

# Finite time linear quadratic based optimal control of BLDC motor employing distributed parameters modeling

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Original scientific paper

An optimal control theory for linear quadratic finite time horizon problem is presented and combined with distributed parameters model of the BLDC (BrushLess Direct-Current) motor. Method appropriateness for minimization of the phase current control error and energy delivered to the drive is proven. The paper focuses on finding the best weighting configuration of the objective function. Presented control strategy is performed and presented employing the numerical computations.

**Key words:** BLDC motor, Numerical analysis, Optimal control

**Upravljanje bezkolektorskim DC motorom s distribuiranim parametrima zasnovano na linearnom kvadratičnom regulatoru s konačnim horizontom.** U ovom radu je predstavljen problem optimalnog upravljanja modelom bezkolektorskog DC motora s distribuiranim parametrima zasnovan na linearnom kvadratičnom regulatoru na konačnom horizontu. Metoda je prikladna za minimizaciju regulacijske pogreške fazne struje i utrošene energije za pokretanje pogona. Fokus rada je na traženju najboljih koeficijenata funkcije cilja. Predstavljena strategija upravljanja je realizirana i validirana u simulacijama.

**Ključne riječi:** bezkolektorski DC motor, numerička analiza, optimalno upravljanje

## 1 INTRODUCTION

BLDC (BrushLess Direct-Current) motors are commonly used in the industry. The improvement of power switching electronic elements, especially integrated circuits, has led to development and improvement in control strategies [8,14,16,17,19,20]. In literature, there are many works related to the BLDC motor control including optimal control [11,13,18,24,27]. The adaptive control has been studied in [1], where the current reshaping control algorithm has been developed and applied to reduce torque ripples. Another work has been proposed in [2], in this approach the robustness problem of the motor drive has been evaluated. Significant papers are also related to the determination of the optimal switching sequence i.e. the determination of the control parameters such as the turn-on to turn-off angle or the current amplitude [21,22], the mapping of the performance quantities of the interest to a designer or the curve fitting technique to model the flux linkage curves and obtain the optimal waveform of the exciting current [23]. Researchers also proposed the control methods employing algorithms for ripple reduction and precise speed tracking [9,12], sliding mode control or sensorless methods [2] and finally the neural networks, fuzzy logic or other artificial intelligence based techniques [3,7,24-26].

The paper proposes the application of the well-known LQR (Linear-Quadratic Regulator) control method for the BLDC motor. The success of the control methodology depends on precise modeling of the system, especially on plant control accuracy. If there is a mismatch due to the model inaccuracy (model parameters), plant changes (changes in device, speed or power level) or nonlinearities then the resulting controller will degrade and the system may become unstable.

The really good modeling may be performed using a distributed parameters model, where the state depends not only on time, but also space configuration. Hence, it is possible to take the end winding effect, cogging torque or magnetic saturation into account, for instance. It guarantees not only stability of the system but also stability margins [15]. The LQR control is calculated using a linear model of the plant under control. Employing the distributed, FE (Finite Element) based model of the motor, we obtain an exact plant model and so the controller might be optimal.

The idea of the presented paper deals with the finite time linear quadratic based control of the BLDC motor considering its distributed parameter finite element model [11,13]. The control problem consists of finding the op-

timal voltage control subject to minimization of the performance index [18] and motor dynamics. The aim of the paper is to optimally minimize current control error considering energy delivered to the motor. Moreover, the relationship between quadratic forms that describe energy lost in the motor and energy delivered to the device is analyzed. The influence of reference current shape on motor dynamics condition is studied as well. Presented methodology is confirmed with the numerical analysis of the problem.

## 2 CONTROL PLANT MODEL

The BLDC motor is modeled in 3D domain by finite element technique. The magnetic field behavior is described by the Maxwell equations. The electric circuit equations are also considered. The Galerkin method is used to obtain the set of describing equations to be solved numerically. The Euler backward procedure is applied to solve the coupled field and circuit equations. Field - circuit equations are strongly coupled by the common variable  $\mathbf{A}$  – the magnetic vector potential. The strong coupling is obtained by considering 27-node element of discretization with the first order shape functions [4,5]. The global matrix system of the field – circuit equations is as follows:

$$\mathbf{C}(\mu)\mathbf{A}(t) + \mathbf{P}\mathbf{I}(t) = \mathbf{M}(\omega(t)) \quad (1)$$

$$\mathbf{Q} \frac{d\mathbf{A}(t)}{dt} + \mathbf{R}\mathbf{I}(t) = \mathbf{U}(t) \quad (2)$$

where  $\mathbf{A} \in \mathbb{R}^n$ ,  $\mathbf{I} \in \mathbb{R}^m$  is the winding current vector,  $\mathbf{U} \in \mathbb{R}^m$  is the input voltage applied to the windings,  $\mathbf{C} \in \mathbb{R}^{m \times n}$  is the matrix related to the permeability,  $\mathbf{M} \in \mathbb{R}^m$  denotes the magnetization vector of movable permanent magnets with speed  $\omega$ ,  $\mathbf{P} \in \mathbb{R}^{m \times m}$  is the matrix related to the node currents,  $\mathbf{Q} \in \mathbb{R}^{m \times n}$  is the matrix related to the linkage flux and  $\mathbf{R} \in \mathbb{R}^{m \times m}$  is the resistance matrix. Dimension  $m$  denotes the number of motor windings and  $n$  the number of discrete grid nodes describing the electromagnetic field.

The continuous equation system (1)-(2) includes two unknown vectors  $\mathbf{A}$  and  $\mathbf{I}$ . Using time – space discretization in 3D domain, its discrete form is as follows [5,24]:

$$\begin{bmatrix} \mathbf{C}(\mu) & \mathbf{P} \\ \frac{\mathbf{Q}}{\Delta t} & \mathbf{R} \end{bmatrix} \begin{bmatrix} \mathbf{A}^t \\ \mathbf{I}^t \end{bmatrix} = \begin{bmatrix} \mathbf{M}(\omega^{t+\Delta t}) \\ \mathbf{U}^{t+\Delta t} + \frac{\mathbf{Q}}{\Delta t} \mathbf{A}^{t+\Delta t} \end{bmatrix} \quad (3)$$

Presented system of equation (3) is large and sparse. It is solved iteratively with time step  $\Delta t$  employing bi-conjugate gradient algorithm BiCG [28].

The fixed grid technique is applied to consider the movement during field calculation [11]. The grid of discretization is independent of rotor position. At each time step, the electromagnetic torque is computed via Maxwell stress tensor. The force is evaluated along surface defined

in the airgap between rotor and stator accordingly to the eggshell approach [6,10].

The new rotor position and speed are computed by solution of the discrete movement equation:

$$\begin{bmatrix} 1 & -\Delta t \\ 0 & 1 + \Delta t \frac{b}{J} \end{bmatrix} \begin{bmatrix} \Theta^{t+\Delta t} \\ \omega^{t+\Delta t} \end{bmatrix} = \begin{bmatrix} \Theta^t \\ \omega^t + \Delta t \frac{1}{J} T^{t+\Delta t} \end{bmatrix}, \quad (4)$$

where  $\Theta$  is the rotor displacement,  $J$  is the rotor inertia,  $b$  is the damping coefficient,  $T = T_E - T_L$  is the difference between the electromagnetic and the load torque.

## 3 FINITE TIME HORIZON CONTROL PROBLEM

The aim of the authors is to compute the optimal control that minimizes the current control error and energy delivered to the three phase Y-connected electric circuit [18,27] of the motor. The problem consists of finding a voltage vector  $\mathbf{U} = [u_1 \ u_2 \ u_3]^T$  which transfers winding currents  $\mathbf{I} = [i_1 \ i_2 \ i_3]^T$  from initial state  $\mathbf{I}(0)$  to the reference current  $\mathbf{I}_0 = \mathbf{I}(T)$  with specified control time  $T$ . The problem is to minimize the objective function

$$J(\mathbf{U}) = \frac{1}{2} \mathbf{E}^T(T) \mathbf{G} \mathbf{E}(T) + \dots + \frac{1}{2} \int_0^T (\alpha \cdot \mathbf{E}^T \mathbf{E} + \beta \cdot \mathbf{U}^T \mathbf{U}) dt, \quad (5)$$

where  $\mathbf{E} \in \mathbb{R}^m$  is the current control error vector,  $\mathbf{U}$  is a function of the voltage control,  $\alpha$  and  $\beta$  are the weighting factors and the form  $\frac{1}{2} \mathbf{E}^T(T) \mathbf{G} \mathbf{E}(T)$  stands for energy scrap function at time  $T$ , where  $\mathbf{G} \in \mathbb{R}^{m \times m}$  is a coefficient matrix defining system energy at the moment  $T$ . The form  $\frac{1}{2} \mathbf{E}^T(T) \mathbf{G} \mathbf{E}(T)$  is assumed to be zero because the error  $\mathbf{E}(T)$  is assumed to be zero.

From the circuit equation (2), the matrix of self and mutual inductances is computed. In case of the BLDC motor for the high-speed operation the inductances are constant and independent of rotor position. Moreover, the back electromotive force  $\mathbf{EMF} \in \mathbb{R}^m$  is computed considering total cross sectional areas of the go and return side of the stator windings. Then, for the control purpose, the circuit equation (2) is considered in the form:

$$\mathbf{L} \frac{d\mathbf{I}}{dt} + \mathbf{R}\mathbf{I} + \mathbf{EMF} = \mathbf{U}. \quad (6)$$

or in the state space form, where  $\mathbf{I}$  is the state vector, that inform about kinetic energy change in the plant windings:

$$\frac{d\mathbf{I}}{dt} = -\mathbf{L}^{-1} \mathbf{R}\mathbf{I} + \mathbf{L}^{-1} (\mathbf{U} - \mathbf{EMF}), \quad (7)$$

where  $\mathbf{L} \in \mathbb{R}^{m \times m}$  is a constant matrix related to the self and mutual inductance and  $\mathbf{R}$  is a resistance matrix. Assuming that the error  $\mathbf{E} = \mathbf{I}_0 - \mathbf{I}$  is a new state variable, then equation (7) takes the following form:

$$\frac{d\mathbf{E}}{dt} = -\mathbf{L}^{-1}\mathbf{R}\mathbf{E} + \mathbf{L}^{-1}\mathbf{U}_*, \quad (8)$$

where  $\mathbf{U}_* = \mathbf{R}\mathbf{I}_0 - \mathbf{U} + \mathbf{EMF}$  may be treated as a new excitation, but only  $\mathbf{U}$  vector is under control. Employing the linear quadratic control methodology and considering objective function (5) subject to (8), the control law is as follows [29]:

$$\mathbf{U}_* = \beta^{-1}\mathbf{L}^{-1}\mathbf{\Gamma}(\mathbf{I}_0 - \mathbf{I}) + \mathbf{R}\mathbf{I}_0 + \mathbf{EMF}. \quad (9)$$

Using the calculus of variations, the feedback matrix gain  $\mathbf{\Gamma} \in \mathbb{R}^{m \times m}$  is computed from Riccati differential equation [18]:

$$\frac{d\mathbf{\Gamma}}{dt} = -(\mathbf{L}^{-1}\mathbf{R})^T\mathbf{\Gamma} - \mathbf{\Gamma}\mathbf{L}^{-1}\mathbf{R} + \mathbf{\Gamma}\mathbf{L}^{-1}\beta^{-1}\mathbf{L}^{-1}\mathbf{\Gamma} + \alpha\hat{\mathbf{I}}, \quad (10)$$

where  $\hat{\mathbf{I}} \in \mathbb{R}^{m \times m}$  is an identity matrix. The matrix gain  $\mathbf{\Gamma}$  depends on time, its values for all  $T/\Delta t$  steps have to be calculated. The transversality condition may be stated that  $\mathbf{\Gamma}(T) = \mathbf{G}$ , applying the Euler method for derivative solution approximation, the Riccati equation can be solved iteratively using backward procedure from  $\mathbf{\Gamma}(T)$  to  $\mathbf{\Gamma}(0)$ .

#### 4 NUMERICAL EXPERIMENT

The applicability of the proposed control theory is investigated on a numerical 3D finite element based distributed parameter model of the real BLDC motor, which parameters are presented in Tab. 1. The motor cross sectional diagram is presented in Fig. 1. It has 6 stator slots and 4 rotor poles [24].

The motor numerical model is simplified in a way that the eddy current problem and material magnetic nonlinearities are neglected. In addition, the motor is free of the load torque. However, for the sake of control law accuracy, the back electromotive force originated from rotating permanent magnets is taken into account. The back electromotive force EMF has sinusoidal course dependent on rotor pole position and its amplitude in steady state for the motor speed of 300 rad/s equals 289 mV. The mesh of the motor includes the rotor and the stator discretization, the thin coil system, the air gap and includes 16 290 nodes (Figure 2). The model has 32760 unknown variables, where 9720 are unknown  $A_r$ , 8640 are unknown  $A_\varphi$ , 14400 are unknown  $A_z$ . The BiCG accuracy is set to  $1,0 \cdot 10^{-3}$ .

In this numerical experiment, the outlined control technique is applied to the above-described motor model. The proposed linear quadratic current regulator aims at minimization of the current control error and energy delivered

Table 1. Motor parameters

Nominal voltage	36 V
Rated speed	4000 RPM
Rated torque	0,43 Nm
Rated power	180 W
Outer diameter of stator	27,5 mm
Inner diameter of stator	16,25 mm
Diameter of rotor	15 mm
Motor length	90 mm
Resistance/phase	1,0 $\Omega$
Self-inductance/phase	0,4 mH
Mutual inductance	0,14 mH
Rotor inertia	1,098e-6 kgm <sup>2</sup>
Damping	3e-5 Nms

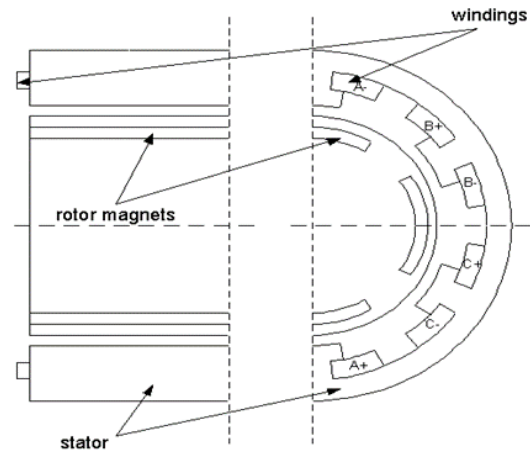


Fig. 1. Geometry of the BLDC motor

to the motor without loss of the motor dynamics. Desired minimization of the objective function (5) is achieved by selecting accurate  $\{\alpha, \beta\}$  configuration and time horizon  $T$  of the control. Choice of the time horizon is dependent on the electric circuit time constant. Hence, it is possible to obtain the control that the reference current may be reached faster than at time related to the electric time constant.

To obtain the control law, firstly the time dependent Riccati equation (10) is solved for  $\mathbf{\Gamma}$ . This is achieved by backward iterative procedure in  $T/\Delta t$  steps starting from  $\mathbf{\Gamma}(T) = \mathbf{G}$  to  $\mathbf{\Gamma}(0)$  of the discrete Riccati equation:

$$\mathbf{\Gamma}^{t-\Delta t} = \mathbf{\Gamma}^t - \Delta t \left[ -(\mathbf{L}^{-1}\mathbf{R})^T\mathbf{\Gamma}^t - \mathbf{\Gamma}^t\mathbf{L}^{-1}\mathbf{R} + \dots + \mathbf{\Gamma}^t\mathbf{L}^{-1}\beta^{-1}\mathbf{L}^{-1}\mathbf{\Gamma}^t + \alpha\hat{\mathbf{I}} \right] \quad (11)$$

The matrix  $\mathbf{\Gamma}$  has equal diagonal coefficients values and

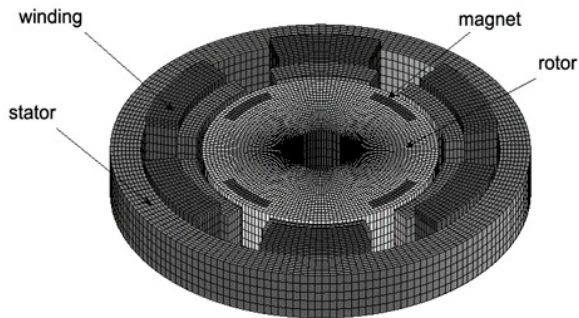


Fig. 2. Discrete model of the BLDC motor

equal non-diagonal coefficients values, which courses in time horizon T presents Fig. 3. These values are used to calculate control law at time t.

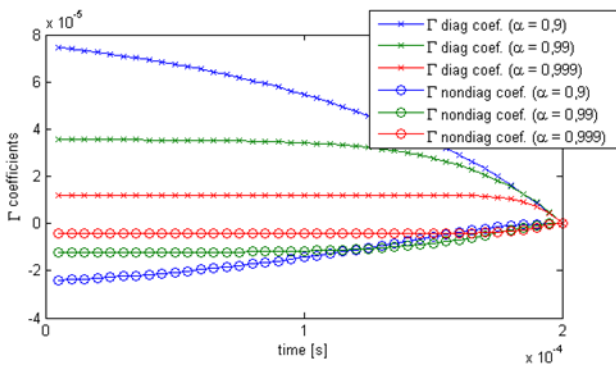


Fig. 3. Solution of the Riccati equation

The relevance of the optimal control approach is demonstrated by a computation assuming the reference current equals to 3,6A. For the purpose, three sets of  $\{\alpha, \beta\}$  weighting factors are investigated aiming to explain how the performance index configuration influences the control objectives. These sample sets are  $\{0,999; 0,001\}$ ,  $\{0,99; 0,01\}$  and  $\{0,9; 0,1\}$ , giving ratio  $\alpha/\beta$  equal 1000, 100 and 10, respectively. The control time is assumed  $T=0,2$  ms, as a half of the electric time constant (0,4 ms).

Figures 4.-9. present phase voltages and currents for the different factors  $\alpha$  and  $\beta$  defined in the objective function (5).

To demonstrate the influence of proposed control technique, i.e. control parameters on rotor speed and torque, figures 10.-11. present speed trajectory and torque trajectory (including cogging torque) for all configurations of weighting factors  $\alpha$  and  $\beta$ .

Numerical analysis reveals the impact of the  $\alpha/\beta$  relationship which has on control objective accomplishment:

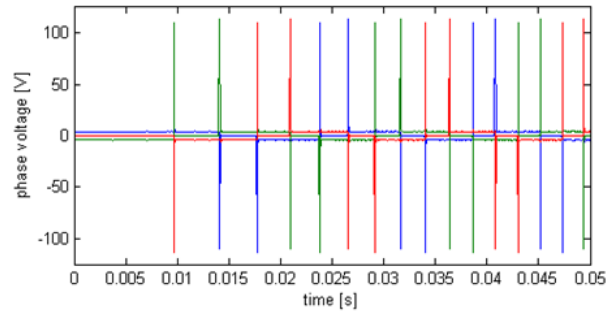


Fig. 4. Motor controls for  $\alpha=0,999 \beta=0,001$

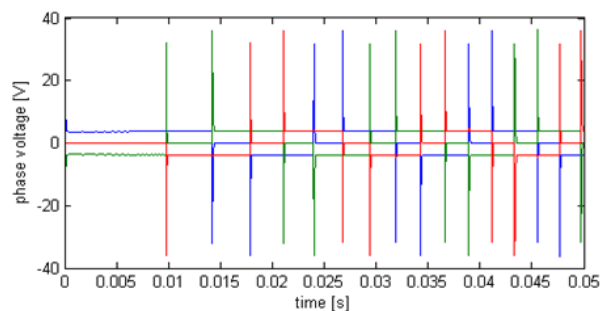


Fig. 5. Motor controls for  $\alpha=0,99 \beta=0,01$

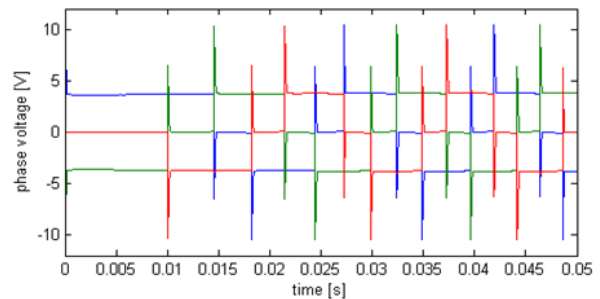


Fig. 6. Motor controls for  $\alpha=0,9 \beta=0,1$

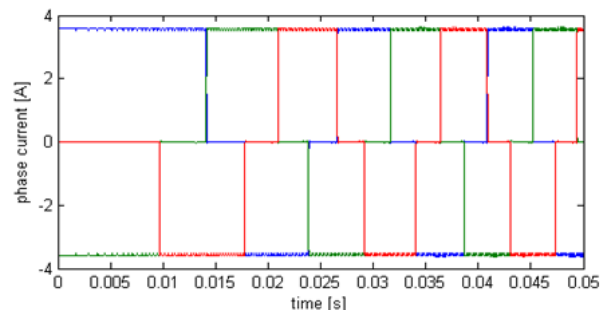


Fig. 7. Motor phase currents for  $\alpha=0,999 \beta=0,001$

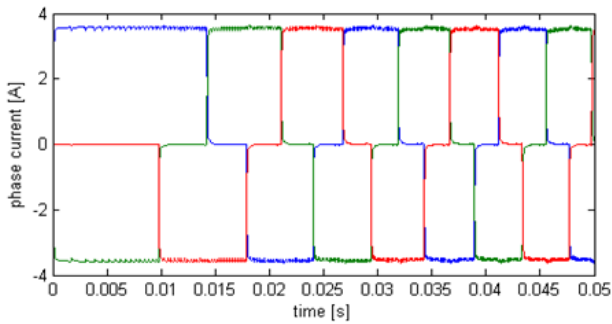


Fig. 8. Motor phase currents for  $\alpha=0,99 \beta=0,01$

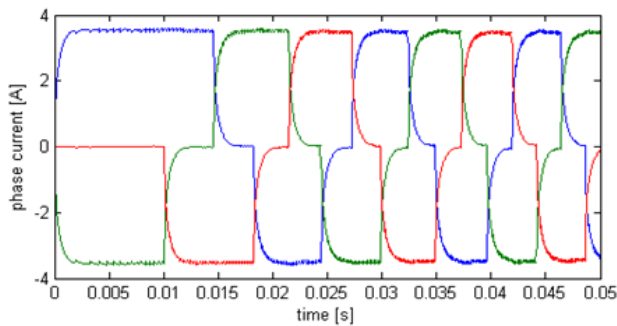


Fig. 9. Motor phase currents for  $\alpha=0,9 \beta=0,1$

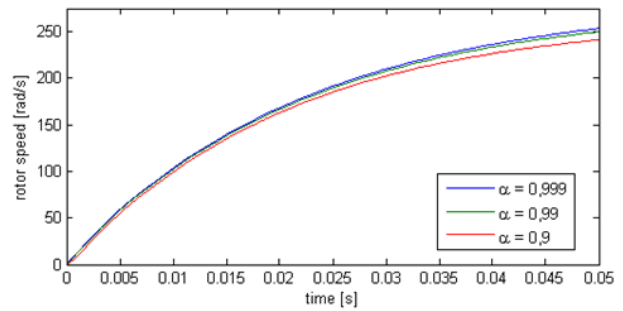


Fig. 10. Rotor speed

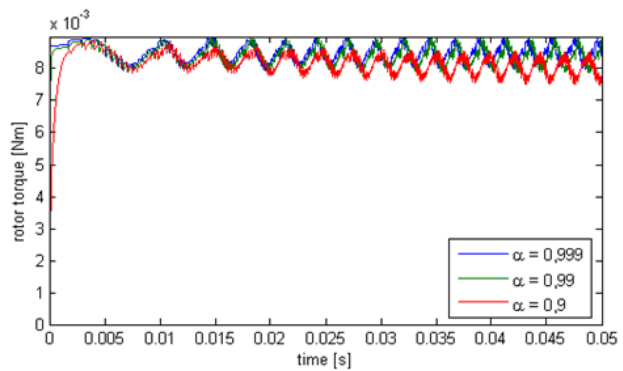


Fig. 11. Electromagnetic torque

the higher value of the relationship, the smaller the control error. The advantage of  $\alpha$  over  $\beta$  directs the performance index to control error minimization, since the weighting factor  $\alpha$  stands by  $\alpha \cdot \mathbf{E}^T \mathbf{E}$  quadratic form. On the other hand, with bigger  $\beta$  values, then the energy delivered to the system is reduced and voltage peaks at phase commutations are reasonably diminished, leading however to larger settling times and control steady state errors. Table 2. presents the quantitative comparison of system performance subject to different objectives configuration.

Table 2. Comparison of average control error, average value of phase voltage and current rising time

$\{\alpha, \beta\}$ configuration	Average control error by $I_{ref} = 3,6A$	Average value of phase voltage	Current rising time
{0,9;0,1}	0,391 A	3,91 V	0,29 ms
{0,99;0,01}	0,107 A	4,20 V	0,06 ms
{0,999;0,001}	0,034 A	4,31 V	0,03 ms

The average control error falls down with higher  $\alpha / \beta$  as expected, thanks to bigger amount of energy fed on windings - the average value of phase voltage rises. This, as well, has an impact on the current rising time which

drops down when the objective function is oriented on transient state shortening. The time horizon of 0,2 ms for the optimal control is met for  $\{0,999; 0,001\}$  and  $\{0,99; 0,01\}$ . When  $\alpha$  is only one order larger than  $\beta$ , the time constant is larger than assumed. In comparison to infinite time horizon optimal control, the finite horizon control gives the same or improved quality indicators in all considered cases.

Lastly, the influence of the reference current shape on motor dynamics is studied. Reference current of 3,6 A is compared to 1,6 A and 5,2 A. Table 3. presents the average control error and Tab. 4. outlines average values of the voltage fed on windings for different  $\{\alpha, \beta\}$  weighting configurations of the performance index.

Table 3. Comparison of average control error for different reference currents

$\{\alpha, \beta\}$ configuration	$I_{ref} = 1,6A$	$I_{ref} = 3,6A$	$I_{ref} = 5,2A$
{0,9;0,1}	0,095 A	0,391 A	0,774 A
{0,99;0,01}	0,035 A	0,107 A	0,208 A
{0,999;0,001}	0,011 A	0,034 A	0,059 A

The above results show that the smallest control error is achieved by small reference current and rises linearly with an increase of the set point. Again, the weighting

Table 4. Comparison of average value of voltage controls for different reference currents

$\{\alpha, \beta\}$ configuration	$I_{ref} = 1,6A$	$I_{ref} = 3,6A$	$I_{ref} = 5,2A$
{0,9;0,1}	1,70 V	3,91 V	5,74 V
{0,99;0,01}	1,78 V	4,20 V	6,31 V
{0,999;0,001}	1,82 V	4,31 V	6,49 V

configuration of {0,999;0,001} provides the best objective achievement in terms of control error reduction. The finite time optimal control system's stability for different reference current values is proofed.

## 5 CONCLUSION

The paper presents a finite time linear quadratic optimal controller, which minimizes the adopted objective function. Obtained optimal voltage control leads to the minimization of the current control error subject to electric energy delivery reduction. The researchers focused on finding the best weighting configuration of the performance index and examined the influence of the reference current shape on system dynamics and stability. Presented control applicability is confirmed on distributed parameter finite element model of the existing BLDC drive.

It might be concluded that the higher the value of the  $\alpha / \beta$  relationship, the better the system performs. The best configuration seems to be {0,99; 0,01} by which the control error was reasonably small and control time horizon constraint fulfilled. Additionally, the phase voltage peaks at commutation change equal 35V by 3,6A set point which is achievable with the market available power stages, contrary to 155 V peaks present by the configuration of {0,999; 0,001}. The finite time optimal current control proves, hereby, its applicability for the BLDC drives applications.

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