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Thermodynamical aspect of definitions »CAPE« and »TCAPE«

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The paper analyses the thermodynamical aspect of the terms CAPE and TCAPE, defined in the standard way according to Brunt (1941) and in another way in Renno and Ingersol's paper (1996). Based on this, in this paper it is pointed out that the thermodynamical foundations for the difference existing between the two definitions. The analyses itself is based on the presentation of all the relevant terms in thermodynamics diagrams and relations for internal-equilibrium processes and processes with internal friction.

The paper shows the internal friction which Renno and Ingersol take into consideration and make their definition of CAPE and TCAPE different from the standard one and not equivalent to it. In this connection, only according to the standard definition CAPE and TCAPE represent the maximum work which is real in advance calculable measure of convective circulation intensity.

Keywords: maximum work, CAPE, TCAPE, energy dissipation

1. Introduction

In the article Renno and Ingersoll (1996) convection processes are analysed as the consequence of temperature non-equilibrium in the troposphere, situated between the ground of higher temperature and the stratosphere lower layer of lower temperature. The temperature non-equilibrium causes the occurrence of other forms of non-equilibrium, such as mechanical and concentration ones (*e.g.* change of water vapour content in the air). Out of the above-mentioned non-equilibrium, the mechanical non-equilibrium causes, initiates and maintains the convection motion in the atmosphere. Its numerical measure is at the same time the convection intensity value. In thermodynamics, this measure is presented by maximum work that can be obtained from the existing non-equilibrium. In the atmosphere, this non-equilibrium is both mechanical and thermal (temperatural), and the maximum possible work derived from it would be »exergy« of the warm and humid ground-level air. Under the circumstances of atmospheric convection the possibility of heat exchange is relatively negligible, so the maximum work in the thermodynamic sense (exergy) boils down to maximum adiabatic work, Ninić (2006). Such work that might be done by the low-level air in non-equilibrium atmosphere is in fact the convection intensity measure. In the case when convection consists of raising the air from the ground, the maximum work equals the buoyancy force work. It can be obtained in an imaginary process of adiabatic equilibrium air rising to the maximum available height »H_{max}« in particular circumstances. Therefore:

$$e_{\uparrow} = \int_{0}^{H_{\max}} (\rho_a - \rho)\, \mathrm{v}\, g\, dz = \int_{0}^{H_{\max}} \left(\frac{\rho_a}{\rho} - 1 \right) g\, dz \tag{1}$$

By introducing specific volumes in place of densities, (1) gets the form well known in thermodynamics of flow processes:

$$e_{\uparrow} = - \int_{p_a(0)}^{p_a(H_{\max})} (\mathrm{v} - \mathrm{v}_a)\, dp_a \tag{2}$$

Namely $-\mathrm{v}dp_a$ is the convecting air flow technical work while rising up in the atmosphere, and $-\mathrm{v}_a\, dp_a$ is part of that work »spent« on increasing its potential energy in the gravity field (see further on following the equation (3)). The difference in (2) is the work the air can perform besides rising, therefore creating the kinetic energy i.e. the convection intensity. The designation $e_{\uparrow}$ points out the adiabatic part of the updraft flow maximum work (i.e. of its exergy). $e_{\uparrow}$ further on referred simple to as updraft work potential.

2. Convection intensity measure in meteorology

In the definition of low-level air maximum work $e_{\uparrow}$, first proposed in meteorology by Brunt (1941), the updraft movement conditioned by downdraft, also having its maximum work $e_{\downarrow}$, is taken into consideration. We need to point out that the downdraft movement of air is created and maintained by rejecting heat to the height H_{max}. Because of rejecting the heat, the air parcel which from the ground comes to the height H_{max}, gains a new »buoyancy force«, however in the opposite direction (downward), due to its heat rejection and density increase. If the updraft work capacity $e_{\uparrow}$ according to (2) (in meteorology called CAPE) is added the downdraft i.e. $e_{\downarrow}$, (or DCAPE calculated analogously with (2)), updraft–downdraft or total work capacity – TCAPE can be obtained. If there was no separation of condensed phase the air parcel is a closed system and forms a thermodynamic cycle in gravitational field. So the sum of the two adiabatic works $e_{\uparrow}$ and $e_{\downarrow}$ could be interpreted as elements of Carnot reversible cycle between temperatures on the ground and at the maximum height. In this case the sum $e_\uparrow$ and $e_\downarrow$ is the maximum work connected with temperature difference between ground and H_{max} level and with Carnot cycle efficiency. From such integral point of view this temperature difference is the primary cause of the circulation occurrence.

The total work capacity was first derived by Brunt in the example of air movement as an ideal gas in the Carnot cycle. The updraft–downdraft moving air in fact performs the Brayton cycle, equivalent to the Carnot. It consists of two adiabatic curves (updraft or »warm« and downdraft or »cold«) and two isobars. The first isobar is »near-ground« one, during which the air receives heat from the ground. The second isobar is at the height H_{max}, during which the air transmits the heat lowering its own temperature in the surrounding atmosphere at $H = H_{max}$. All these processes are shown in solid lines 1–2–3–4 in p–v diagram in Figure 1.

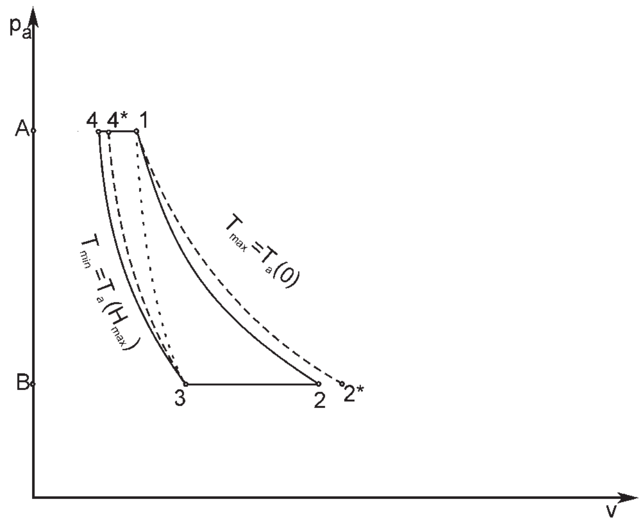

Figure 1. Brayton equilibrium cycle of updraft–downdraft air movement.

The near-ground air state is presented by point »1«. Process 1–2 represents the equilibrium adiabatic expansion where the air rises up to H_{max}. In this case $T_2 > T_3 = T_a(H_{max})$. The convecting air delivers heat at that height, in which process it cools to $T_3 = T_a(H_{max})$. By equilibrium adiabatic (reversible) downdraft (process 3–4), the convecting air returns to the near-ground pressure, to state 4. Points 1 and 3 show the surrounding atmosphere states in the diagram in Figure 1., i.e. the dotted curve 1–3 represents the relationship per height for the atmosphere states. The latter is in accordance with the fact that the atmosphere must obviously be unstable.

Let us now interpret in Figure 1. the notions regarding the definitions $e_{\uparrow}$ and $e_{\downarrow}$, i.e. CAPE and DCAPE according to Brunt (1941). The area A_{A13B}, represents potential energy at the height H_{max}, i.e.

$$A_{A13B} = e_p(H_{max}) = gH_{max} \tag{3}$$

The evidence for this is the following. Let us take 1–3 as an imaginary equilibrium diabatic flow process, with air rising. In such a case the area A_{A13B}, will represent the technical work done by that air. As the process was without buoyancy force, with $v(z) = v_a(z)$, the work as a whole was spent on increasing its potential energy. Thus, if the total warm air technical work at its rising represents the area A_{A12B}, then the triangle shape area A_{123} represents the work which is the updraft air flow intensity measure, which is also contained in (2), i.e.:

$$e_{\uparrow} = A_{123} = CAPE \tag{4}$$

In the Appendix, there is a new and simple CAPE numerical calculation method for humid air. The method works with the real moist air and arbitrary density distribution per height $\rho_a(z)$.

Analogously (4), the surface A_{413} in Figure 1. represents the downdraft movement work. This work is at the same time the downdraft movement intensity measure. Therefore, if the convection intensity at a location is influenced by updraft and downdraft flow, it is then justifiable to take TCAPE:

$$TCAPE = e_{\uparrow} + e_{\downarrow} = A_{1234} \tag{5}$$

as total intensity measure.

Although Brunt (1941) introduces TCAPE based on the Carnot instead of more realistic Brayton cycle in Figure 1., actually there is no essential difference if they are, as already said, equivalent. Namely, if the Carnot cycle lower and upper temperatures are mean thermodynamic temperatures corresponding to the Brayton cycle isobares, then between them there is no difference in efficiency.

3. Energy dissipation influence

Emanuel (1986, 1989) touches on the interesting problem of mechanical energy dissipation influence to dissipation intensity itself. Namely, the CAPE definition according to (2) can be maintained, provided that pressure function v under the integral is substituted by v-function in a real process, which takes into consideration energy dissipation. Let us dwell on now on such an alternative understanding of CAPE* and TCAPE* which we mark with asterisks and which, to a certain extent, differ from the original according to Brunt. Unlike the Brayton and Carnot cycle, the one with internal friction in Renno and Ingersoll's paper (1996) is internally non-equilibrium. As a modification of equilibrium Brayton cycle, that cycle is achieved so that Brayton cycle is added internal friction i.e. mechanical energy dissipation. The internal friction is contained in adiabatic compression and expansion processes. By application of the above mentioned modifications the equilibrium Brayton cycle transits »in figure (1).« into cycle 1–2*–3–4*.

Approaching thermodynamic analysis of such a (real) cycle of the convecting air, we start from the First Law of Thermodynamics, and this for an observer connected to the Earth. As for this observer there is no transfer of work with external bodies, for all process parts, the first law boils down to the following:

$$dq = dh_{tot} \tag{6}$$

with

$$h_{tot} = h + \frac{v^2}{2} + gz \tag{7}$$

Here, q is added heat, and h is humid air specific enthalpy. The value of total enthalpy h_{tot} represents the total humid air specific energy of fluid flow. Enthalpy h is specified by its temperature, pressure and moisture content. For example in the case of $t > 0$ °C

$$h \cong c_p t + Lr \tag{8}$$

where L is latent heat of evaporation and r is mass of water vapour per mass of pure air (»mixing ratio«).

The law on conservation of energy (6) is applicable on both equilibrium and non-equilibrium processes. Non-equilibrium processes with internal friction, a subject matter herein, represent a particular case.

Such non-equilibrium processes can be modelled with the equivalent equilibrium processes, according to the method hereinafter presented. According to this method, the friction work appears explicitly in (6) in the form of two additional terms. The first such term is added to the right side of the expression (6) and represents fictive reversible work – extracted and in the same time added as »friction heat«: $dw_{fr} = dq_{fr}$. So, the »friction heat« is the other supplementary term added to the left side (6). In such a way friction does not change total enthalpy, but does change entropy. The equilibrium model of a non-equilibrium process is supplemented by equation (6), to obtain the following form:

$$dq + dq_{fr} = dh_{tot} + dw_{fr} \tag{9}$$

where dw_{tr} is the internal friction work expressed in equilibrium form

$$dw_{fr} = -\nu \, dp_{fr} \tag{10}$$

with dq_{fr} expressed also in equilibrium form:

$$dw_{fr} = dq_{fr} = Tds_{fr} \tag{11}$$

$$dq + dq_{fr} = Tds \tag{12}$$

dp_{fr} and ds_{fr} in (10) and (11) are pressure and entropy differential changes due to the friction process. Symbols d in dq, dq_{fr} and dw_{fr} doesn't mean exact differential of state functions because heat, friction work and a friction heat are – as presented here – real or fictive external influences. So, dp_{fr} and ds_{fr} are parts of differentials of state functions p and s.

According to the same equilibrium model, this time for the thermodynamic observer connected to a small portion of moving air, is

$$dq + dq_{fr} = dh - \mathrm{v}dp \tag{13}$$

where $dq+dq_{fr}$ is total fictive reversible heat for this observer. In the same way $-\mathrm{v}dp$ is total reversible performed work for him (part of this work is consumed for changes of kinetic and potential energy and part as friction work).

Treatment of real processes expressed by the equations (9) to (13) is known in technical thermodynamics. The first author himself has some additional arguments concerned with the application and development of this modelling method.

Renno and Ingersoll (1996) according to Emanuel (1986, 1989) substitute in (9) the approximate expression for humid air enthalpy (8) and in accordance with (12) they obtain

$$Tds = d\left(c_p t + Lr + \frac{v^2}{2} + gz\right) + dw_{fr} \tag{14}$$

4. Alternative expression for TCAPE

Integrating (14) and with (11) along real convection updraft–downdraft cycle and taking into consideration that h is a state function – the following is derived:

$$\oint Tds = -\oint \mathrm{v}dp \tag{15}$$

Integral $-\oint \mathrm{v}dp$ represents work of potential energy and kinetic energy changes and internal friction per cycle. As for a cycle a potential and kinetic energy zero, then:

$$-\oint \mathrm{v}dp = w_{fr.cy} \tag{16}$$

This is in accordance with (13) and (15). Formally, it is also easy to present

$$-\oint \mathrm{v}dp = \oint pd\mathrm{v} = \oint Tds \tag{17}$$

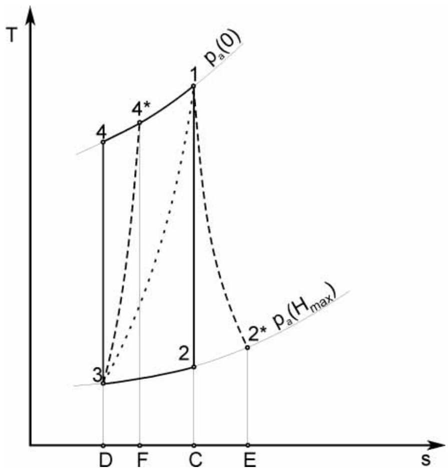

Figure 2. Internal equilibrium and internal non-equilibrium Brayton cycle of updraft–downdraft airflow.

as in any case cycle areas in rectangular p–v and T–s diagram are always equal. The latter (17) coincides with Renno and Ingersoll's (1996) equation (3).

The cycle with friction 1–2*–3–4* is not the same as the cycle 1–2–3–4 in Figure 1., where CAPE and TCAPE are defined and interpreted graphically. In Figure 2. there is the alternative presentation of both cycles in T–s diagram.

According to the model supporting the equations (8) and (14) all real processes may be formally considered internally equilibrium. According to (16) and (17) all friction work done returns to the air as »heat« q_{fr} in the processes 1–2* and 3–4*, and the total »added« heat equals:

$$"q_{in}" = A_{34^*12^*ED} \tag{18}$$

Rejected heat is at the same time actually rejected

$$q_{out} = A_{32^*ED} \tag{19}$$

while actually added heat is only:

$$q_{in} = q_{4^*1} = A_{4^*1CF} \tag{20}$$

Total friction work is therefore

$$w_{fr.cy} = "q_{in}" - q_{out} = A_{4^*12^*3} \tag{21}$$

with

$$q_{in} = q_{out} \quad \text{i.e.} \quad A_{4^*1CF} = A_{32^*ED} \tag{22}$$

According to the same model and the equations (16) and (17), friction work is the real cycle area in the rectangular T–s and p–V diagram. The value $w_{fr.cy}$ presents real friction work during the convection cycle, and obviously differs from supposed TCAPE according to (5). Physical interpretation of this difference is as follows: in the processes with internal friction there occurs loss of work or mechanical energy dissipation, and, at the same time there is the increase of the air technical work capacity in the continuation of the process (due to the existence of »added heat« from internal friction work – »reheat factor«). For the previously stated reasons, the expression (17) occurring with Renno and Ingersoll (1996), represents the convective cycle actual work dissipation

$$TCAPE^* = \oint Tds = \oint pd\mathrm{v} \qquad (23)$$

However, TCAPE does not represent actual friction work, but total work capacity (maximum work) for energy dissipation. It can only be pre-calculated out of the unstable atmospheric conditions.

Renno and Ingersoll (1996) introduce TCAPE* under the name TCAPE and give it the sense for a »boundary layer convection as a heat engine«, having in mind the cycles with humid air without precipitation (below cloud base). They extend the same conclusions to a more general case of »deep convection as a heat engine«. The same authors, however, do not make the difference herein emphasised between TCAPE* and the standard meteorological definition TCAPE. They, also, interpret TCAPE* according to (23) as a value derived for a reversible heat engine, which obviously is not the case, as the internal friction in the equations (13) to (23) is incompatible with reversibility.

5. Conclusion

Analysing the thermodynamic aspect of convection intensity, we can conclude that the integral in (23) according to Renno and Ingresoll (1996) represents the actual friction work in a convection cycle, i.e. mechanical energy dissipation. The authors do not point out the difference according to the standard meteorological definition TCAPE, or its physical basis. They call the real cycle with internal friction equilibrium one, which actually it is not. According to what is presented in this analysis, TCAPE* is not a criterion for the evaluation of convection intensity based on the atmospheric state preceding or accompanied by during the convection. It is only the dissipation actual work value and incalculable as the actual process – symbolised by the states 2* and 4* in the diagrams – is not known in advance. The convection intensity criterion for given atmospheric conditions must be total work capacity, therefore reversible, i.e. without internal friction. In a sense, the standard meteorological definition CAPE and TCAPE is better and, besides it is in single-valued manner determined by the atmospheric conditions.

The new, alternative procedure of CAPE calculus presented in the Appendix has not been compared with the existing calculation methods. It is particular for fully taking in account the actual characteristics of humid air, it is physically transparent and, besides the *h–r* diagram, the mere use of calculator is sufficient.

Nomenclature

A [m^2]	– area in thermodynamic diagram
CAPE $\left[\frac{J}{kg}\right]$	– updraft flow maximum work
c_p [J/kgK]	– specific heat capacity of the humid air
DCAPE $\left[\frac{J}{kg}\right]$	– downdraft flow maximum work
$e_p \left[\frac{J}{kg}\right]$	– specific potential energy
$e_{\downarrow} \left[\frac{J}{kg}\right]$	– the same as DCAPE
$e_{\uparrow} \left[\frac{J}{kg}\right]$	– the same as CAPE
g [m/s^2]	– acceleration of gravity
h [J/kg]	– specific enthalpy of humid air
H_{max} [m]	– maximum height (at troposphere level)
h_{tot} [J/kg]	– specific total enthalpy
L [kJ/kg]	– latent heat of evaporation
p [Pa]	– pressure
q [J/kg]	– specific heat exchanged
r [kg$_v$/kg$_{d.a.}$]	– mixing ratio (in general including water vapour, liquid and ice)
$\boldsymbol{R}$ [J/kmolK]	– universal gas constant
s [J/kgK]	– specific entropy
T [K]	– absolute temperature
t [°C]	– temperature

TCAPE $\left[\frac{J}{kg}\right]$ – total maximum work

v [m^3/kg] – specific volume of air

v [m/s] – speed

w [J/kg] – specific work exchanged

z [m] – height

Greek symbols

ρ_a [kg/m^3] – density of atmospheric air

ρ [kg/m^3] – density of convecting air

Subscript

a – atmospheric

cond – condensation

cy – cycle

fr – friction

in – input

n – new numerical step (in appendix)

out – output

p – previous numerical step (in appendix)

v – vapour

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Appendix: CAPE Calculus Based on the Humid Air State Diagram

Direct numeric CAPE calculus according to (2) is possible without any simplifications of humid air characteristics, if five individually simple calculus steps are combined. The first three steps are directed at finding the connection of the convecting air specific volume with the pressure at its adiabatic rising. It is the function $v = v(p_a)$ in (2). The fourth step regards subtraction of known atmospheric height profile $v_a(p_a)$. These four steps show changes of all state properties over one rising stage. The final step is addition the differences $v(p_a) - v_a(p_a)$ overall rising stages as integral approximation in (2).

The first three steps are determined by means of the Mollier *h–r* diagram for moist air, Bošnjakovic F., Blackshare P. L. (1965), or – what is fully equivalent to this diagram – »psychrometric chart«, group of authors (1997). Although any state of air is determined by three variables (t, r and $p = p_a(z)$), it is a unique diagram in which a state is determined by a point (t, x), plus one parametric curve dependable on pressure. This dependency is so simple that having *h–r* at your disposal just for one (standard atmospheric) pressure, it is able to adjust the same diagram to any other pressure. This is achieved by a short conversion, which only moves the parametric curve – »line of humid air saturation«. Besides the Mollier *h–r* diagram, for finding CAPE it is necessary to know the function $v = v(t, r, p_a)$:

$$v = \frac{\left(\frac{1}{29} + \frac{r_v}{18}\right)\boldsymbol{R}T}{p_a(1+r_v)} \cdot \frac{r_v}{r_v + r_{cond}} \qquad \text{(i)}$$

where: r_v content of humidity as vapour, r_{cond} content of humidity as condensed phase, and $\boldsymbol{R}$ universal gas constant.

The first step in the calculus $v(p_a)$ at equilibrium adiabatic air rising is the relation

$$\Delta h = v(p_a)\Delta p_a \,, \qquad \text{(ii)}$$

derived from (13) for equilibrium adiabate. The calculus begins from the given near-ground air state, where the pressure changes with the adopted numerical step Δp_a. This step has a fully determined height equivalent Δz for rising stage. After the first such step the enthalpy is $h_n = h_p + \Delta h$ and the new pressure $p_{an} = p_{ap} + \Delta p$, while the humidity content r remains the same. The fore-mentioned is the same in case of vapour condensation occurrence, if the condensate does not precipitate from the air.

The second step is finding the new state, as a point in *h–r* diagram for the given h, r and p_a. Reading from the diagram all the necessary in (i) data are gathered for the third step calculus – namely, for the new specific volume calculation. There follows the repetition of the same three steps for the following rising stage as the new state etc. for all lower and lower pressures, i.e. for all higher co-ordinates z. After reaching the pressure at the maximum height $p_{a\ min} = p_a\ (H_{max})$ the obtained dependency v(p_a), together with the given function $v_a(p_a)$, is substituted in the sum replacing the integral in (2) and further on CAPE. The procedure for air lowering and DCAPE calculus is analogous. The whole procedure is adjusted for computer application, for which it was only necessary to build in the *h–r* diagram into the memory and make the whole described procedure algorithmic.

SAŽETAK

Termodinamički aspekt definicija »CAPE« i »TCAPE«

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U radu se analizira termodinamički aspekt pojmova CAPE i TCAPE, definiranih na standardni način i na način u radu *Rennoa i Ingersola* (1996). Svi procesi koji su poslužili za definiranje tih pojmova ilustriraju se u termodinamičkim dijagramima i opisuju se točnim termodinamičkim relacijama. U prvom slučaju oni su po definiciji potpuno ravnotežni tj. reverzibilni. Nadalje, u radu se pokazuje da unutarnje trenje, koje *Renno* i *Ingersol* uzimaju u obzir, čini njihovu definiciju CAPE i TCAPE različitom od standardne. Naime, CAPE i TCAPE po njihovoj definiciji predstavljaju rad disipacije pri konvekciji, a ne njen radni potencijal.

Ključne riječi: maksimalni rad, CAPE, TCAPE, disipacija energije

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