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Fan chart or Monte Carlo simulations for assessing the uncertainty of inflation forecasts in Romania?

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Fan chart or Monte Carlo simulations for assessing the uncertainty of inflation forecasts in Romania?

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The objective of this paper is to make a comparison between two methodologies used to assess the forecasts of uncertainty: a numerical method based on Monte Carlo simulations and a graphical representation represented by the fan charts. For the inflation rate predictions made for Romania over the period Q4:2012– Q4:2013, a fan chart based on BVAR models with non-informative priors presents a lower degree of uncertainty compared with a fan chart using VAR models. The numerical procedure is based on forecasts that use auto-regressive models and the Monte Carlo method. In this case, the probabilities that the inflation forecasts are greater than the National Bank of Romania's (NBR's) target and the previous value increased from a quarter to the next. Therefore, this method of assessing the forecast uncertainty is a better tool than the fan charts. Moreover, the simple NBR methodology that did not take into account the probability distribution of the forecasts should be replaced by the fan charts. The forecast's uncertainty assessment is necessary for the establishment of monetary policy.

Keywords: uncertainty; fan chart; BoE methodology; two piece normal density; Monte Carlo method

JEL classification: E37, C54

1. Introduction

In literature there are two ways of assessing a forecast's uncertainty: a numerical approach and a graphical representation. In this article, our objective is to make a comparison between the two approaches. The numerical approach consists of using the Monte Carlo simulation to determine the probability of having a predicted inflation rate greater than a fixed value (usually the target established by the Central Bank). The graphical representation refers to the construction of a fan chart that was introduced by the Bank of England. Global output (GDP), inflation and the interest rate are variables for which the forecast's uncertainty is mostly evaluated. According to Bratu (2012b) the researchers are more interested, in the context of the recent crisis, in getting the most suitable strategy for decreasing the degree of uncertainty of macroeconomic forecasts.

Our research is oriented around the quarterly predictions made for the inflation rate in Romania, a country that uses the inflation targeting system. The fan chart was not used until now by the National Romanian Bank because of the difficult methodology – the methodology used by the bank having many poor points. Our proposed methodology based on fan charts and Monte Carlo simulations is a more accurate mirror of an

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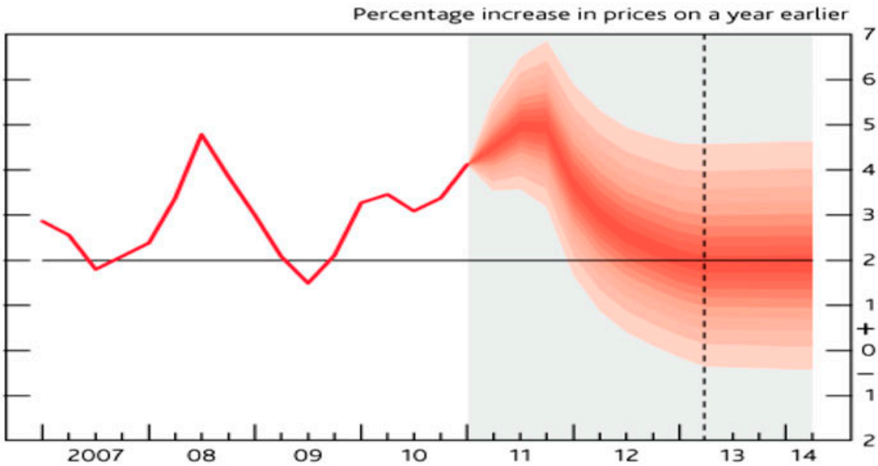

Figure 1. Fan chart for the consumer price index forecasts made by the Bank of England in 2011.[1]
Source: Bank of England http://www.bankofengland.co.uk/publications/inflationreport/irfanch.htm.

inflation-predicting process from Romania. In Figure 1 we can see the representation of a fan chart for England in 2011.

2. Literature review

For a central bank, the forecasting process is a key element for establishing best policies. However, all predictions are affected by uncertainty. Therefore, the main objective is to reduce the degree of uncertainty. The fan chart is a map of different possible evolutions of a phenomenon, and is based on probability distribution. Ericsson (2001) shows that although the literature uses the expression 'forecast uncertainty', the correct one is 'the uncertainty of forecasting errors', because certain values for a future phenomenon are given, but we do not know what the error associated with predictions is.

Ericsson (2001) made a detailed description of the forecast uncertainty problem, providing important information, such as establishing the definition of forecast uncertainty, the main measures of evaluation and its consequences. The author defined the concept of uncertainty as the variance that different results registered for certain indicators with respect to predicted values. In other words, the uncertainty reflects the difference between the actual recorded values and the projected ones. According to Bratu (2012a) the assessment of forecast uncertainty for macroeconomic indicators is necessary for the improvement of the decisional process made in the establishment of monetary and other governmental policies.

According to the study of Vega (2003), the literature in the forecasts domain has begun to pay particular attention to density forecasts over the last 20 years. Given the asymmetry of risk, it was considered necessary the pass from point forecasts to a complete representation of the probability distribution.

Novo and Pinheiro (2003) point out that a first measurement of uncertainty was achieved by the Bank of England in 1996 by publishing estimates for the probability distribution of expected values for inflation and GDP. After the Bank of England initiative, many central banks represent the density forecast using a graphic called the 'width chart' or 'fan chart'. Basically, the fan chart shows the probability distribution of the forecast variable or many prediction intervals determined for different probabilities.

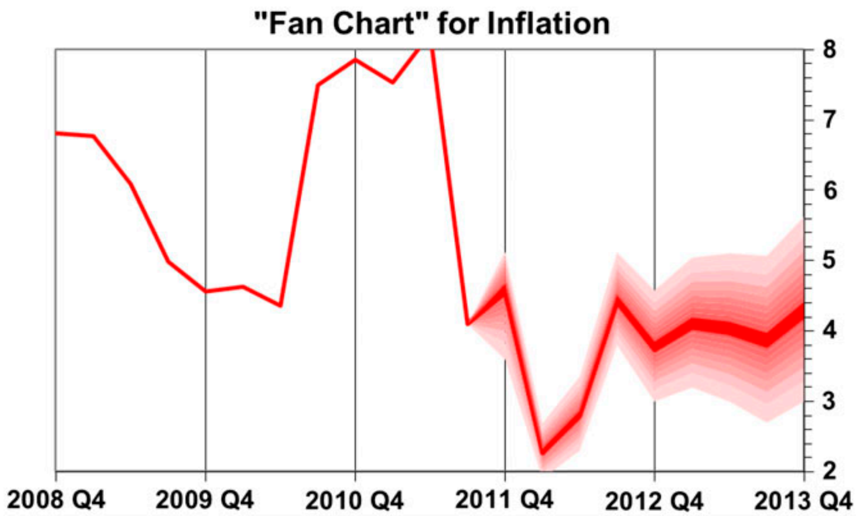

Figure 2. Fan chart for projected rates of inflation in Romania based on a VAR model (Q4: 2012–Q4:2013).
Source: own computations.

Descriptions of statistical methods used by the Bank of England (BoE) and Sweden to build fan charts are made by Britton, Fisher, and Whitley (1998). In Figure 2 a fan chart for inflation rate in Romania based on a VAR model is displayed.

The depth of the shadow is associated to the probability of the density function. The darkest portion of the band covers about 10% of probability, including the central projection. Over time, the uncertainty increases and the band widens. Each successive pair of bands must cover about 10% of probability, in aggregate not exceeding 90% of probability.

For building densities, some parametric methods are used, with the measure of risk or uncertainty being given by the value of the density forecast parameters. The problem of measuring risk and uncertainty was most approached in the context of inflation targeted by central banks. In this case, the risk is associated with the probability that forecast inflation be higher or lower than three reference measures: core forecast, targeted inflation, and targeted inflation approximations.

Since the introduction of fan charts in 1996, in an inflation report by the Bank of England, these charts have been studied by many authors. For example, Wallis and Hatch made detailed researches about fan charts and the Bank of England forecasts.

Although the methodology proposed by Britton, Fisher, and Whitley (1998) is the best known, Cogley, Morozov, and Sargent (2003) used the minimum entropy method to obtain essential information in predicting inflation, and they compared densities forecasts with the fan charts made by the Bank of England. Cogley, Morozov, and Sargent (2003) started from the forecast densities generated using a BVAR model with stochastic variances and coefficients of deviation. Then, these densities are modified by introducing additional information obtained using a relative entropy method proposed by Robertson et al. (2005).

Garratt, Lee, Pesaran, and Shin (2000) calculated, using a small macro-econometric model, the densities forecast for England's inflation and output.

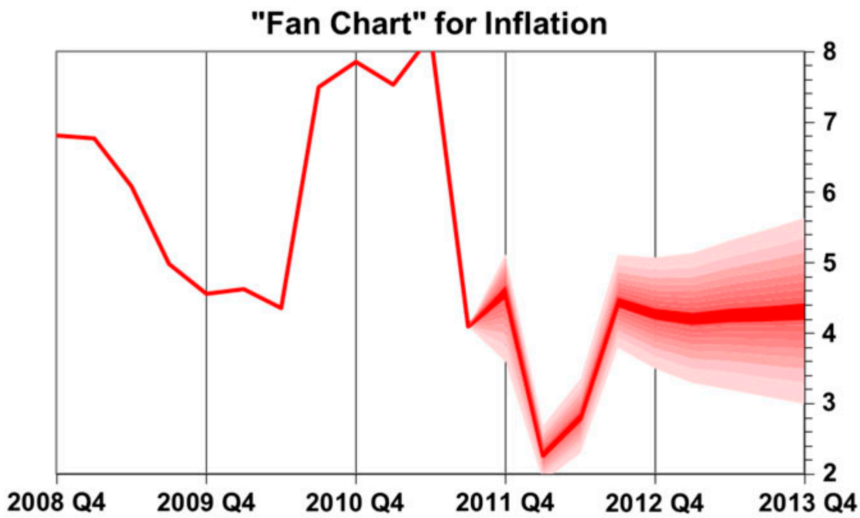

Figure 3. Fan chart for projected rates of inflation in Romania based on a BVAR model (Q4: 2012–Q4:2013).
Source: own computations.

In Figure 3 the representation of a fan chart for inflation rate in Romania based on a BVAR model can be seen. The method used to build a fan chart developed by the Bank of Sweden, Riskbank is based on a non-diagonal covariance matrix. Between conditioning variables, linear correlations appear that do not influence the asymmetry of predicted variable distributions. For small dimensions there is also the case when the correlation influences the asymmetry. Novo and Pinheiro (2003) made two key changes in the assumptions of BoE methodology, showing two deficiencies: a linear combination of the modal values of input variables is a poor approximation of the forecast errors mode, when the initial distribution is asymmetric and the hypothesis of independence of errors is too restrictive.

Wallis (2004) evaluated the density for the forecasts of the Bank of England, as well as for those of the National Institute of Economic and Social Research. The author concludes that for both institutions the forecasts' central tendency is biased and the forecasts' density overestimates their uncertainty.

Wallis (2004) and Clements concluded that if a forecast horizon is only a year, the probability for a high inflation rate is overestimated. Elder, Kapetanios, Taylor, and Yates (2005) showed that this probability was overestimated for GDP even if the horizons are smaller than a year. Furthermore, from their research, there resulted an overestimation of the variance of forecasted GDP. Gneiting and Ranjan (2011) observed an overestimation of uncertainty. Gneiting, Balabdaoui, and Raftery (2007) study the probabilistic calibration property. Although the Bank of England did not take into account the resolution analysis, Mitchell and Wallis (2011) consider it very important. Dowd (2008) analysed the fan charts built for GDP and concluded that for a short forecast horizon the risk is less captured. Since 2008, the European Central Bank has represented its inflation using fan charts. Osterholm (2008) built a VAR model for the Swedish economy, with parameter estimation being based on the Bayesian technique in order to formalise the prediction uncertainty. The major advantage of the Bayesian approach is given by the fact that the posterior forecast density interpretation is equivalent to that of a fan chart.

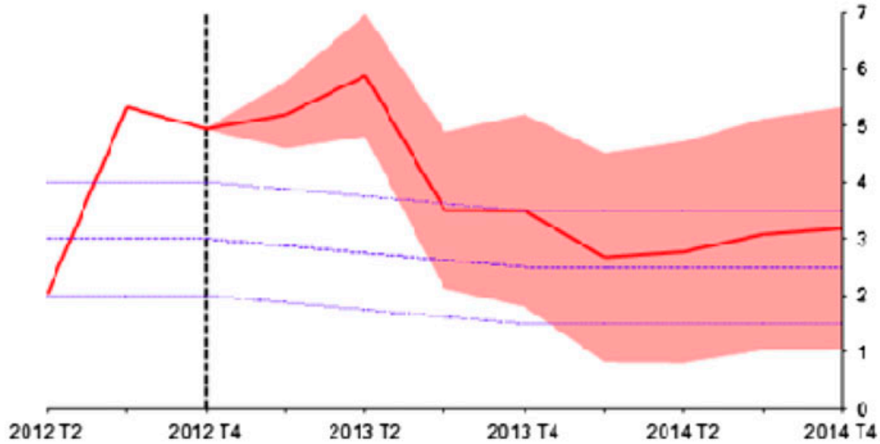

Figure 4. The interval of uncertainty for projections of inflation rate in Romania (from 2012–Q1 2014 Q4).
Source: National Bank of Romania (www.bnr.ro).

In Figure 4 the graph proposed by NBR is displayed in order to suggest the degree of uncertainty. Galbraith and van Norden (2011) evaluated the forecast probabilities using the densities published by the Bank of England and they made their graphical representation, measuring how much they exceeded a threshold. The authors evaluated their resolution and calibration, showing the relative performance of forecasts also using the low resolution for output (GDP) predictions.

Important contributions in the literature related to the fan charts can be found in King (2004), who builds these graphics for longevity, and Scherbov and Sanderson (2004), who use fan charts to represent the life expectancy. Beyond the frame of inflation evaluation, fan charts are used to describe the probability density for future survival rates for men in a study made by Blake, Cairns, and Dowd (2008).

The Monte Carlo method is actually often used in uncertainty analysis. It is a sampling method that supposes to generate the input distribution. The simulations' values can be analysed as probability distributions or can be transformed in order to get reliability forecasts, confidence intervals, tolerance areas or error bars.

Buhlmann (2002) showed that the bootstrap technique is another method of generating a sample distribution that can be used when the type of repartition is not known. The bootstrap technique supposes the replacements of elements from the sample, with each observation having the same probability of being selected. The means of all generated samples are registered. A larger normally distributed population is chosen and its parameters are estimated and the repartition of sample means are determined. Bilan, Gazda, and Godziszewski (2012) show a high uncertainty for the output gap forecasts in Poland and Ukraine.

Lanser and Kranendonk (2008) modelled four sources of uncertainty, first theoretically, for each model, specifying the corresponding disturbance by probability density. After the theoretical presentation, the authors assess the sources of uncertainty for the Saffier model, the quarterly macroeconomic model of the Dutch Bureau for Economic Policy Analysis. This institution has assessed, since 1991, the quality of its macroeconomic forecasts, based on stochastic simulations, and produced many works about the exogenous variables, parameters and error models uncertainty. In Table 1 a description of notations is presented.

Table 1. Notations used to build the fan chart.

Current No.	Notation	Significance
1.	h	Forecast horizon
2.	X_t^i	Factor that affects the inflation rate
3.	μ_t^i	More likely value for the ith factor at time t
4.	p_t^i	Balance of risk for the ith factor at time t
5.	σ_t^i	Forecast error standard deviation for the ith factor at time t
6.	n2	Number of factors for which p_t^i = 0,5
7.	π_t^i	Inflation rate at time t
8.	μ_t^π	More likely inflation rate at time t
9.	p_t^π	Balance of risk for the inflation rate at time t
10.	σ_t^π	Forecast error standard deviation for the inflation rate at time t
11.	φ_j^i	Response of π_{t+j} to an impulse in X_t^i
12.	ξ_t^i	Bias indicator of ith factor at time t
13.	ξ_t^π	Bias indicator of the inflation rate at time t
14.	γ_t^i	Inverse bias indicator of ith factor at time t
15.	γ_t^π	Inverse bias indicator of the inflation rate at time t

Source: Author's own calculation.

3. The Bank of England (BoE) approach for fan charts

For variables that must be forecasted, variables called 'input variables', we determine the probability distribution, which will be aggregated. Two types of errors are measured, errors that Novo and Pinheiro (2003) classify as:

- errors of conditioning variables (rate of inflation, interest rate, consumption etc.);
- pure errors occurring in variables (they are calculated eliminating the first category of errors from the all measured errors).

The assumptions on which the BoE approach is based are, according to Britton, Fisher, and Whitley (1998):

- the representation of error prediction as a linear combination of the variables to forecast;
- input variables are independent;
- marginal probability distributions of input variables consist of two normally distributed parts, an aspect described in literature by the expression 'two-piece normal distribution' (TPN) or split normal distribution;
- TPN distribution has three parameters: mode (μ) and two standard deviations, to the right and left (σ_1 *şi* σ_2).

Since inflation rates are not symmetrically distributed around the most probable value, Britton, Fisher, and Whitley (1998) justify the need of the split normal distribution (TPN), which shows that the prediction error works only in one way.

Banerjee and Das (2011) show that in order to build the forecast distributions, the specification of three parameters is necessary: a measure of central tendency, an estimate of degree of uncertainty, a presentation of the balance of risk. In Table 2 the two types of probabilities are displayed for Q4:2012-Q4:2013.

Table 2. The probabilities of getting inflation rates greater than some reference values in Romania.

Quarter	Probability *P*	Probability *P*'
Q4:2012	0.5023	0.5145
Q1:2013	0.5112	0.5155
Q2:2013	0.513	0.5162
Q3:2013	0.5132	0.5171
Q4:2013	0.514	0.535

Source: own computations.

(a) an appropriate measure of central tendency

The modal value is often chosen, because it is the most likely value to maximise the probability density. However, the mode uses only a part of the information contained in the database and it does not have the average asymptotic significance, a fact that generates problems in achieving the inference when the sample distribution is not known. When there are multiple modal values, only one of these will be chosen, which limits the efficiency in using the mode.

(b) Banerjee and Das (2011) recommended the use of dispersion to quantify the degree of uncertainty (how different the forecasted values are from the central value) at the expense of average absolute error or interquartile deviation.

(c) The risk can be symmetrically distributed around the central tendency or it can be unbalanced, when the mode and the average differ.

In the following, the methodology proposed by the BoE will be analysed and described by Novo and Pinheiro (2003), following two directions: the linear combinations and the TPN aggregation.

3.1. Linear combinations from BoE approach

If y is the variable for which the forecast is realised, and the forecasting horizon is H, then the forecast value will be denoted by y_{t+H}. The vectors of $(1\times K)$ dimension for different paths of the conditioning variables are: x_{t+H}, $h = 1,2,...,H$. The central forecast for the 0 version is: $\hat{y}(x^0_{t+1}, x^0_{t+2}, ..., x^0_{t+H})$. This prediction results from a variety of econometric models. A local linear approximation is influenced by changes in conditioning variables: $\tilde{y}(x^1_{t+H}, ..., x^1_{t+1}) = \hat{y}(x^0_{t+H}, ..., x^0_{t+1}) + \beta'_0(x^1_{t+H} - x^0_{t+H}) + \beta'_1(x^1_{t+H-1} - x^0_{t+H-1}) + ... + \beta'_{H-1}(x^1_{t+1} - x^0_{t+1})$ where $\{x^1_{t+1}, x^1_{t+2}, ..., x^1_{t+H}\}$ is an alternative to conditional variances. The estimated effects of y_{t+h} to the change of different factors included in x_{t+h-i} are called interim multipliers denoted with $\beta_i, i = 0, ..., H-1$. The pure forecast error is $\varepsilon_{t+H} = y_{t+H} - \tilde{y}(x_{t+H}, x_{t+H-1,...,} x_{t+1})$. This error, according to Novo and Pinheiro (2003), aggregates the following components:

- estimated errors interim multipliers;
- errors generated by the approximation of a nonlinear model with a linear one;
- errors of misspecification;
- economic shocks in the forecasting horizon.

Taking into account the first relation, the total forecast error is: $e_{t+H} = y_{t+H} - \hat{y}(x_{t+H}, x_{t+H-1,...,} x_{t+1}) = \beta'_0(x^1_{t+H} - x^0_{t+H}) + \beta'_1(x^1_{t+H-1} - x^0_{t+H-1}) + ... + \beta'_{H-1}(x^1_{t+1} - x^0_{t+1}) + \varepsilon_{t+H}$. If y_{t+h} is a vector of independent variables, then the first relation is written (Γ_i is the matrix of coefficients of final form linear combinations): $\tilde{y}(x^1_{t+H}, \ldots, x^1_{t+1}) = \hat{y}(x^0_{t+H}, \ldots, x^0_{t+1}) + \Gamma_0(x^1_{t+H} - x^0_{t+H}) + \Gamma_1(x^1_{t+H-1} - x^0_{t+H-1}) + \ldots + \Gamma_{H-1}(x^1_{t+1} - x^0_{t+1})$. Using a similar reasoning we obtain: $e_{t+H} = y_{t+H} - \hat{y}(x_{t+H}, x_{t+H-1,\ldots,}x_{t+1}) = \Gamma_0 (x^1_{t+H} - x^0_{t+H}) + \Gamma_1(x^1_{t+H-1} - x^0_{t+H-1}) + \ldots + \Gamma_{H-1}(x^1_{t+1} - x^0_{t+1}) + \varepsilon_{t+H}$ If we consider a dynamic model with simultaneous equations in structural form, the residual vector, u_{t+H}, is an approximation of the central forecast. As a consequence of the shocks appearing in the residual variable after i periods, the predictors modify because of the shocks in the residuals. The changes in predictors are measured by the matrix. In these conditions, the vector of pure errors can be written as: $\varepsilon_{t+H} = \psi_0 u_{t+H} + \psi_1 u_{t+H\text{-}1} + \ldots + \psi_{H\text{-}1} u_{t+1}$. Finally, we reach the following relation, which is crucial from the perspective that it breaks down the total error in the error of conditioning variables and the pure error: $e_{t+H} = \psi_0 u_{t+H} + \psi_1 u_{t+H-1} + \ldots + \psi_{H-1} u_{t+1} + \Gamma_0(x^1_{t+H} - x^0_{t+H}) + \Gamma_1(x^1_{t+H-1} - x^0_{t+H-1}) + \ldots + \Gamma_{H-1}(x^1_{t+1} - x^0_{t+1}) + \varepsilon_{t+H}$ The importance of these linear equations is determined by the fact that on their bases, marginal probability distributions are determined and fan charts and confidence bands are drawn.

3.2. TPN aggregation in BoE methodology

A random variable, z, has a two-piece normal distribution (TPN) with parameters (μ, σ_1, σ_2). If its probability density function exists, it verifies: $f(z) = \alpha_1 \varphi(z/\mu, \sigma_1^2),\ z \leqslant \mu$, where $\varphi(z/\mu, \sigma_i^2)$is the probability density of a normal distribution with parameters (μ, σ_i) and $\alpha_i = \frac{\sigma_i}{\sigma_1+\sigma_2}$. John (1982) shows the following relations for average, variance and the moment of order 3:

$$E(z) = \mu + (\sigma_2 - \sigma_1)\sqrt{\frac{2}{\pi}}, \tag{1}$$

$$Var(z) = \left(1 - \frac{2}{\pi}\right)(\sigma_2 - \sigma_1)^2 + \sigma_1\sigma_2, \tag{2}$$

$$M_3 = (\sigma_2 - \sigma_1)\sqrt{\frac{2}{\pi}}\left[\left(\frac{4}{\pi} - 1\right)(\sigma_2 - \sigma_1)^2 + \sigma_1\sigma_2\right]. \tag{3}$$

If the standard deviations are different, the distribution is asymmetrical, and in the case of equal deviations classical normal distribution results, which is symmetric.

In vectorial terms, if $\boldsymbol{e}$ is the total error, $\boldsymbol{z}$ the vector of input variables and $\boldsymbol{a}$ the vector of coefficients, we can write the linear combination using the relation: $\boldsymbol{e} = \boldsymbol{az}$. Modal values of input variables are zero.

We consider the variance and the mode quantile, denoted, Var(z_n) and $P(z_n) \leqslant Mo(z_n)$ respectively. In order to calculate the average of input variables it is necessary to determine the standard deviations, which are obtained by solving the following equation system:

After solving the above equation system we can determine the mean of the input variables:

$$E(z_n) = (\sigma_{2n} - \sigma_{1n})\sqrt{\frac{2}{\pi}} \tag{4}$$

The total forecast error mean is zero. Given the assumption of independence, the variance of error e is obtained by summing the weighted variances of the input variables.

We approximate error distribution by TPN, the values of standard deviations being determined by solving the following system:

$$\sum_n a_n E(z_n) = (\sigma_2 - \sigma_1)\sqrt{\frac{2}{\pi}} \tag{5}$$

$$\sum_n a_n^2 Var(z_n) = (\sigma_2 - \sigma_1)^2 \left(1 - \frac{2}{\pi}\right) + \sigma_1 \cdot \sigma_2 \tag{6}$$

There are several parameterisations of the split normal distribution (TPN). Banerjee and Das (2011) proposed two equivalent parameterisations. In a first variant of parameterisation, the probability is calculated as:

$$\begin{gathered} P(li \leqslant x \leqslant ls) = \frac{2\sigma_1}{\sigma_1 + \sigma_2}\left[\varphi\left(\frac{ls-\mu}{\sigma_1}\right) - \varphi\left(\frac{li-\mu}{\sigma_1}\right)\right],\ pentru\ li < ls \leqslant \mu \\ P(li \leqslant x \leqslant ls) = \frac{2\sigma_1}{\sigma_1 + \sigma_2}\left[\varphi\left(\frac{ls-\mu}{\sigma_2}\right) - \varphi\left(\frac{li-\mu}{\sigma_2}\right)\right],\ pentru\ \mu \leqslant li < ls \\ P(li \leqslant x \leqslant ls) = \frac{2\sigma_1}{\sigma_1 + \sigma_2}\left[\sigma_2 \cdot \varphi\left(\frac{ls-\mu}{\sigma_2}\right) - \sigma_1 \cdot \varphi\left(\frac{li-\mu}{\sigma_1}\right) + \frac{\sigma_1 - \sigma_2}{2}\right],\ pentru\ li \leqslant \mu < ls \end{gathered} \tag{7}$$

We will specify the version of parameterisation in which the distribution has three parameters: the mode (Mo), the uncertainty or the standard deviation (σ) and the skewness or asymmetry (Y). The probability density has the following form:

$$\begin{gathered} f_X(x; Mo, \sigma, Y) = \frac{A}{\sigma\sqrt{2\pi}} e^{-\frac{1-\gamma}{2\sigma^2}(x-Mo)^2},\ x \leqslant Mo \\ f_X(x; Mo, \sigma, Y) = \frac{A}{\sigma\sqrt{2\pi}} e^{-\frac{1+\gamma}{2\sigma^2}(x-Mo)^2},\ x > Mo \end{gathered} \tag{8}$$

$-1 < Y < 1$ is the inverse of the skewness coefficient, and A is a normalisation constant. The formulas to calculate the standard deviations of a split normal distribution are given by:

$$\sigma_1 = \sqrt{\frac{\sigma^2}{1-\gamma}} \text{ and } \sigma_2 = \sqrt{\frac{\sigma^2}{1+\gamma}} \tag{9}$$

(1) *Dacă* $Y > 0$, $\sigma_1 > \sigma_2$ *= > biased to the left distribution*
(2) *Dacă* $Y < 0$, $\sigma_1 < \sigma_2$ *= > biased to the right distribution*
(3) *Dacă* $Y = 0$, $\sigma_1 = \sigma_2$ *= > normal distribution*

$$\begin{aligned} \text{The balance of risk}: \ p = \frac{\sigma_1}{\sigma_1 + \sigma_2} = \frac{\frac{\sigma}{\sqrt{1-\gamma}}}{\frac{\sigma}{\sqrt{1-\gamma}} + \frac{\sigma}{\sqrt{1+\gamma}}} = \frac{1}{1 + \sqrt{\frac{1-\gamma}{1+\gamma}}} = \ > \frac{1}{p} = 1 + \sqrt{\frac{1-\gamma}{1+\gamma}} \\ = \ > 1 + \frac{1-\gamma}{1+\gamma} = \frac{(1-p)^2}{p^2} + 1 = \ > \frac{2}{1+\gamma} = \frac{1 - 2p + 2p^2}{p^2} = \ > \gamma \\ = \frac{2p-1}{2p^2 - 2p + 1}. \xi \text{ is the skewness indicator.} \end{aligned}$$

$$Thus,\ \xi = \overline{Mo} - Mo = (\sigma_2 - \sigma_1)\sqrt{\frac{2}{\pi}} = \left(\sqrt{\frac{\sigma^2}{1+\gamma}} - \sqrt{\frac{\sigma^2}{1-\gamma}}\right)\cdot\sqrt{\frac{2}{\pi}},\ \beta = \xi^2\frac{\pi}{2\sigma^2} \quad (10)$$

$$\begin{aligned} \gamma &= -\sqrt{1-\left(\frac{-1+\sqrt{1+2\beta}}{\beta}\right)^2},\ for\ \xi > 0 \\ \gamma &= \sqrt{1-\left(\frac{-1+\sqrt{1+2\beta}}{\beta}\right)^2},\ for\ \xi < 0 \end{aligned} \quad (11)$$

Limits of the BoE methodology for fan charts are: the choice of the mode implies a too restrictive loss function, building confidence intervals around the mode asymmetry affects the method used to determinate the skewness of the distribution.

These limits include the fact that the fan charts are used to evaluate the risk and the uncertainty in the economy, which will be taken into account in establishing the economic policies. The banks' forecasting activity is based on this graphical representation. Economic analysis will take into account the possible shocks that may occur in the economy. In making predictions, knowledge regarding uncertainty and of risk balance is essential.

We start from the standard form of a VAR model of order p:

$$y_t = A_1 y_{t-1} + A_2 y_{t-2} + \cdots + A_p y_{t-p} + \mu + \varepsilon_t \quad (12)$$

y_t- vector of endogenous variables (dimension: $n*1$)
μ- vector of constant terms (dimension: $n*1$)
ε_t- vector of errors (dimension: $n*1$)
The errors are independently, identically and normally distributed.
The matrices of coefficients have the dimension $n*n$.

For this type of model, Ciccarelli and Rebucci (2003) identified the over-fitting problem (the number of parameters to estimate $n(np + 1)$ increases geometrically by the number of variables while the increase is proportional to the number of lags). The Bayesian approach is more suitable because we do not know if some coefficients are null or not. For the vector of parameters we can associate probability distributions. The estimation supposes the knowledge of prior distribution and of the information in the data. Litterman (1986) made several observations regarding the use of BVAR models for macroeconomic data:

- most of the macroeconomic time series include a trend;
- the recent lags have a major influence;
- the own lags of a variable influence it to a larger proportion than the lags of other variables in the model.

Litterman (1986) started from a multivariate random-walk to define the prior distribution. Actually, the prior repartition is centred on the random walk ($y_{n,t} = \mu_n + y_{n,t-1} + \varepsilon_{n,t}$).

The following are properties for standard priors:

- the priors are flat (non-informative) for deterministic variables;
- the priors are independent and normally distributed for the lags of endogenous variables;

- the means of prior distributions are set to zero, excepting the first lag of the dependent variable in each equation (they are set to one).

Other priors have to be set for variances. The standard error of the estimate corresponding to variable j in equation i with the lag l is

$$S(j, i, l)\colon S(j,\ i,\ l) = \frac{[\delta_g(l) f(j,i)] s_i}{s_j}. \tag{13}$$

- δ –hyper-parameter (overall tightness of the prior)

where
$g(l)$ = tightness of lag *1* compared with lag l ($g(l)$ decreases harmonically, $g(l) = l^d$). If the lag length increases, the tightness around the prior mean will increase.
$f(j,i)$ = tightness of the prior on variable j compared with variable i in equation i.
$f(j,i) = w_{ij}$, for $i \neq j$
$f(j,i) = 1$, for $i=j$
In this research we will use the non-informative priors to make quarterly inflation rate forecasts on the horizon Q4:2012–Q4:2013.

4. The assessment of uncertainty for the inflation rate forecasts

4.1. A fan chart for assessing the inflation forecasts uncertainty in Romania

In order to build a Fan Chart, we departed from the NBR (National Bank of Romania) inflation report. We used the data from 2005:Q3–2012:Q3 and we made forecasts for 2012:Q4–2013:Q4.

We used the following notations while computing the fan chart:

To compute the fan chart we have to determine $(\mu^{\pi}_{t+h|t}, \sigma^{\pi}_{1,t+h|t}, \sigma^{\pi}_{2,t+h|t})$, where h = 1,2,..,9. To calculate this triplet, we have to follow two steps:

(1) Determination of more likely inflation forecast path $(\mu^{\pi}_{t+h|t})$;
(2) Computation of fan chart, which supposes the determination of $(\sigma^{\pi}_{1,t+h|t}, \sigma^{\pi}_{2,t+h|t})$.
(3) Inflation rate forecasts
   - we identify the factors that may affect the inflation rate (X^i_t) over the forecasting horizon and we determine their more likely path $(\mu^{\pi}_{t+h|t})$;
   - we compute the more likely short-term inflation prediction knowing the factors' paths and the data series for inflation up to time t. In this case we use some different models;
   - we compute the forecast error standard deviation of the inflation rate, which is the sum of two components: historical forecast error standard deviation estimation and an uncertainty multiplier.
(4) Fan chart computation
   - we classify the factors according to the balance of risks and we select only those factors for which $p^i_{t+h|h} \neq 0,5$ for at least one point in time in the forecast horizon;
   - we compute the forecast error standard deviation of the factors and the response of the inflation rate to one unit impulse in each factor φ^i_{h-1};

- we transform the forecast error standard deviation and the factor's balance of risks into skewness indicators using the formulas;
- The factors' skewness indicators are transformed into inflation rate skewness indicators by using the impulse response function;
- The inflation rate forecast error standard deviation and its skewness indicators are transformed into left and right standard deviations using some of the above formulas;
- we compute the percentiles of the inflation forecast;
- we compute the probability table, the expected forecasts and the median.

The programme used to build the fan chart is developed using MS Office Excel 2003 in Visual Basic.

A VAR model with two variables, the yearly rate of inflation and the yearly rate of devaluation measured quarterly was estimated. The response of inflation to the rate of devaluation is positive, short lived, and statistically significant.

The responses correspond to one SD innovation so we will have to change units to responses to 1 percentage point. The variations in exchange rate are due to inflation modification and to changes in exchange rate.

For the factor uncertainty, the smoothed forecast RMSE of the devaluation rate that arises from the forecasts of the VAR was introduced.

The historical prediction errors for the exchange rate are computed by rolling the estimation of the same VAR model and forecasting the exchange rate nine quarters ahead each time a quarter is added. Finally, the forecast of the model for the next nine quarters is compared to the outlook and, by a rule of thumb, the balance of risks for these factors is determined. We also assessed the uncertainty using, this time, a BVAR model with non-informative prior.

In the last quarter of 2013 it is more likely to have an inflation rate around 4%, a value far from the target of 2.5% fixed by NBR. The indicator is located in the interval [3%; 5.5%].

For the fan chart based on a BVAR model with non-informative prior we have a better assessment of uncertainty, the stability of the predictions being more clearly highlighted compared with the fan chart based on VAR forecasts.

Providing an evaluation of uncertainty is related to the effectiveness with which an institution fails to influence the economic activity. The methodology used by BNR is a simple one, for example the measure of global medium uncertainty for the rate on inflation based on its macroeconomic short-term forecast model is used as the mean absolute error (MAE-mean absolute error). This synthetic indicator includes all effects of unanticipated past shocks that led to the deviation of the expected values from the registered ones. Based on this type of error prediction, forecasting intervals are built, with NBR numbering several advantages of its methodology:

- it considers all the previous shocks that have affected the rate of inflation;
- it determines a classification of the deviations from the actual values in the history of projections: deviations that determined an overestimation of the projected inflation and deviations that generated an underestimation;
- the methodology excludes any arbitrary assumption about the action of individual risk factors;
- it allows the adjustment of intervals of uncertainty, so that they reflect the assessments of different agents regarding the magnitude of the future uncertainty in relation to the one of previous periods.

The interval of uncertainty built by BNR is a very simple one, far from the complex methodology proposed by the BoE. This aspect can be noticed from the chart below. Unlike the fan chart, where the shadow depth is assessed, the BNR chart makes no distinction between degrees of uncertainty and it does not consider the forecast distribution. It does not consider the uncertainty in terms of probabilities, but it evaluates only an indicator of prediction accuracy. As a synthetic indicator, the MAE is not able to identify the most important factors in the forecasting horizon. Some methodological notes are necessary in building the interval of uncertainty:

- prediction errors are calculated as the difference between predicted values based on the forecasting model and actual values on the medium term of the rate of inflation rate for forecasting horizons of 1 to 8 quarters;
- values of the inflations are the mean quarterly ones;
- the obtained values were logarithmically adjusted to eliminate the irregular trend of concentration of uncertainty at different forecasting horizons, but also to smooth the intervals' limits.

If we compare this graph with the fan chart, we can see that the NBR predicts a lower inflation rate (less than 4% in the last quarter of 2014).

Unlike the root mean square error (RMSE) indicator, the indicator for forecasting error MAE is less sensitive to large prediction errors. If the dataset is small, MAE is recommended, but most institutions use RMSE, as its unit of measurement is the same as the one of the indicator that is calculated. RMSE is always at least equal to the MAE. Equality occurs if the errors have the same magnitude. The difference between the MAE and the RMSE is higher, the greater the variability of the data series. RMSE is affected by generalised variance, the interpolation, the errors in the phase and by the presence of outliers.

4.2. The assessment of inflation rate forecasts uncertainty using Monte Carlo simulations

The forecasts are made starting from an autoregressive model (AR) for a stationary data series. We made one-step-ahead forecasts based on econometric models. Simulations are made starting from these models, and new forecasts obtained. Supposing we have a model AR of order p:

$$X_t = a_0 + a_1 \cdot X_{t-1} + a_2 \cdot X_{t-2} + \cdots + a_p \cdot X_{t-p} + \varepsilon_t \tag{14}$$

The application of the Monte Carlo method supposes several steps:

(1) The econometric model estimation (an AR (p) model in this case)
(2) The average and the standard deviation of the parameters are determined
(3) A normal distribution is generated for each parameter, knowing the average and the standard deviation (we chose 1000 replications)
(4) The simulated values of the dependent variable are computed knowing the values of the parameter's distribution and the observed values.
(5) The average and the standard deviation of the simulated values for dependent variable are computed.
(6) An indicator of reliability is computed, starting from a critical value randomly selected by the researcher (q^*):

$$R = \frac{q * -m}{\sigma} \tag{15}$$

(7) The probability that the predicted inflation rate is greater than the target is:

$$P = 1 - \varphi(R) \tag{16}$$

where ρ is the probability of R in a normal standard repartition.

(8) The reliability indicator can be based on another reference value (the previous value of the inflation rate) and it is denoted by R'. The associated probability is P'.

Franses, Kranendonk, and Lanser (2011) used Monte Carlo simulation to assess four sources of uncertainty in forecasts based on the Saffier model.

Some valid models were built for Romania (AR(2) models for the quarterly inflation rate), for which the errors are not correlated, the distribution is a normal one and the homoscedasticity hypothesis is checked according to the White test without cross terms.

The Monte Carlo (MC) method was used to construct one-step-ahead forecasts for inflation rate in Romania (Q4:2012–Q4:2013). The parameters used to generate the MC simulations are the average and the standard deviation of the parameters of AR(2) models. One thousand replications were chosen and their average represents the new point forecast.

The critical values (q*) used to calculate the reliability indicators are: the difference between the targeted inflation in Romania and the predicted value and the difference between the predicted value and the inflation rate registered in the previous quarter. According to Siok and Har (2012), the inflation targeting became frequently used, starting in the 1990s in the context of price stability. But Arestis and Sawyer (2013) showed that during the recent financial crisis there were many doubts regarding the target inflation regime.

The degree of uncertainty grew over the mentioned horizon. It is very likely that the predicted inflation rate in each quarter will be greater than the target fixed by the NBR. However, it seems that we have a higher probability of registering higher forecasts value than the previous effective inflation rates in each quarter.

If we compare this numerical procedure based on the MC method with the fan chart representation, we obtained for this particular case a decrease in the degree of uncertainty from one quarter to another. On the other hand, for the fan chart the uncertainty increases in time, the probability of guaranteeing the forecasted value being lower because of the remoteness compared with the prediction origin. Therefore, the best solution would be to accompany the fan chart by this procedure, based on Monte Carlo simulations.

5. Conclusions

In this paper, we proposed two methodologies for assessing the inflation forecasts in Romania: the numerical method of computing the probability that the forecast be greater than a benchmark value (method based on Monte Carlo simulations) and a graphical method (fan chart).

The fan charts are one of the suggested ways to evaluate the uncertainty of macroeconomic forecasts based on the models, with the best known methodology being the one of the Bank of England. Although scientists have pointed to certain weaknesses of it, the fan chart continues to be used by some central banks, with some researchers bringing improvements to the assumptions used in building this type of graphic. In the case of Romanian quarterly inflation forecasts, according to uncertainty evaluation, there are more chances to have an inflation rate that is higher than the reference value. This approach shows a lower degree of uncertainty compared with fan charts for which the probabilities of having a certain value for the inflation forecast decrease over time.

The methodology used by the NBR to build the interval of uncertainty for inflation is very simple and it could be improved or replaced with the methodology of building fan charts, because it is not based on a probabilistic approach but on mean absolute error. A lower degree of uncertainty was obtained by constructing a fan chart using BVAR models with non-informative priors compared with classical VAR models.

In conclusion, the numerical procedure proposed in this article is a better tool of assessing forecast uncertainty compared with fan charts, but this remains a good measure of highlighting the inflation forecast uncertainty compared with the NBR approach. The evaluation of a forecast's uncertainty is very useful for the establishment of the monetary policy of central banks.

Note

1. Targeted inflation is defined by the Bank of English as RPIX inflation (Retail Prices Index).

References

Arestis, P., & Sawyer, M. (2013). Moving from inflation targeting to prices and incomes policy. *Panoeconomicus, 60*, 1–17.

Banerjee, N., & Das, A. (2011). *Fan chart: Methodology and its application to inflation forecasting in India*. Mumbai: Department of Economic and Policy Research.

Blake, D., Cairns, A. J. G., & Dowd, K. (2008). Modelling and management of mortality risk: A review. *Scandinavian Actuarial Journal, 108*, 79–113.

Bilan, Y., Gazda, J., & Godziszewski, B. (2012). Derivative variables as indicators of output gap in Poland and Ukraine, *Actual Problems of Economics, 129*, 331–339.

Bratu, M. (2012a, June). The reduction of uncertainty in making decisions by evaluating the macroeconomic forecasts performance in Romania. *Economic Research- Scientific Journal, 25*, 239–262. Original Scientific Paper.

Bratu, M. (2012b). *Accuracy assessment of short run macroeconomic forecasts in Romania, Economics & Sociology, 5*, 26–38.

Britton, E., Fisher, P., & Whitley, J. (1998, February). The inflation report projections: Understanding the fan chart. *Bank of England Quarterly Bulletin, 38*, 30–37.

Buhlmann, P. (2002). Sieve bootstrap with variable length – Markov chains for stationary categorical time series (with discussions). *Journal of America Statistics Association, 97*, 443–455.

Ciccarelli, M., Rebucci, A. (2003). *Bayesian VARs: A survey of the recent literature with an application to the European Monetary System* (IMF Working Papers 03/102). International Monetary Fund.

Cogley, T., Morozov, S., & Sargent, T. J. (2003). *Bayesian fan charts for U.K. inflation: Forecasting and sources of uncertainty in an Evolving Monetary System* (CFS Working Paper Series 2003/44). Center for Financial Studies.

Dowd, K. (2008). Backtesting the RPIX inflation fan charts. *The Journal of Risk Model Validation, 1*, 29–47.

Elder, R., Kapetanios, G., Taylor, T., & Yates, T. (2005). Assessing the MPC's fan charts. *Bank of England Quarterly Bulletin Autumn, 41*, 326–345.

Ericsson, N. R. (2001). *Forecast uncertainty in economic modeling* (Vol. 697). Board of Governors of the Federal Reserve System.

Franses, P. H., Kranendonk, H. C., & Lanser, D. (2011). One model and various experts: Evaluating Dutch macroeconomic forecasts. *International Journal of Forecasting, 27*, 482–495.

Galbraith, J. W., & van Norden, S. (2011). *GDP and inflation probability forecasts derived from the Bank of England fan charts*. Montreal: Department of Economics, McGill University.

Garrat, A., Lee, K., Pesaran, M. H., & Shin, Y. (2000). *Forecast uncertainties in macroeconometric modelling: An application to the UK economy* (Cambridge Working papers in Economics 0004). Faculty of Economics, University of Cambridge.

Gneiting, T., Balabdaoui, F., & Raftery, A. E. (2007). Probabilistic forecasts, calibration and sharpness. *Journal of the Royal Statistical Society Series B: Statistical Methodology, 69*, 243–268.

Gneiting, T., & Ranjan, R. (2011). Comparing density forecasts using threshold and quantile weighted proper scoring rules. *Journal of Business and Economic Statistics, 29*, 411–422.

King, M. (2004). *What fates impose: Facing up to uncertainty*. The Eighth British Academy Annual Lecture 2004.

Lanser, D., & Kranendonk, H. (2008, September 23). Investigating uncertainty in macroeconomic forecasts by stochastic simulation (Discussion Paper, No. 112). London: CPB.

Litterman, R. B. (1986). Forecasting with Bayesian vector autoregressions – Five years of experience. *Journal of Business & Economic Statistics, 4*, 25–38.

Mitchell, J., & Wallis, K. (2011). Evaluating density forecasts: Forecast combinations, model mixtures, calibration and sharpness. *Journal of Applied Econometrics, 26*, 1023–1040.

Novo, A., & Pinheiro, M. (2003). *Uncertainty and risk analysis of macroeconomic forecasts: Fan charts revised*. Economic Research Department, Banco de Portugal.

Osterholm, P. (2008). A structural Bayesian VAR for model-based fan charts. *Applied Economics, 40*, 1557–1569.

Robertson, J. C., Tallman, E. W., & Whiteman, C. H. (2005). Forecasting using relative entropy. *Journal of Money, Credit and Banking, Blackwell Publishing, 37*, 383–401.

Sanderson, W., Scherbov, S. (2004). A new perspective on population aging. *Demographic Research, 16*, 27–58.

Siok, K. S., & Har, W. M. (2012). Does inflation targeting work in emerging East-Asian economies? *Panoeconomicus, 5*, 599–608.

Vega, M. (2003). *Policy makers priors and inflation density forecasts*. (Working Paper). Banco Central de la Reserva del Perú.

Wallis, K. F. (2004). *An assessment of Bank of England and National Institute Inflation Forecast Uncertainties*. Coventry: Department of Economics, University of Warwick.