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## The four-factor asset pricing model on the Polish stock market

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In the three-factor asset pricing model a cross sectional portfolio returns variation is explained by the excess return of the market portfolio (RM), stock capitalisation and a factor associated with the book-to-market (B/M) ratio. This model, however, does not explain the momentum effect. Since this effect is present on many stock markets, the three-factor model is augmented by the momentum factor. This article presents the study of the four-factor asset pricing model on the Warsaw Stock Exchange (WSE) which is one of the largest stock markets in Central and Eastern Europe. The empirical analysis is based on monthly data from the period April 2003–December 2012 which includes different stages of the business cycle. This article shows that momentum is a significant factor on the WSE and the four-factor model describes the returns variation much better than the three-factor model.

**Keywords:** asset pricing models; four-factor model; momentum; value premium; emerging markets

**JEL classification:** G12.

### 1. Introduction

One of the most commonly used tools in a portfolio asset pricing is Fama and French's (1993) model where the cross-sectional variation in average portfolio returns is explained by three factors: excess return (RM) of a market portfolio, and stock's size and stock's book-to-market (B/M) ratio. Although, the three-factor model explains various anomalies observed on stock markets (e.g. Fama & French, 1996) it does not explain the momentum effect described by Jegadeesh and Titman (1993). Jegadeesh and Titman show the profitability of a momentum strategy, i.e. the strategy of buying stocks that have performed well in the previous three to 12 months and selling stocks that have performed poorly in the past. As showed additionally later by Jegadeesh and Titman (2001) the momentum effect has continued into the 1990s.

The existence of the momentum phenomenon has been extensively examined in recent years on a variety of stock markets. Rouwenhorst (1998) proves that momentum strategies were profitable on 12 European stock markets. Griffin, Ji, and Martin (2003) confirm the existence of momentum phenomenon on different countries all around the world while Chui, Titman, and Wei (2000) found momentum effect on Asian Stock markets except Japan and South Korea. The fact that profitability of momentum strategy is common on many markets justified the construction of a momentum factor and its application in asset pricing. Carhart (1997) extends the Fama and French model by

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adding a fourth factor, 'winners minus losers' (WML) which is equal to the difference between returns on one-year winners (i.e. stocks with the highest returns in the previous 12 months) and returns on one-year losers (i.e. stocks with the lowest returns in the previous 12 months).

Since then, the four-factor model has been applied in developed markets, e.g. by Fama and French (2010, 2012) and has been proved to describe better cross-sectional variation of portfolio returns than classical three-factor model. However, Fama and French (2014) have also considered a five-factor model.

Emerging markets are characterised by relatively higher returns and higher volatility when compared to developed markets and there is still no consensus which asset pricing model explains better stock returns on emerging markets. In the literature there are some proposals of asset pricing models that could be appropriate for emerging markets. However, only a few of them concentrate on Central and Eastern European countries in transition. Borys (2011) analyses Capital Asset Pricing Model (CAPM) and so-called macroeconomic factor models in the Visegrad countries: Czech Republic, Hungary, Poland and the Slovak Republic in the period 1993–2003. Borys concludes that a model including excess market return, industrial production, inflation and term structure explains variation of stock returns.

Foye, Mramor, and Pahor (2013) use a three-factor model to explain cross-sectional returns in new members of the European Union from June 2005 to July 2012. In their analysis Foye et al. use weekly data of about 150 stocks from eight Eastern European stock markets. The majority of them (about 70%) are listed on the Warsaw Stock Exchange (WSE). The results of Foye et al. confirm poor explanatory ability of 'small minus big' (SMB) on emerging markets. Thus they propose to replace the market value of equity by net income/cash flow from operating activities (NI/CFO) which is a proxy for earnings management. However, even after this adjustment a three-factor model still poorly fits to the data and thus it inadequately explains cross-sectional variation in stock returns.

The usefulness of four-factor model proposed by Carhart (1997) is studied by Cakici and Tan (2012). They examine size, value and momentum effects on the basis of the monthly data of 18 emerging countries grouped into three regions: Asia, Latin America and Eastern Europe. The region of Eastern Europe includes the Czech Republic, Hungary, Russia, Turkey and Poland. Cakici and Tan find a strong evidence of the value effect in all the emerging regions but find no momentum effect in Eastern Europe. One of the possible explanations of the lack of momentum effect is that the study is performed on the basis of the data from January 1990 to December 2011. In the case of Eastern European economies this period is too long because stock markets in this region started mainly in the early 1990s. Moreover, their structures have been gradually developed (for example, a continuous trading on the WSE started in 1996). Additionally, the period under study is not homogeneous due to the accession of the Czech Republic, Hungary and Poland to the European Union.

Asset pricing models exclusively on the Polish stock market are examined for example by Czapkiewicz and Skalna (2010), Urbański (2012), Waszczuk (2013) and Zaremba (2014). In general, they confirm that the three-factor model explains portfolio returns variation when stocks are sorted by size and value, but fails to explain momentum effect. Even the four-factor model constructed by Zaremba (2014) incorrectly describes cross-sectional returns variation of portfolios sorted by size and momentum.

The aim of this article is a detailed analysis of cross-sectional returns variation on the WSE which is one of the largest and most important stock markets in Central and

Eastern Europe.<sup>1</sup> The empirical analysis of the Fama and French three-factor model and Carhart's (1997) four-factor model for securities listed on the WSE is performed on the basis of monthly data from April 2003 to December 2012. The period under study contains the final part of Poland accession process to the European Union and first few years of Polish membership in the EU.

The article is organised as follows. In section two the four-factor model is described in detail and some methodological issues are discussed. Section three presents data, factors and portfolios applied in the study. Section four contains results of empirical analysis. A short summary and conclusions are presented in the final section.

## 2. Model and methods

According to Cochrane (2001) the central asset-pricing formula may be expressed as:

$$E_{t-1}(M_t R_t) = E(M_t R_t | I_{t-1}) = 1, \tag{1}$$

where  $R_t$  denotes portfolio returns at time  $t$ ,  $I_t$  is the information set at time  $t$ ,  $M_t$  is a stochastic discount factor – the same for each portfolio under study. The most popular specification of  $M_t$  in empirical studies is a linear function of a set of  $K$  proxies such that:

$$M_t = a + \beta_1 f_{1,t} + \dots + \beta_K f_{K,t} = a + \beta^T F_t, \tag{2}$$

where  $F_t = (f_{1,t}, \dots, f_{K,t})^T$  is a vector of factors' values at time  $t$ ,  $\beta = (\beta_1, \dots, \beta_K)$  denotes vector of risk factor sensitivities or loadings. The discussion in this article focuses on the case where  $\beta$  is the same throughout the entire sample period. Under this assumption the conditional model (1) is equivalent to the unconditional model.

In the classical CAPM  $K$  is equal to 1 and the underlying factor is the RM of the market portfolio, denoted as  $RM_t$ . In the three factor model of Fama and French (1993) the factors are: the RM of the market portfolio ( $RM_t$ ), the difference between returns on small and big stock portfolios ( $SMB_t$ ) and the difference between returns on value and growth stocks portfolios ( $HML_t$ ). In the four-factor model proposed by Carhart (1997) the discount factor has the following form:

$$M_t = a + \beta_{RM} RM_t + \beta_{SMB} SMB_t + \beta_{HML} HML_t + \beta_{WML} WML_t \tag{3}$$

where  $WML_t$  is a momentum factor, i.e. it is the difference between portfolios of stocks with the highest and lowest returns in the previous year. Hence, in this paper we consider the model:

$$E(R_t) = \gamma_o + \gamma_{RM} \beta_{RM} + \gamma_{SMB} \beta_{SMB} + \gamma_{HML} \beta_{HML} + \gamma_{WML} \beta_{WML} \tag{4}$$

where  $i = 1, \dots, N$

$$R_{i,t} = \alpha_i + \beta_{RM,i} RM_t + \beta_{SMB,i} SMB_t + \beta_{HML,i} HML_t + \beta_{WML,i} WML_t + \varepsilon_{i,t}. \tag{5}$$

The parameters  $\gamma_{RM}$ ,  $\gamma_{SMB}$ ,  $\gamma_{HML}$ ,  $\gamma_{WML}$  are risk premiums associated with risk factors i.e. they stand for the reward for bearing the risk of a given factor. If loadings with respect to the risk factors are important determinants of average returns then the  $\gamma_{RM}$ ,  $\gamma_{SMB}$ ,  $\gamma_{HML}$ ,  $\gamma_{WML}$  parameters are significant.

Models (4) and (5) are appropriate to asset pricing on a given market when their betas correctly capture the cross-sectional variation of RMs for all  $i = 1, \dots, N$ , i.e. when intercepts  $\alpha_i$  are simultaneously equal to zero for all  $i = 1, \dots, N$ . According to

Huberman and Kandel (1987) when this condition is satisfied then risk factors span the *ex ante* minimum variance tangency portfolio that can be created from all assets (Fama & French, 2012). Hence, in order to verify the conjecture that factors generate efficient portfolios the null hypothesis

$$H_0 : \alpha = 0$$

should be tested, where  $\alpha = [\alpha_1, \dots, \alpha_N]^T$  is the vector of all intercepts from (1) or (2). This can be verified by GRS test statistic proposed by Gibbons, Ross, and Shanken (1989):

$$GRS = \left(\frac{T}{N}\right) \left(\frac{T - N - K}{T - K - 1}\right) \left[ \frac{\alpha^T \hat{\Sigma}^{-1} \hat{\alpha}}{1 + \hat{\mu}^T \hat{V}^{-1} \hat{\mu}} \right] \tag{6}$$

where  $T$  is the sample size,  $N$  is the number of portfolios,  $K$  is the number of factors,  $\hat{\alpha}$  is a vector of regression intercepts,  $\hat{\Sigma}$  is the residual covariance matrix and  $\hat{V}$  is the sample covariance matrix of the factors. When the disturbances  $\varepsilon_{i,t}$  are temporally independent and jointly normal with zero mean, GRS statistic has F distribution with  $N$  and  $T - N - K$  degrees of freedom.

The parameters of the model (5) can be estimated by General Least Square (GLS) or General Method of Moments (GMM). However, in the case of GLS the errors-in-variables problem occurs. When error terms  $\varepsilon_{i,t}$  in (5) follow the multidimensional normal distribution, it is possible to correct the variance of the estimates using the formulas due to Shanken (1992). However, when this assumption is not fulfilled then the two-step GMM procedure suggested by Cochrane (2001) can be applied. This approach does not require assumption about normality of returns and it is robust to both conditional heteroscedasticity and serial correlation in model residuals as well as in factors. Hence, in this article the parameters of the model (4) are estimated as follows.<sup>2</sup>

Consider the partition of unknown parameter vector  $\varphi = (\alpha^T, \beta_1^T, \dots, \beta_K^T, \Gamma)$  into two sub-vectors  $\varphi = (\varphi_1, \varphi_2)$  where  $\varphi_1 = (\alpha^T, \beta_1^T, \dots, \beta_K^T)$  and  $\varphi_2 = \Gamma$  is a vector of unknown parameters in (4). The constructed consistent estimate  $\hat{\varphi}_2$  obtained via two step GMM estimator satisfies the minimum of the function:

$$\min_{\varphi_2} (\bar{R} - [1_N, \hat{\beta}] \varphi_2)^T W_{2T} (R - [1_N, \hat{\beta}] \varphi_2) \tag{7}$$

Where

$$W_{2T} = S_{2T}^{-1} \quad \text{and} \quad S_{2T} = \Omega_0 + \sum_{j=1}^m w(j, m) [\Omega_j + \Omega_j^T] \tag{8}$$

and

$$\Omega_j = \frac{1}{T} \sum_{t=j+1}^T h_t h_{t-j}^T \quad \text{where} \quad h_t = \hat{\beta}(F_t - F) + \varepsilon_t (j = 0, 1, \dots, m). \tag{9}$$

The  $w(j, m)$  in (8) is Parzen kernel (Andrews, 1991). The matrix  $([1_N, \hat{\beta}]^T W_{2T} [1_N, \hat{\beta}])^{-1}$  is considered as the asymptotic covariance matrix. The two-step strategy decreases the efficiency of the GMM estimates. In order to attain the efficiency bound of the GMM estimator, computed estimate  $\hat{\varphi}_2$  is used in the simulation study as a starting point for the iteration.

### 3. Data and factors

Our study is based on monthly data of all stocks quoted on the Main List of the WSE. The Main List is a regulated market. This ensures the availability of all necessary information about companies. Stock prices as well as firms' fundamental data cover the period from April 2003 to December 2012, however the data from 2003 are applied only to examine momentum effect and to assign stocks to winners or losers portfolios at the beginning of 2004. Because in each month we use data of all stocks quoted on the main market of the WSE our study does not suffer from the survivor bias. All data are collected from the *Emerging Markets Information Service* (EMIS). The period under study is rather short when compared with other asset pricing model studies, especially those concerning the US stock market. However, as it is mentioned in the Introduction, the length of the period is mainly influenced by the accession of Poland to the European Union in May 2004 and integration of Polish economy with the EU members' markets.

#### 3.1. Factors

The factors *SMB*, *HML* and *WML* are constructed on the basis of capitalisation, B/M ratio and on the basis of monthly log-returns. We define factors in a similar manner as Fama and French (1996) with slightly modifications mainly due to number of securities in our data-set. We construct factors and portfolios on the basis on the most recent data available in a formation moment.

The RM of the market portfolio is approximated by the difference between monthly return on WIG (the main index of the WSE) and the 52-week Polish treasury bill rate.

At the end of each month, stocks with a positive book value are divided into six groups as follows. First, the stocks are divided into two groups: small (*S*) and big (*B*) stocks. The group of big stocks contains the largest stocks with the total log-capitalisation equal to 50% of aggregated log-capitalisation of the whole market and the group of small stocks contains all remaining stocks. The number of stocks in these groups increases in the whole period under study from 72 to 217 and from 39 to 125 for small and big stock groups, respectively. The stocks are also independently sorted by B/M ratio and then they are divided into three groups of stocks with low (*L*), medium (*M*) and high (*H*) values of B/M according to the 30th and 70th percentiles. As a result of the intersection of these independent divisions according to size and B/M six portfolios (*SL*, *SM*, *SH*, *BL*, *BM* and *BH*) are formed. For each portfolio the equally-weighted and the value-weighted monthly returns are computed. In the case of equally-weighted returns *SMB* is taken as the difference between the average return of small stocks from *SL*, *SM* and *SH* and the average return of big stocks from *BL*, *BM* and *BH*. For value-weighted returns *SMB* is the difference between average of returns on the small stock portfolios (i.e. *SL*, *SM* and *SH*) and the average of returns on the big stock portfolios (i.e. *BL*, *BM* and *BH*). *HML* is computed in a similar way as the difference between the average returns on portfolios with the high B/M (i.e. *SH* and *BH*) and with the low B/M (*SL* and *BL*). The size effect for portfolios with low and high B/M is described by *SMB<sub>Low</sub>* and *SMB<sub>High</sub>*, respectively. The value effect for small and big portfolios is described by *HML<sub>S</sub>* and *HML<sub>B</sub>*. To compute the factors and to divide stocks into portfolios we apply the actual firms' book value and market capitalisation instead of data from the previous years as in Fama and French (2012) for example.

In the momentum literature (Jegadeesh, 1990; Jegadeesh & Titman, 1993; Szyszka, 2006) the most pronounced momentum effect is observed when portfolios are formed

on the basis of returns from the previous 12 months. Hence, in order to create the fourth factor, *WML*, all stocks are divided into three groups (losers, neutral and winners) according to their returns from the 12 months prior to the portfolio formation moment.<sup>3</sup> Because our data starts in April 2003 thus first momentum portfolio is formed at the end of March 2004. The losers portfolio contains 30% of stocks with the lowest last-year returns, while 30% of stocks with the highest past returns are assigned to the winners portfolio. As before, the independent sort on size and momentum results in six portfolios (*SL*, *SN*, *SW*, *BL*, *BN*, *BW*). The *WML* factor is computed as the difference between the average of returns on winners portfolios (*SW* as well as *BW*) and losers portfolios (*SL* and *BL*). The size effect for losers and winners portfolios described by *SMB<sub>L</sub>* and *SMB<sub>W</sub>*, and momentum effect for small and big portfolios describe by *WML<sub>S</sub>* and *WML<sub>B</sub>* are also analysed.

In the analysis of size, value and momentum effects and cross-sectional patterns in average returns on the WSE we follow Fama and French (2012). Table 1 presents averages, standard deviations and results of t-tests for risk factors under study. The results of significance tests are presented mainly in the sake of clarity. The more detailed discussion about the effects of factors under study is presented in following sections.

The insignificantly negative value of *SMB* mean (-0.14% per month) suggests that in the period 2004–2012 there was no visible size effect on the WSE. It is in line with literature that documented a weakening size effect since its discovery in the early 1980s (e.g. Fama & French, 2012). The significant size effect is visible only for stocks with low B/M ratio while it is insignificant for stocks with high B/M or the losers and winners stock portfolios. The significantly negative value of *SMB<sub>Low</sub>* mean (-0.89%) indicates that the stocks with low B/M are responsible for the negative value of *SMB* mean.

The average of *HML* is 0.29% per month but the standard deviation is large (3.8%), thus the value effect is also insignificant. However, the comparison of *HML<sub>S</sub>* and *HML<sub>B</sub>* indicates the existence of significantly positive value effect for small stocks (0.84%). It is in line the results for global and regional factors of Fama and French (2012) who show that the larger value premium for small stocks is typical. However, in our study this effect is not observable when the value-weighted portfolios are taken into consideration. The results of Table 1 indicate that the only significant factor is *WML* with the

Table 1. Summary statistics for risk factors: April 2004–December 2012.

	Equally-weighted portfolios			Value-weighted portfolios		
	Average	Std. dev.	t-statistics	Average	Std. dev.	t-statistics
<i>RM</i>				<b>0.03</b>	<b>6.39</b>	<b>0.05</b>
<i>SMB</i>	-0.14	3.01	-0.47	-0.04	4.62	-0.08
<i>HML</i>	0.29	3.80	0.77	-0.24	4.68	-0.51
<i>WML</i>	1.22*	4.97	2.52	1.43*	5.75	2.55
<i>HML<sub>S</sub></i>	0.84*	4.31	1.99	0.32	5.03	0.66
<i>HML<sub>B</sub></i>	-0.51	4.54	-1.14	-0.79	7.25	-1.12
<i>SMB<sub>Low</sub></i>	-0.89*	3.60	-2.52	-0.38	4.54	-0.87
<i>SMB<sub>High</sub></i>	0.46	4.46	1.05	0.73	8.30	0.90
<i>WML<sub>S</sub></i>	0.72	5.10	1.45	1.51*	5.49	2.82
<i>WML<sub>B</sub></i>	2.12*	6.28	3.45	1.35	8.49	1.63
<i>SMB<sub>L</sub></i>	0.68	5.27	1.32	-0.26	7.06	-0.38
<i>SMB<sub>W</sub></i>	-0.71	3.86	-1.89	-0.10	5.24	-0.20

Notes: \* - significance at 5% level.

Source: Authors' calculation.

monthly average equal to 1.22% (for the equally weighted portfolios) or 1.43% (for the value-weighted portfolios). In the case of the equally-weighted portfolios, momentum effect is much more visible for big stocks (2.12% per month) than for small stocks (0.72%). This is mainly due to the nature of momentum strategy which gives positive returns despite the market phase. On the other hand, in the case of the value-weighted portfolios *WML* for small and for big stocks are close to each other, but only for small stocks momentum effect is significant. The comparison of the values of *WML* for the equally-weighted and value-weighted portfolios indicates that the insignificance of momentum effect in the small stocks portfolios is caused by the behaviour of the smallest stocks with low capitalisation.

### 3.2. Portfolios

Following asset pricing literature (e.g. Fama & French, 2012) two cases of portfolios construction are considered. The stocks are sorted on size and value and on size and momentum. This procedure is similar to procedure of the factors formation described above. At the end of each month all stocks with positive book value are sorted independently into four groups on size, value or momentum. These sorts are performed on the basis of the appropriate data computed in the formation day (i.e. log-capitalisation, B/M ratio and returns from last 12 months). The intersection of the 4 × 4 sorts on size and B/M and the intersection of the 4 × 4 sorts on size and momentum form 16 size-value and 16 size-momentum portfolios. At the end of the next month this procedure is repeated and each portfolio is updated on the basis of new data. As a result, 105 monthly data for each portfolio are computed.

The number of stocks on the WSE is rather small when compared with, e.g. developed markets so all stocks are allocated into 16 portfolios. Table 2 presents averages and standard deviations of monthly portfolio returns for size-value (Panel A) and size-momentum (Panel B) equally-weighted portfolios<sup>4</sup>.

Panel A of Table 2 confirms the results of Table 1 that value effect is noticeable only for small stocks portfolios. The average monthly returns increase monotonically from significant parameter -1.49% for growth stocks (with low *B/M*) to insignificant

Table 2. Averages and standard deviation for monthly returns on equally-weighted portfolios constructed on the basis of capitalisation, size and momentum, April 2004–December 2012.

	Average				Standard deviation			
	low B/M	2	3	high B/M	low B/M	2	3	high B/M
<i>Panel A: size-value portfolios</i>								
small	-1.49	-0.97	-0.26	-0.22	8.14	8.17	8.27	9.23
2	-0.82	-0.48	-0.59	-0.74	7.52	7.34	8.17	9.27
3	-0.17	-1.19	-0.44	-0.73	6.67	7.04	7.74	9.30
big	-0.16	-0.33	-0.28	-0.37	5.88	7.13	7.80	9.33
<i>Panel B: size-momentum portfolios</i>								
	losers	2	3	winner	Losers	2	3	winner
small	-1.25	-0.06	-0.02	-0.90	9.72	7.94	8.16	8.12
2	-1.40	-0.80	0.36	0.06	9.97	8.22	8.06	7.45
3	-2.05	-0.59	-0.23	0.40	9.88	7.96	7.67	6.53
big	-1.66	-0.60	-0.10	1.17	12.33	8.25	7.65	7.90

Source: Authors' calculation.



parameter  $-0.22\%$  per month for value stocks (with high  $B/M$ ). In the other size quartiles changes in averages are not monotonically.

Panel B of Table 2 presents averages and standard deviations for the portfolios constructed on the basis of sorts on size and momentum. Generally, the monotonic changes from small to big portfolios are not observed. The only exception is the winners quartile where the reverse size effect is observed and average returns increase monotonically from  $-0.9\%$  for the small winners portfolios to  $1.17\%$  for the big winners portfolio. This is opposite to results for developed markets (Fama & French, 2012) where the size effect for winners and the highest average for small winners portfolio are typical. Momentum effect (i.e. higher average returns for winners than for losers) can be observed in all size quartiles. Moreover, this effect gets stronger when stock size increases and it is particularly noticeable in third and fourth size quartiles, where average returns increase monotonically from losers to winners.

#### 4. Asset pricing

From the results presented in above section it can be concluded that the cross-sectional patterns in average portfolio returns on the WSE are quite different from patterns on developed markets described in Fama and French (2012). Nevertheless, the momentum effect dominates on the WSE whereas the size and value effects are less important. Therefore, the question occurs whether size, value and momentum are sufficient factors for the correct assets pricing on the Polish stock market.

This article tests the implication of the four-factor model for the WSE. The tests focus on two basic implications of the model: whether the intercepts in simultaneous regression model (5) are jointly equal to zero and whether there exists linear relation (4) between systematic risks and expected returns.

##### 4.1. Asset pricing for size-value portfolios

First we consider simultaneous regression model (5) to explain excess returns of the size-value portfolios. This model explains excess returns of 16 portfolios by  $RM$ ,  $SMB$ ,  $HML$  and  $WML$ . Estimation results are presented in Table 3. The unknown parameters of the model are estimated by means of two methods: OLS method and GMM. However, in the case of this study, the use of both methods gives comparable results. We also present the results of GRS test for the significance of regression intercepts. In order to compare the ability of three- and four-factor model to describe excess returns on the WSE the results of GRS tests for both models are summarised at the bottom of Table 3.

The analysis of results presented in Table 3 reveals some interesting patterns. First of all, it can be noticed that all  $\beta_{RM}$ 's are significantly positive at 1% level. However, they almost do not differ and all values are very close to 1 what indicates that market portfolio does not explain cross-sectional returns variability. The variability of parameters is observed among estimates of  $\beta_{SMB}$ . Within each value quartile (except the second one), the estimates of size loadings  $\beta_{SMB}$  decrease monotonically as firm size increases from values greater than one for the smallest stocks to even negative values for the big stock portfolios. It means that small stocks are more sensitive on  $SMB$  values and thus more risky than big stocks. Estimates of  $\beta_{HML}$  also show some cross-sectional pattern. Within each size quartile,  $\beta_{HML}$  increase monotonically from growth to value stock portfolios. In fact, in two first value quartiles all  $\beta_{HML}$ 's are negative, whereas in other two quartiles all estimates of  $B/M$  loadings are positive. It follows that portfolios with high

Table 3. The estimation results for the simultaneous equation model  $R_{i,t} = \alpha_i + \beta_{RM,i}RM_t + \beta_{SMB,i}SMB_t + \beta_{HML,i}HML_t + \beta_{WML,i}WML_t + \varepsilon_{i,t}$  for size-value portfolios.

	low	2	3	high	Low	2	3	high
	$\beta_{RM}$				$\beta_{SMB}$			
small	0.84*	0.91*	0.99*	0.91*	1.58*	1.16*	1.31*	1.33*
2	0.97*	0.87*	0.92*	0.95*	1.02*	1.13*	0.76*	0.73*
3	0.86*	0.87*	0.93*	0.92*	0.40*	0.21	0.03	0.27
big	0.85*	1.02*	1.04*	1.01*	0.18	0.24**	-0.11	-0.70*
	$\beta_{HML}$				$\beta_{WML}$			
small	-0.42**	-0.21	0.05	0.71*	-0.03	-0.23	0.09	0.10
2	-0.75*	-0.06	0.27	0.59*	-0.17	0.13	-0.08	-0.20
3	-0.01	-0.04	0.29	0.78*	0.09	-0.11	-0.08	-0.09
big	-0.15	-0.20	0.19	0.68*	0.06	-0.15	-0.14	0.02
	$\alpha$				$R^2$			
small	-0.015*	-0.009**	-0.006	-0.008**	0.69	0.75	0.84	0.88
2	-0.007	-0.009**	-0.009**	-0.010**	0.74	0.74	0.76	0.83
3	-0.007	-0.014*	-0.009	-0.012**	0.68	0.68	0.72	0.71
big	-0.006**	-0.005	-0.006	-0.011	0.79	0.85	0.83	0.65

Three-factor model: GRS = 1.85,  $p$ -value = 0.037.  
 Four-factor model: GRS = 1.21,  $p$ -value = 0.277.

Note: \*, \*\* – significance at 1% and 5% level.

Critical value for GRS tests for 5% significance level is 1.764 and 1.763 for four- and three-factor models, respectively.

Source: Authors' calculation.

B/M values are more sensitive to HML and they are more risky than portfolios with low B/M values. However the values of  $\beta_{HML}$  for growth stocks are negative, so the portfolios with low B/M values have tendency to give opposite returns to HML values, especially small firms portfolios where  $\beta_{HML}$  are significantly negative. When loadings  $\beta_{WML}$  are analysed, there is no regularity observed.

In order to verify whether factors under study generate efficient portfolios the hypothesis that all intercepts are together zeros is verified. The GRS test statistic and associated  $p$ -value are presented in the last row of Table 3. As a comparison, the results of GRS test for Fama and French three-factor model are also displayed. For three-factor models (without WML) the results of the GRS test fails the null hypothesis that the intercepts for all portfolios under study are equal zero (GRS = 1.85 with  $p$ -value = 0.03). Hence RM, SMB and HML do not generate efficient portfolios that can be created from all assets. The addition of the fourth factor describing momentum effect significantly improves efficiency of the asset pricing model. The GRS statistics (GRS = 1.21 with  $p$ -value = 0.28) indicate that null hypothesis cannot be rejected and at any reasonable significance level.

#### 4.2. Asset pricing tests for size-momentum portfolios

Similar analysis is performed for size-momentum portfolios. The estimation results of model (2) parameters are presented in Table 4. Similarly to Table 3, market portfolio does not explain cross-sectional returns variability because all  $\beta_{RM}$ 's values are very close to 1. Similarly, majority of B/M loadings  $\beta_{HML}$  are insignificant and there is no visible variability between them.

Table 4. The estimation results for the simultaneous equation model  $R_{i,t} = \alpha_i + \beta_{RM,i}RM_t + \beta_{SMB,i}SMB_t + \beta_{HML,i}HML_t + \beta_{WML,i}WML_t + \varepsilon_{i,t}$  for size-momentum portfolios.

	losers	2	3	winners	losers	2	3	winners
	$\beta_{RM}$				$\beta_{SMB}$			
small	0.94*	0.88*	0.92*	0.90*	1.32*	1.09*	1.30*	1.33*
2	0.91*	0.91*	0.85*	0.99*	0.81*	0.80*	1.08*	1.13*
3	0.95*	0.93*	0.94*	0.78*	0.06	0.22	0.61*	0.38**
big	1.08*	1.05*	1.10*	0.94*	-0.55**	-0.08	-0.01	0.25
	$\beta_{HML}$				$\beta_{WML}$			
small	-0.20	0.07	-0.13	-0.01	-0.75*	-0.15	-0.01	0.26*
2	0.19***	-0.04	0.18	-0.22	-0.75*	-0.34	0.16	0.20*
3	0.03	-0.02	-0.15	0.23***	-0.93*	-0.35***	-0.05	0.32*
big	0.10	0.03	-0.26	-0.02	-0.78**	-0.22	-0.16	0.28
	$\alpha$				$R^2$			
small	-0.006	-0.002	-0.003	-0.018*	0.85	0.82	0.73	0.69
2	-0.010**	-0.008**	-0.002	-0.005***	0.81	0.75	0.64	0.81
3	-0.016*	-0.007**	-0.006	-0.006	0.77	0.72	0.69	0.62
big	-0.016***	-0.010**	-0.004	0.005	0.51	0.75	0.84	0.53

Three factor model: GRS = 2.74,  $p$ -value = 0.00.  
 Four-factor model: GRS = 1.36,  $p$ -value = 0.18.

Note: \*, \*\* – significance at 1, 5 and 10% level.  
 Critical value for GRS tests for 5% significance level is 1.764 and 1.763 for four- and three-factor models, respectively.  
 Source: Authors’ calculation.

For size-momentum portfolios cross-sectional patterns are observed only for estimates of  $\beta_{SMB}$  and  $\beta_{WML}$ . Size loadings  $\beta_{SMB}$  vary from -0.55 for big losers portfolio to 1.33 for small winners portfolio. Moreover, within each momentum quartile, analogously to Table 3,  $\beta_{SMB}$  ‘s decrease monotonically from values greater than one for the small stock portfolios to even significantly negative values for the portfolios of large stocks. In the same manner,  $\beta_{WML}$  ‘s vary from -0.93 to 0.32. Within each size quartile momentum loadings  $\beta_{WML}$  increase monotonically from negative values for losers portfolios to positive values for winners portfolios.

Similarly to size-value portfolios, in order to verify whether factors under study generate efficient portfolios the hypothesis that all intercepts are jointly equal to zero is verified. For three-factor model (without  $WML$ ) results of GRS test definitely reject the null hypothesis that the intercepts for all considered portfolios are together zero (GRS = 2,74 with  $p$ -value = 0.00). In the case of four-factor model the GRS is equal to 1.36 with  $p$ -value = 0.18.

### 4.3. The cross sectional regression

The results presented in the previous sections indicate that the four-factor model holds empirically and it has an explanatory power. The next step of our analysis is a calculation of risk premiums in model (4) which offset the investors for taking one unit of risk associated with corresponding risk factor. To estimate risk premiums related to  $RM$ ,  $SMB$ ,  $HML$  and  $WML$  factors we apply the GMM method.

Table 5 presents results for the cross-sectional regression (4) for size-value and size-momentum portfolios.<sup>5</sup> First, it should be noted that parameter  $\gamma_o$  is insignificant for

Table 5. Risk premium estimation results of the model  $E(R_t) = \gamma_o + \gamma_{RM}\beta_{RM} + \gamma_{SMB}\beta_{SMB} + \gamma_{HML}\beta_{HML} + \gamma_{WML}\beta_{WML}$ .

	$\gamma_o$	$\gamma_{RM}$	$\gamma_{SMB}$	$\gamma_{HML}$	$\gamma_{WML}$	$R_c^2$
<i>Four factor model</i>						
size-value portfolios	-0,035	0,031	-0,002	0,001	0,026	60%
t-statistics	-1,623	1,519	-0,671	0,195	2,295	
(p-value)	(0,105)	(0,129)	(0,502)	(0,845)	(0,022)	
size-momentum portfolios	0,023	-0,028	-0,003	-0,016	0,013	73%
t-statistics	1,002	-1,185	-0,785	-1,624	2,497	
(p-value)	(0,316)	(0,236)	(0,432)	(0,104)	(0,013)	

Source: Authors' calculation.

both set of portfolios. It means that the variation of average returns of portfolios depends only on variation of systematic risks related to underlying factors. On the other hand, the  $\gamma_{SMB}$  and  $\gamma_{HML}$  parameters are insignificant for both size-value and size-momentum portfolios and the risks associated with size and value effect are negligible on the WSE in the period under study. The loadings on  $WML$  represent an important cross-sectional determinant of average returns for both set of portfolios. The risk premium connected with momentum effect equals to 2.6% per month for size-value portfolios and 1.3% per month for size-momentum portfolios.

To describe the goodness of fit of these models we use the cross-sectional  $R_c^2$  measure employed by Jagannathan and Wang (1998) and Lettau and Ludvigson (2001). It is a measure of unconditional deviations from the model under study and shows the fraction of cross-sectional variation in average returns that is explained by the model. For portfolios sorted on size and value  $R_c^2$  is equal to 60%, whereas for portfolios sorted on size and momentum effect  $R_c^2 = 73\%$ . Taking into account that the null hypothesis that all intercepts in equations (5) are equal to 0 is not rejected we re-estimate model (4)-(5) with restrictions  $\alpha_i = 0$  for  $i = 1, \dots, N$ . Then  $R_c^2$  is equal to 76% and 73% for size-value and size momentum portfolios, respectively. The illustration of the above cross-sectional regressions is Figure 1 which presents the relationship between the observed averages of portfolios monthly returns (y-axis) and the expected portfolios monthly returns (x-axis) predicted by the model (4) for both sets of portfolios: constructed according to size and value (left panel) and constructed according to size and momentum (right panel).

If the asset pricing model perfectly describes the realised average returns, all the portfolios should fall on the line. The plot indicates that generally the pricing errors are not too big, and the model explains the size-value portfolios. Only portfolios of small

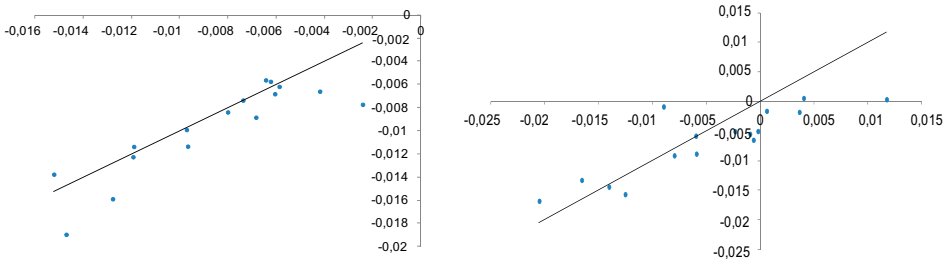


Figure 1. Portfolios average monthly returns (x-axis) and portfolios predicted monthly returns (y-axis): size-value portfolios (left panel) and size-momentum portfolios (right panel). Source: Authors' calculation.

stocks with low B/M and portfolios of big stocks with high B/M are explained rather poorly. Similar conclusion is obtained when the size-momentum portfolios are analysed. The four factor model explains portfolio of small winners and portfolio with big winners rather poorly.

## 5. Summary and conclusions

Most studies concerning asset pricing analyse the size, value and momentum effects only on developed markets. This issue has not been thoroughly examined in respect to emerging markets, especially to stock markets in Eastern Europe. The aim of this article is then the analysis of the four-factor asset pricing model on the WSE which is one of the largest stock market in the region. In the four-factor model *RM*, *SMB*, *HML* and *WML* are considered as explanatory variables. *RM* is the excess market return, *SMB* is the difference between returns on small and big stock portfolios, *HML* is the difference between the returns on portfolios of stocks with high and low B/M ratio. The fourth factor, *WML*, is the difference between returns on winners and losers portfolios according to their returns from the 12 months prior to the portfolio formation day. The empirical verification of the four-factor model is performed on the basis of the monthly data of securities quoted on the WSE in 2003–2012.

The analysis presented in this article indicates that the three factor-model including only *RM*, *SMB* and *HML* does not generate efficient portfolios. The inclusion of the fourth factor describing the momentum effect significantly improves the efficiency of the asset pricing model. Moreover, after the inclusion of *WML* into the model, the risk premium parameters corresponding to *SMB* and *HML* are insignificant for both size-value and size-momentum portfolios. Only loadings on *WML* represent an important cross-sectional determinant of average returns for both sets of the portfolios. The risk premium connected with the momentum effect equals 2.5% per month for the size-value portfolios and 1.4% per month for the size-momentum portfolios. The study implies that the four-factor asset pricing model describes the cross-sectional variation of the returns of the portfolios more accurately than the classical three-factor model and thus it can be applied to explain differences between the returns of diversified equity portfolios on the Polish stock market.

The momentum effect is the most pronounced and dominates over the size and value effects. However, comparison with other studies indicates that the results are sensitive to way of constructing of factors. This article confirms that four-factor models satisfactory describes returns variation on the WSE when the factors are constructed on the basis of the most recent data.

## Notes

1. According to Federation of European Securities Exchanges (FESE) monthly report total capitalisation of the WSE was 126 680.7mln € at the end of April 2013. In this part of the Europe only stock exchanges in Istanbul and Moscow have larger capitalisation than the WSE.
2. We do not present technical details that are described in Cochrane (2001).
3. Following the momentum literature we do not consider any lag between computation of momentum effect and portfolio formation.
4. The study has been done also for a value-weighted portfolio, but the results are worse than results for equally weighted portfolios.
5. We do not present here results for three-factor model because the GRS test rejects the null hypothesis.

## References

- Andrews, D. W. K. (1991). Heteroskedasticity and autocorrelation consistent covariance matrix estimation. *Econometrica*, 59, 817–858.
- Borys, M. (2011). Testing multi-factor asset pricing models in the visegrad Countries. *Czech Journal of Economics and Finance*, 61, 118–139.
- Cakici, N., & Tan, S. (2012). *Size, value, and momentum in emerging market stock returns* (Working Paper). Fordham University.
- Carhart, M. (1997). On persistence in mutual fund performance. *Journal of Finance*, 52, 57–82.
- Chui, A. C. W., Titman, S., & Wei, K. C. J. (2000). *Momentum, legal systems, and ownership structure: an analysis of Asian stock markets* (Working Paper.) NBER.
- Cochrane, J. (2001). *Asset pricing*. Princeton, NJ: Princeton University Press.
- Czapkiewicz, A., & Skalna, I. (2010). The fama-french model for the polish market. *Przegląd Statystyczny*, 57, 128–141.
- Fama, E. F., & French, K. R. (1993). Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics*, 33, 3–56.
- Fama, E. F., & French, K. R. (1996). Multifactor explanations of asset pricing anomalies. *Journal of Finance*, 51, 55–84.
- Fama, E. F., & French, K. R. (2010). Luck versus skill in the cross-section of mutual fund returns. *Journal of Finance*, 65, 1915–1947.
- Fama, E. F., & French, K. R. (2012). Size, value, and momentum in international stock returns. *Journal of Financial Economics*, 3, 457–472.
- Fama, E. F., & French, K. R. (2014). *A five-factor asset pricing model* (Working Paper).
- Foye, J., Mramor, D., & Pahor, M. (2013). A respecified fama french three-factor model for the new european union member states. *Journal of International Financial Management & Accounting*, 24, 3–25.
- Gibbons, M. R., Ross, S. A., & Shanken, J. (1989). A test of the efficiency of a given portfolio. *Econometrica*, 57, 1121–1152.
- Griffin, J. M., Ji, X., & Martin, J. S. (2003). Momentum investing and business cycle risk: Evidence from pole to pole. *Journal of Finance*, 58, 2515–2547.
- Huberman, G., & Kandel, S. (1987). Mean-variance spanning. *Journal of Finance*, 42, 873–888.
- Jagannathan, R., & Wang, Z. (1998). An asymptotic theory for estimating beta-pricing models using cross-sectional regression. *Journal of Finance*, 53, 1258–1309.
- Jegadeesh, N. (1990). Evidence of predictable behavior of security return. *Journal of Finance*, 45, 881–898.
- Jegadeesh, N., & Titman, S. (1993). Returns to buying winners and selling losers: Implications for stock market efficiency. *Journal of Finance*, 48, 65–91.
- Jegadeesh, N., & Titman, S. (2001). Profitability of momentum strategies: An evaluation of alternative explanations. *Journal of Finance*, 56, 699–720.
- Lettau, M., & Ludvigson, S. (2001). Resurrecting the (C)CAPM: A cross-sectional test when risk premia are time-varying. *Journal of Political Economy*, 109, 1238–1287.
- Rouwenhorst, K. G. (1998). International momentum strategies. *Journal of Finance*, 53, 267–284.
- Shanken, J. (1992). On the estimation of beta-pricing models. *The Review of Financial Studies*, 5, 1–33.
- Szyska, A. (2006). Zjawisko kontynuacji stóp zwrotu na Giełdzie Papierów Wartościowych w Warszawie [Momentum at the Warsaw Stock Exchange]. *Bank i Kredyt*, 8, 37–49.
- Urbański, S. (2012). Multifactor explanations of returns on the warsaw stock exchange in light of the ICAPM. *Economic Systems*, 36, 552–570.
- Waszczuk, A. (2013). A risk-based explanation of return patterns - Evidence from the Polish stock marke. *Emerging Markets Review*, 15, 186–210.
- Zaremba, A. (2014). *Cross-sectional asset pricing models for the polish market* (Working Paper). Retrieved from <http://dx.doi.org/10.2139/ssrn.2396884>.