APPLICATION OF “EINSTEIN'S RIDDLE” IN SOLVING CONSTRUCTION MACHINE ALLOCATION PROBLEMS

Scientiffic paper / Znanstveni rad

Borna Dasović
Josip Juraj Strossmayer University of Osijek, Faculty of Civil Engineering, univ.bacc.ing.aedif.
Marko Čorak
Josip Juraj Strossmayer University of Osijek, Faculty of Civil Engineering, univ.bacc.ing.aedif.
Mario Galić
Josip Juraj Strossmayer University of Osijek, Faculty of Civil Engineering, PhD
Corresponding author: mgalic@gfos.hr
Uroš Klanšek
University of Maribor, Faculty of Civil Engineering, Transportation Engineering and Architecture, Slovenia, Associate Professor

Abstract: “Einstein’s riddle” is a popular example of constraints satisfaction problem. Since its introduction, different forms and variations of the riddle have been presented. Regardless of the variant of the riddle, its solution is considered a tough challenge for humans. Researchers have developed and are still developing mathematical models, as well as computational simulation models for solving it. In this article, the authors have modified a previously published mathematical model and developed a computational spreadsheet model for solving the riddle, which provides a unique solution for the riddle. The model was also tested in a small and medium-scaled form for solving constraint satisfaction problems regarding the allocation of construction machines. The authors have also highlighted the model’s limitations for solving such problems and made suggestions regarding necessary modifications in the model to solve more complex problems in the same domain.

Keywords: Einstein’s riddle, zebra riddle, construction, machine allocation, combinatorial optimization

PRIMJENA „EINSTEINOVE ZAGONETKE“ PRI RJEŠAVANJU PROBLEMA ALOKACIJE GRAĐEVINSKIH STROJEVA

Sažetak: „Einsteinova zagonetka“ je prepoznatljiv primjer kombinatornog problema ispunjenja ograničenja. Ova zagonetka je imala više verzija, no bez obzira na formulaciju, uglavnom se smatra vrlo teškim zadatkom. Znanstvenici su razvijali i dalje razvijaju matematičke modele, a potom i računalne simulacijske modele za rješavanje spomenutog problema. Autori su u ovome radu modificirali ranije predstavljeni matematički model, a potom prema njemu izradili računalni model, koristeći se proračunskim tablicama kako bi riješili zagonetku. Model je ponudio jedinstveno rješenje u vrlo kratkom vremenu, a potom je ispitan pri rješavanju sličnog problema u građevinskoj praksi. Istaknuta su ograničenja u primjeni modela u obliku kojim je riješena „Einstenova zagonetka“ te koje su modifikacije nužne za aplikaciju pri rješavanju kompleksnijih problema u istoj domeni.

Ključne riječi: Einsteinova zagonetka, zebra zagonetka, građevinarstvo, alokacija strojeva, kombinatorna optimizacija

1 INTRODUCTION

Combinatorial optimization problems are ubiquitous everyday tasks, and as challenging tasks, they have been and still are interesting in several scientific fields. One of the most popular examples in the field of combinatorial optimization, ascribed to Albert Einstein, is “Einstein’s riddle” [1-4]. In literature, this problem is also known as the “Zebra Puzzle” [5-7] and is often associated with Lewis Carroll [8-10].

Even though it is not fully clear if it was originally Einstein’s formulation of the problem, it is broadly considered as a textbook example and is a frequently studied combinatorial optimization problem (i.e., constraint satisfaction problem), owing to Einstein’s great popularity, scientific achievements, and charisma. The above-mentioned problem and its variations have attracted great interest from the scientific community over the years. The aim of this paper is to explore the application potential of such optimization tasks in the field of construction. After a brief introduction of constraints satisfaction problems and the origin of Einstein’s riddle, its formulations and solutions, a spreadsheet solution of the riddle is presented. In the third chapter, the authors introduce an application of Einstein’s riddle in the field of construction, as also the extent to which the shown spreadsheet solution is useable, and modifications that are necessary to solve more complex problems. The final part of the paper gives the concluding remarks and recommendations for further researches.

2 EINSTEIN’S RIDDLE

2.1 Constraint satisfaction problem

A constraint satisfaction problem (CSP) is defined in literature as a mathematical problem having a given set of variables with associated domains whose every possible solution must satisfy all unary or binary constraints[11, 12]. The complexity of a constraint satisfaction problem depends largely on the tightness of constraints, or as Barbara M. Smith [13] explained it: “… *when the constraints are loose, there are many solutions and it is easy to find one; when the constraints are tight, it is easy to prove that there are no solutions.*” Complexity of constrained problems depends on the number and type of variables and constraints. However, when solutions are in range from near 0 to near 1 CSPs are found to be NP-complete problems.

2.2 Background of “Einstein’s riddle”

It is not clear whether the original formulation of the riddle was by the great Albert Einstein as a boy, who opined that approximately only 2% of the world’s population could solve it, or by Lewis Carroll who coined the “Zebra Puzzle”. As the cigarette brands mentioned in the puzzle were not available during Einstein’s youth and Einstein himself never explicitly claimed to have formulated the riddle, most scientists are of the view, that it could not have been him, who formulated this riddle. Nevertheless, ever since it was presented, it has attracted the interest of both general and scientific communities. In literature, regardless of how it is addressed, the riddle is often presented as an example of combinatorial optimization and constraints satisfaction problem. The solution of this puzzle is considered to be a difficult challenge for humans, something that has been elaborated and quantitatively proved [14].

2.3 Riddle formulation

The most common formulation of the riddle, published in the Life magazine in 1962, is given as a list of related facts and constraints connecting five people [15-17] having different nationalities, preferring different beverages, living in houses having specific colors, preferring different brands of cigarettes, and having different pets. Facts and constraints that form the riddle are given in the following list:

- The British lives in the red house.
- The Swedish keeps dogs as pets.
- The Dane drinks tea.

- The green house is on the left of the white house.
- The green house’s owner drinks coffee.
- The owner who smokes Pall Mall rears birds.
- The owner of the yellow house smokes Dunhill.
- The owner living in the center house drinks milk.
- The Norwegian lives in the first house.
- The owner who smokes Blend lives next to the one who keeps cats.
- The owner who keeps the horses lives next to the one who smokes Dunhill.
- The owner who smokes Bluemaster drinks beer.
- The German smokes Prince.
- The Norwegian lives next to the blue house.
- The owner who smokes Blend lives next to the one who drinks water.

The riddle is given as the question, which of the five people keeps fish as pets?

Previous researchers have proved that there is a unique solution to the problem, regardless of how the variables and the domains are named. Because of the nature of listed constraints, it is arguable if this is an entirely logical problem. However, to complete the puzzle and gain a unique solution, it is necessary to satisfy all the given constraints. In order to formulate it as a CSP, it is necessary to mathematically define the variables, domains, and the constraints.

2.4 Mathematical model of the riddle

The authors have modified the model formulation presented by Yeomans [16] and structured a mathematical model of the problem (Table 1) as a matrix of binary variables with 5 associated domains (i.e. nationality denoted as “N_i”, house colour “HC_i”, beverage “B_i”, cigarette brand “CB_i”, pets “P_i”) forming the rows; with “i” variables for each domain (i=1, 2,…, 5); while order of houses is denoted as “H_j”, and form the column “j” variables (j=1, 2,…,5). The solution is constrained by the condition that the sum of variables of each row equals 1 and that sum of each column referred to its domain, too equals 1. As such, the mathematical model consists of a total of 125 binary variables.

The set of conditions, given as the list of 15 constraints and facts in the puzzle, are mathematically formulated and added to the mathematical model (Table 2).

Table 1 Mathematical model of the riddle

Domains and Variables			House H_1	House H_2	House H_3	House H_4	House H_5	Constraints
Nationality N_i	N_1	Norwegian	N_{11}	N_{12}	N_{13}	N_{14}	N_{15}	$N_{ij} \in [0,1]$
	N_2	Swedish	N_{21}	N_{22}	N_{23}	N_{24}	N_{25}	$\sum_{i=1}^{5} N_{ij} = 1$
	N_3	German	N_{31}	N_{32}	N_{33}	N_{34}	N_{35}	
	N_4	Dane	N_{41}	N_{42}	N_{43}	N_{44}	N_{45}	$\sum_{j=1}^{5} N_{ij} = 1$
	N_5	British	N_{51}	N_{52}	N_{53}	N_{54}	N_{55}	
House color HC_i	HC_1	Yellow	HC_{11}	HC_{12}	HC_{13}	HC_{14}	HC_{15}	$HC_{ij} \in [0,1]$
	HC_2	Green	HC_{21}	HC_{22}	HC_{23}	HC_{24}	HC_{25}	$\sum_{i=1}^{5} HC_{ij} = 1$
	HC_3	Blue	HC_{31}	HC_{32}	HC_{33}	HC_{34}	HC_{35}	
	HC_4	Red	HC_{41}	HC_{42}	HC_{43}	HC_{44}	HC_{45}	$\sum_{j=1}^{5} HC_{ij} = 1$
	HC_5	White	HC_{51}	HC_{52}	HC_{53}	HC_{54}	HC_{55}	
Beverage Bi	B_1	Coffee	B_{11}	B_{12}	B_{13}	B_{14}	B_{15}	$B_{ij} \in [0,1]$
	B_2	Beer	B_{21}	B_{22}	B_{23}	B_{24}	B_{25}	$\sum_{i=1}^{5} B_{ij} = 1$
	B_3	Tea	B_{31}	B_{32}	B_{33}	B_{34}	B_{35}	
	B_4	Water	B_{41}	B_{42}	B_{43}	B_{44}	B_{45}	$\sum_{j=1}^{5} B_{ij} = 1$
	B_5	Milk	B_{51}	B_{52}	B_{53}	B_{54}	B_{55}	
Cigarette brand CBi	CB_1	Dunhill	CB_{11}	CB_{12}	CB_{13}	CB_{14}	CB_{15}	$CB_{ij} \in [0,1]$
	CB_2	Blend	CB_{21}	CB_{22}	CB_{23}	CB_{24}	CB_{25}	$\sum_{i=1}^{5} CB_{ij} = 1$
	CB_3	Blue Master	CB_{31}	CB_{32}	CB_{33}	CB_{34}	CB_{35}	
	CB_4	Prince	CB_{41}	CB_{42}	CB_{43}	CB_{44}	CB_{45}	$\sum_{j=1}^{5} CB_{ij} = 1$
	CB_5	Pall Mall	CB_{51}	CB_{52}	CB_{53}	CB_{54}	CB_{55}	
Pet Pi	P_1	Birds	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}	$P_{ij} \in [0,1]$
	P_2	Horses	P_{21}	P_{22}	P_{23}	P_{24}	P_{25}	$\sum_{i=1}^{5} P_{ij} = 1$
	P_3	Cats	P_{31}	P_{32}	P_{33}	P_{34}	P_{35}	
	P_4	Dogs	P_{41}	P_{42}	P_{43}	P_{44}	P_{45}	$\sum_{j=1}^{5} P_{ij} = 1$
	P_5	Fishes	P_{51}	P_{52}	P_{53}	P_{54}	P_{55}	

Table 2 Constraints in the mathematical model of the riddle

Description	Mathematical model	
The British lives in the red house.	$\sum_{j=1}^{5} j \cdot N_{5j} = \sum_{j=1}^{5} j \cdot HC_{4j}$	(1)
The Swedish keeps dogs as pets.	$\sum_{j=1}^{5} j \cdot N_{2j} = \sum_{j=1}^{5} j \cdot P_{4j}$	(2)
The Dane drinks tea.	$\sum_{j=1}^{5} j \cdot N_{4j} = \sum_{j=1}^{5} j \cdot B_{3j}$	(3)
The green house is on the left of the white house.	$\sum_{j=1}^{5} j \cdot HC_{2j} < \sum_{j=1}^{5} j \cdot HC_{5j}$	(4)
The green house's owner drinks coffee.	$\sum_{j=1}^{5} j \cdot HC_{2j} = \sum_{j=1}^{5} j \cdot B_{1j}$	(5)
The owner who smokes Pall Mall rears birds.	$\sum_{j=1}^{5} j \cdot CB_{5j} = \sum_{j=1}^{5} j \cdot P_{1j}$	(6)
The owner of the yellow house smokes Dunhill.	$\sum_{j=1}^{5} j \cdot HC_{1j} = \sum_{j=1}^{5} j \cdot CB_{1j}$	(7)
The owner living in the center house drinks milk.	$B_{53} = 1$	(8)
The Norwegian lives in the first house.	$N_{11} = 1$	(9)
The owner who smokes Blend lives next to the one who keeps cats.	$\sum_{j=1}^{5} j \cdot CB_{2j} - \sum_{j=1}^{5} j \cdot P_{3j} = 1 - (2 \cdot D_1)$ $D_1 \in [0,1] \rightarrow 0 = right; 1 = left$	(10) (11)
The owner who keeps the horses lives next to the one who smokes Dunhill.	$\sum_{j=1}^{5} j \cdot P_{2j} - \sum_{j=1}^{5} j \cdot CB_{1j} = 1 - (2 \cdot D_2)$ $D_2 \in [0,1] \rightarrow 0 = right; 1 = left$	(12) (13)
The owner who smokes Bluemaster drinks beer.	$\sum_{j=1}^{5} j \cdot CB_{3j} = \sum_{j=1}^{5} j \cdot B_{2j}$	(14)
The German smokes Prince.	$\sum_{j=1}^{5} j \cdot N_{3j} = \sum_{j=1}^{5} j \cdot CB_{4j}$	(15)
The Norwegian lives next to the blue house.	$HC_{32} = 1$	(16)
The owner who smokes Blend lives next to the one who drinks water.	$\sum_{j=1}^{5} j \cdot CB_{2j} - \sum_{j=1}^{5} j \cdot B_{4j} = 1 - (2 \cdot D_3)$ $D_3 \in [0,1] \rightarrow 0 = right; 1 = left$	(17) (18)

Constraints (8) and (9) are given as facts, and hence, mathematically defined. For conditions (10), (12), and (17) additional binary variables were added and denoted as “D_1”, “D_2”, and “D_3” in order to model the relative direction of the first mentioned variable from the nesting second variable (i.e., 0 marks the location on the right side, while 1 marks the left side). The equality constraint (16) has as only one possible solution: if N_{11}= 1 (Norwegian lives in the first house) then HC_{32} = 1 (the blue house is the second in the row). The objective function of the optimization problem is given as the target value of 25, which can also be considered as a final constraint. The sum of all variables has to be equal to 25 and all domain variables are applied to each house exactly once:

$$\sum_{j=1}^{5}\sum_{i=1}^{5} N_{ij} + \sum_{j=1}^{5}\sum_{i=1}^{5} HC_{ij} + \sum_{j=1}^{5}\sum_{i=1}^{5} B_{ij} + \sum_{j=1}^{5}\sum_{i=1}^{5} CB_{ij} + \sum_{j=1}^{5}\sum_{i=1}^{5} P_{ij} = 25 \quad (19)$$

2.5 Spreadsheet solution of the riddle

The optimization problem was structured as a spreadsheet model in Microsoft Excel 2016 [18] and solved using Solver add-in. Excel's Solver is suited for solving optimization problems structured with a maximum of 200 decision variables, as has been noted in previous researches [19, 20]. This is greater than the number of binary variables of the formulated problem. The model is based on formulations suggested in [15], but modified according to the mathematical model presented in the previous section. For solving the aforementioned problem, the authors used a PC with a 64-bit processor running at 3.00GHz and having 4GB RAM. The Solver provided a solution (Table 3) in 10 seconds using the "Simplex LP" solving method, which employs the Branch and Bound method [21, 22] for solving the discrete linear problem. The answer to the question "who owns the fish?" is the German keeps fish as pets.

Table 3 Spreadsheet solution of "Einstein's riddle"

Domains	House H_1	House H_2	House H_3	House H_4	House H_5
Nationality	Norwegian	Dane	British	German	Swedish
House color	Yellow	Blue	Red	Green	White
Beverage	Water	Tea	Milk	Coffee	Beer
Cigarette brand	Dunhill	Blend	Pall Mall	Prince	Blue Master
Pet	Cats	Horses	Birds	Fish	Dogs

3 APPLICATION OF "EINSTEIN'S RIDDLE" IN CONSTRUCTION

3.1 Problem 1: Spreadsheet solution of a small scale CSP in construction

3.1.1 Formulation of problem 1

CSPs in construction can be identified in small scale to complex problems having numerous and different types of variables and constraints. This section presents a solution to a small-scale problem common to construction sites: the optimal allocation of machines for simultaneous serial operations in several ongoing projects, or in the same project but at different locations. Previous work on such tasks [23] have revealed, that even in relatively small-scale problems the number of feasible combinations and parameters exceeds human imagination and experience. Modeling and simulation, are therefore, considered as appropriate tools for complex decision-making, which ensure optimal solutions.

Following is an example of an allocation problem for two different types of machines, loaders "L_i" and trucks "T_i". Three machines of each type (i = *1, 2, 3*) are used for loading and transportation of the excavated soil from three construction sites "CS_j" (j= 1, 2, 3) in such a manner, that each site is served by exactly one loader and one truck. With the number of machines required equaling those available, this is an example of a balanced model, for which an exact solution for the problem is achievable. Input parameters for the machines are listed in table 4. These are machine capacity "q_i", average cycle time "t_i", average machine efficiency "e_i", and average hourly expenses for each machine for each construction site "c_{ij}", which were generated from intervals taken from literature [24, 25].

Table 4 Input parameters of problem 1

Machine	q_i[m^3]	t_i[s]	e_i[m^3/h]	c_{i1}[€/h]	c_{i2}[€/h]	c_{i3}[€/h]
Loader L_1	0.6	41	27.18	55.00	55.06	49.84
Loader L_2	0.6	40	28.73	55.26	50.39	50.61
Loader L_3	0.8	45	33.41	53.82	49.62	50.39
Truck T_1	10.0	2178	15.21	41.97	39.40	41.40
Truck T_2	12.0	2396	16.43	37.76	39.62	38.85
Truck T_3	14.0	2318	19.72	39.08	43.85	34.51

3.1.2 Mathematical model

The mathematical formulation of the problem is shown in Table 5. The selection of machines from both domains is executed by binary variables L_{ij}, T_{ij} є [0,1]. The constraint lies in solving the problem in a manner, that exactly one machine of each type is allocated to each site.

Table 5 Matrix formulation of problem 1

		Construction sites CS_j			
		CS_1	CS_2	CS_3	Constraints
Loaders L_i	L_1	L_{11}	L_{12}	L_{13}	$\sum_{i=1}^{3} L_{ij} = 1$
	L_2	L_{21}	L_{22}	L_{23}	
	L_3	L_{31}	L_{32}	L_{33}	$\sum_{j=1}^{3} L_{ij} = 1$
Trucks T_i	T_1	T_{11}	T_{12}	T_{13}	$\sum_{i=1}^{3} T_{ij} = 1$
	T_2	T_{21}	T_{22}	T_{23}	
	T_3	T_{31}	T_{32}	T_{33}	$\sum_{j=1}^{3} T_{ij} = 1$

Each site has to be served by only one machine from each domain, which implies that there are 27 feasible combinations or scenarios “S_n” defined by binary variables (Eqn. 20 and 21). Each scenario generates two optimization parameters: combined expenses (unit costs of loader “$c_{L,ij}$” and truck “$c_{T,ij}$”) of employed machines on each construction site “$E_{n,j}$”, and the usage of the employed server (loader) “$\rho_{n,L,i}$”, which in queuing theory is a well-known parameter. This is defined as the ratio of the average entry into the system “λ”, dependent on the trucks’ cycle time, and the average number of serving operations in a unit of time "μ", dependent on loaders efficiency and trucks’ capacity (Eqn. 22 and 23).

$$L_{ij} + T_{ij} \rightarrow S_n;\ n = 1, \dots, 27;\ S_n \in [0,1] \tag{20}$$

$$S_n = \begin{cases} 0,\ L_{ij} \wedge T_{ij} = 0 \\ 1,\ L_{ij} \vee T_{ij} = 1 \end{cases}, i, j = 1,2,3 \tag{21}$$

$$E_{n,j} = c_{L,ij} + c_{T,ij} \tag{22}$$

$$\rho_{n,L,i} = \frac{\lambda}{\mu} \propto e_{n,Ti} \tag{23}$$

Owing to the fact that the Excel solver can find the optimal solution on the basis of only one objective function and that the usage of loaders depends on the efficiency of the trucks, the ratio “$R_{n,j}$” of combined expenses of employed machines “$E_{n,j}$” on each construction site and employed truck efficiency “$e_{T,i}$” was determined to be an objective function (Eqn. 24).Thus, the optimal solution is the scenario which generates the smallest value of “$R_{n,j}$”.

$$\min Z = \sum_{j=1}^{3} \sum_{i=1}^{3} \frac{E_{n,j}}{e_{T,i}} = \sum_{j=1}^{3} R_{n,j} \tag{24}$$

https://doi.org/10.13167/2016.13.2

3.1.3 *Spreadsheet solution*

The problem was structured as a spreadsheet model and the same PC as well as the same search algorithm for solving the problem was applied. The authors initially used an “evolutionary” solving method to obtain the solution and then verified it using the “Simplex LP” method. Either way, the solver provided an optimal solution in matter of seconds. The R_j value was 15.725 (Table 6), while the total hourly expenses were 265.18 €/h.

Table 6 Optimal allocation of machines in problem 1

Construction site CS_1	Construction site CS_2	Construction site CS_3	
L_3+T_2	L_2+T_1	L_1+T_3	
$R_1 = 5.543$	$R_2 = 5.905$	$R_3 = 4.277$	$\rightarrow \Sigma R_j=15.725$

3.2 Problem 2: Spreadsheet solution of a medium scaled CSP in construction

3.2.1 *Formulation of problem 2*

The second example of CSP in construction is an expanded problem 1. An additional construction site requiring the same type of construction machines and two more machines of each type in the pool of available machines were added to the existing problem. The source and values of input parameters of the machines (Table 7) are the same as those in problem 1.

Table 7 Input parameters of problem 2

Machines	q_i[m³]	t_i[s]	e_i[m³/h]	c_{i1}[€/h]	c_{i2}[€/h]	c_{i3}[€/h]	c_{i4}[€/h]
Loader L_1	0.6	41	27.18	55.00	55.06	49.84	50.28
Loader L_2	0.6	40	28.73	55.26	50.39	50.61	51.05
Loader L_3	0.8	45	33.41	53.82	49.62	50.39	45.83
Loader L_4	0.7	42	29.60	52.15	50.83	50.17	52.17
Loader L_5	0.6	41	27.18	57.00	50.06	50.28	49.95
Truck T_1	10.0	2178	15.21	41.97	39.40	41.40	44.62
Truck T_2	12.0	2396	16.43	37.76	39.62	38.85	39.84
Truck T_3	14.0	2318	19.72	39.08	43.85	34.51	41.40
Truck T_4	12.0	2355	17.18	40.00	42.40	42.18	38.29
Truck T_5	10.0	2150	16.10	36.15	42.29	42.73	44.40

Problem 2 addresses five available loaders “L_i” and five trucks “T_i” (i=1, 2,…, 5) for loading and transportation of the excavated soil from four different construction sites “CS_j” (j=1, 2,…, 4) in such a manner, that each of them is served by exactly one loader and one truck. Hence, one machine of each type will remain unassigned.

3.2.2 *Mathematical model*

The problem formulation, as shown in table 8, is no longer quadratic (i.e., available machines outnumber required machines). This means that there will be unassigned machines in each of feasible 100 scenarios “*Sn*”. As in the previous instance, the employment of machines from both domains and scenarios are defined using binary variables (Eqn. 25 and 26).

Table 8 Matrix formulation of problem 2

		Construction sites CS_j				
		CS_1	CS_2	CS_3	CS_4	Constraints
Loaders L_i	L_1	L_{11}	L_{12}	L_{13}	L_{14}	$L_{ij} \in [0,1]$
	L_2	L_{21}	L_{22}	L_{23}	L_{24}	$\sum_{i=1}^{5} L_{ij} = 1$
	L_3	L_{31}	L_{32}	L_{33}	L_{34}	
	L_4	L_{41}	L_{42}	L_{43}	L_{44}	$\sum_{j=1}^{4} L_{ij} = 1$
	L_5	L_{51}	L_{52}	L_{53}	L_{54}	
Trucks T_i	T_1	T_{11}	T_{12}	T_{13}	T_{14}	$T_{ij} \in [0,1]$
	T_2	T_{21}	T_{22}	T_{23}	T_{24}	$\sum_{i=1}^{5} T_{ij} = 1$
	T_3	T_{31}	T_{32}	T_{33}	T_{34}	
	T_4	T_{41}	T_{42}	T_{43}	T_{44}	$\sum_{j=1}^{4} T_{ij} = 1$
	T_5	T_{51}	T_{52}	T_{53}	T_{54}	

The combined expenses of employed machines on each construction site "$E_{n,j}$" and usage of employed server "$\rho_{n,L,i}$" are formulated in Eqn. 27 and 28.

$$L_{ij} + T_{ij} \rightarrow S_n;\ n = 1, \dots, 100;\ S_n \in [0,1] \tag{25}$$

$$S_n = \begin{cases} 0,\ L_{ij} \wedge T_{ij} = 0 \\ 1,\ L_{ij} \vee T_{ij} = 1 \end{cases},\ i = 1,2,3,4,5;\ j = 1,2,3,4 \tag{26}$$

$$E_{n,j} = c_{L,ij} + c_{T,ij} \tag{27}$$

$$\rho_{n,L,i} = \frac{\lambda}{\mu} \propto e_{n,Ti} \tag{28}$$

In this case, four additional constraints (Table 9) were added to simulate the following real-time requirements: i) one loader cannot be transported to a specific construction site; ii) one loader is not compatible with a certain truck; iii) one loader has to be employed on a certain construction site; iv) two specified trucks cannot be employed on two adjacent construction sites.

Table 9 Additional constraints in mathematical model

Description	Mathematical model	
Loader 1 is not eligible for construction site 3.	$L_{13} = 0$	(29)
Truck 2 has to be employed along with Loader 3.	$\sum_{j=1}^{4} j \cdot T_{2j} = \sum_{j=1}^{4} j \cdot L_{3j}$	(30)
Loader 4 cannot be paired with truck 3.	$\sum_{j=1}^{4} L_{4j} \cdot T_{3j} = 0$	(31)
Trucks 2 and 3 cannot be employed on two adjacent construction sites.	$\left\lvert \sum_{j=1}^{4} j \cdot T_{2j} - \sum_{j=1}^{4} j \cdot T_{3j} \right\rvert > 1$	(32)

The optimal scenario is one which simultaneously shows minimal total expenses (Eqn. 33) and maximal trucks' efficiencies (Eqn. 34):

$$\min Z = \sum_{j=1}^{4} E_{n,j} \tag{33}$$

$$\max Z = \sum_{i=1}^{4} e_{n,T,i} \tag{34}$$

3.2.3 Spreadsheet solution

The spreadsheet model of the problem was developed using Microsoft Excel, as in previous cases. However, Excel's solver could not find a solution by employing any of its solving methods. Hence, the authors used Microsoft Solver Foundation (MSF) [26], a more advanced add-in solver to solve the problem. MSF is capable of solving an optimization problem having more than one objective function. For this scenario, two objective functions were formulated (minimization of total hourly expenses and maximization of trucks' efficiency) having the same priority, as was defined in the mathematical model of the problem explained in the previous section. After the search process was run, MSF provided an optimal solution within the user-specified time of 120 s (Table 10). Unassigned machines were found to be loader L_2 and truck T_4. The total hourly expenses were 370.29 €/h; while the sum of the trucks' efficiencies was 67.46 m^3/h.

Table 10 Optimal allocation of machines in problem 2

Construction site CS_1	**Construction site CS_2**	**Construction site CS_3**	**Construction site CS_4**	
L_3+T_2	L_1+T_1	L_4+T_5	L_5+T_3	
E_1=91,58 €/h	E_2=94,46 €/h	E_3=92,90 €/h	E_4=91,35 €/h	→ ΣE_j=370.29 €/h
e_{T2}=16,43 m^3/h	e_{T1}=15,21 m^3/h	e_{T5}=16,10 m^3/h	e_{T3}= 19,72 m^3/h	→ Σe_{Ti}=67.46 m^3/h

4 DISSCUSSION AND CONCLUSSIONS

This paper addressed the applicability of a well-known example of CSP, “Einstein's riddle”, for solving the problem of allocating construction machines. Following a brief introduction, the authors gave the background of the general formulation of a CSP and “Einstein's riddle”. Owing to the broad acceptance of the riddle, its employability in construction operations was analyzed. In the second section, the authors presented the riddle in its commonly known form and introduced the general mathematical problem formulation as well as the spreadsheet solution of the problem obtained using Microsoft Excel's add-in solver. The solver provided a unique solution of the riddle, which matches the one obtained by previous researchers. Using the same methodology in the third section, a small-scale problem of simultaneous allocation of construction machines on various construction sites was considered. The problem comprised of allocating three loaders and three trucks on three different construction sites in such a manner, that each site has to be assigned exactly with one loader and one truck. The optimization objective was to minimize the total hourly expenses of the assigned machines and to maximize the loaders' employment. Owing to the solver's limitation of optimizing problems with only one objective function, the authors introduced a unique optimization parameter by connecting the two aforementioned objective functions. The solver provided a quick and optimal solution to the problem. Problem 1 was used in structuring the medium-scale problem 2 involving an additional construction site and additional two available machines of each type. As Excel's solver could not solve this problem, the authors used the more advanced Microsoft Solver Foundation add-in solver, which is capable of optimizing more than one objective function. MSF provided a relatively quick optimal solution to the problem.

It can be concluded that allocation of construction machines can be formulated as CSPs. However, the choice of the solver for the spreadsheet solution of the problem depends on constraints and variable types, the problem size, and the number of objective functions. For further development of the suggested approach, the authors will upgrade the model by including other types of machines and different construction works, as well as multi-channel relationships among employed machines. For solving more complex instances of addressed problems, it is suggested that a neural network based optimization and employment of more advanced software should be considered. However, in terms of potential for commercial usage of the proposed approach, the authors wish to emphasize the necessity for simplification of input data, as well as the control of results.

References

[1] Plenert, G. J. 2014: Supply Chain Optimization Through Segmentation and Analytics, CRC Press, London.
[2] Friesen, D. D.; Patterson, M. C.; Harmel, B. 2013: A genetic algorithm spreadsheet model for optimizing a combinatorial problem, International Journal of Management Research and Reviews, 3 (3), pp. 2497-2512,
[3] Schwitter, R. 2008: Reconstructing Hard Problems in a Human-Readable and Machine-Processable Way. In: Ho T-B, Zhou Z-H, editors. Pacific Rim International Conference on Artificial Intelligence: Springer; pp. 1046-1052.
[4] Hindriks, K.; Vromans, J. 2007: Design and Evaluation of Formal Representations: An Incremental Approach using Logic Grid Puzzles. In: Geertzen J, Thijsse E, Bunt H, editors. International Workshop on Computational Semantics - IWCS'07. Tilburg, The Netherlands pp. 334–338.
[5] Gregor, M.; Zábovská, K.; Smataník, V. 2015: The zebra puzzle and getting to know your tools. Intelligent Engineering Systems (INES), 2015 IEEE 19th International Conference on: IEEE; pp. 159-164.
[6] Salavati, S.; Hajjarzadeh, S.; Mazloom, M. 2009: An Optimized Method for Solving Zebra Puzzle. 2009 Second International Conference on Computer and Electrical Engineering: IEEE; pp. 448-451.
[7] Bakó, M. 2002: Why we need to teach logic and how can we teach it, International Journal for Mathematics Teaching and Learning, pp. 1473-0111,
[8] Crabtree, J.; Zhang, X. 2015: Recognizing and Managing Complexity: Teaching Advanced Programming Concepts and Techniques Using the Zebra Puzzle, Journal of Information Technology Education: Innovations in Practice, 14 pp. 171-189,
[9] Little, J.; Gebruers, C.; Bridge, D.; Freuder, E. 2002: Capturing constraint programming experience: A case-based approach. CP-02 Workshop on Reformulating Constraint Satisfaction Problems.
[10] Bučko, B.; Hanušniak, V.; Joštiak, M.; Zábovská, K. 2015: Inferring over functionally equivalent ontologies. Digital Information Processing and Communications (ICDIPC), 2015 Fifth International Conference on: IEEE; pp. 201-206.
[11] Kumar, V. 1992: Algorithms for constraint-satisfaction problems: A survey, AI magazine, 13 (1), pp. 32-44,
[12] Mackworth, A. K.; Freuder, E. C. 1985: The complexity of some polynomial network consistency algorithms for constraint satisfaction problems, Artificial intelligence, 25 (1), pp. 65-74, http://dx.doi.org/10.1016/0004-3702(85)90041-4
[13] Smith, B. M. 2001: Constructing an asymptotic phase transition in random binary constraint satisfaction problems, Theoretical Computer Science, 265 (1), pp. 265-283, https://doi.org/10.1016/S0304-3975(01)00166-9
[14] Browne, C. 2013: Deductive search for logic puzzles. Computational Intelligence in Games (CIG), 2013 IEEE Conference on: IEEE; pp. 1-8.
[15] Patterson, M. C.; Harmel, B.; Friesen, D. 2007: A spreadsheet solution to Einstein's Riddle, International Journal of Technology, Policy and Management, 7 (1), pp. 49-67, https://doi.org/10.1504/IJTPM.2007.012239
[16] Yeomans, J. S. 2003: Solving “Einstein's Riddle” Using Spreadsheet Optimization, INFORMS Transactions on Education, 3 (2), pp. 55-63, https://doi.org/10.1287/ited.3.2.55
[17] Life Magazine. 1962. Who owns the zebra? 17 (95), Life International magazine.
[18] Microsoft: Excel 2016. Office 2016 Professional.
[19] Baños, R.; Gil, C.; Reca, J.; Montoya, F. G. 2010: A memetic algorithm applied to the design of water distribution networks, Applied Soft Computing, 10 (1), pp. 261-266, https://doi.org/10.1016/j.asoc.2009.07.010
[20] Mélard, G. 2014: On the accuracy of statistical procedures in Microsoft Excel 2010, Computational statistics, 29 (5), pp. 1095-1128, https://doi.org/10.1007/s00180-014-0482-5
[21] Land, A. H.; Doig, A. G. 1960: An automatic method for solving discrete programming problems, Econometrica, 28 (3), pp. 497-520, https://doi.org/10.2307/1910129
[22] Klanšek, U. 2011: Optimizacija v operativnem gradbeništvu,Univerza v Mariboru, Fakulteta za gradbeništvo, Maribor, Slovenija. (in Slovenian)
[23] Galić, M.; Završki, I.; Dolaček-Alduk, Z. 2016: Scenario simulation model for optimized allocation of construction machinery, Građevinar, 68 (2), pp. 105-112, https://doi.org/10.14256/JCE.1462.2015
[24] Institut Građevinarstva Hrvatske IGH. *Standardna kalkulacija radova u visokogradnji - Bilten III*. Đukan, P. Zagreb, 2015. (in Croatian)
[25] König, H. 2008: Maschinen im Baubetrieb,2nd,Springer, Wiesbaden, Germany.
[26] Microsoft Solver Foundation: Solver Foundation