# Asset allocation and regime switching on Croatian financial market

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Abstract. It has been known for quite some time now that financial markets exhibit changes in regimes over time. A majority of the literature tends to support that financial markets undergo regimes of bull and bear markets. This characteristic should be modeled in a proper way as investors are always interested in beating the market: either by achieving better returns than others, or by minimizing their portfolio risks. There exist many mathematical and statistical models that are used as tools to achieve the mentioned goals. Introducing the regime switching methodology in existing models has proven to be facilitate achieving such goals. Therefore, the objective of this study is to utilize the regime switching methodology on the Croatian financial market to ascertain its usefulness for Croatian investors. Multivariate regime switching and non-switching models were estimated using daily data from the period 2 January 2007 to 31 December 2015. The assumption is that the investor is interested in stock and bond markets. The results from the MGARCH and regime switching MGARCH models are then compared in order to give answers as to whether the respective methodology applied to the Croatian market is useful and how it may benefit investors. Most of the results support the presumption of incorporating this particular methodology in financial modeling for the Croatia markets. This is the first research that applies the regime switching MGARCH methodology in Croatia (including the Balkan region), hence we expect that this will be a significant contribution to existing methodologies in literature.

 ${\bf Keywords:}$  MGARCH, regime switching, portfolio, Croatian financial market, nonlinear models

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## 1. Introduction

Questions by investors regarding the successful portfolio management on financial markets have existed for some time. Numerous mathematical, statistical and econometric models and methods have been developed (and are

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still under development) and aim to answer some of these questions. In the last 30 years, there has been an explosion of research in the field of time series analysis, especially in the field of financial econometrics. Initial models were quite simple based on assumptions such as linear relationship between variables. However, practice has shown that the relationship between financial variables are not linear. Consequently, empirical estimations based on wrong assumptions result in distorted outcomes. Portfolio management seeks valid estimations and forecasts for sound decision making. On the other hand, poor decisions may affect not only individual portfolios (leading to capital losses) but also entire markets (precipitating a financial crisis).

Thus, today the problem posed by economic theories and practices is the inability of linear and numerous nonlinear models to successfully capture real economic trends. Some of these problems are: structural changes such as oil price shocks [21], sticky prices and exchange rate bands [34], asymmetry behavior of output and unemployment over time [16], specific characteristics of financial markets and time series<sup>‡</sup> [17], changing correlations [2], financial crises [3], etc. Perhaps the most typical trait is that financial markets exhibit regime switching (RS henceforward) behavior. This means that markets alternate between phases of bull and bear markets; i.e. greater and smaller volatility, return and correlations. If this is true, investors should seek to exploit this kind of characteristic to achieve the best possible results (in terms of return and/or risk). It is also natural to think that economies and markets undergo different regimes of better and worse performance [3].

Until the late 1980s, the standard procedure in financial modeling was to use univariate GARCH models, when MGARCH<sup>§</sup> models were introduced. Up to the present time, these models have been the most popular approach in financial modeling. However, research has intensified providing evidence that even the (M)GARCH methodology is unable to fully explain the behavior of financial markets and assets. Not all asymmetry can be explained using the standard models [6]; standard models predict greater persistency of volatility than is the case [15]; they cannot capture structural breaks properly [13, 24]; they are based on the false assumption that distributions of higher moments of return do not change over time [1], etc. The Croatian financial market has been subject to these phenomena. Given that this specific market has not been sufficiently explored, opportunities exist to further research this topic. Such research would considerably increase the quality of investment decisions. Moreover, it can assist shifting a stagnating market into a recovery, which is still recuperating from the 2008 financial crisis. The main topic in this paper will be to address the standard models used in finance and econometrics but which are unable to capture real trends at the level of specific nonlinear models. On the other hand,

<sup>&</sup>lt;sup>‡</sup> Such as asymmetric behavior, outliers, volatility clustering, etc.

<sup>&</sup>lt;sup>§</sup> Multivariate Generalized Autoregressive Conditional Heteroskedasticity.

we will argue that RS models are more accurate. These models can form optimal portfolios resulting in desirable earnings, maximizing utility and/or managing risk. Specifically, we will focus on the RS methodology as a superior tool for portfolio management.

Hence, objective of this paper is to show that the RS methodology is superior for portfolio management in terms of basic performance measures compared to non-switching models already in use on the Croatian market. This paper is structured to address certain topics. The second section deals with relevant results from previous research. The methodology used here is explained in the third section, whilst the results stemming from empirical research are provided in the fourth section. The final and fifth section provides conclusions.

#### 2. Previous research

This section presents results from previous research that incorporates the RS methodology in the MGARCH models. Most papers focus on developed markets, which is not surprising. Moreover, this methodology is still under development, where many of these papers are neither strictly theoretical nor empirical. Most researchers develop a models and test them over the course of time. Initial research dates from the late 1990s. Ramchand and Susmel [30] estimated univariate RS models (a basic form of the Bollerslev model [10]) and bivariate VAR(1) models on selected pairs of countries based on the assumption of two regimes: high and low volatility and return correlations. They showed that regimes can be exploited by investors, although they failed to check all of the necessary diagnostics. Edwards and Susmel [14] use the same methodology to explore several developing markets. They estimate returns using ARMA and VAR(1) models under the assumption of the existence of RS risks. The results are similar as in [30]. Next, they focused on interest rates [15] in the same markets. They also modelled three regimes given that the diagnostic tests implied the need for three regimes.

More complex research emerged after the publishing of these initial papers. Cappiello and Fearnley [11] estimated the ICAPM<sup>\*\*</sup> model for developed markets as a RS BEKK. However, they failed to account for the entire theoretical basis. The RS models were better in forecasting than non-switching models. Billio and Pelizzon [8, 9] estimated the VaR for Italian stocks and the transmission of volatility between selected European markets. Several RS specifications were applied<sup>††</sup>. Not surprisingly, these were superior to non-switching models. Hence, they recommended incorporating switching methodology in existing models.

<sup>\*\*</sup> International CAPM.

 $<sup>^{\</sup>dagger\dagger}$  Univariate models, beta model, factor and multivariate switching.

A more complete theoretical methodology of the multivariate RS was done in Pelletier [29]. In his model, he assumes that correlations are constant within the regime. It is a transition from the  $CCC^{\ddagger}$  to the  $DCC^{\$\$}$  model. The empirical part of the paper focused on interest rates. He showed that his model was superior for making forecasts compared to the DCC model. Billio and Caporin [7] extended his model under the assumption that correlations change within the regime. They successfully test it on several developed capital markets. Baele [6] took it a step further and introduced certain diagnostic tests. He focused on the integration of western European capital markets. The regimes were shown to be economically and statistically significant. Lee and Yoder [25, 26] were the first to developed the RS BEKK model, but they only considered the bivariate case. Hedging opportunities were better in the RS model. Nomikos, Alizadeh and Pouliasis [27] showed that RS BEKK was suitable for futures markets (based on USA data).

A significant contribution was provided by Haas and Mittnik [19]. They thoroughly defined a multivariate generalization of the RS GARCH model (diagonal VEC model). However, they assumed that the same latent variable governs regimes with different returns. Statistical diagnostics showed that their model compared better to other models used in the study. In the following year, Chen [12] focused on the CCC model. He elaborated that in the model in [19], it was difficult to estimate the correlations. The reason for this is that when a correlation changes, the degree of change in the variance of one return or the other affecting the correlation is unknown. He did solve this problem. Furthermore, he assumed that each return series has its own latent variable governing RS behavior. Given that he solved numerous issues from previous research, we will use the Chen [12] model in the empirical section of this paper.

In the last couple of years, several papers have emerged focusing on specific markets. Some studies include Sheu and Lee [31], Otranto [28], Haas and Liu [18], Sheu, Lee and Lai [32], etc. Such research emphasizes the importance of incorporating the RS methodology when modeling risk and return on financial markets.

Having gained insight into research dealing with the Croatian market and with reference to this methodology, the conclusion is that only several papers have address this specific issue. All of these papers are much simpler in terms of methodology compared to research abroad. For example, Arnerić and Erjavec [4] were the first to use RS on Croatian data. The univariate RS model was superior to other models for diagnostics purposes. Kunovac [23] took account of asymmetry in the behavior of risk and return on the Zagreb Stock Exchange and several other European markets. He also addressed univariate cases. However, he did post-estimation analysis, something that is not addressed in [4].

<sup>&</sup>lt;sup>‡‡</sup> Constant conditional correlation.

<sup>§§</sup> Dynamic conditional correlation

Certain basic portfolio management simulations were also conducted. RS portfolios gave greater returns than other portfolios, but the author did not analyze the relevant risks. Visković, Arnerić and Rozga [36] estimated several univariate RS models for 6 European countries. They recommend using this methodology to solve problems concerning structural breaks in data. Škrinjarić [33] used the RS CAPM model for 21 stocks on the Croatian market. She discovered that this model was better than the original linear model, where beta could nonetheless be used as a measure of risk. Arnerić and Škrabić [5] made a similar research as [36] for similar countries. As can be seen, research is scarce, oversimplified and does not provide post-estimation analysis which could otherwise be useful for portfolio management purposes and investors. It is our expectation that this paper will bridge this gap. The next section provides information on the methodology used in empirical research.

#### 3. Methodology

As mentioned in the previous section, the main model we followed is Chen's [12]. The reason is that the model overcame some of the pitfalls of previously developed models. It is more realistic to assume that each financial asset has its own latent variable governing RS dynamics. This means that volatilities follow individual RS behavior. If all financial markets behaved the same way, no possibility of diversification would exist. In addition, this model is somewhat simpler to establish than previous ones. This is also a desirable for investors.

The CCC model provided by Bollerslev [10] is the baseline for modeling. Correlations are constant within a particular regime, but vary when comparing different regimes. Covariance is modeled as a three-component process: it depends on individual volatilities, as well as on the correlation between them. Chen [12] observed stock and bond market interactions. We too will observe these same two markets in Croatia. We shall designate  $V_{s(t),t}^{i}$  for the volatility of the *i*-th return in state s(t) in time t,

$$\begin{split} &i \in \left\{ stock, bond \right\}, \quad s \in \left\{ 1, 2 \right\}, \quad t \in \left\{ 1, 2, ..., T \right\}, \mathbf{R}_{s(t)} = \begin{bmatrix} R_{s(t)}^{stock} & R_{s(t)}^{bond} \end{bmatrix}' \\ &\text{return vector,} \\ & \boldsymbol{\mu}_{s(t)} = \begin{bmatrix} \mu_{s(t)}^{stock} & \mu_{s(t)}^{bond} \end{bmatrix} \end{split}$$

conditional mean vector and  $\boldsymbol{\varepsilon}_t = \begin{bmatrix} \varepsilon_t^{stock} & \varepsilon_t^{bond} \end{bmatrix}$  residual vector. The main model is defined as:

$$\begin{aligned} \boldsymbol{R}_{s(t)} &= \boldsymbol{\mu}_{s(t)} + \boldsymbol{\varepsilon}_{s(t)} \\ \boldsymbol{\varepsilon}_{s(t)} &\sim N(\boldsymbol{0}, \boldsymbol{\Xi}_{s(t)}) \end{aligned} \tag{1}$$

with  $\mathbf{0} = \begin{bmatrix} 0 & 0 \end{bmatrix}'$  and

$$\Xi_{s(t)} = \begin{bmatrix} V_{s(t),t}^{stock} & Corr(stock, bond)_s \cdot \sqrt{V_{s(t),t}^{stock} V_{s(t),t}^{bond}} \\ Corr(stock, bond)_s \cdot \sqrt{V_{s(t),t}^{stock} V_{s(t),t}^{bond}} & V_{s(t),t}^{bond} \end{bmatrix} .$$
(2)

The return vector  $\mathbf{R}_{s(t)}$  in (1) is modeled as a mean vector under the assumption of a constant return in each regime. This is the simplest version of the mean vector form. Other economic variables can be included to describe the return series as well. The conditional variance and covariance matrix  $\mathbf{\Xi}_{s(t)}$  consists of changing individual volatilities

$$(V_{s(t),t}^{stock} \text{ and } V_{s(t),t}^{bond})$$

and a changing covariance

$$(Corr(stock, bond)_s \cdot \sqrt{V_{s(t),t}^{stock}V_{s(t),t}^{bond}}).$$

Individual volatilities are modeled as GARCH(1,1) processes in each regime. For example, stock volatility is modeled in the following way:

$$\begin{array}{c} \textit{regime1:} \quad \textit{V}_{l,t}^{\textit{stock}} = \alpha_{l,0}^{\textit{stock}} + \alpha_{l,1}^{\textit{stock}} s_{l,t-1}^{\textit{stock}} + \beta_{l,1}^{\textit{stock}} \textit{V}_{l,t-1}^{\textit{stock}} \\ \textit{stock} \\ \textit{volatility} \\ \textit{regime2:} \quad \textit{V}_{2,t}^{\textit{stock}} = \alpha_{2,0}^{\textit{stock}} + \alpha_{2,1}^{\textit{stock}} s_{2,t-1}^{\textit{stock}} + \beta_{2,1}^{\textit{stock}} \textit{V}_{2,t-1}^{\textit{stock}} \end{array}$$

The transition probability matrix consists of the probabilities of transitioning from one regime to another. Probabilities are defined as

$$P(s(t) = i \mid s(t-1) = j) = p_{ij}, \ i, j \in \{1, 2\}, \ 0 \le p_{ij} \le 1, \ \sum_{j=1}^{2} p_{ij} = 1, \ \forall i.$$

The model is estimated in two stages using the maximum likelihood method. First, the return series are filtered using an adequate ARMA(p,q) model. This allows us to estimate the return vector  $\mathbf{R}_{s(t)}$  as given in (1). Then follows the two-step estimation procedure. The first step involves estimating the univariate RS GARCH(1,1) models and obtaining their coefficients. These coefficients are fixed in the second step where the remaining parameters in the model (1)-(2) are estimated. Details on the maximum likelihood estimation of this model are given in Chen [12:26-27]. We stated the stationarity condition for the each RS process, with the details given in Chen [12:27-28] and Haas, Mittnik and Paollela [20:500-501]. For the non-switching univariate case in the GARCH methodology, the sum of alpha and beta parameters is calculated when observing the stationarity of the variance. A similar procedure is performed in the case of multivariate RS. Consider the following matrix:

206

Asset allocation and regime switching on Croatian financial market

$$\boldsymbol{M} = \begin{bmatrix} \boldsymbol{M}_{11} & \boldsymbol{M}_{21} \\ \boldsymbol{M}_{12} & \boldsymbol{M}_{22} \end{bmatrix}, \tag{3}$$

where

$$\begin{split} \boldsymbol{M}_{ij} &= p_{ij} \left( \boldsymbol{\beta} + \boldsymbol{\alpha}_{1} \boldsymbol{e'}_{i} \right), \ \boldsymbol{\beta} = diag \left( \beta_{1,1}^{n}, \ \beta_{2,1}^{n} \right), \ \boldsymbol{\alpha}_{1} = \begin{bmatrix} \alpha_{1,1}^{n} & \alpha_{2,1}^{n} \end{bmatrix}', \text{ and } \boldsymbol{e}_{i} \text{ is a } 2 \times 1 \\ \text{vector with the } i\text{-th element equal to 1 and the other 0, } n \in \{stock, bond\}. \text{ The model is stationary if the largest eigenvalue of } \boldsymbol{M}, \text{ eig}(\boldsymbol{M}), \text{ is less than the unit value. Accordingly, individual conditional volatilities can be inverted in each regime using the expression given in [20:501].} \end{split}$$

Ang and Bekaert (2000) defined a regime classification measure RCM, which shows the efficiency of the chosen RS model in classifying observations of a process:

$$RCM = 100k^2 \frac{1}{T} \sum_{t=1}^{T} \left( \prod_{i=1}^{k} p_{i,t} \right),$$

where  $p_{i,t}$  is the smoothed regime probability. It is a normalized measure (ranging from 0 to 100); with its lower values indicating that the model is more successful in regime classification. This measure will be used in this study.

#### 4. Empirical results

For empirical research purposes, daily data on CROBEX stock index and CROBIS bond index was collected for the period from 2 January 2007 to 31 December 2015 from the ZSE [36]. The entire estimation was performed using Time Series Modelling 4 software. Returns on stock and the bond market were calculated as compound returns. The aim was to estimate an RS MGARCH model described in the previous section and its non-switching counterpart. Furthermore, a comparison of these models will be done subject to diagnostics and portfolio optimization possibilities. Both return series were found to be stationary on usual levels of significance. Firstly, the appropriate ARMA(p,q)models were used on raw return data in order to filter the data. The CROBEX was modeled as an ARMA(1,1) and CROBIS as an AR(1) process<sup>\*\*\*</sup>. Then, filtered data was used to estimate the univariate RS GARCH(1,1) models, under the assumption that the two regimes exist. This assumption is based on previous empirical research on the Croatian financial market where all existing studies support the existence of two regimes. The estimation of the results is given in Table 1. It is apparent that both returns are greater in regime 1 than in regime 2. Moreover, the reactions of individual volatilities to market shocks and

207

<sup>&</sup>lt;sup>\*\*\*</sup> Results from stationarity tests and ARMA modeling have been omitted due to lack of space but are available upon request. The appropriate ARMA model was chosen based on BIC, AIC and HQ information criteria, as well as on statistical significance of estimated parameters.

volatility persistence are greater in regime 2. It is for this reasons that we call regime 1 a bull market and regime 2 a bear market. The duration of regime 1 for CROBEX is 22.72 days and for 3.16 days for CROBIS (duration = $1/(1-p_{11})$ ). However, on average, the stock market finished with losses in both regimes. The stationarity condition is fulfilled for both assets. Next, matrices  $M_{ii}$  and M for both assets and the corresponding eigenvalues were calculated. It is evident that the stationarity conditions are met<sup>†††</sup>. In the second step, we fixed the parameters from the upper part of Table 1 and estimated the rest of the model. The results are shown in the lower part of Table 1. It becomes apparent that the correlation is smaller in regime 1 than in regime 2. Neither is it statistically significant. This is in favor of diversification purposes for investors. The result indicating that the correlation is greater in a bear market is not surprising and is in line with existing literature. In addition, the duration of regime 1 subject to such a correlation is greater than that of regime 2 (4.5)days vs. 1.3 days), thus quite possibly providing more advantages for portfolio management. Finally, the RCM measure was calculated to ascertain the accuracy of this model in classifying regimes. A value of 30.7 was the final result, means that the model is moderately good.

Estimated parameters Univariate models	CROBEX	CROBIS					
Regime 1							
$\hat{\mu}_1$	$0.00021 \ (0.143)$	0.00002 (0.601)					
$\hat{lpha}_{1,0}$	$0.00271 \ (0.000)$	$0.00065 \ (0.000)$					
$\hat{lpha}_{1,1}$	$0.076\ (0.000)$	$0.047 \ (0.007)$					
$\hat{oldsymbol{eta}}_{\mathrm{l},\mathrm{l}}$	$0.896\ (0.000)$	$0.496\ (0.000)$					
Regime 2							
$\hat{\mu}_2$	$0.0022 \ (0.684)$	$0.00003 \ (0.76)$					
$\hat{lpha}_{2,0}$	$0.011 \ (0.000)$	$0.0001 \ (0.000)$					
$\hat{lpha}_{2,1}$	$0.921 \ (0.354)$	0.192(0.001)					
$\hat{eta}_{2,1}$	$0.824\ (0.000)$	0.926 (0.000)					
$p_{11}$	0.956	0.684					
$p_{22}$	0.010	0.351					
Log L	7494.93	11459.2					
eig(M)	0.99	0.99					

<sup>&</sup>lt;sup>†††</sup> We have omitted some of the details, but they are available upon request.

Multivariate model:						
	Regime 1	Regime 2				
Correlation coefficient	0.016(0.488)	0.108(0.037)				
$p_{11}$	0.780					
$p_{22}$	0.235					
Log L	20809.8					
Roots of MA System	0.896; 0.496					

 Table 1: Estimation of the results from univariate and multivariate regime switching

 models

Note: *p*-values are given in brackets. Log L stands for log likelihood. eig(M) stands for the largest eigenvalue of matrix M.

Next, a CCC model without regimes was estimated for comparing the two. This enabled us to ascertain whether there truly exists a need for regimes. The results are given in Table 2. The value of the log likelihood function is less than that of the RS model. This is the first sign that this assumption should be included in the analysis. The stock return alpha is greater than the bonds return, as is the case for the RS model. However, the correlation coefficient is an average of the two regimes (0.058 compared to 0.016 and 0.108), which may be misleading.

Estimated parameters	CROBEX	CROBIS	
$\hat{\mu}$	$0.0002 \ (0.181)$	$0.000004 \ (0.901)$	
$\hat{lpha}_0$	$0.000001 \ (0.000)$	$4.7 \cdot 10^{-8} (0.000)$	
$\hat{lpha}_1$	0.108(0.000)	$0.076\ (0.000)$	
$\hat{oldsymbol{eta}}_1$	0.890(0.000)	0.916(0.000)	
Correlation coefficient	0.058 (0.002)		
Log L	18541.09		

Table 2: Estimation of results from multivariate CCC model without regimes

Given the possibility of using this methodology in portfolio management, some applications have already being performed. Kroner and Ng [22] derived the optimal portfolio weights for two assets using the expression

$$w_{1,t}^* = \frac{\sigma_{1,t}^2 - \sigma_{12,t}}{\sigma_{1,t}^2 - 2\sigma_{12,t} + \sigma_{2,t}^2},$$

where  $w_{l,t}^*$  denotes the optimal portfolio weight for the first asset,  $\sigma_{l,t}^2$  the first asset variance,  $\sigma_{2,t}^2$  the second asset variance and  $\sigma_{12,t}$  the covariance in time

#### Tihana Škrinjarić and Boško Šego

t. Conditional variances and covariances were estimated for each day in both models, and the optimal portfolio weights calculated. Figure 1 shows the differences in the optimal weight of CROBEX. The non-switching model constantly includes more stocks in the portfolio over time. Both models however, show that fewer stocks should have been included in the portfolio during the crisis period. In the last few years, stock weight has increased, given that the market has stabilized and stocks are providing greater returns.



Figure 1: Optimal portfolio weight for CROBEX, non-switching model (black line) and regime switching model (gray line)

A test for differences in average portfolio weights was also conducted, and for usual levels of statistical significance, the two differed (the z-test value was 29.86 and the p-value was 0.000). Thus, the expected returns and risks of the two portfolios using these models might well differ. Before comparing them, the efficient frontiers from the Markowitz portfolio theory were constructed for both models. As can be seen in Figure 2, the differences are substantial. Assuming no regimes, achieving positive returns was not possible. However, this changes if the investor takes advantage of the two-regime model. When regime 1 occurs, positive returns might be achievable. When regime 2 occurs, it might be wise to invest in other markets (exchange rate, metals, etc.) so as to avoid loses. Accordingly, such strategies may be formed in order to invest in the stock-bond portfolio in regime 1 and achieve positive returns. When forecasting regime 2, investing in other assets might be desirable so as to utilize more favorable opportunities.

210



Figure 2: Efficient frontiers, non-switching model (dashed line) and regime switching (black line - regime 1, gray line - regime 2)

Descriptive	Expected		Portfolio		Standardized	
statistics	return		risk		return	
	No regime	2 Regimes	No	2 Regimes	No regime	2 Regimes
			regime			
Mean	$2.22 \cdot 10^{-6}$	$7.33 \cdot 10^{-6}$	$4.14 \cdot 10^{-6}$	$3.52 \cdot 10^{-6}$	13.18961	17.94320
Median	$1.18 \cdot 10^{-5}$	$1.09 \cdot 10^{-5}$	$2.23 \cdot 10^{-6}$	$1.07 \cdot 10^{-6}$	4.418038	5.170249
Max	0.01770	0.01731	$5.92 \cdot 10^{-5}$	0.000142	3577.093	2992.546
Min	0.013664	0.014355	$7.44 \cdot 10^{-7}$	$4.74 \cdot 10^{-7}$	4971.165	2773.165

 Table 3: Descriptive statistics for multivariate models with and without 2 regimes

 Note: bolded numbers denote better portfolio performance.

Based on Figure 1, expected returns and risks for both portfolios were calculated for the observed period. As shown in Table 3, on average, no regime portfolio performs better in terms of expected return. This is not surprising given the greater weight of stocks (see Figure 1) in the portfolio. However, investors are interested in risks as well, even more so than returns. The RS model is better in terms of risk minimization. In addition, it appears to be a better model when observing both return and risk together (i.e. standardized return). Finally, a simulation of portfolio rebalancing was carried out for the observed period. We invested into CROBEX and CROBIS as shown in Figure 1 each trading day. Cumulative returns<sup>‡‡‡</sup> are calculated and compared in Figure 3. Most of the time the RS model was superior to the non-switching model. Even if only investing in CROBEX or CROBIS (a passive strategy) alone, the results are not as good as

 $<sup>\</sup>ddagger \ddagger \ddagger$  The returns are standardized to take into consideration both risk and return.

in the RS model (comprehensive results available upon request). Furthermore, the RS portfolio had greater returns 51.76% of the time with lesser risks 77.73% of the time.



Figure 3: Cumulative returns, non-switching model (gray line) and regime switching (black line)

#### 5. Conclusion

This paper had several objectives. One was to familiarize readers with problems associated with quantitative finance and existing models. The inability of these models to capture real situations led to the development of regime switching models. Existing research is relatively scarce if focusing on multivariate models. The main reason is the technical difficulty in estimate such models. However, empirical research that applies this methodology has shown to be superior compared to other non-switching models. This study has focused on the Croatian financial market due to a lack of similar studies. The RS model assumes 2 regimes: a bull and a bear market, a usual assumption in existing research. Estimations were in line with foreign literature distinguishing the two states. More importantly, this model was superior to its non-switching counterpart when comparing portfolio performances. This means that the RS methodology improves portfolio management and reduces overall risks. Information resulting from the RS models can be very important and useful to (potential) investors. Some of the pitfalls of the study were as follows: the absence of transaction costs, focusing solely on daily data and two assets, focusing on basic performance calculations, etc. Hence, future research should eliminate these inadequacies. Furthermore, we will compare other RS models and different trading strategies, as well as try to answer other questions that investors may be interested in. These questions include important topics such as what drives the regimes on Croatian financial market, is it possible to find better models to achieve even better results, and so on. However, this is the first research that applies the RS MGARCH methodology in Croatia (as well as the Balkan region), hence we expect to have made a contribution to current research.

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