(I_1)}{\partial (I_1)^2} - 1\right](2q_2 - \gamma q_1)}{\left[-q_1\frac{\partial^2 c_1(I_1)}{\partial (I_1)^2} - 1\right](2-\gamma^2)}\right| < q_1.$$

We therefore have $\frac{\partial \pi_2}{\partial \gamma} = -q_1 q_2 - \gamma q_2 \frac{\partial q_1}{\partial \gamma} < 0$. Conclusions are therefore achieved and the proof is complete. ■

Proof of Propositions 4

Here we show Proposition 4. By virtue of (17)–(19), we immediately have the following system of equations, which are the first order optimal conditions of (17)–(19).

$$\frac{\partial \pi_1^B}{\partial p_1} = \frac{\partial \pi_1^C}{\partial q_1}\frac{\partial q_1}{\partial p_1} + \frac{\partial \pi_1^C}{\partial q_2}\frac{\partial q_2}{\partial p_1} + \frac{\partial \pi_1^C}{\partial I_1}\frac{\partial I_1}{\partial p_1} = 0, \frac{\partial \pi_1^B}{\partial I_1} = -q_1\frac{\partial c_1(I_1)}{\partial I_1} - I_1 = 0,$$

$$\frac{\partial \pi_2^B}{\partial p_2} = \frac{\partial \pi_1^C}{\partial q_1}\frac{\partial q_1}{\partial p_2} + \frac{\partial \pi_1^C}{\partial q_2}\frac{\partial q_2}{\partial p_2} + \frac{\partial \pi_1^C}{\partial I_1}\frac{\partial I_1}{\partial p_2} = 0.$$

Using the above equation, (2), (5) and (14) yield the relationship $\left.\frac{\partial \pi_1^C}{\partial q_2}\right|_{(q_1^{B,*},q_2^{B,*},I_1^{B,*})} = -\gamma q_1, \left.\frac{\partial q_2}{\partial p_1}\right|_{(q_1^{B,*},q_2^{B,*},I_1^{B,*})} = \frac{\gamma}{1-\gamma^2}, \left.\frac{\partial q_1}{\partial p_1}\right|_{(q_1^{B,*},q_2^{B,*},I_1^{B,*})} = \frac{-1}{1-\gamma^2}$. and $\left.\frac{\partial \pi_1^C}{\partial I_1}\right|_{(q_1^{B,*},q_2^{B,*},I_1^{B,*})} = 0$. By virtue of the equation $\left.\frac{\partial \pi_1^B}{\partial p_1}\right|_{(q_1^{B,*},q_2^{B,*},I_1^{B,*})} = 0$, we therefore have $\left.\frac{\partial \pi_1^C}{\partial q_1}\right|_{(q_1^{B,*},q_2^{B,*},I_1^{B,*})} < 0$. According to the concavity of π_1^C, $\left.\frac{\partial \pi_1^C}{\partial q_1}\right|_{(q_1^{B,*},q_2^{B,*},I_1^{B,*})} < 0$, and $\left.\frac{\partial \pi_1^C}{\partial q_1}\right|_{(q_1^{C,*},q_2^{C,*},I_1^{C,*})} = 0$, we immediately have the relationship $q_1^{C,*} \leq q_1^{B,*}$. Similarly, we have $q_2^{C,*} \leq q_2^{B,*}$.

From (18), $q_1^{C,*} \leq q_1^{B,*}, q_2^{C,*} \leq q_2^{B,*}$. and the convexity of $c_1(I_1)$ yields. $I_1^{c,*} \leq I_1^{B,*}$. $q_1^{C,*} \leq q_1^{B,*}$, $q_2^{C,*} \leq q_2^{B,*}, I_1^{c,*} \leq I_1^{B,*}$. and (2) imply $p_1^{C,*} \geqslant p_1^{B,*}$ and $p_2^{C,*} \geqslant p_2^{B,*}$.

$p_1^{C,*} \geqslant p_1^{B,*}, p_2^{C,*} \geqslant p_2^{B,*}$, $q_1^{C,*} \leq q_1^{B,*}, q_2^{C,*} \leq q_2^{B,*}$ and $I_1^{c,*} \leq I_1^{B,*}$. Induce $SW^{C,*} \leqslant SW^{B,*}$.

Combining with the conclusion of Proposition 5, we have.$SW^{B,*} \leq \bar{S}\bar{W}$.

Conclusions are therefore achieved and the proof is complete. ■