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Pregledni znanstveni članak / Review

# Preliminary Determination of Spatial Geodetic Monitoring Accuracy for Free Station Method

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*ABSTRACT. In this paper, one of the modern spatial geodetic monitoring method was described: a free station method. This method is effectively used during monitoring of displacements for buildings and constructions under dynamic load. Theoretical prove for rigorous preliminary computing of displacements determination accuracy using free station method was done. The analysis of errors, which influence on the accuracy of spatial monitoring, was presented. It was found out that errors of initial survey network, errors of free station coordinates and errors of spatial monitoring targets coordinates were influencing on the accuracy of measured displacements using the free station method. Using the covariance transformation rules, rigorous formulas for preliminary calculation of displacement determination accuracy were obtained. Experimental calculations were carried out using these rigorous formulas, deploying practical results of spatial geodetic monitoring of transport structure situated in the zone of metro line underground construction. Executed calculations proved that chosen method of spatial deformation monitoring and equipment accuracy conformed the design requirements.*

*Keywords: free station, geodetic spatial monitoring, deformation target, covariance matrix, accuracy calculation, initial data errors.*

## 1. Introduction

Spatial geodetic monitoring of civil constructions is one of the most complicated tasks of engineering geodesy. Modern survey equipment allow to simplify many technological processes. At the same time, new equipment led to invention of advanced spatial monitoring methods. Nowadays, accurate and precise total stations with automatic target recognition (ATR) and reflectorless modes have become quite common (Reda and Bedada 2012, Lutes 2002). Exceptionally popular are motorized total stations (Paar et al. 2005). Motorized total stations are often used

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in combination with others measuring technics (Erdoğan and Gülal 2010, Kopáčik et al. 2016, Kopáčik et al. 2013, Kopáčik et al. 2011, van Dosselaer et al. 2012) or deployed as a part of automatic monitoring systems, like in case of Leica GeoMos, Trimble 4D etc.

The traditional spatial geodetic monitoring method, executed from fixed network points, was superseded by the free station method, which makes possible to carry out spatial monitoring from any suitable point around or inside of building (Braun and Štroner 2014). If necessary requirements are followed, the free station method provides adequate accuracy of spatial geodetic monitoring, while its efficient speed is much higher in comparison with other methods. This method is especially effective for spatial geodetic monitoring of transport structures experiencing dynamic load: bridges, flyovers, viaducts (Beshr 2015, Cetl et al. 2006). Spatial monitoring of these constructions deformations is carrying out in kinematic mode using total stations (Mill et al. 2015, Milovanović et al. 2011, Psimoulis and Stiros 2013). In many cases of geodetic monitoring, we have to execute observations from unstable points. It can be due to obstructions or demands for proper measurement accuracy. We have to understand that in such case the total station coordinates will be changing permanently. Therefore, from the technical point of view, it is a traditional geodetic monitoring, but as the total station coordinates changing permanently, we have free station observations in each cycle. Varity of different measurements in the free station method is exceeding traditional observations and, therefore, preliminary calculation of spatial monitoring accuracy executed by the free station method needs to be done. Presently, such computations are carrying out using non-rigorous methods (Lapaine et al. 2014), which do not consider the influence on accuracy of measurements of all parameters, as seen from (Beshr and Elnaga 2011). The research's aim is to study the features of modern survey equipment deployment for spatial monitoring in a case of the free station method with getting of rigorous formulas for preliminary accuracy calculation. Further, worked-out mathematical equations were probated in the spatial geodetic monitoring of flyover project.

## 2. Theory of free station method accuracy

The basic condition for using the free station method is existence of initial survey network with fixed points situated out of the deformation process zone. We will discuss only 3D survey networks, therefore, will consider only spatial 3D coordinates as definition of "coordinate" and "displacement". Determination of the free station coordinates is carrying out with reference to the fixed points of survey network (Kougia et al. 1986, Braun and Štroner 2014). Further, the obtained free station coordinates will be used as a fixed base for deformation targets (reflectors), these displacements will be monitored. Therefore, final coordinates of deformation targets contain three types of errors: errors of initial survey network, errors of free station coordinates determination and errors of deformation targets displacement measurements. As can be seen, accumulation of errors and their influence on each other has place. We offer the error of spatial monitoring target coordinates, determined by using the free station method write as:

$$\mathbf{K}_D = \mathbf{K}_{GN} + \mathbf{K}_{FS} + \mathbf{K}_M \, , \tag{1}$$

where:

- $\mathbf{K}_{GN}$ – covariance matrix of initial survey network errors influence;
- $\mathbf{K}_{FS}$ – covariance matrix of errors of free station coordinates determination which computed with errorless initial data (survey network coordinates do not contains errors);
- $\mathbf{K}_{M}$ – covariance matrix of deformation target coordinates measurements with reference to the free station.

Covariance matrix of errors for free station coordinates determination $\mathbf{K}_{FS}$ is obtained from adjustment of 3D-resection. The structure of this matrix depends on resection geometry, and values of its elements depend from accuracy of distances, vertical and horizontal angles measurements (Osada et al. 2010). This matrix has a following well-known structure:

$$\mathbf{K}_{FS} = \begin{bmatrix} m^2_{X_{FS}} & k_{X_{FS}Y_{FS}} & k_{X_{FS}Z_{FS}} \\ k_{X_{FS}Y_{FS}} & m^2_{Y_{FS}} & k_{Z_{FS}Y_{FS}} \\ k_{X_{FS}Z_{FS}} & k_{Z_{FS}Y_{FS}} & m^2_{Z_{FS}} \end{bmatrix}, \tag{2}$$

where:

- $m_{X_{FS}}$, $m_{Y_{FS}}$, $m_{Z_{FS}}$ – Root Mean Square (RMS) errors of abscissa, ordinate and applicate of free station;
- $k_{X_{FS}Y_{FS}}$ – correlation moments.

To find out covariance matrix of initial survey network errors influence $\mathbf{K}_{GN}$, covariance matrix $\mathbf{K}$ is used, which is obtained from initial survey network adjustment results. Generally, matrix $\mathbf{K}$ is:

$$\mathbf{K} = \begin{bmatrix} m^2_{X_1} & k_{X_1Y_1} & k_{X_1Z_1} & \cdots & k_{X_1Z_n} \\ k_{X_1Y_1} & m^2_{Y_1} & k_{Z_1Y_1} & \cdots & k_{Y_1Z_n} \\ k_{X_1Z_1} & k_{Z_1Y_1} & m^2_{Z_1} & \cdots & k_{Z_1Z_n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ k_{X_1Z_n} & k_{Y_1Z_n} & k_{Z_1Z_n} & \cdots & m^2_{Z_n} \end{bmatrix}, \tag{3}$$

where:

- $m_{X_i}$, $m_{Y_i}$, $m_{Z_i}$ – RMS errors of abscissa, ordinate and applicate $i$-th point;
- $k_{X_iY_j}$ – correlation moments;
- $n$ – number of network points.

There are coordinates dispersions of initial survey network points in the main diagonal of matrix (3).

The main issue is how to modify matrix $\mathbf{K}$ to matrix $\mathbf{K}_{GN}$. It is a complex task to compute the matrix $\mathbf{K}_{GN}$. In general, influence of initial survey network errors can be written as (Kougia et al. 1986):

$$\mathbf{K}_{GN} = \mathbf{B}\mathbf{K}\mathbf{B}^T, \tag{4}$$

where:

**K** – covariance matrix of initial survey network points from adjustment and

**B** – matrix of partial derivatives from measured point (free station) by fixed points coordinates of initial survey network. Generally, matrix **B** can be presented as (Kougia et al. 1986):

$$\mathbf{B}=\begin{bmatrix} \frac{\partial X_{FS}}{\partial X_1} & \frac{\partial X_{FS}}{\partial Y_1} & \frac{\partial X_{FS}}{\partial Z_1} & \cdots & \frac{\partial X_{FS}}{\partial X_n} & \frac{\partial X_{FS}}{\partial Y_n} & \frac{\partial X_{FS}}{\partial Z_n} \\ \frac{\partial Y_{FS}}{\partial X_1} & \frac{\partial Y_{FS}}{\partial Y_1} & \frac{\partial Y_{FS}}{\partial Z_1} & \cdots & \frac{\partial Y_{FS}}{\partial X_n} & \frac{\partial Y_{FS}}{\partial Y_n} & \frac{\partial Y_{FS}}{\partial Z_n} \\ \frac{\partial Z_{FS}}{\partial X_1} & \frac{\partial Z_{FS}}{\partial Y_1} & \frac{\partial Z_{FS}}{\partial Z_1} & \cdots & \frac{\partial Z_{FS}}{\partial X_n} & \frac{\partial Z_{FS}}{\partial Y_n} & \frac{\partial Z_{FS}}{\partial Z_n} \end{bmatrix}. \tag{5}$$

It should be reminded that free station coordinates are computed from 3D-resection. To find out partial derivatives in the matrix (5), we will use method of numerical differentiation. Partial derivatives are computed numerically using the following formulas:

$$\frac{\partial X_{FS}}{\partial X_1}\approx\frac{X_{FS}^0-X_{FS}}{\Delta X_1}=\frac{\Delta X_{FS}}{\Delta};\ \frac{\partial Y_{FS}}{\partial X_1}\approx\frac{Y_{FS}^0-Y_{FS}}{\Delta X_1}=\frac{\Delta Y_{FS}}{\Delta};\ \frac{\partial Z_{FS}}{\partial X_1}\approx\frac{Z_{FS}^0-Z_{FS}}{\Delta X_1}=\frac{\Delta Z_{FS}}{\Delta}. \tag{6}$$

Let us look how to make the calculations by formulas (6). Using obtained from adjustment of free station coordinates $X_{FS}, Y_{FS}, Z_{FS}$ with condition that network fixed points do not contain errors of initial data, in formulas (6) free station coordinates $X_{FS}^0, Y_{FS}^0, Z_{FS}^0$ calculated considering the condition that coordinate $X_1$ of the first fixed point was distorted by adding correction Δ, which can be practically accepted equal to 50 mm. Therefore, gradually changing fixed points coordinates, one by one, all columns of matrix (5) can be computed and meaning of covariance matrix $\mathbf{K}_{GN}$ can be found.

When covariance matrix of free station position is computed, considering the influence of initial data, we can calculate covariance matrix $\mathbf{K}_M$ of determination of deformation targets coordinates. 3D-polar method is used to get the coordinates of deformation targets. It could be considered that accuracy of coordinates will depend on accuracy of free station and accuracy of the initial survey network points. Under accuracy of the initial survey network points we mean point used for orientation of the total station (back sight). Then, formula for matrix $\mathbf{K}_M$ can be written as:

$$\mathbf{K}_M=\mathbf{K}_P+\mathbf{K}_N, \tag{7}$$

where:

$\mathbf{K}_P$ – covariance matrix of deformation targets errors with exception of initial data errors;

$\mathbf{K}_N$ – covariance matrix of initial data errors (errors of free station and orientation point).

Covariance matrix $\mathbf{K}_P$ can be computed as (Beshr and Elnaga 2011):

$$\mathbf{K}_P = \mathbf{AMA}^{\mathbf{T}}, \tag{8}$$

where:

**A** – matrix of partial derivatives from equations for coordinates calculation using 3D-polar method;

**M** – 3x3 diagonal matrix. Diagonal elements of matrix **M** are dispersions of angles measurements and distances measured by total station.

After some modifications the matrix **A** of partial derivatives will look:

$$\mathbf{A} = \begin{bmatrix} S\cos z\cos(\beta+\alpha) & -S\sin z\sin(\beta+\alpha) & \sin z\cos(\beta+\alpha) \\ S\cos z\sin(\beta+\alpha) & S\sin z\cos(\beta+\alpha) & \sin z\sin(\beta+\alpha) \\ -S\sin z & 0 & \cos z \end{bmatrix}, \tag{9}$$

where:

$\alpha$ – directional angle of initial direction;

$\beta$ – horizontal angle;

$z$ – zenith distance;

$S$ – distance.

Matrix of dispersions of angle and distance measurements without considering of correlation is:

$$\mathbf{M} = diag\left[m_z^2 \quad m_\beta^2 \quad m_S^2\right], \tag{10}$$

where:

$m_z$ – RMS error of zenith distance measurement;

$m_\beta$ – RMS error of horizontal angle measurement;

$m_S$ – RMS error of distance measurement.

These values can be taken from total station passport or can be calculated like in (Bird 2009).

Now we have to get covariance matrix of initial data errors. For this, we will use the described above approach. Finally, covariance matrix of initial data errors $\mathbf{K}_N$ will be:

$$\mathbf{K}_N = \mathbf{B}_N \mathbf{K}_{FS/GN} \mathbf{B}_N^T. \tag{11}$$

Covariance matrix (11) describes cumulative influence of initial survey network errors and errors of free station coordinates determination with reference to the initial survey network. Covariance matrix $\mathbf{K}_{FS/GN}$ has block diagonal structure and contains two blocks. Block of free station errors, which we offer to compute from the equation:

$$\mathbf{K}_{ST} = \mathbf{K}_{GN} + \mathbf{K}_{FS}\,, \tag{12}$$

which proves that initial network errors are influence on whole error measurements for deformation target's positions via errors of free station coordinates.

Second block is a covariance matrix containing errors of orientation point for free station. This covariance matrix is extracted form full covariance matrix of survey network $\mathbf{K}$. Then matrix $\mathbf{K}_{FS/GN}$ can be written as:

$$\mathbf{K}_{FS/GN} = \begin{bmatrix} m^2_{X_{ST}} & k_{X_{ST}Y_{ST}} & k_{X_{ST}Z_{ST}} & 0 & 0 & 0 \\ k_{X_{ST}Y_{ST}} & m^2_{Y_{ST}} & k_{Y_{ST}Z_{ST}} & 0 & 0 & 0 \\ k_{X_{ST}Z_{ST}} & k_{Y_{ST}Z_{ST}} & m^2_{Z_{ST}} & 0 & 0 & 0 \\ 0 & 0 & 0 & m^2_{X_i} & k_{X_iY_i} & k_{X_iZ_i} \\ 0 & 0 & 0 & k_{X_iY_i} & m^2_{Y_i} & k_{Y_iZ_i} \\ 0 & 0 & 0 & k_{X_iZ_i} & k_{Y_iZ_i} & m^2_{Z_i} \end{bmatrix}. \tag{13}$$

Elements of matrix $\mathbf{B}_N$ can be computed in the same manner as elements of matrix $\mathbf{B}$ were, and matrix $\mathbf{B}_N$ will be:

$$\mathbf{B}_N = \begin{bmatrix} \frac{\partial X_M}{\partial X_{ST}} & \frac{\partial X_M}{\partial Y_{ST}} & \frac{\partial X_M}{\partial Z_{ST}} & \frac{\partial X_M}{\partial X_i} & \frac{\partial X_M}{\partial Y_i} & \frac{\partial X_M}{\partial Z_i} \\ \frac{\partial Y_M}{\partial X_{ST}} & \frac{\partial Y_M}{\partial Y_{ST}} & \frac{\partial Y_M}{\partial Z_{ST}} & \frac{\partial Y_M}{\partial X_i} & \frac{\partial Y_M}{\partial Y_i} & \frac{\partial Y_M}{\partial Z_i} \\ \frac{\partial Z_M}{\partial X_{ST}} & \frac{\partial Z_M}{\partial Y_{ST}} & \frac{\partial Z_M}{\partial Z_{ST}} & \frac{\partial Z_M}{\partial X_i} & \frac{\partial Z_M}{\partial Y_i} & \frac{\partial Z_M}{\partial Z_i} \end{bmatrix}. \tag{14}$$

Partial derivatives can be computed numerically using the following formulas:

$$\frac{\partial X_M}{\partial X_{ST}} \approx \frac{X^0_M - X_M}{\Delta X_{ST}} = \frac{\Delta X_M}{\Delta}; \frac{\partial Y_M}{\partial X_{ST}} \approx \frac{Y^0_M - Y_M}{\Delta X_{ST}} = \frac{\Delta Y_M}{\Delta}; \frac{\partial Z_M}{\partial X_{ST}} \approx \frac{Z^0_M - Z_M}{\Delta X_{ST}} = \frac{\Delta Z_M}{\Delta}. \tag{15}$$

Like in the first case in formulas (6), using coordinates $X_M, Y_M, Z_M$ of deformation targets, obtained from polar measurements, with condition that free station and base orientation point do not contains error of initial data, in formulas (15) coordinates of deformation target $X^0_M, Y^0_M, Z^0_M$ are computed considering the condition that the deviation $\Delta$ of 50 mm is added to the free station coordinate $X_{ST}$. Consequently, gradually changing free station coordinates and orientation point all columns of matrix (14) can be computed and the covariance matrix $\mathbf{K}_N$ can be found.

Therefore, initial equation (1) is transformed. We substitute (8) and (11) in (7) and obtain equation for determination of error of deformation target coordinate using free station method:

$$\mathbf{K}_D = \mathbf{K}_M = \mathbf{AMA^T} + \mathbf{B}_N \mathbf{K}_{FS/GN} \mathbf{B}_N^T, \tag{16}$$

Matrix (2), (4), (8) and (11) are used for computation the final errors matrix (16). Presented preliminary calculation of accuracy for determination of deformation target coordinates using the free station method is rigorous and considers all parameters, which influences on accuracy of measurements with count of correlation relations. In order to prove the proposed approach, the experimental computation of accuracy in a case of measurement of displacements using the free station method during spatial geodetic monitoring of transport structure will be carried out.

## 3. Object of research

The free station method was implemented for organizing the flyover spatial geodetic monitoring. The object of research is situated in construction zone of Kiev underground metro line. It was decided to carry out the spatial monitoring after considerable movement of ground surface nearby the flyover. The biggest sediments was observed nearby flyover (Figure 1.a) and cracks in its structures had occurred (Figure 1.b).

a) b)

Figure 1. *Features of flyover deformations.*

The next program of measurements was chosen for spatial monitoring. Initial survey network (points 1–6) around the object were established and observed from points T and P. After this motorized total station was left in the construction zone approximately in the same position where was point T. It was carried out measuring every 5 minutes, firstly initial points to define own coordinates using the free station method and then, using 3D-polar method, deformation targets fixed on the flyover were observed with reference to the free station position (Figure 2).

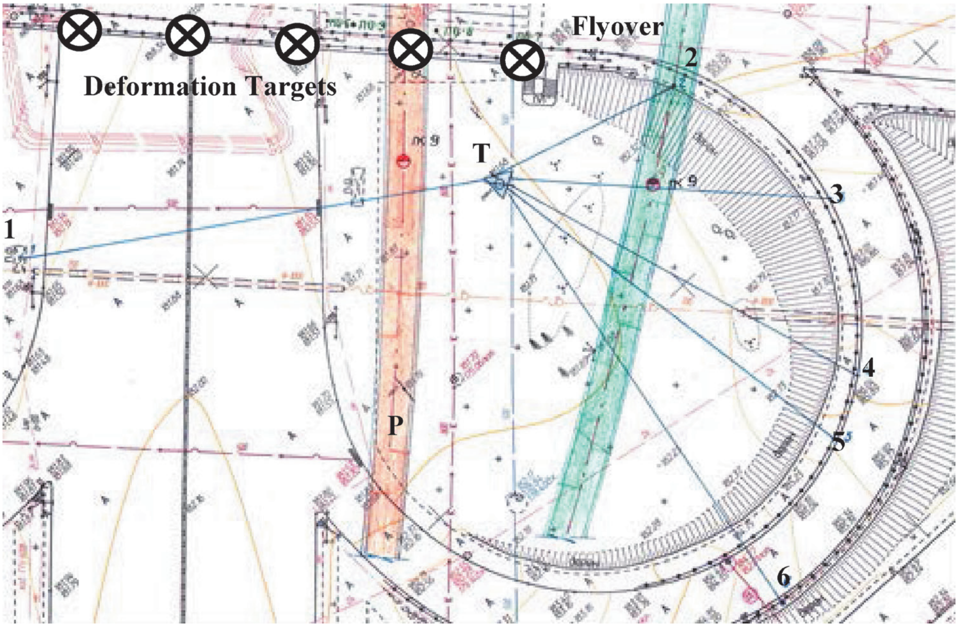


Figure 2. *Topographic plan of spatial monitoring project.*

Therefore, the described measurement program confirms the theory of free station measurements, cited above. Despite that the total station is firmly fixed on metallic pillar, which is situated in the construction site zone, total station coordinates are changing continuously. Hence, free station coordinates are changing from one cycle to another.

As per method statement for spatial geodetic monitoring at the project, displacements of flyover constructions have to be measured with ±2 mm accuracy.

## 4. Experimental research for spatial geodetic monitoring accuracy with use of free station method

To carry out pre-calculation of accuracy using proposed method, the scheme of measurement with measured values and resection geometry is presented in Figure 3.

On figure above is shown: initial survey network points (1–6), near each point the conventional height is shown; total station point (free station T) with permanently changing coordinates; distances and angles from free station (T) to network points (1–6); observation area (D); axis of left (A) and right (B) underground tunnel lines.

The total station, deployed for survey network measurements and defining of deformation target coordinates, had the next specifications: $m_z$ – 1 sec; $m_\beta$ – 1 sec; $m_S$ (–1 mm + 1 ppm).

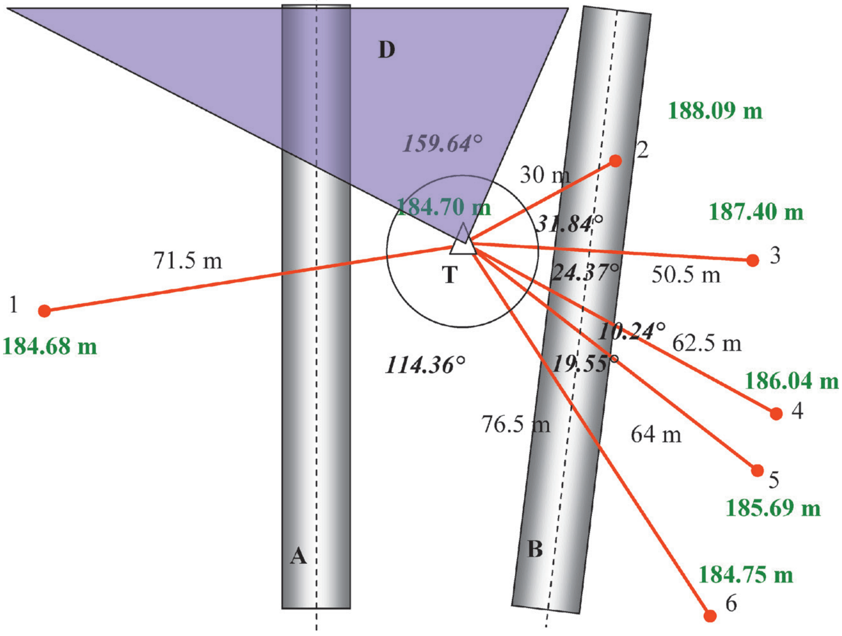


Figure 3. *Geometry of resection and measured values.*

First, will get covariance matrix of errors of initial survey network coordinates. Initial survey network point's coordinates were obtained from two fixed points; one of them was used as a free station (T). Scheme of measurement for defining of initial survey network points coordinates is shown on Figure 4.

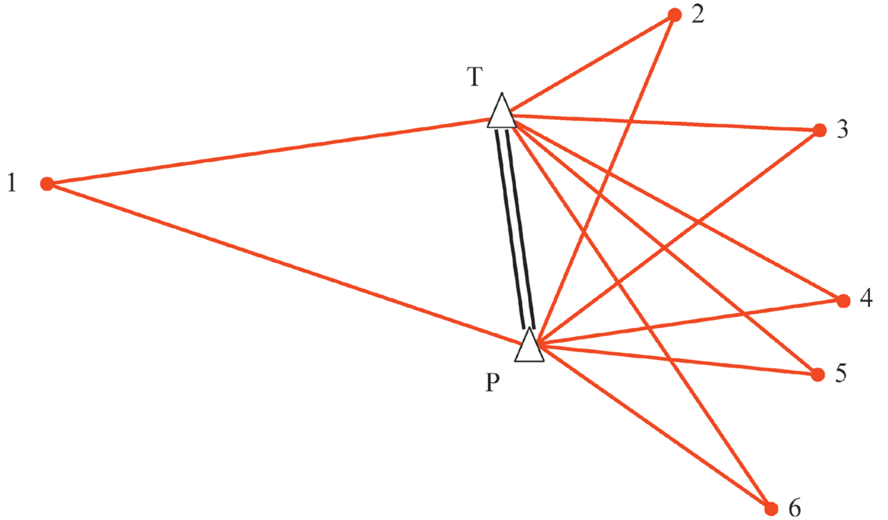


Figure 4. *Scheme of measurements in initial survey network.*

As a computation result, coordinates of initial points were obtained and their RMS errors are presented in the Table 1. Unit weight error 1.00 was used for all calculations.

Table 1. *Coordinates and RMS of base survey network points.*

| Point | X [m] | Y [m] | Z [m] | $m_X$ [mm] | $m_Y$ [mm] | $m_Z$ [mm] |
|---|---|---|---|---|---|---|
| P | 1001.4540 | 1016.1320 | 181.7500 | fixed | fixed | Fixed |
| T | 1031.0350 | 1041.4210 | 181.6800 | fixed | fixed | Fixed |
| 1 | 1070.8258 | 982.4283 | 183.9794 | 0.5 | 0.5 | 0.2 |
| 2 | 1022.3221 | 1069.7007 | 188.9398 | 0.2 | 0.4 | 0.2 |
| 3 | 994.1269 | 1074.6381 | 188.2997 | 0.3 | 0.8 | 0.2 |
| 4 | 972.3745 | 1059.2068 | 186.9997 | 0.4 | 0.4 | 0.2 |
| 5 | 967.5753 | 1050.2699 | 186.6996 | 0.4 | 0.4 | 0.2 |
| 6 | 955.6498 | 1027.5980 | 185.7495 | 0.6 | 0.3 | 0.2 |

Scheme of initial survey network with fixed points error ellipses and common error ellipses is shown in Figure 5.

Covariance matrix $\mathbf{K}$ of initial survey network will be:

$$\mathbf{K}=\begin{bmatrix}
0.254 & -0.127 & 0.007 & -0.017 & -0.011 & -0.001 & -0.001 & -0.006 & 0.0 & 0.010 & -0.040 & -0.003 & 0.019 & -0.038 & -0.003 & 0.035 & -0.026 & -0.003 \\
-0.127 & 0.243 & -0.007 & -0.026 & 0.041 & 0.010 & -0.047 & -0.019 & 0.001 & -0.039 & -0.016 & 0.002 & -0.032 & -0.023 & 0.001 & -0.012 & -0.037 & 0.0 \\
0.007 & -0.007 & 0.061 & 0.0 & -0.001 & 0.0 & 0.001 & 0.0 & 0.0 & 0.001 & 0.0 & 0.0 & 0.001 & 0.0 & 0.0 & 0.001 & 0.0 & 0.0 \\
-0.017 & -0.026 & 0.0 & 0.038 & -0.039 & -0.010 & 0.024 & 0.011 & 0.0 & 0.014 & 0.020 & 0.0 & 0.007 & 0.023 & 0.001 & -0.007 & 0.025 & 0.001 \\
-0.011 & 0.041 & -0.001 & -0.039 & 0.190 & 0.040 & -0.035 & -0.013 & 0.001 & -0.036 & 0.004 & 0.003 & -0.034 & -0.003 & 0.002 & -0.025 & -0.019 & 0.002 \\
-0.001 & 0.010 & 0.0 & -0.010 & 0.040 & 0.025 & -0.008 & -0.003 & 0.0 & -0.008 & 0.0 & 0.0 & -0.007 & -0.002 & 0.0 & -0.005 & -0.005 & 0.0 \\
-0.001 & -0.047 & 0.001 & 0.024 & -0.035 & -0.008 & 0.119 & -0.030 & -0.010 & 0.033 & 0.016 & -0.001 & 0.026 & 0.022 & -0.001 & 0.008 & 0.033 & 0.0 \\
-0.006 & -0.019 & 0.0 & 0.011 & -0.013 & -0.003 & -0.030 & 0.606 & 0.057 & 0.013 & 0.009 & 0.0 & 0.009 & 0.011 & 0.0 & 0.001 & 0.015 & 0.0 \\
0.0 & 0.001 & 0.0 & 0.0 & 0.001 & 0.0 & -0.010 & 0.057 & 0.039 & -0.001 & 0.0 & 0.0 & -0.001 & 0.0 & 0.0 & 0.0 & -0.001 & 0.0 \\
0.010 & -0.039 & 0.001 & 0.014 & -0.036 & -0.008 & 0.033 & 0.013 & -0.001 & 0.169 & -0.085 & -0.015 & 0.032 & 0.003 & -0.002 & 0.023 & 0.018 & -0.001 \\
-0.040 & -0.016 & 0.0 & 0.020 & 0.004 & 0.0 & 0.016 & 0.009 & 0.0 & -0.085 & 0.148 & 0.014 & -0.013 & 0.039 & 0.003 & -0.031 & 0.030 & 0.002 \\
-0.003 & 0.002 & 0.0 & 0.0 & 0.003 & 0.0 & -0.001 & 0.0 & 0.0 & -0.015 & 0.014 & 0.037 & -0.003 & 0.002 & 0.0 & -0.003 & 0.0 & 0.0 \\
0.019 & -0.032 & 0.001 & 0.007 & -0.034 & -0.007 & 0.026 & 0.009 & -0.001 & 0.032 & -0.013 & -0.003 & 0.195 & -0.089 & -0.018 & 0.029 & 0.009 & -0.002 \\
-0.038 & -0.023 & 0.0 & 0.023 & -0.003 & -0.002 & 0.022 & 0.011 & 0.0 & 0.003 & 0.039 & 0.002 & -0.089 & 0.129 & 0.012 & -0.027 & 0.033 & 0.002 \\
-0.003 & 0.001 & 0.0 & 0.001 & 0.002 & 0.0 & -0.001 & 0.0 & 0.0 & -0.002 & 0.003 & 0.0 & -0.018 & 0.012 & 0.035 & -0.003 & 0.001 & 0.0 \\
0.035 & -0.012 & 0.001 & -0.007 & -0.025 & -0.005 & 0.008 & 0.001 & 0.0 & 0.023 & -0.031 & -0.003 & 0.029 & -0.027 & -0.003 & 0.376 & -0.053 & -0.027 \\
-0.026 & -0.037 & 0.0 & 0.025 & -0.019 & -0.005 & 0.033 & 0.015 & -0.001 & 0.018 & 0.030 & 0.0 & 0.009 & 0.033 & 0.001 & -0.053 & 0.078 & 0.005 \\
-0.003 & 0.0 & 0.0 & 0.001 & 0.002 & 0.0 & 0.0 & 0.0 & 0.0 & -0.001 & 0.002 & 0.0 & -0.002 & 0.002 & 0.0 & -0.027 & 0.005 & 0.038
\end{bmatrix}$$

There are dispersions of point coordinates on the matrix diagonal. To define influence of these dispersions and correlation moments on the free station coordinates dispersions, using formulas (4)–(6). We compute matrixes $\mathbf{B}$ and $\mathbf{K}_{GN}$ :

$$\mathbf{B}=\begin{bmatrix}
0.300 & 0.044 & 0.034 & -0.020 & 0.208 & -0.118 & 0.140 & 0.046 & -0.002 & 0.216 & -0.072 & 0.028 & 0.224 & -0.104 & 0.032 & 0.134 & -0.116 & 0.024 \\
0.352 & 0.102 & 0.034 & -0.466 & 0.780 & -0.076 & 0.072 & 0.074 & 0.020 & 0.042 & 0.002 & 0.014 & 0.016 & 0.008 & 0.008 & -0.026 & 0.040 & -0.002 \\
0.072 & -0.028 & 0.096 & -0.268 & 0.062 & 0.410 & 0.024 & 0.004 & 0.174 & 0.056 & -0.008 & 0.122 & 0.058 & -0.010 & 0.112 & 0.050 & -0.012 & 0.080
\end{bmatrix},$$

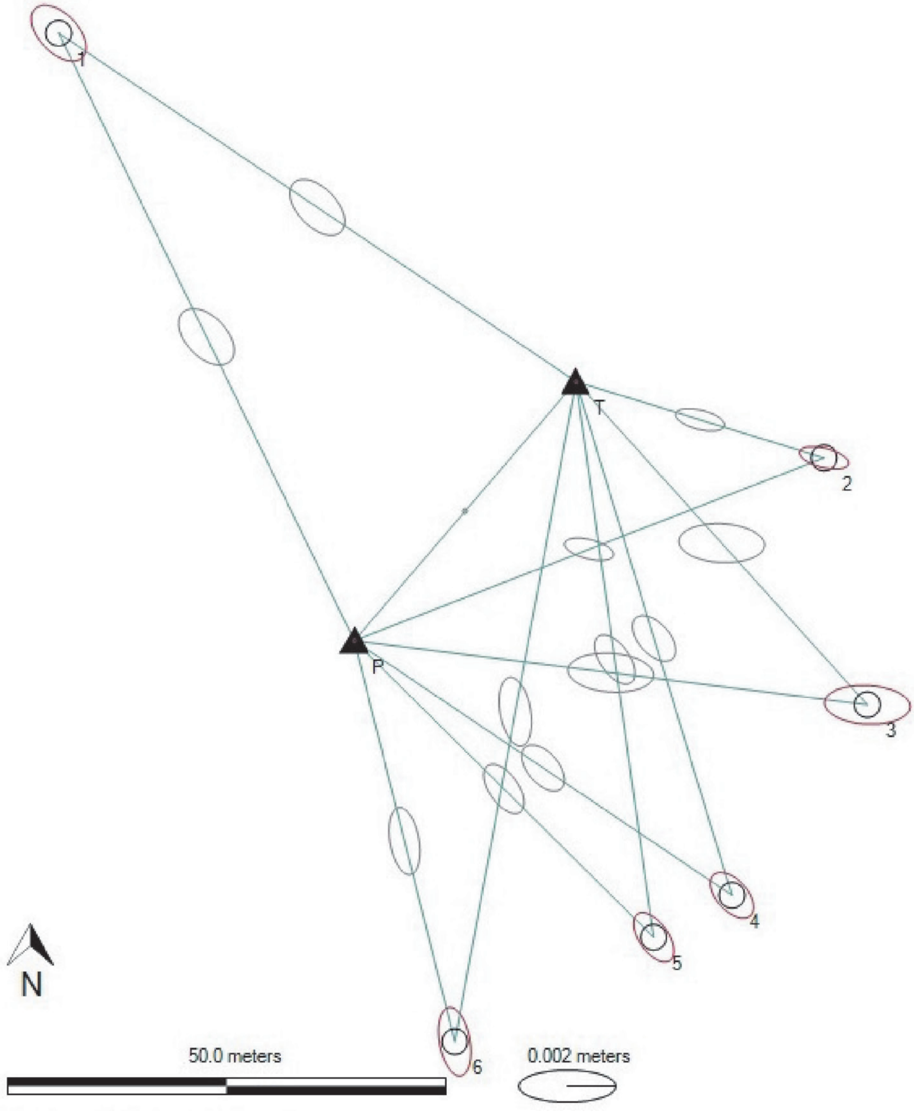


Figure 5. *Scheme of initial survey network with error ellipses.*

$$\mathbf{K}_{GN} = \begin{bmatrix} 0.077 & 0.045 & 0.023 \\ 0.045 & 0.169 & 0.040 \\ 0.023 & 0.040 & 0.019 \end{bmatrix}.$$

Then, calculate $\mathbf{K}_{FS}$ covariance matrix (2) of free station coordinates defining without considering the initial data influence. Scheme of free station 3D-resection with error ellipses is presented in Figure 6.

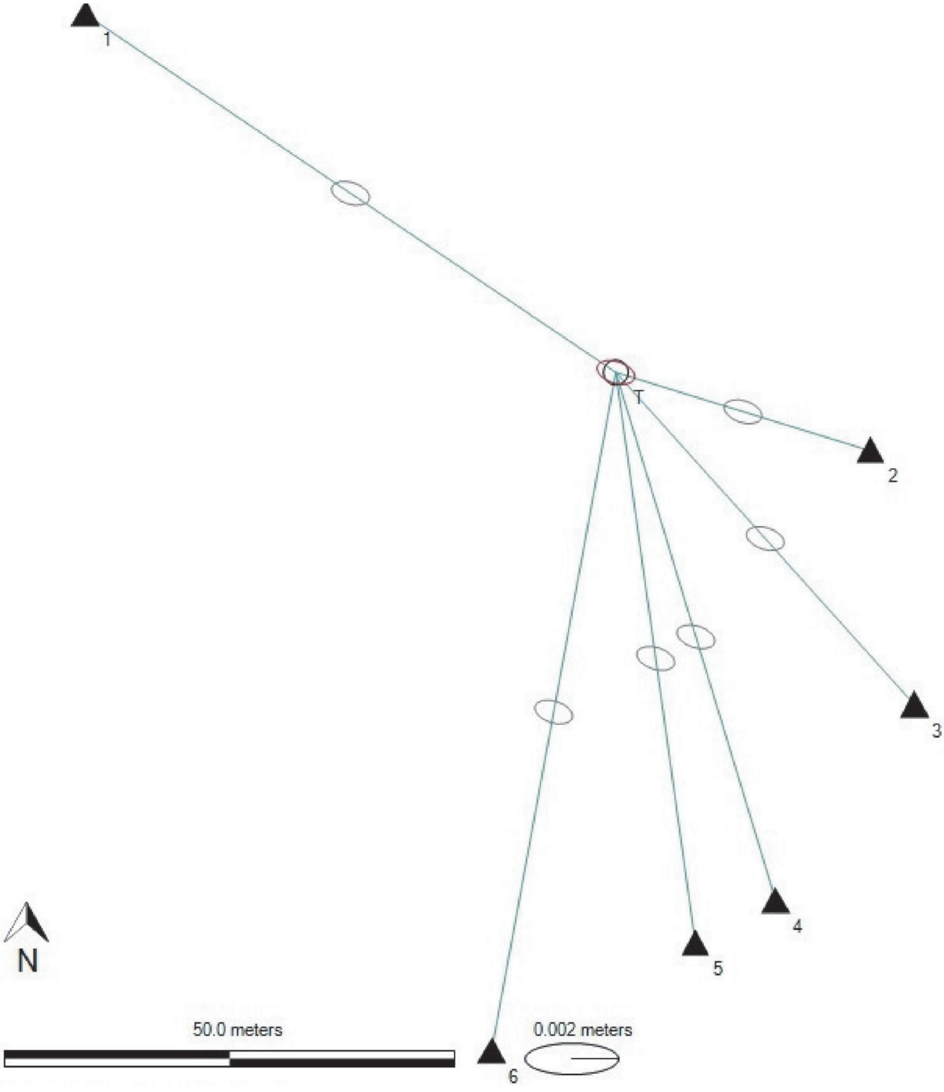


Figure 6. *Scheme of 3D-resection for free station with error ellipses.*

Free station coordinates and RMS errors of coordinates are shown in Table 2.

Table 2. *Free station Coordinates and RMS.*

| **Point** | **X [m]** | **Y [m]** | **Z [m]** | $m_X$ **[mm]** | $m_Y$ **[mm]** | $m_Z$ **[mm]** |
|---|---|---|---|---|---|---|
| **T** | 1031.0350 | 1041.4210 | 184.7000 | 0.1 | 0.2 | 0.1 |

Covariance matrix of free station coordinates defining is:

$$\mathbf{K}_{FS} = \begin{bmatrix} 0.020 & -0.011 & -0.003 \\ -0.011 & 0.051 & 0.007 \\ -0.003 & 0.007 & 0.011 \end{bmatrix}.$$

At next step we will compute covariance matrix of initial data errors $\mathbf{K}_N$ . Firstly, calculate matrix $\mathbf{K}_{ST}$ using (12):

$$\mathbf{K}_{ST} = \mathbf{K}_{GN} + \mathbf{K}_{FS} = \begin{bmatrix} 0.098 & 0.034 & 0.021 \\ 0.034 & 0.221 & 0.047 \\ 0.021 & 0.047 & 0.030 \end{bmatrix}.$$

As orientation point will choose point 1. Then covariance matrix $\mathbf{K}_{FS/GN}$ will be:

$$\mathbf{K}_{FS/GN} = \begin{bmatrix} 0.098 & 0.034 & 0.021 & 0 & 0 & 0 \\ 0.034 & 0.221 & 0.047 & 0 & 0 & 0 \\ 0.021 & 0.047 & 0.030 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.254 & -0.127 & 0.007 \\ 0 & 0 & 0 & -0.127 & 0.243 & -0.007 \\ 0 & 0 & 0 & 0.007 & -0.007 & 0.061 \end{bmatrix}.$$

Suppose that we need to compute accuracy of deformation target coordinates defining. This target position are defined by the next measured values: $z$ = 85 deg, $\beta$ = 45 deg, $\alpha$ = 303.9664 deg, $S$ = 50 m.

Using formulas (14) – (15) will compute matrix $\mathbf{B}_N$ :

$$\mathbf{B}_N = \begin{bmatrix} 0.089 & -0.074 & 0 & 0.110 & 0.074 & 0 \\ -0.564 & 0.620 & 0 & 0.564 & 0.380 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}.$$

And using formula (11) we will get finally covariance matrix of initial data $\mathbf{K}_N$ .

$$\mathbf{K}_N = \mathbf{B}_N \mathbf{K}_{FS/GN} \mathbf{B}_N^T = \begin{bmatrix} 0.004 & 0.000 & -0.002 \\ 0.000 & 0.154 & 0.017 \\ -0.002 & 0.017 & 0.030 \end{bmatrix}.$$

To compute the last covariance matrix $\mathbf{K}_P$ , we will use the following parameters of measurements:

$$\mathbf{M} = diag\begin{bmatrix} \frac{1}{\rho^2} & \frac{1}{\rho^2} & 1 \end{bmatrix}.$$

Then, using (8) will get:

$$\mathbf{K}_P = \mathbf{AMA}^{\mathrm{T}} = \begin{bmatrix} 0.526 & -0.526 & -0.058 \\ -0.526 & 0.526 & 0.058 \\ -0.058 & 0.058 & 0.066 \end{bmatrix}.$$

Finally:

$$\mathbf{K}_D = \mathbf{K}_P + \mathbf{K}_N = \begin{bmatrix} 0.530 & -0.526 & -0.060 \\ -0.526 & 0.680 & 0.075 \\ -0.060 & 0.075 & 0.096 \end{bmatrix}.$$

Using elements of this matrix, we calculate RMS error for defining the 3D-displacements of points. In order to calculate 3D displacement accuracy, we have to take diagonal elements of $\mathbf{K}_D$ matrix, sum them and multiple this sum by 2. Therefore, we will get:

$$m_{\Delta D} = \sqrt{2\left(K_{D_{11}} + K_{D_{22}} + K_{D_{33}}\right)} = 1.6 \text{ mm}.$$

Obtained accuracy of displacement measurements conforms the design requirements (1.6 mm ≤ 2 mm). Therefore, it is possible to compute expected accuracy of displacements defining from free station using proposed equations.

## 5. Conclusions

Use of contemporary robotic total stations has increased effectiveness of spatial geodetic monitoring. Deploying total stations, even in case of fixed network points, spatial monitoring often is carried out by the free station method. Therefore, it is required to know the accuracy of displacements measurements in advance. Knowing a priori accuracy of displacements determination makes it possible to carry out a probabilistic analysis of measurement results with high rank of reliability. To solve this problem, rigorous equations for preliminary calculation of spatial monitoring observations accuracy (executed by the free station method) were obtained and presented in this paper. Effectiveness of these equations was proved in the case of spatial geodetic monitoring for determination of civil construction displacements.

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# Preliminarno određivanje točnosti geodetskog monitoringa primjenom metode slobodnog stajališta

*SAŽETAK. U radu je opisana jedna od modernih metoda geodetskog monitoringa: metoda slobodnog stajališta. Ta se metoda učinkovito koristi tijekom monitoringa pomaka na zgradama i građevinama pod dinamičkim opterećenjem. Izveden je teorijski dokaz za temeljito preliminarno računanje točnosti određivanja pomaka koristeći metodu slobodnog stajališta. Prikazana je analiza pogrešaka koje utječu na točnost geodetskog monitoringa. Utvrđeno je da su pogreške inicijalne geodetske mreže, pogreške koordinata slobodnog stajališta i pogreške koordinata meta geodetskog monitoringa utjecale na točnost izmjerenih pomaka primjenom metode slobodnog stajališta. Koristeći pravila transformacije kovarijanci, dobivene su točne formule za preliminarni izračun točnosti određivanja pomaka. Eksperimentalni izračuni provedeni su primjenjujući navedene formule te uz upotrebu praktičnih rezultata geodetskog monitoringa prometnih objekata smještenih u zoni podzemne građevine linije metroa. Provedena računanja su dokazala da su izabrana metoda monitoringa prostornih deformacija i točnost uređaja bili u skladu sa zahtjevima dizajna.*

*Ključne riječi: metoda slobodnog stajališta, geodetski monitoring, meta deformacije, matrica kovarijance, računanje točnosti, pogreške početnih podataka.*

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