Economic Research-Ekonomska Istraživanja

ISSN: 1331-677X (Print) 1848-9664 (Online) Journal homepage: http://www.tandfonline.com/loi/rero20

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To cite this article: Tomas Baležentis & Alvydas Baležentis (2016) Dynamics of the total factor productivity in Lithuanian family farms with a statistical inference: the bootstrapped Malmquist indices and Multiple Correspondence Analysis, Economic Research-Ekonomska Istraživanja, 29:1, 643-664, DOI: 10.1080/1331677X.2016.1193946

To link to this article: http://dx.doi.org/10.1080/1331677X.2016.1193946

Published online: 30 Jun 2016.

ECONOMIC RESEARCH-EKONOMSKA ISTRAŽIVANJA, 2016
VOL. 29, NO. 1, 643–664
http://dx.doi.org/10.1080/1331677X.2016.1193946

Routledge
Taylor & Francis Group

OPEN ACCESS

Dynamics of the total factor productivity in Lithuanian family farms with a statistical inference: the bootstrapped Malmquist indices and Multiple Correspondence Analysis

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ABSTRACT
The paper combines the bootstrapped Malmquist productivity index and the Multiple Correspondence Analysis to measure the changes in the total factor productivity. The bootstrapped Malmquist productivity index enables us to identify insignificant change in the total factor productivity, whereas the Multiple Correspondence Analysis relates the estimates to the environmental variables. A sample of Lithuanian family farms is utilised to test the proposed framework. Specifically, the research covers 200 family farms and the period of 2004–2009. The analysis showed that the total factor productivity decreased by some 15–18% during 2004–2009 depending on the farming type. Multiple Correspondence Analysis suggested that all of the farming types exhibited change in the total factor productivity close to the average, although the crop farming was located in the more stochastic area.

ARTICLE HISTORY
Received 5 November 2013
Accepted 4 February 2016

KEYWORDS
Total factor productivity; bootstrapped Malmquist index; Multiple Correspondence Analysis; family farms

JEL CLASSIFICATIONS
C43; D24; Q12

1. Introduction

The Lithuanian agricultural sector is influenced by both economic and demographic transitions, which, indeed, are pertinent to many Central and East European countries (cf. Gorton & Davidova, 2004) due to de-collectivisation and European integration. Accession to the European Union (EU) in 2004 induced certain variations in support policies and trade policies. Therefore, it is important to ascertain whether Lithuanian farmers managed to exploit the new possibilities or, conversely, these changes resulted in the emergence of a more hostile business environment during the post-accession period. Indeed, this paper focuses on Lithuanian family farms, as opposed to corporate farms, which produced over 70% of the agricultural output in Lithuania during 2004–2009. The post-accession period features structural changes in terms of farm size and specialisation. Specifically, an increase in the mean farm size due to the exits of small-scale farms has been observed alongside a switching to crop farming in lieu of animal farming (Baležentis, Kriščiukaitienė, & Baležentis, 2014). However, the average farm size is still below that in other countries with similar farming possibilities (e.g.,

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Denmark). This implies a need for identification of factors causing efficiency and productivity growth in Lithuanian agriculture.

Growth in productive efficiency constitutes the major driver of business profitability and sustainability. In its essence, the very term efficiency refers to the distance between an observed production plan and the production frontier (production possibility frontier). However, the production frontier can move inwards or outwards from the origin point depending on the technological development underlying the observed productive system. Thus, one needs to measure not only efficiency, but also the total factor productivity (TFP) change, which tackles both firm-specific catch-up and system-wide technical change. For the latter purpose, the productivity indices are usually employed (Caves, Christensen, & Diewert, 1982). These can be Malmquist, Luenberger, Hicks–Moorsteen, Färe–Primont etc. This study focuses on the Malmquist productivity index.

The Malmquist productivity index can be estimated by means of the distance functions based either on parametric (e.g., stochastic frontier analysis) or non-parametric (e.g., data envelopment analysis) estimates. The generic non-parametric methods do not account for the statistical noise. Therefore, the bootstrapping approach was offered by Simar and Wilson (1998b, 2000) for the data envelopment analysis (DEA) and the Malmquist productivity indices (Simar & Wilson, 1999). Wilson (2008) also developed the *FEAR* package to facilitate these computations.

The latter methodology has been widely employed for the productivity analyses. As for agriculture and fisheries, Hoff (2006) analysed the fishing activity by the means of the bootstrapped Malmquist indices; Odeck (2009) applied the bootstrapped Malmquist indices to the Norwegian grain industry; Balcombe, Davidova, and Latruffe (2008) researched into the productivity of the Polish family farms, whereas Rezitis, Tsiboukas, and Tsoukalas (2009) focused on Greek livestock farms. The other sectors were also analysed by means of the bootstrapped Malmquist indices. For instance, Perelman and Serebrisky (2012) analysed the efficiency and productivity of Latin American airports. Jaraitė and Di Maria (2012) employed the bootstrapped Malmquist indices for analysis of power generation in the European Union. Horta, Camanho, Johnes, and Johnes (2013) analysed the performance of the construction industry. Arjomandi, Valadkhani, and Harvie (2011) utilised the bootstrapped Malmquist indices for analysis of the Iranian banking sector. Zhou, Ang, and Han (2010) employed the bootstrapped Malmquist indices for the analysis of carbon emissions with weak disposability. Parteka and Wolszczak-Derlacz (2013) assessed productivity in higher education by the means of the bootstrapped Malmquist indices. Essid, Ouellette, and Vigeant (2014) applied the bootstrapped Malmquist along with quasi-fixed inputs to measure the educational productivity.

Given that the Lithuanian agricultural sector faces multiple transformations, the data regarding performance of farms might be perturbed in various ways. Therefore, one needs to employ statistical methods to identify significant changes in efficiency and productivity. The bootstrapped Malmquist index allows for such an analysis and thus can be considered as an appropriate tool for analysis of agricultural productivity change. This paper applies the bootstrapped Malmquist productivity index to a sample of the Lithuanian family farms in order to estimate the dynamics of the total factor productivity there. Furthermore, Multiple Correspondence Analysis (MCA) is employed to visualise the underlying patterns of the total factor productivity change. Indeed, the bootstrapped Malmquist indices have not been applied to the Lithuanian agricultural sector up to now. This paper, thus, aims at identifying the sources and factors of growth in the total factor productivity in Lithuanian family farms. Whereas the techniques used, namely bootstrapped Malmquist index and MCA, are well-established ones, their combination is fairly new. Accordingly, the proposed approach might be relevant for further applied research.

The paper proceeds as follows: Section 2 treats the preliminaries for the bootstrapped Malmquist productivity index. Section 3 presents Multiple Correspondence Analysis. An empirical application of the bootstrapped Malmquist index and MCA is given in Section 4. Finally, Section 5 draws the conclusions.

2. Productive technology and Malmquist index

This section presents the main concepts of efficiency and productivity. The first sub-section describes the very definition of efficiency, whereas the second one presents the Malmquist productivity index. The Malmquist productivity index enables one to quantify the changes in firm-specific efficiency as well as the global shift in the production frontier.

2.1. Measures of efficiency

In order to relate the Debreu–Farrell measures to the Koopmans definition of efficiency, and to relate both to the structure of production technology, it is useful to introduce some notation and terminology (Fried, Lovell, & Schmidt, 2008). Let producers use inputs $x = (x_1, x_2, \ldots, x_m) \in \mathfrak{R}_+^m$ to produce outputs $y = (y_1, y_2, \ldots, y_n) \in \mathfrak{R}_+^n$. Production technology then can be defined in terms of the production set:

$$T = \{(x, y)|x \text{ can produce } y\} \tag{1}$$

Thus, Koopmans efficiency holds for an input–output bundle $(x, y) \in T$ if, and only if, there is no ordered pair $(x', y') \in T$, such that $x' \leq x$ and $y' \geq y$.
Technology set can also be represented by input requirement set, $I(y)$, and output correspondence set, $O(x)$:

$$I(y) = \{x|(x, y) \in T\} \tag{2}$$

and

$$O(x) = \{y|(x, yt) \in T\} \tag{3}$$

The isoquants or efficient boundaries of the sections of T can be defined in radial terms as follows (Farrell, 1957). Every $y \in \mathfrak{R}_+^n$ has an input isoquant:

$$isoI(y) = \{x|x \in I(y), \lambda x \notin I(y), \lambda < 1\} \tag{4}$$

Similarly, every $x \in \mathfrak{R}_+^m$ has an output isoquant:

$$isoO(x) = \{y|y \in O(x), \lambda y \notin O(x), \lambda > 1\} \tag{5}$$

In addition, decision-making units (DMUs) might be operating on the efficiency frontier defined by equations (4) and (5), albeit still using more inputs to produce the same output if compared with another efficient DMU. In this case, the former DMU experiences a slack in inputs. The following subsets of the boundaries $I(y)$ and $O(x)$ describe Pareto-Koopmans efficient firms:[1]

$$effO(x) = \{y | y \in O(x), y' \notin O(x), \forall y' \geq y, y' \neq y\} \quad (6)$$

Note that $effO(x) \subseteq isoO(x) \subseteq O(x)$.

There are two distinctive types of efficiency measures, namely the Shepard distance function, and Farrell distance function. These functions yield the distance between an observation and the efficiency frontier. Shepard (1953) defined the following output distance function:

$$D_O(x, y) = \min\left\{\theta \middle| \left(x, y/\theta\right) \in T\right\} \quad (7)$$

Similarly, the following equations hold for the Farrell output-oriented measure:

$$TE_O(x, y) = \max\left\{\varphi \middle| \left(x, \varphi y\right) \in T\right\} \quad (8)$$

$$TE_O(x, y) = 1/D_O(x, y) \quad (9)$$

where $TE_O(x, y) \geq 1$ for $y \in O(x)$, and $TE_O(x, y) = 1$ for $y \in isoO(x)$.

2.2. The Malmquist productivity index

Measurement of the total factor productivity (TFP) of a certain DMU involves measures for both technological and firm-specific developments. As Bogetoft and Otto (2011) put it, firm behaviour changes over time should be explained in terms of special initiatives as well as technological progress. The benchmarking literature (Bogetoft & Otto, 2011; Coelli, Rao, O'Donnell, & Battese, 2005; Ramanathan, 2003) suggests the Malmquist productivity index is the most celebrated TFP measure. Hence, this section describes the preliminaries of Malmquist index.

Färe, Grosskopf, and Margaritis (2008) first describe productivity as the ratio of output y over input x. Thereafter, the productivity can be measured by employing the output distance function of Shepard (1953):

$$D^t_{ov}(x^s, y^s) = \min\left\{\theta : (x^s, y^s/\theta) \in T^t\right\} \quad (10)$$

where T^t stands for the technology set (production possibility set) of the period t and s denotes the same or an adjacent time period, index ov denotes an output distance under variable returns to scale. In the case where $s = t$, equation (10) is called a contemporaneous distance function. This particular function is equal to unity if and only if certain input and output set belongs to production possibility frontier.

Let P^t be the convex cone (with vertex at the origin) spanned by T^t; then $T^t \subseteq P^t$ (Simar & Wilson, 1998a). If a technology T^t exhibits constant returns to scale everywhere, then it implies a mapping $x \to y$ that is homogeneous of degree 1, i.e., $(x, y) \in T^t \Rightarrow (\lambda x, \lambda y) \in T^t, \forall \lambda > 0$. In this case, $T^t = P^t$. If T^t does not exhibit constant returns to scale everywhere, then $T^t \subset P^t$. The Shepard efficiency measure for the constant returns to scale, denoted by index oc, technology can thus be given as

$$D^t_{oc}(x^s, y^s) = \min\left\{\theta : (x^s, y^s/\theta) \in P^t\right\} \quad (11)$$

Generally, $0 < D_{oc}^{t}\left(x^{t}, y^{t}\right) \leq D_{ov}^{t}\left(x^{t}, y^{t}\right) \leq 1$. In case a DMU is efficient under the assumed technology, $D_{ov}^{t}(x^{s}, y^{s})$ or $D_{oc}^{t}(x^{s}, y^{s})$ equals unity. In case $s \neq t$, no upper bound exists.
The Malmquist productivity index (Malmquist, 1953) can be employed to estimate TFP changes of a single firm over two periods (or vice versa), across two production modes, strategies, locations etc. In this study we shall focus on output–oriented Malmquist productivity index and apply it to measure period–wise changes in TFP. The output–oriented Malmquist productivity index due to Caves et al. (1982) is defined as

$$M_{o} = \left(M_{o}^{0} \cdot M_{o}^{1}\right)^{1/2} = \left(\frac{D_{oc}^{0}(x^{1}, y^{1})}{D_{oc}^{0}(x^{0}, y^{0})} \frac{D_{oc}^{1}(x^{1}, y^{1})}{D_{oc}^{1}(x^{0}, y^{0})}\right)^{1/2} \tag{12}$$

with indexes 0 and 1 representing respective periods and sub-index c denoting the constant returns to scale (CRS) assumption. The two terms in brackets follows the structure of Fisher's index. Note that all the distances in equation (12) are based on the CRS technology, otherwise the Malquist index would not feature its interpretation as a productivity index. There have been a number of ways to decompose the Malmquist index offered (Färe, Grosskopf, Lindgren, & Roos, 1992; Färe, Grosskopf, Norris, & Zhang, 1994; Ray & Desli, 1997; Simar & Wilson, 1998a; Wheelock & Wilson, 1999) with each of them allowing to account for different sources of TFP change. For instance, Färe et al. (1992) proposed decomposing TFP change in terms of efficiency change (EC or catching up) and technical change (TC or shifts in the frontier):

$$M_{o} = EC \cdot TC \tag{13}$$

where

$$EC = D_{oc}^{1}(x^{1}, y^{1}) \big/ D_{oc}^{0}(x^{0}, y^{0}) \tag{14}$$

and

$$TC = \left(\frac{D_{oc}^{0}(x^{1}, y^{1})}{D_{oc}^{1}(x^{1}, y^{1})} \frac{D_{oc}^{0}(x^{0}, y^{0})}{D_{oc}^{1}(x^{0}, y^{0})}\right)^{1/2} \tag{15}$$

EC captures the movement relative to the frontier induced by changes in a production plan (i.e. catching up). Specifically, EC exceeds unity whenever a firm gets closer to the frontier as the time passes. TC represents the movement of the frontier within the neighbourhood of a firm (i.e. technical change). Under technological progress, the frontier moves further from the point of origin and TC exceeds unity. This situation implies that a greater amount of outputs can be produced by consuming fewer resources under the new technology. Consequently, the values of Malmquist index exceeding unity represent a positive TFP growth and those below unity represent a negative TFP growth for a particular observation. Figure 1 presents a graphical interpretation of the input Malmquist productivity index. Here, the point A denotes an initial production plan in period t, whereas point B stands for another production plan during period t+1. Meanwhile, the two isoquants, $isoO^{t}$ and $isoO^{t+1}$, represent the efficient technology during periods t and t+1, respectively. The two points A and B are projected onto efficiency frontiers at the points A^{t} and B^{t} or A^{t+1} and B^{t+1} depending on the reference period. After achieving the full efficiency, a decision-making unit (DMU) would move from point A towards point A^{t}. The change in inputs, however,

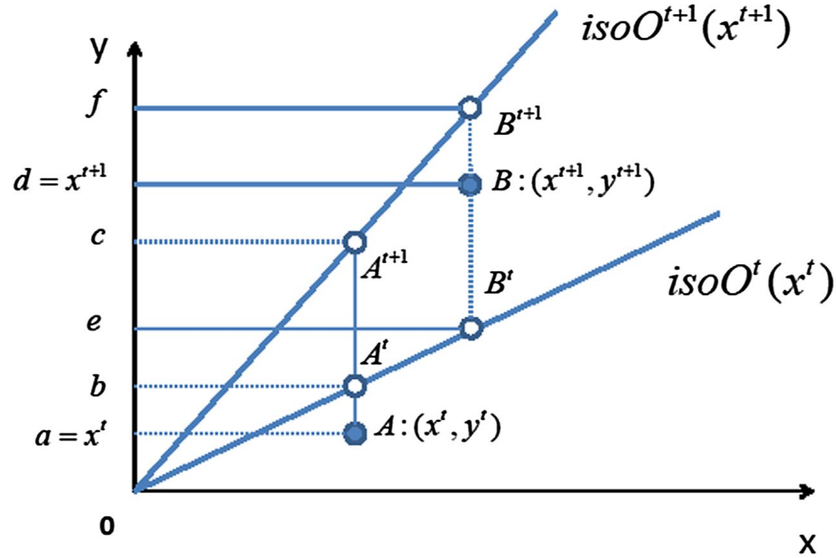

Figure 1. A graphical interpretation of the output-oriented Malmquist productivity index.Source: designed by the authors.

makes the DMU move along the efficiency frontier towards point B^t. It is the technological innovation that makes the frontier shift and thus the point B^{t+1} is achieved. Meanwhile, the DMU experiences certain technical inefficiency and remains operating in point B^{t+1}. The Malmquist productivity index quantifies both the frontier shift and inefficiency change.

Specifically, the two components of the Malmquist productivity index, EC and TC, can be explained in terms of Figure 1. The Malmquist productivity index can be obtained as follows (Färe et al., 2008):

$$M_o = \left(\frac{0d/0e}{0a/0b} \frac{0d/0f}{0a/0c} \right)^{1/2} \tag{16}$$

Similarly, its components for efficiency change and technical change are given by:

$$EC = \frac{0d/0f}{0a/0b} \tag{17}$$

$$TC = \left(\frac{0d/0e}{0d/0f} \frac{0a/0b}{0a/0c} \right)^{1/2} \tag{18}$$

As mentioned before, the Malmquist productivity index can be decomposed in a number of ways thus accounting for the different factors of changes in the total factor productivity. Färe et al. (1994), for instance, further decomposed the EC term, i.e. the global efficiency change, into the two components, namely pure technical efficiency change (PEC) and scale efficiency change (SEC):

$$M_o = EC \cdot TC \equiv PEC \cdot SEC \cdot TC \tag{19}$$

The latter two components measure the performance of a firm in terms of both variable returns to scale (VRS) and CRS technologies. Specifically, the PEC component is obtained by considering the change in pure technical efficiency (i.e. VRS efficiency), whereas the SEC component relies on distance from both CRS and VRS frontiers:

$$M_o = \underbrace{\frac{D^1_{ov}(x^1,y^1)}{D^0_{ov}(x^0,y^0)}}_{PEC} \cdot \underbrace{\left(\frac{D^1_{oc}(x^1,y^1)/D^1_{ov}(x^1,y^1)}{D^0_{oc}(x^0,y^0)/D^0_{ov}(x^0,y^0)}\right)}_{SEC} \cdot \underbrace{\left(\frac{D^0_{oc}(x^1,y^1)}{D^1_{oc}(x^1,y^1)}\frac{D^0_{oc}(x^0,y^0)}{D^1_{oc}(x^0,y^0)}\right)^{1/2}}_{TC} \tag{20}$$

where $PEC > 1$ indicates catch-up of a certain DMU in terms of pure technical efficiency, $PEC = 1$ indicates no change, and $PEC < 1$ indicates a negative catch-up effect; $SEC > 1$ indicates that a DMU gets closer to its optimal scale of operation, $SEC = 1$ indicates no change in scale efficiency, and $SEC < 1$ implies that a DMU moves further from the optimal scale. As one can note, the TC component in equation (20) is the same as that in equation (15).

In case a certain DMU keeps its efficiency at the same level throughout the two periods under consideration, the CRS frontier remains unchanged and the only change is the shift in the VRS frontier; the TC component will not identify these developments. As a remedy to this shortcoming, an additional decomposition of the Malmquist productivity index was offered by Simar and Wilson (1998a). Whereas the EC component was further decomposed by Färe et al. (1994), Simar and Wilson (1998a) introduced a decomposition of the TC term into the pure technology change (PTC) and changes in scale of the technology (STC). Therefore, the Malmquist productivity index can be decomposed into the four components:

$$M_o = EC \cdot TC \equiv PEC \cdot SEC \cdot PTC \cdot STC \tag{21}$$

The latter two terms refer to VRS and both VRS and CRS technologies, respectively. Indeed, these computations follow the spirit of the EC decomposition offered by Färe et al. (1994). The following computations then lead to estimation of the Malmquist productivity index (Simar & Wilson, 1998a):

$$M_o = \underbrace{\frac{D^1_{ov}(x^1,y^1)}{D^0_{ov}(x^0,y^0)}}_{PEC} \cdot \underbrace{\left(\frac{D^1_{oc}(x^1,y^1)/D^1_{ov}(x^1,y^1)}{D^0_{oc}(x^0,y^0)/D^0_{ov}(x^0,y^0)}\right)}_{SEC} \cdot \underbrace{\left(\frac{D^0_{ov}(x^1,y^1)}{D^1_{ov}(x^1,y^1)}\frac{D^0_{ov}(x^0,y^0)}{D^1_{ov}(x^0,y^0)}\right)^{1/2}}_{PTC} \cdot \underbrace{\left(\frac{D^0_{oc}(x^1,y^1)/D^0_{ov}(x^1,y^1)}{D^1_{oc}(x^1,y^1)/D^1_{ov}(x^1,y^1)}\frac{D^0_{oc}(x^0,y^0)/D^0_{ov}(x^0,y^0)}{D^1_{oc}(x^0,y^0)/D^1_{ov}(x^0,y^0)}\right)^{1/2}}_{STC} \tag{22}$$

where PEC and SEC feature the same interpretations as in equation (20); $PTC > 1$ means that the VRS frontier moves outwards due to a technical progress, $PTC = 1$ implies no change, and $PTC < 1$ indicates an inward movement of the VRS frontier associated with a technological regress; $STC > 1$ suggests that the underlying technology increases its curvature and approaches VRS; $STC = 1$ means that the technology exhibits no change in its shape, and $STC < 1$ implies a flattening of the technology and a movement towards CRS.

2.3. Estimation and bootstrapping of the Malmquist index

As a non-parametric deterministic method, DEA defines the empirical production frontier (Titko, Stankevičienė, & Lāce, 2014), which, in turn, suffers from certain caveats. Basically, the production frontier is defined in terms of the observed data sample, which can lack the most productive DMUs peculiar for the original population. As a result, the production frontier might be biased inwards, thus affecting the efficiency estimates. Simar and Wilson (1998b, 1999, 2000), therefore, developed the bootstrapping methodology for DEA and Malmquist productivity indices as a remedy to the sampling bias. The bootstrap approach enables us to test the hypotheses about the population distributions of the estimates under analysis. Specifically, one can obtain a confidence interval for the Malmquist index in order to test whether it significantly differs from unity.

The key idea of bootstrapping is to stimulate the original case study for B times with recalculation of the parameters for each of the iterations (Hoff, 2006). These computations will lead to B estimates (realisations) of the parameters of interest, which, in turn, enable us to estimate the distributional properties thereof. Given the data sample cannot be taken from the population for B times, a re-sampling with replacement from the observed data sample is facilitated in order to mimic the underlying Data Generation Process (DGP).

The Malmquist productivity index is estimated with respect to distance function estimates. The distance functions rely on the production frontiers defined by the data points in the observed data sample. These data points, thus, need to be adjusted in order to estimate the true production frontier. The observed input–output bundles for the kth DMU, (x_k^t, y_k^t), are generated by some unknown DGP (Hoff, 2006). In the case of the output-oriented DEA model, a certain data point (x_k^t, y_k^t) might be located on the production frontier (isoquant) or somewhere along the ray connecting the initial observation and its projection on the frontier, (x_k^t, Y_k^t), where Y_k^t is a linear combination of the observed output values in the sample defining an optimal point for the observed data point. The observation can thus be assumed to be generated conditionally on the input by the random factor $\theta_k^t \in (0, 1]$ (cf. equation (7)), i.e. $(x_k^t, y_k^t) = (x_k^t, Y_k^t\theta_k^t)$. The distance function given in equation (7) is bounded to the interval $(0, 1]$, therefore Simar and Wilson (1999) employed the smoothed empirical density function and the reflection method for the efficiency scores. When dealing with the longitudinal data, a bivariate smoothing procedure must also be used to maintain the inter-temporal correlation between the efficiency scores of a certain DMU.

The bootstrap procedure offered by Simar and Wilson (1999) relies on re-sampling from the observed set of DEA efficiency scores, $\left\{ \left(\theta_k^0, \theta_k^1\right)_{k=1,2,\ldots,K} \right\}$, where K is the number of

DMUs. The procedure begins with estimation of the distance functions given in equations (10) and (11). The following linear programming problem is specified for the contemporaneous distance functions under the CRS technology (cf. equation (10)):

$$\begin{aligned} &\left(D_{oc}^{t}(x_k^t, y_k^t)\right)^{-1} = \max_{\hat{\theta}_k^t, \lambda_k} \hat{\theta}_k^t \\ &\text{s. t.} \\ &\sum_{k=1}^{K} \lambda_k x_{i,k}^t \leq x_{i,k}^t, \quad i = 1, 2, \dots, m; \\ &\sum_{k=1}^{K} \lambda_k y_{j,k}^t \geq \hat{\theta}_k^t y_{j,k}^t, \quad j = 1, 2, \dots, n; \\ &\lambda_k \geq 0, \quad k = 1, 2, \dots, K; \\ &t \in \{0, 1\}; \\ &\hat{\theta}_k^t \text{unrestricted} \end{aligned} \tag{23}$$

where indexes $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$ denote certain inputs and outputs, respectively. Similarly, the inter-temporal distance functions under the CRS technology are estimated by employing the following linear programming model:

$$\begin{aligned} &\left(D_{oc}^{t}(x_k^s, y_k^s)\right)^{-1} = \max_{\hat{\theta}_k^{t,s}, \lambda_k} \hat{\theta}_k^{t,s} \\ &\text{s. t.} \\ &\sum_{k=1}^{K} \lambda_k x_{i,k}^t \leq x_{i,k}^s, \quad i = 1, 2, \dots, m; \\ &\sum_{k=1}^{K} \lambda_k y_{j,k}^t \geq \hat{\theta}_k^{t,s} y_{j,k}^s, \quad j = 1, 2, \dots, n; \\ &\lambda_k \geq 0, \quad k = 1, 2, \dots, K; \\ &t \in \{0, 1\}; s \in \{0, 1\}; t \neq s; \\ &\hat{\theta}_k^t \text{unrestricted} \end{aligned} \tag{24}$$

The respective VRS efficiency estimates (cf. equation (11)) are obtained by the virtue of the following linear programming problem:

$$\begin{aligned} &\left(D_{ov}^{t}(x_k^s, y_k^s)\right)^{-1} = \max_{\hat{\vartheta}_k^{t,s}, \lambda_k} \hat{\vartheta}_k^{t,s} \\ &\text{s. t.} \\ &\sum_{k=1}^{K} \lambda_k x_{i,k}^t \leq x_{i,k}^s, \quad i = 1, 2, \dots, m; \\ &\sum_{k=1}^{K} \lambda_k y_{j,k}^t \geq \hat{\vartheta}_k^{t,s} y_{j,k}^s, \quad j = 1, 2, \dots, n; \\ &\sum_{k=1}^{K} \lambda_k = 1; \\ &\lambda_k \geq 0, \quad k = 1, 2, \dots, K; \\ &t \in \{0, 1\}; s \in \{0, 1\}; \\ &\hat{\vartheta}_k^t \text{unrestricted} \end{aligned} \tag{25}$$

The distance functions needed for the Malmquist productivity index and its components (equation (22)) are now defined in equations (23)–(25). The DEA estimate of the Malmquist productivity index is then obtained for each DMU:

$$\begin{aligned} \hat{M}_o^k &= \widehat{PEC}_o^k \cdot \widehat{SEC}_o^k \cdot \widehat{PTC}_o^k \cdot \widehat{STC}_o^k \\ &\equiv \underbrace{\frac{D_{ov}^1\left(x_k^1, y_k^1\right)}{D_{ov}^0\left(x_k^0, y_k^0\right)}}_{\widehat{PEC}_o^k} \cdot \underbrace{\left(\frac{D_{oc}^1\left(x_k^1, y_k^1\right)/D_{ov}^1\left(x_k^1, y_k^1\right)}{D_{oc}^0\left(x_k^0, y_k^0\right)/D_{ov}^0\left(x_k^0, y_k^0\right)}\right)}_{\widehat{SEC}_o^k} \\ &\cdot \underbrace{\left(\frac{D_{ov}^0\left(x_k^1, y_k^1\right)}{D_{ov}^1\left(x_k^1, y_k^1\right)} \frac{D_{ov}^0\left(x_k^0, y_k^0\right)}{D_{ov}^1\left(x_k^0, y_k^0\right)}\right)^{1/2}}_{\widehat{PTC}_o^k} \\ &\cdot \underbrace{\left(\frac{D_{oc}^0\left(x_k^1, y_k^1\right)/D_{ov}^0\left(x_k^1, y_k^1\right)}{D_{oc}^1\left(x_k^1, y_k^1\right)/D_{ov}^1\left(x_k^1, y_k^1\right)} \frac{D_{oc}^0\left(x_k^0, y_k^0\right)/D_{ov}^0\left(x_k^0, y_k^0\right)}{D_{oc}^1\left(x_k^0, y_k^0\right)/D_{ov}^1\left(x_k^0, y_k^0\right)}\right)^{1/2}}_{\widehat{STC}_o^k} \end{aligned} \tag{26}$$

The CRS efficiency scores entailed by equation (23) are then used to project the observed data points on the production frontier so that the new values, $(\tilde{x}_k^t, \tilde{y}_k^t)$, are obtained by projecting the original values on the CRS frontier: $(\tilde{x}_k^t, \tilde{y}_k^t) = (x_k^t, y_k^t \cdot \hat{\theta}_k^t)$ for each k. A pseudo sample of the sets of efficiency scores, $\left\{ \left(\theta_k^0, \theta_k^1\right)_{k=1,2,\ldots K} \right\}$, is obtained for each DMU by drawing with replacement from the DEA estimates of efficiency, $\left\{ \left(\hat{\theta}_k^0, \hat{\theta}_k^1\right)_{k=1,2,\ldots,K} \right\}$. The bivariate smoothed kernel is involved in the latter stage. The new pseudo sample is subsequently utilised to establish a new set of pseudo observations: $(x_{k,b}^t, y_{k,b}^t) = (\tilde{x}_k^t, \tilde{y}_k^t / \tilde{\theta}_k^t)$, i.e. the observations are pushed away from the CRS frontier by the virtue of the pseudo score, $\tilde{\theta}_k^t$. As a result, the bootstrap replicates of the DEA efficiency scores are obtained by solving the following linear programming problems:

$$\begin{aligned} &\left({}_b^*D_{oc}^t(x_k^t, y_k^t)\right)^{-1} = \max_{\theta_{k,b}^{t*}, \lambda_k} \theta_{k,b}^{t*} \\ &\text{s. t.} \\ &\sum_{k=1}^K \lambda_k x_{i,k,b}^t \le x_{i,k}^t, \quad i = 1, 2, \ldots, m; \\ &\sum_{k=1}^K \lambda_k y_{j,k,b}^t \ge \theta_{k,b}^{t*} y_{j,k}^t, \quad j = 1, 2, \ldots, n; \\ &\lambda_k \ge 0, \quad k = 1, 2, \ldots, K; \\ &t \in \{0, 1\}; \\ &\hat{\theta}_k^t \text{unrestricted} \end{aligned} \tag{27}$$

for contemporaneous CRS technology; and

$$\begin{aligned}
&\left({}_b^*D_{oc}^t(x_k^s, y_k^s)\right)^{-1} = \max_{\theta_{k,b}^{t,s*},\lambda_k} \theta_{k,b}^{t,s*} \\
&\text{s. t.} \\
&\sum_{k=1}^{K} \lambda_k x_{i,k,b}^t \le x_{i,k}^s, \quad i = 1, 2, \ldots, m; \\
&\sum_{k=1}^{K} \lambda_k y_{j,k,b}^t \ge \theta_{k,b}^{t,s*} y_{j,k}^s, \quad j = 1, 2, \ldots, n; \\
&\lambda_k \ge 0, \quad k = 1, 2, \ldots, K; \\
&t \in \{0, 1\}; s \in \{0, 1\}; t \ne s; \\
&\hat{\theta}_k^t \text{unrestricted}
\end{aligned} \tag{28}$$

For inter-temporal CRS technology; and

$$\begin{aligned}
&\left({}_b^*D_{ov}^t(x_k^s, y_k^s)\right)^{-1} = \max_{\vartheta_{k,b}^{t,s*},\lambda_k} \vartheta_{k,b}^{t,s*} \\
&\text{s. t.} \\
&\sum_{k=1}^{K} \lambda_k x_{i,k,b}^t \le x_{i,k}^s, \quad i = 1, 2, \ldots, m; \\
&\sum_{k=1}^{K} \lambda_k y_{j,k,b}^t \ge \vartheta_{k,b}^{t,s*} y_{j,k}^s, \quad j = 1, 2, \ldots, n; \\
&\sum_{k=1}^{K} \lambda_k = 1; \\
&\lambda_k \ge 0, \quad k = 1, 2, \ldots, K; \\
&t \in \{0, 1\}; s \in \{0, 1\}; \\
&\hat{\vartheta}_k^t \text{unrestricted}
\end{aligned} \tag{29}$$

for VRS technology. Note that the observed production plans, (x_k^t, y_k^t), are projected onto the bootstrap frontier defined by the bootstrap sample, $(x_{k,b}^t, y_{k,b}^t)$. The bootstrap estimates of the Malmquist productivity index and its components are then given by:

$$\begin{aligned}
\hat{M}_o^{*k,b} &= \widehat{PEC}_o^{*k,b} \cdot \widehat{SEC}_o^{*k,b} \cdot \widehat{PTC}_o^{*k,b} \cdot \widehat{STC}_o^{*k,b} \\
&\equiv \underbrace{\frac{{}_b^*D_{ov}^1\left(x_k^1, y_k^1\right)}{{}_b^*D_{ov}^0\left(x_k^0, y_k^0\right)}}_{\widehat{PEC}_o^{*k,b}} \cdot \underbrace{\left(\frac{{}_b^*D_{oc}^1\left(x_k^1, y_k^1\right) / {}_b^*D_{ov}^1\left(x_k^1, y_k^1\right)}{{}_b^*D_{oc}^0\left(x_k^0, y_k^0\right) / {}_b^*D_{ov}^0\left(x_k^0, y_k^0\right)}\right)}_{\widehat{SEC}_o^{*k,b}} \\
&\cdot \underbrace{\left(\frac{{}_b^*D_{ov}^0\left(x_k^1, y_k^1\right)}{{}_b^*D_{ov}^1\left(x_k^1, y_k^1\right)} \frac{{}_b^*D_{ov}^0\left(x_k^0, y_k^0\right)}{{}_b^*D_{ov}^1\left(x_k^0, y_k^0\right)}\right)^{1/2}}_{\widehat{PTC}_o^{*k,b}} \\
&\cdot \underbrace{\left(\frac{{}_b^*D_{oc}^0\left(x_k^1, y_k^1\right) / {}_b^*D_{ov}^0\left(x_k^1, y_k^1\right)}{{}_b^*D_{oc}^1\left(x_k^1, y_k^1\right) / {}_b^*D_{ov}^1\left(x_k^1, y_k^1\right)} \frac{{}_b^*D_{oc}^0\left(x_k^0, y_k^0\right) / {}_b^*D_{ov}^0\left(x_k^0, y_k^0\right)}{{}_b^*D_{oc}^1\left(x_k^0, y_k^0\right) / {}_b^*D_{ov}^1\left(x_k^0, y_k^0\right)}\right)^{1/2}}_{\widehat{STC}_o^{*k,b}}
\end{aligned} \tag{30}$$

The described procedure is reiterated B times for each $k = 1, 2, \ldots, K$ and $b = 1, 2, ..., B$. As a result, the sets of bootstrap values are obtained for the Malmquist index and its components: $\left\{ \hat{M}_o^{*k,b}, \widehat{PEC}_o^{*k,b}, \widehat{SEC}_o^{*k,b}, \widehat{PTC}_o^{*k,b}, \widehat{STC}_o^{*k,b} | b = 1, 2, ..., B, k = 1, 2, ..., K \right\}$. The bias-corrected estimate of $\hat{M}_o^k$ can thus be estimated as follows:

$$\begin{aligned} \hat{\hat{M}}_o^k &= \hat{M}_o^k - \widehat{bias}_B(\hat{M}_o^k) \\ &\equiv \hat{M}_o^k - \left(\frac{1}{B} \sum_{b=1}^{B} \hat{M}_o^{*k,b} - \hat{M}_o^k \right) \\ &= 2\hat{M}_o^k - \frac{1}{B} \sum_{b=1}^{B} \hat{M}_o^{*k,b} \end{aligned} \quad (31)$$

whereas $\hat{M}_o^k$ can be substituted by either of its components, namely $\widehat{EC}_o^k, \widehat{TC}_o^k, \widehat{PEC}_o^k, \widehat{SEC}_o^k, \widehat{PTC}_o^k, \widehat{STC}_o^k$. The bias-corrected estimate entailed by equation (31) however, might exhibit a higher mean-square error than the original estimator, unless $s_{*o}^2 < \frac{1}{3}\left(\widehat{bias}_B(\hat{M}_o^k) \right)^2$, where s_{*o}^2 is the variance of the bootstrap estimates $\left\{ \left(\hat{M}_o^{*k,b} \right)_{b=1,2,...,B} \right\}$ (Simar & Wilson, 1999). In case the latter condition is not fulfilled one should employ the original estimators.

Let M_o^k be the true estimate of the Malmquist productivity index, which is unobserved. The confidence intervals for the Malmquist indices are estimated by assuming that the unknown distribution of $\left(\hat{M}_o^k - M_o^k \right)$ can be approximated by the distribution of $\left(\hat{M}_o^{*k,b} - \hat{M}_o^k \right)$ conditioned on the original data sample, $S = \left\{ (x_k^t, y_k^t) | k = 1, 2, \ldots, K; t = \{0, 1\} \right\}$. Therefore, the values a_α and b_α such that

$$\Pr\left(-b_\alpha \le \hat{M}_o^k - M_o^k \le -a_\alpha \right) = 1 - \alpha \quad (32)$$

where α is a small value of, for instance, 0.05 or 0.1, can be replaced by a_α^* and b_α^* such that

$$\Pr\left(-b_\alpha^* \le \hat{M}_o^{*k,b} - \hat{M}_o^k \le -a_\alpha^* | S \right) = 1 - \alpha \quad (33)$$

The values a_α^* and b_α^* are found by sorting the values $\left(\hat{M}_o^{*k,b} - \hat{M}_o^k \right)$, $b = 1, 2, ..., B$, in an ascending order; deleting $B\alpha/2$ of these values at either end of the sorted array; and setting $-b_\alpha^*$ and $-a_\alpha^*$ equal to the endpoints of the truncated array, such that $a_\alpha^* \le b_\alpha^*$. Therefore, we have

$$\Pr\left(-b_\alpha^* \le \hat{M}_o^k - M_o^k \le -a_\alpha^* | S \right) \approx 1 - \alpha \quad (34)$$

which implies that a $(1 - \alpha)$% confidence interval for M_o^k is defined as

$$\hat{M}_o^k + a_\alpha^* \le M_o^k \le \hat{M}_o^k + b_\alpha^* \quad (35)$$

In case the interval obtained by the virtue of equation (35) does not include the unity, one can conclude that the change in the Malmquist productivity index is significant at the significance level of α. The same routine can be generalised for the components of the Malmquist productivity index, i.e. $\widehat{EC}_o^k, \widehat{TC}_o^k, \widehat{PEC}_o^k, \widehat{SEC}_o^k, \widehat{PTC}_o^k, \widehat{STC}_o^k$.

3. Multiple Correspondence Analysis

Multiple Correspondence Analysis (MCA) is an ordination technique that aims to identify the relationships among multiple categorical and quantitative variables (Abdi & Valentin, 2007). To illustrate the approach, assume there are K categorical (nominal) variables with J_K levels associated with each of them; therefore, the total number of variables is defined as $\sum_{k=1}^{K} J_k = J$. In addition, assume there are I observations. This setting renders an $I \times J$ indicator matrix, denoted as $\mathbf{X}$. The elements of the latter matrix are binary values describing the relationships among observations and certain values of the categorical variables. MCA proceeds by projecting the observations onto factorial axes thus mimicking the principal component analysis. Denoting the overall sum of the elements of $\mathbf{X}$ by N, one can obtain the probability matrix, $\mathbf{Z}$, as follows: $\mathbf{Z} = N^{-1}\mathbf{X}$. Let $\mathbf{1}$ be a vector of ones, the dimension whereof is determined by the dimensions of $\mathbf{Z}$ so that the row and column totals are, respectively, $\mathbf{r} = \mathbf{Z1}$ and $\mathbf{c} = \mathbf{1}^{\mathbf{T}}\mathbf{Z}$. The resulting totals are rearranged into diagonal matrices, $\mathbf{D_r} = \mathrm{diag}(\mathbf{r})$ and $\mathbf{D_c} = \mathrm{diag}(\mathbf{c})$. The diagonal matrices are then exploited for singular value decomposition:

$$\mathbf{D_r}^{-\frac{1}{2}}\left(\mathbf{Z} - \mathbf{rc}^{\mathbf{T}}\right)\mathbf{D_c}^{-\frac{1}{2}} = \mathbf{P\Delta Q}^{\mathbf{T}} \tag{36}$$

where singular values are arranged into diagonal matrix $\mathbf{\Delta}$ and $\mathbf{\Lambda} = \mathbf{\Delta}^2$ is the matrix of eigenvalues. Subsequently, factor scores for rows and columns are computed via

$$\mathbf{F} = \mathbf{D_r}^{-\frac{1}{2}}\mathbf{P}\Delta \tag{37}$$

and

$$\mathbf{G} = \mathbf{D_c}^{-\frac{1}{2}}\mathbf{Q\Delta} \tag{38}$$

The supplementary observations or variables can be included in the analysis. They differ from categorical variables mentioned above in that the former ones do not contribute to the overall inertia and thus are not considered when constructing the factors. However, they can be used as explanatory variables and projected onto the factors. Assuming that $\mathbf{i}_{\mathrm{sup}}^{\mathbf{T}}$ and $\mathbf{j}_{\mathrm{sup}}$ are the supplementary rows and columns, the following procedure entails the associated coordinates, $\mathbf{f}_{\mathrm{sup}}$ and $\mathbf{g}_{\mathrm{sup}}$:

$$\mathbf{f}_{\mathrm{sup}} = \left(\mathbf{i}_{\mathrm{sup}}^{\mathbf{T}}\mathbf{1}\right)^{-1}\mathbf{i}_{\mathrm{sup}}^{\mathbf{T}}\mathbf{G}\Delta^{-1} \tag{39}$$

and

$$\mathbf{g}_{\mathrm{sup}} = \left(\mathbf{i}_{\mathrm{sup}}^{\mathbf{T}}\mathbf{1}\right)^{-1}\mathbf{j}_{\mathrm{sup}}^{\mathbf{T}}\mathbf{F\Delta}^{-1}. \tag{40}$$

4. Application of the bootstrapped Malmquist index to Lithuanian farms

The technical efficiency (TE) was assessed in terms of the input and output indicators commonly employed for agricultural efficiency and productivity analyses. More specifically, the utilised agricultural area (UAA) in hectares was chosen as the land input variable, and annual work units (AWU) as the labour input variable, intermediate consumption in Litas, and total assets in Litas as a capital factor. On the other hand, the three output indicators represent crop, livestock, and other outputs in Litas, respectively. Indeed, the three output indicators enable us to tackle the heterogeneity of production technology across different farms.

The data for 200 farms selected from the Farm Accountancy Data Network sample cover the period of 2004–2009. Thus, a balanced panel of 1200 observations is employed for analysis. The analysed sample covers relatively large farms (mean UAA – 244 ha). As for labour force, the average was 3.6 AWU. The data were analysed in a cross–section way.

In order to quantify the change in productivity across different farming types, the farms were classified into the three groups in terms of their specialisation. Specifically, farms peculiar with crop output larger than 2/3 of the total output were considered as specialised crop farms, whereas those specific with livestock output larger than 2/3 of the total output were classified as specialised livestock farms. The remaining farms fell into the mixed farming category.

The bootstrapped Malmquist index was employed to estimate the changes in the total factor productivity in 200 Lithuanian family farms during 2004–2009. As mentioned in the preceding section, the bootstrapped Malmquist enables us to identify the significant changes in the total factor productivity. The analysed sample, therefore, was classified into the three groups, which encompassed farms that featured a significant decrease, no change, or a significant increase in the Malmquist productivity indices. Given the bias-corrected estimates cannot be used unless variance of the bootstrap estimates is three times lower than the squared bias of the original estimate, the original estimates are usually reported. Consequently, the indices that did not differ from unity at $\alpha = 0.1$ were equal to unities for the further analysis. Hereafter, these variables will be referred to as the adjusted ones.

Table 1 reports the numbers of farms that experienced total factor productivity changes (output-oriented Malmquist index), whether positive, negative, or insignificant. As one can note, half of the recorded changes in the TFP were negative ones, one third were positive

Table 1. The numbers of farms that experienced changes in total factor productivity (Malmquist index), 2004–2009.

	Number of farms				Percentage		
Farm type	Increase	Decrease	No change	Total	Increase	Decrease	No change
Crop	**246**	**397**	**103**	**746**	**33**	**53**	**14**
2004–2005	27	100	31	158	17	63	20
2005–2006	27	102	17	146	18	70	12
2006–2007	125	10	7	142	88	7	5
2007–2008	55	65	33	153	36	42	22
2008–2009	12	120	15	147	8	82	10
Livestock	**38**	**54**	**24**	**116**	**33**	**47**	**21**
2004–2005	11	4	3	18	61	22	17
2005–2006	5	13	7	25	20	52	28
2006–2007	18	7	6	31	58	23	19
2007–2008	3	11	6	20	15	55	30
2008–2009	1	19	2	22	5	86	9
Mixed	**44**	**67**	**27**	**138**	**32**	**49**	**20**
2004–2005	8	8	8	24	33	33	33
2005–2006	2	21	6	29	7	72	21
2006–2007	23	1	3	27	85	4	11
2007–2008	10	12	5	27	37	44	19
2008–2009	1	25	5	31	3	81	16
Total	**328**	**518**	**154**	**1000**	**33**	**52**	**15**

Source: designed by the authors.

Table 2. Descriptive statistics for numbers of farms that experienced changes in total factor productivity (Malmquist index), 2004–2009.

Farm type	Increase	Decrease	No change
	Coefficient of variation		
Crop	0.97	0.55	0.50
Livestock	0.80	0.57	0.41
Mixed	1.03	0.64	0.42
	Range		
Crop	80	75	17
Livestock	57	64	21
Mixed	82	77	22

Source: designed by the authors.

ones and some 15% were insignificant, i.e. the TFP change did not differ from unity. The largest share of observations associated with a decrease in the TFP was observed for the crop farms (53%). The two remaining farming types featured higher shares of observations associated with no productivity change.

The shares of farms experiencing respective changes in the TFP varied to different extents across different farming types and time periods. Table 2 presents the coefficients of variation as well as ranges for the period of 2004–2009. As one can note, it was the crop and mixed that experienced the highest variation in shares of farms featuring TFP change. As for the directions of the TFP change, the lowest variation was observed for the category associated with no change in the TFP.

The Herfindahl–Hirschman Index (HHI) was computed for each farming type in order to assess the degree of farm concentration in each direction of the TFP change. In this case, the maximal value, 10,000, implies that all of the observations feature the single direction of the TFP change, whereas the lower values are associated with higher variation across the directions. The average index values of 5,652, 4,872, and 5,382 were observed for crop, livestock, and mixed farms, respectively. Therefore, the livestock farms tended to be more heterogeneous in terms of the TFP change, whereas the crop farms were peculiar with the highest homogeneity.

The efficiency change (EC) component of the Malmquist productivity index measures whether a farm decreased its distance to the observed production frontier (catch-up effect). As Table 3 suggests, the positive efficiency change was prevailing amongst the mixed and crop farms to a higher extent (29% and 25%, respectively), if compared with the livestock farms. Indeed, the mixed farms usually featured no change in efficiency (73% of the respective observations). The crop farms exhibited the highest share of observations associated with a decrease in efficiency (40%), whereas the latter share was lower for both the mixed farms (29%) and the crop farms (25%).

The mixed farms exhibited the highest variation of directions of the efficiency change throughout 2004–2009 (Table 4). The highest variation of the share of farms exhibiting an increasing efficiency, though, was observed for the livestock farms. As in the case of the TFP change, the share of farms associated with insignificant efficiency change was peculiar with the lowest variation.

The values of the HHI induced that the livestock farms exhibited the highest homogeneity in terms of the direction of the efficiency change (HHI=6232). The mixed and crop farms were specific with HHI values of 4561 and 3852, respectively. Thus, the livestock farms might be considered as the most homogeneous ones in terms of efficiency change.

Table 3. The numbers of farms that experienced efficiency changes (component of the Malmquist index), 2004–2009.

	Number of farms				Percentage		
Farm type	Increase	Decrease	No change	Total	Increase	Decrease	No change
Crop	**190**	**298**	**258**	**746**	**25**	**40**	**35**
2004–2005	36	51	71	158	23	32	45
2005–2006	13	89	44	146	9	61	30
2006–2007	72	26	44	142	51	18	31
2007–2008	35	69	49	153	23	45	32
2008–2009	34	63	50	147	23	43	34
Livestock	**16**	**15**	**85**	**116**	**14**	**13**	**73**
2004–2005	4	3	11	18	22	17	61
2005–2006		6	19	25	0	24	76
2006–2007	10	3	18	31	32	10	58
2007–2008		3	17	20	0	15	85
2008–2009	2		20	22	9	0	91
Mixed	**40**	**40**	**58**	**138**	**29**	**29**	**42**
2004–2005	11	4	9	24	46	17	38
2005–2006		19	10	29	0	66	34
2006–2007	17	3	7	27	63	11	26
2007–2008	4	11	12	27	15	41	44
2008–2009	8	3	20	31	26	10	65
Total	**246**	**353**	**401**	**1000**	**25**	**35**	**40**

Source: designed by the authors.

Table 4. Descriptive statistics for the farms that experienced efficiency changes (component of the Malmquist index), 2004–2009.

Farm type	Increase	Decrease	No change
	Coefficient of variation		
Crop	0.60	0.40	0.18
Livestock	1.03	0.69	0.20
Mixed	0.86	0.83	0.35
	Range		
Crop	42	43	15
Livestock	32	24	33
Mixed	63	56	39

Source: designed by the authors.

The technical change mostly affected crop and mixed farms (Table 5). Specifically, 26% of the crop farm observations were associated with an increase in technology, whereas 36% of these with a decrease therein. Meanwhile, the mixed farms exhibited the values of 17% and 31%, respectively. The livestock farms were the least dynamic ones in terms of the technical change, with 66% of respective observations being associated with insignificant technical change. Indeed, the largest share of the livestock farm observations describing the change in efficiency or technology featured the insignificant changes, whereas the shares of these farms associated with significant changes in the TFP were much greater (33% for an increase in the TFP and 47% for a decrease).

Although the largest share of the livestock farms were specific with no technical change, the numbers of these farms associated with either positive or negative technical change varied substantially (coefficients of variation, 1.5 and 1.6, were the highest two if compared with those specific for the remaining farming types) (see Table 6). The mixed farms exhibited the lowest variation in shares of farms associated with the expansion of the technology.

Table 5. The numbers of farms that experienced technical changes (component of the Malmquist index), 2004–2009.

	Number of farms				Percentage		
Farm type	Increase	Decrease	No change	Total	Increase	Decrease	No change
Crop	**193**	**266**	**287**	**746**	**26**	**36**	**38**
2004–2005	2	98	58	158	1	62	37
2005–2006	36	36	74	146	25	25	51
2006–2007	120		22	142	85	0	15
2007–2008	35	5	113	153	23	3	74
2008–2009		127	20	147	0	86	14
Livestock	**13**	**26**	**77**	**116**	**11**	**22**	**66**
2004–2005	6	1	11	18	33	6	61
2005–2006	7	4	14	25	28	16	56
2006–2007		2	29	31	0	6	94
2007–2008			20	20	0	0	100
2008–2009		19	3	22	0	86	14
Mixed	**23**	**43**	**72**	**138**	**17**	**31**	**52**
2004–2005		9	15	24	0	38	63
2005–2006	10	3	16	29	34	10	55
2006–2007	8		19	27	30	0	70
2007–2008	5		22	27	19	0	81
2008–2009		31		31	0	100	0
Total	**229**	**335**	**436**	**1000**	**23**	**34**	**44**

Source: designed by the authors.

Table 6. Descriptive statistics for the farms that experienced technical changes (component of the Malmquist index), 2004–2009.

Farm type	Increase	Decrease	No change
	Coefficient of variation		
Crop	1.33	1.06	0.66
Livestock	1.51	1.60	0.52
Mixed	0.97	1.36	0.61
	Range		
Crop	85	86	60
Livestock	33	86	86
Mixed	34	100	81

Source: designed by the authors.

The HHI for the livestock farms was a quite high one (namely, 7098). Indeed, most of the livestock farms had been associated with insignificant technical change during 2004–2009. The crop and mixed farms were specific with HHI values of 6000 and 6493, respectively. The latter two farming types, therefore, exhibited more versatile patterns of the technical change.

The means of the adjusted Malmquist indices are given in Table 7. As one can note, the three farming types did not differ significantly in terms of the cumulative mean TFP change: these values fluctuated between 0.82 and 0.85 across the farming types. This finding implies that the TFP had decreased by some 15–18% throughout 2004–2009. The negative TFP changes were observed for crop farms during all of the analysed periods save that of 2006–2007. Both the livestock and the mixed farms exhibited positive changes in 2004–2005 also. The steepest cumulative decrease in efficiency, represented by the EC component, was observed for the crop farms. Specifically, efficiency there decreased by some 21%. The inward movement of the production frontier, identified by the TC component, negatively

Table 7. The Malmquist productivity index and its decomposition across farming types, 2004–2009.

Farm type	M	EC	TC	PEC	SEC	PTC	STC
Crop	**0.82**	**0.79**	**0.95**	**0.77**	**0.96**	**0.95**	**1.00**
2004–2005	0.89	0.97	0.92	0.93	1.03	0.93	0.99
2005–2006	0.79	0.83	0.95	0.87	0.96	0.92	1.03
2006–2007	1.66	1.13	1.39	1.06	1.02	1.38	0.99
2007–2008	0.96	0.92	1.02	0.92	0.99	1.04	0.97
2008–2009	0.73	0.94	0.78	0.97	0.97	0.77	1.01
Livestock	**0.82**	**0.97**	**0.82**	**0.93**	**0.99**	**0.73**	**1.09**
2004–2005	1.19	1.02	1.12	1.02	1.00	1.02	1.11
2005–2006	0.88	0.93	0.96	0.95	0.99	0.91	1.03
2006–2007	1.13	1.06	0.99	1.01	1.01	1.03	0.98
2007–2008	0.92	0.95	1.00	0.96	0.99	0.97	1.01
2008–2009	0.76	1.02	0.77	0.99	1.00	0.79	0.97
Mixed	**0.85**	**0.94**	**0.79**	**0.92**	**0.90**	**0.56**	**1.50**
2004–2005	1.01	1.05	0.96	1.05	0.99	0.70	1.40
2005–2006	0.84	0.82	1.01	0.87	0.97	0.98	1.04
2006–2007	1.28	1.12	1.07	1.01	1.03	1.10	0.98
2007–2008	0.98	0.95	1.01	1.00	0.93	0.99	1.03
2008–2009	0.80	1.03	0.75	1.01	0.99	0.75	1.02

Notes: the geometric means of the adjusted estimates are presented; the annual data represent productivity changes, whereas farming type-specific heading rows exhibit the cumulative changes for 2004–2009.
Source: designed by the authors.

affected the mixed farms: the TFP decreased by 21% due to the negative technical change. The livestock farms also experienced the same decrease in technology, which amounted to some 18%.

The two terms, EC and TC, can be further decomposed to analyse the sources of changes in efficiency and technology itself. The decomposition of the efficiency change term, EC, into the two components revealed that the scale efficiency change, SEC, did not play an important role for either of the farming types. It can thus be concluded that the underlying technology was CRS. The mixed farms though, exhibited some features of a VRS technology. The highest decrease in pure efficiency (PEC) was observed for the crop farms (23%), whereas livestock and mixed farms experienced much lower decreases of 7–8%. Decomposition of the TC component induced that the pure technical change, PTC, decreased the productivity of the crop and mixed farms by 27% and 44%, respectively, whereas the crop farms did not suffer from a decrease in technology. However, the negative effect on the mixed farms was alleviated by increasing convexity of the technology: the STC component indicated a 50% increase in productivity. Therefore, the mixed farms diverged in their scale, particularly in the period of 2004–2005.

The multivariate analysis was carried out in order to reveal the underlying patterns of the productivity change across farming types and time periods. Specifically, the multiple correspondence analysis (MCA) was applied to identify the relations between farming types, years, and TFP changes. The package *FactoMineR* (Husson, Lê, & Pages, 2010) was utilised to implement MCA. The MCA enables us to explore the relations between the categorical variables by the means of the χ^2 distance.

In our case we distinguished the three categories for estimates of the bootstrapped Malmquist productivity index and its components, namely (i) increase, (ii) no change, and (iii) decrease in TFP. Therefore, the seven variables, $\widehat{M}_o^k, \widehat{EC}_o^k, \widehat{TC}_o^k, \widehat{PEC}_o^k, \widehat{SEC}_o^k, \widehat{PTC}_o^k, \widehat{STC}_o^k$, were classified into the three groups by means of the bootstrap confidence intervals. The two supplementary variables, year and farm type, were also considered in order to better describe

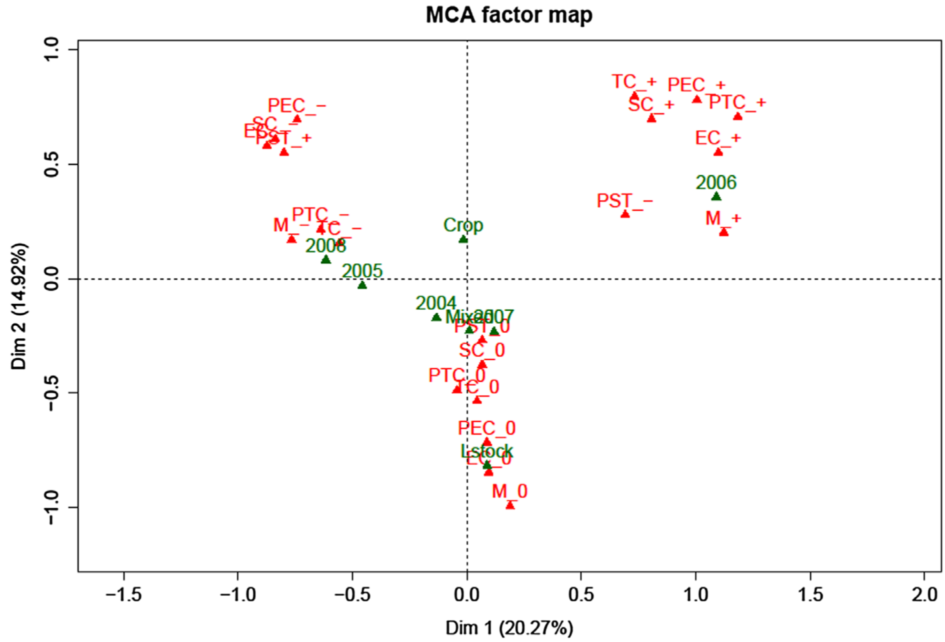

Figure 2. The MCA plot describing relationships between Malmquist indices and supplementary variables. Source: designed by the authors.

the productivity change patterns. The resulting MCA plot is depicted in Figure 2. The first two components explain some 35% of the total inertia.

As one can note, the three groups of productivity change indices emerged. Indeed, they were associated with a positive (NE part of the plot), negative (NW), or insignificant (S) change in productivity, respectively. Clearly, the first component axis presented a gradient of productivity change, i.e. the TFP increased going along the latter axis. The second component axis discriminated the variables associated with a more stochastic TFP change pattern from those related to insignificant changes. Note that the positive STC was associated with negative changes in TFP. The latter finding implies that technological progress was related to CRS technology, whereas technological regress featured the increasing convexity of the production frontier (i.e. VRS technology). A cluster of negative efficiency change components (EC, SC, PEC) was located further away from the origin point, thus indicating that a decrease in efficiency occurred without decrease in other terms of the Malmquist productivity index.

The decrease in TFP was mainly associated with technical regress represented by TC and PTC components. Increasing convexity of the technology was stronger related to efficiency change terms if opposed to technical change ones. This finding implies that negative efficiency change was mainly associated with production and productivity changes among highly specialised (in terms of input/output structure and scale) farms and inefficient farms.

However, positive TFP changes were mainly driven by efficiency change (EC) and scale of technology change (PST). This implies that CRS frontier shifts had less impact upon productivity growth if compared with flattening of the VRS frontier and efficiency change. Accordingly, it might be concluded that TFP growth most frequently occurred due to efficiency change in the sub- or supra-optimal regions of VRS technology (i.e. among highly specialised farms).

All of the farming types exhibited change in the TFP close to the average, although the crop farming was located in the more stochastic area, whereas the livestock farms appeared to be the most stable in terms of the TFP change. Given all of the farming types exhibited a similar level of the TFP change, the livestock farms can be considered as those better performing. The MCA plot does also confirm that the period of 2006–2007 was that of an increase in the TFP, whereas the periods of 2005–2006 and 2008–2009 were associated with a decrease.

5. Conclusions

The Malmquist productivity index enables us to identify the efficiency and total factor productivity gains and sources thereof. However, the estimates of the Malmquist productivity index obtained by the means of data envelopment analysis do not contain any information about the significance of the observed changes in the total factor productivity. Accordingly, stagnation in the total factor productivity can be mistakably considered as a sort of significant change. The bootstrapped Malmquist index constitutes a remedy to the latter issue. Anyway, the large datasets are hard to analyse without additional techniques. This paper, therefore, employed multiple correspondence analysis to identify the underlying patterns of the total factor productivity change. The proposed framework enables us to analyse the relations between the productivity change indices and supplementary (i.e. environmental) variables.

The current study presented an empirical application of the bootstrapped Malmquist index and multiple correspondence analysis for analysis of the Lithuanian family farm performance during 2004–2009. The analysis showed that the total factor productivity decreased by some 15–18% during 2004–2009 depending on the farming type (insignificant changes were eliminated). The crop farms exhibited the steepest decrease in efficiency (21%), whereas the mixed farms featured the negative technical change of the same margin. The multiple correspondence analysis suggested that all of the farming types exhibited change in the total factor productivity close to the average, although the crop farming was located in the more stochastic area, whereas the livestock farms appeared to be the most stable ones in terms of the total factor productivity change. Given all of the farming types exhibited similar levels of the total factor productivity change, the livestock farms can be considered as those better performing. Anyway, a negative technical change poses a need for further research on the possibilities to increase the productivity of the livestock farming.

Multiple Correspondence Analysis implied that zero productivity change was equally associated with all the terms of the Malmquist index. However, different patterns were evident for increase and decrease in productivity. Specifically, an increase in productivity was mainly associated with efficiency gains and flattening of the production frontier (under variable returns to scale). Therefore, an increase in productivity was mainly achieved due to farm-specific improvements. Decreasing productivity was mainly associated with technical regress and, especially for crop farms, these developments were not followed by increasing technical efficiency (relative to frontier movement). Wide-scale support measures (like income smoothing) might therefore be more appropriate to maintain the viability of Lithuanian family farms during a decline of productivity. As regards the productivity-increasing farms, they appeared to remain below the production frontier even after improvement in their productivity. Accordingly, support measures are needed to ensure frontier-pushing innovations among family farms to a greater extent. These could support equipment with serious considerations of power and maintenance costs. It should be noted that, we might also observe a subdued productivity growth in the short run due to influence of adjustment costs even though support measures are appropriate.

Note

1. Inequality in equation (6) has to be read element-wise, therefore we look at the elements of y' which are greater or equal than the corresponding elements of y, yet at least one element must be strictly greater so that the two vectors were not identical.

Disclosure statement

No potential conflict of interest was reported by the authors.

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