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Unbounded regulators with variable gains for a direct-drive robot manipulator

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Original scientific paper

This paper addresses the position-control problem with variable gains for robot manipulators. We present a new regulator based on a hyperbolic-sine structure with tuning rules for control gains. It is demonstrated that the equilibrium point of the closed-loop system is globally, asymptotically stable according to Lyapunov theory. By using a similar methodology, this concept can be extended to other unbounded controllers such as PD and PID. In order to show the usefulness of the proposed scheme and with the purpose of validating its asymptotical stability property, an experimental comparison involving constant gains controllers, for example: simple PD, PID and hyperbolic-tangent schemes vs variable-gains hyperbolic-sine and PD control schemes, was performed by using a three degree-of-freedom, direct-drive robot manipulator.

Key words: Regulator, Variable Gain, Robot manipulators, Control algorithms.

Neograničeni regulatori s promijenjivim pojačanjem za upravljanje robotskim manipulatorima s direktnim pogonom. Ovaj rad se bavi problemom kontrole pozicije s promjenjivim pojačanjem robotskog manipulatora. U radu je predstavljen novi regulator baziran na hiperbolično-sinusnoj stukturi s pravilima ugađanja upravljačkih pojačanja. Pokazano je da je točka ravnoteže sustava u zatvorenoj petlji globalno i asimptotski stabilna prema Lzapunovljevoj teoriji stabilnosti. Korištenjem slilčne metodologije, predstavljeni koncept se može primijeniti na ostale neograničene kontrolere, npr. PD i PID. Kako bi pokazali korisnost predložene sheme i s ciljem provjere asimptotske stabilnosti, provedena je eksperimentalna usporedba između kontolera s konstantnim pojačanjem (npr. jednostavni PD, PID i hiperbolični-tangencijalna shema) i hiperbolično-sinusnih i PD upravljačkih shema s promjenjivim pojačanjem korištenjem robotskog manipulatora s direktnim pogonom i tri stupnja slobode.

Ključne riječi: regulator, promjenjivo pojačanje, robotski manipulator, algoritam upravljanja

1 INTRODUCTION

In this paper, we focus on the position-control problem (also referred to as regulation), which is one of the most relevant issues in robotics. The main goal of position control in joint space is to move the manipulator's end-effector to a fixed, desired target, which is assumed to be constant regardless of its joint position.

This task was used in order to compare the performance obtained with unbounded PD and PID controllers, and a bounded controller (hyperbolic tangent) against the new regulator based on the hyperbolic sine structure (SINH) with variable gains, in this phase we are also using the proposed methodology on variable-gains to PD scheme. All controllers were implemented on a three-degree-of-freedom direct-drive, robot manipulator.

In the literature, most of the papers consider the proportional and derivative gains as constants. In 1953, the concept of *Variable Gain* [1] was first presented and since then, it has been used in many areas like control, automation, medical applications, filters, operational amplifiers, etc.

In robotics and control, many different contributions have been developed using this concept. For instance, in [2], time–varying gains for an adaptive algorithm were presented as well as variations of algorithms proposed in other papers in which time–varying setting gains were introduced. Years later, [3] presented a PD controller with proportional and derivative gains as nonlinear functions of the robot state, for force and contact control; in the same year, [4] analyzed the properties of different nonlinear PI controllers with variable gains; in this work different structures and methods of fuzzy control have been studied, while performing stability tests and determining the most suitable analytical approach. Also, in [5], a fuzzy control MIMO and a nonlinear control PI with variable gains, were compared. In 1998, [6] and [7] studied the controller structure of the Takagi–Sugeno (TS) type, using a new and more simple scheme of rules. In this work, all consequential rules use a common function and also, they are proportional to other conditions, which means that the number of required parameters is reduced for each rule and, by means of a proportionality rule, a TS fuzzy control approach and a nonlinear PI controls with variable gains, were considered.

In 2000, [8] proposed two variable-gain algorithms, using the least squares method in order to achieve high-speed tracking of parameters and a smooth performance in steady state. Later [9] presented the Mamdani fuzzy controllers with state feedback of variable gains, including a demonstration of local stability.

In [10], an universal integral controller with nonlinear gains for nonlinear systems, was proposed. In this work, a sliding-mode controller is designed with the purpose of increasing the performance of the transient response of a second-order system. Stability tests were performed and local, asymptotical stability in the sense of Lyapunov, was demonstrated. This work demonstrated that the nonlinearity of proportional and integral gains reduce the overshoot and enhances the transient response of the robot manipulator.

In [11] this work was extended to fuzzy controllers of the Mamdani type, by studying the two and three-dimensional analytic structure of these controllers and setting the required conditions to become, structurally, PID, PI and PD controllers with variable gains. In other words, this work was aimed at establishing the conditions that enable a fuzzy controller, whose input-output relationship can be explicit or implicitly expressed, to achieve the form of a PID, PI or PD controller with variable gains.

In [12] a PD controller with variable gains applied to a tracking task using Growing Multi–Expert Networks (GMN), was designed. This method improved the performance in size of the neural network.

In 2005 [13], another tracking-control structure for robot manipulators is presented. Such a control structure consists of a sliding mode PID control that considered the total or partial knowledge of all the dynamics of the robot manipulator. It also proved the global, asymptotical stability in the sense of Lyapunov, formulated in terms of a full quadratic form.

Two variable–gain controllers were presented in [14]. The first fuzzy controller is designed in order to cancel the effects of variable disturbance which is, in essence, the deviation of the actual system dynamics from the nominal plant as the system traverses a specific trajectory. In the second one, in a set of all linear subsystems, the disturbance was modeled for each subsystem, considered as a nominal plant, taking into account the effects of its neighbor subsystems in which a control action is computed to locally stabilize each nominal plant.

Another neural-networks control method, based on variable gains, was developed in [15]. The control gains were adapted for a teleoperated system for rehabilitation tasks, ensuring a stable, smooth motion of the slave robot while reducing the disruption caused by spasms of the patient. The gains varied depending on changes of the stiffness and inertia of the environment.

In [16], gains which vary according to a polynomial curve, which enabled a PI controller to remove the overshoot characteristic of this type of control, is designed with the purpose of controlling the speed of an induction motor.

In recent years, [17–19] presented a PD control with variable gains for tracking control of a robot manipulator. In this work, the tuning is a self–organizing, fuzzy-like function algorithm, which depends on the error position. Such a controller was compared with a classic PD controller. By means of a candidate Lyapunov function, the required conditions for stability in a region, were established. In [20], variable-gain integral controllers for a linear motion system was presented and demonstrated on a scanning-motion system.

In this paper, the most important motivation is to design a new control scheme for position problem of robot manipulators that achieves a good performance in terms of position error while simplifying the tuning of the gains, avoiding the saturation of the motors. The main difference with respect to other variable-gains schemes reported in the literature is that the tuning rules of variable gains are based on continuous functions of state variables, this is the proportional gain depends on position error and the derivative gain depends of joint velocity, only. The stability analysis is performed by means of Lyapunov theory and, in order to validate the performance of the proposed variable-gain schemes, experimental results of the controllers applied to a three degree–of–freedom direct drive robot manipulator, are presented.

The organization of this paper is as follow: Section 2 focuses on the description and presentation of useful properties of the robot dynamics of an n–degree–of–freedom, direct-drive robot manipulator; Section 3 presents the PD-type control with variable gains and the control-problem formulation. Section 4 presents a new controller based on the hyperbolic-sine structure with variable gains, the control-problem formulation and the stability analysis. In the Section 5 the robustness of the controller is shown by the position error bound. Section 6 describes the experimental platform and the new scheme of variable gains used for the proposed controllers. Finally, the experimental results are presented in Section 7.

2 ROBOT DYNAMICS

Consider the nonlinear dynamics of a n–degree–of–freedom rigid robot manipulator, and assume that all links are joined together by revolute joints. See [21–23].

$$M(\boldsymbol{q})\ddot{\boldsymbol{q}} + C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) + B\dot{\boldsymbol{q}} = \boldsymbol{\tau} \tag{1}$$

where $\boldsymbol{q}$, $\dot{\boldsymbol{q}}$ and $\ddot{\boldsymbol{q}}$ represent the $n \times 1$ vectors of joint displacements, joint velocities and joint accelerations, respectively; $\boldsymbol{\tau}$ is the $n\times 1$ vector of applied torques; the manipulator's inertia matrix is defined by $M(\boldsymbol{q})$, which is a $n \times n$ symmetric positive definite matrix; $C(\boldsymbol{q},\dot{\boldsymbol{q}})$ is the $n \times n$ matrix of centripetal and Coriolis torques; $\boldsymbol{g}(\boldsymbol{q})$ is the $n\times 1$ vector of gravitational torques obtained as the gradient of the robot potential energy $\mathcal{U}(\boldsymbol{q})$

$$\boldsymbol{g}(\boldsymbol{q}) = \frac{\partial \mathcal{U}(\boldsymbol{q})}{\partial \boldsymbol{q}} \tag{2}$$

and B is the $n \times n$ diagonal matrix of positive entries, denoting the viscous coefficients of each joint. Due to the complex, nonlinear nature of the friction phenomena, a model containing only viscous friction is considered as an acceptable simplification for many robotic applications.

The following are important properties of the dynamic model (1) [24,25],

Property 1 *The matrix $C(\boldsymbol{q},\dot{\boldsymbol{q}})$ and the time derivative of the inertia matrix $\dot{M}(\boldsymbol{q})$ satisfy the following,*

$$\dot{\boldsymbol{q}}^T \left[\frac{1}{2}\dot{M}(\boldsymbol{q}) - C(\boldsymbol{q},\dot{\boldsymbol{q}})\right]\dot{\boldsymbol{q}} = \boldsymbol{0} \quad \forall \boldsymbol{q},\dot{\boldsymbol{q}} \in \mathbb{R}^n$$

$$\dot{M}(\boldsymbol{q}) = C(\boldsymbol{q},\dot{\boldsymbol{q}}) + C(\boldsymbol{q},\dot{\boldsymbol{q}})^T.$$

See [26].

Property 2 *For robots having exclusively revolute joints, there exist a number $k_{c_1} > 0$ such that*

$$\|C(\boldsymbol{q},\boldsymbol{x})\boldsymbol{y}\| \leq k_{c_1}\|\boldsymbol{x}\|\|\boldsymbol{y}\|$$

for all $\boldsymbol{q},\boldsymbol{x},\boldsymbol{y} \in \mathbb{R}^n$

Property 3 *If positive constants k_{pu} and k_{pl} exist such that $k_{pu} \geq k_{pi}(\tilde{q}_i) \geq k_{pl} \forall \tilde{q}_i \in \mathbb{R}$, for $i = 1,\ldots,n$. These represent upper and lower bounds of certain continuously integrable functions as follows,*

$$\frac{1}{2}k_{pl}\|\tilde{\boldsymbol{q}}\|^2 \leq \int_0^{\tilde{\boldsymbol{q}}} \boldsymbol{\sigma}^T K_p(\boldsymbol{\sigma})d\boldsymbol{\sigma} \leq \frac{1}{2}k_{pu}\|\tilde{\boldsymbol{q}}\|^2. \tag{3}$$

See [27–29]

3 PD CONTROL WITH VARIABLE GAINS

Consider a PD-type control scheme with variable gains as presented in [29], which is a control law for motion control. This can be written as

$$\boldsymbol{\tau} = K_p(\tilde{\boldsymbol{q}})\tilde{\boldsymbol{q}} - K_v(\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) \tag{4}$$

where $\tilde{\boldsymbol{q}} = \boldsymbol{q}_d - \boldsymbol{q}$ denotes the $n \times 1$ vector of position error, $K_p(\tilde{\boldsymbol{q}})$ and $K_v(\dot{\boldsymbol{q}})$ are diagonal matrices of $n \times n$ order, whose entries of $K_p(\tilde{\boldsymbol{q}})$ are denoted by $k_{pi}(\tilde{q}_i)$ and the entries of $K_v(\dot{\boldsymbol{q}})$ denoted by $k_{vi}(\dot{q}_i)$, are nonlinear even positive functions.

The closed–loop system is obtained by substituting the control law (4) into the robot dynamics (1). This can be written as

$$\frac{d}{dt}\begin{bmatrix}\tilde{\boldsymbol{q}}\\ \dot{\boldsymbol{q}}\end{bmatrix} = \begin{bmatrix}-\dot{\boldsymbol{q}}\\ M(\boldsymbol{q})^{-1}[K_p(\tilde{\boldsymbol{q}})\tilde{\boldsymbol{q}} - K_v\dot{\boldsymbol{q}} - C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} - B\dot{\boldsymbol{q}}]\end{bmatrix} \tag{5}$$

which is an autonomous nonlinear differential equation whose origin of the state space is the unique equilibrium point.

3.1 Lyapunov function candidate

In order to study the stability of the equilibrium point obtained above, consider the following candidate Lyapunov function used in [29],

$$V(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}}) = \frac{1}{2}\dot{\boldsymbol{q}}^T M(\boldsymbol{q})\dot{\boldsymbol{q}} + \int_0^{\tilde{\boldsymbol{q}}} \boldsymbol{\sigma}^T K_p(\boldsymbol{\sigma})d\boldsymbol{\sigma} - \epsilon\tilde{\boldsymbol{q}}M(\boldsymbol{q})\dot{\boldsymbol{q}} \tag{6}$$

For the complete demonstration see [29].

4 SINH CONTROL WITH VARIABLE GAINS

In this section, the new SINH controller with nonlinear gains matrix as function of the robot configuration, is presented. The proposed control law is described by

$$\boldsymbol{\tau} = K_p(\tilde{\boldsymbol{q}})\begin{bmatrix}\sinh(\tilde{q}_1)\\ \sinh(\tilde{q}_2)\\ \vdots\\ \sinh(\tilde{q}_n)\end{bmatrix} - K_v(\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) \tag{7}$$

where $\tilde{\boldsymbol{q}} = \boldsymbol{q}_d - \boldsymbol{q}$ denotes the $n \times 1$ vector of position error, $K_p(\tilde{\boldsymbol{q}})$ and $K_v(\dot{\boldsymbol{q}})$ are diagonal matrices of $n \times n$ order, whose entries are denoted by $k_{pi}(\tilde{q}_i)$ and $k_{vi}(\dot{q}_i)$, respectively.

The closed–loop system is obtained by substituting the control law (7) into the robot dynamics (1). This can be written as,

$$\frac{d}{dt}\begin{bmatrix}\tilde{\boldsymbol{q}}\\ \dot{\boldsymbol{q}}\end{bmatrix} = \begin{bmatrix} -\dot{\boldsymbol{q}} \\ M(\boldsymbol{q})^{-1}[K_p(\tilde{\boldsymbol{q}})\begin{bmatrix}\sinh(\tilde{q}_1)\\ \sinh(\tilde{q}_2)\\ \vdots \\ \sinh(\tilde{q}_n)\end{bmatrix} - K_v(\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} - \\ -C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} - B\dot{\boldsymbol{q}}] \end{bmatrix} \tag{8}$$

which is an autonomous, nonlinear differential equation and the origin of the state space is the unique equilibrium point.

4.1 Lyapunov function candidate

In order to study the stability of the equilibrium point, consider the candidate Lyapunov function described by,

$$V(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}}) = \frac{1}{2}\dot{\boldsymbol{q}}^T M(\boldsymbol{q})\dot{\boldsymbol{q}} + \int_0^{\tilde{\boldsymbol{q}}} \begin{bmatrix}\sinh(\sigma_1)\\ \sinh(\sigma_2)\\ \vdots \\ \sinh(\sigma_n)\end{bmatrix}^T K_p(\boldsymbol{\sigma})d\boldsymbol{\sigma} - \\ - \epsilon\tilde{\boldsymbol{q}}^T M(\boldsymbol{q})\dot{\boldsymbol{q}} \tag{9}$$

where, for notation purposes, the integral representing

$$\int_0^{\tilde{\boldsymbol{q}}} \begin{bmatrix}\sinh(\sigma_1)\\ \sinh(\sigma_2)\\ \vdots \\ \sinh(\sigma_n)\end{bmatrix}^T K_p(\boldsymbol{\sigma})d\boldsymbol{\sigma} = \sum_{i=1}^{n}\int_0^{\tilde{q}_i} \sinh(\sigma_i)k_{pi}d\sigma_i,$$

$\epsilon > 0$ is defined by,

$$\epsilon = \epsilon(\|\tilde{\boldsymbol{q}}\|) = \frac{\epsilon_0}{1+\|\tilde{\boldsymbol{q}}\|} \tag{10}$$

where ϵ_0 is a positive constant. ϵ must be sufficiently small so as to satisfy,

$$\frac{k_{pl}\lambda_{min}\{M(\boldsymbol{q})\}}{\lambda^2_{Max}\{M(\boldsymbol{q})\}} > \epsilon^2 > 0 \tag{11}$$

k_{pl} and k_{pu} are chosen as design constants and, by using Property 3, sufficient conditions exist to make $V(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}})$ a positive, definite function.

Proof 1 *Consider*

$$\int_0^{\tilde{\boldsymbol{q}}} \begin{bmatrix}\sinh(\sigma_1)\\ \sinh(\sigma_2)\\ \vdots \\ \sinh(\sigma_n)\end{bmatrix}^T K_p(\boldsymbol{\sigma})d\boldsymbol{\sigma} \geq \frac{1}{2}k_{pl}\left\|\begin{bmatrix}\sinh(\tilde{q}_1)\\ \sinh(\tilde{q}_2)\\ \vdots \\ \sinh(\tilde{q}_n)\end{bmatrix}\right\|^2 \\ \geq \frac{1}{2}k_{pl}\|\tilde{\boldsymbol{q}}\|^2. \tag{12}$$

the candidate Lyapunov function (9) *can be bounded as follows,*

$$V(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}}) \geq \frac{1}{2}\lambda_{min}\{M(\boldsymbol{q})\}\|\dot{\boldsymbol{q}}\|^2 + \frac{1}{2}k_{pl}\|\tilde{\boldsymbol{q}}\|^2 - \\ - \epsilon\lambda_{Max}\{M(\boldsymbol{q})\}\|\tilde{\boldsymbol{q}}\|\|\dot{\boldsymbol{q}}\| \tag{13}$$

which is rewritten as,

$$V(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}}) \geq \\ \frac{1}{2}\begin{bmatrix}\|\tilde{\boldsymbol{q}}\|\\ \|\dot{\boldsymbol{q}}\|\end{bmatrix}^T \begin{bmatrix} k_{pl} & -\epsilon\lambda_{Max}\{M(\boldsymbol{q})\} \\ -\epsilon\lambda_{Max}\{M(\boldsymbol{q})\} & \lambda_{min}\{M(\boldsymbol{q})\}\end{bmatrix}\begin{bmatrix}\|\tilde{\boldsymbol{q}}\|\\ \|\dot{\boldsymbol{q}}\|\end{bmatrix} \tag{14}$$

Since ϵ is chosen in such a way that the condition (11) *is satisfied, it is shown that $V(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}})$ is a positive definite function.*

4.2 Time derivative of the candidate Lyapunov function

The time derivative of the candidate Lyapunov function can be written as,

$$\dot{V}(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}}) = \dot{\boldsymbol{q}}^T M(\boldsymbol{q})\ddot{\boldsymbol{q}} - \begin{bmatrix}\sinh(\tilde{q}_1)\\ \sinh(\tilde{q}_2)\\ \vdots \\ \sinh(\tilde{q}_n)\end{bmatrix}^T K_p(\tilde{\boldsymbol{q}})\dot{\boldsymbol{q}} + \\ + \frac{1}{2}\dot{\boldsymbol{q}}^T\dot{M}(\boldsymbol{q})\dot{\boldsymbol{q}} + \epsilon\dot{\boldsymbol{q}}^T M(\boldsymbol{q})\dot{\boldsymbol{q}} - \\ - \epsilon\tilde{\boldsymbol{q}}^T\dot{M}(\boldsymbol{q})\dot{\boldsymbol{q}} - \epsilon\tilde{\boldsymbol{q}}^T M(\boldsymbol{q})\ddot{\boldsymbol{q}} - \dot{\epsilon}\tilde{\boldsymbol{q}}^T M(\boldsymbol{q})\dot{\boldsymbol{q}} \tag{15}$$

where the Leibniz rule for the derivation of integrals has been applied. Previous expression, along the trajectories of the closed–loop equation (8), is expressed by

$$\dot{V}(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}}) = \dot{\boldsymbol{q}}^T K_p(\tilde{\boldsymbol{q}})\begin{bmatrix}\sinh(\tilde{q}_1)\\ \sinh(\tilde{q}_2)\\ \vdots \\ \sinh(\tilde{q}_n)\end{bmatrix} - \dot{\boldsymbol{q}}^T K_v(\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} - \\ - \dot{\boldsymbol{q}}^T B\dot{\boldsymbol{q}} + \frac{1}{2}\dot{\boldsymbol{q}}^T\dot{M}(\boldsymbol{q})\dot{\boldsymbol{q}} - \dot{\boldsymbol{q}}^T C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} - \\ - \begin{bmatrix}\sinh(\tilde{q}_1)\\ \sinh(\tilde{q}_2)\\ \vdots \\ \sinh(\tilde{q}_n)\end{bmatrix}^T K_p(\tilde{\boldsymbol{q}})\dot{\boldsymbol{q}} + \epsilon\dot{\boldsymbol{q}}^T M(\boldsymbol{q})\dot{\boldsymbol{q}} - \\ - \epsilon\tilde{\boldsymbol{q}}^T\dot{M}(\boldsymbol{q})\dot{\boldsymbol{q}} - \epsilon\tilde{\boldsymbol{q}}^T K_p(\tilde{\boldsymbol{q}})\begin{bmatrix}\sinh(\tilde{q}_1)\\ \sinh(\tilde{q}_2)\\ \vdots \\ \sinh(\tilde{q}_n)\end{bmatrix} + \\ + \epsilon\tilde{\boldsymbol{q}}^T K_v(\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \epsilon\tilde{\boldsymbol{q}}^T C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \epsilon\tilde{\boldsymbol{q}}^T B\dot{\boldsymbol{q}} - \\ - \dot{\epsilon}\tilde{\boldsymbol{q}}^T M(\boldsymbol{q})\dot{\boldsymbol{q}} \tag{16}$$

now, by using the skew–symmetry Property 1 and $\dot{M}(\boldsymbol{q}) = C(\boldsymbol{q},\dot{\boldsymbol{q}}) + C(\boldsymbol{q},\dot{\boldsymbol{q}})^T$, the time derivative of the candidate Lyapunov function can be written as,

$$\begin{aligned}\dot{V}(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}}) =& -\dot{\boldsymbol{q}}^T[K_v(\dot{\boldsymbol{q}}) + B - \epsilon M(\boldsymbol{q})]\dot{\boldsymbol{q}}- \\ & -\epsilon\tilde{\boldsymbol{q}}^T K_p(\tilde{\boldsymbol{q}})\begin{bmatrix}\sinh(\tilde{q}_1)\\ \sinh(\tilde{q}_2)\\ \vdots \\ \sinh(\tilde{q}_n)\end{bmatrix} + \epsilon\tilde{\boldsymbol{q}}^T[K_v(\dot{\boldsymbol{q}}) + B]\dot{\boldsymbol{q}}- \\ & -\epsilon\tilde{\boldsymbol{q}}^T C(\boldsymbol{q},\dot{\boldsymbol{q}})^T\dot{\boldsymbol{q}} - \dot{\epsilon}\tilde{\boldsymbol{q}}^T M(\boldsymbol{q})\dot{\boldsymbol{q}}.\end{aligned} \tag{17}$$

where $\dot{\epsilon} = \frac{\epsilon_0}{\|\tilde{\boldsymbol{q}}\|[1+\|\tilde{\boldsymbol{q}}\|]^2}\tilde{\boldsymbol{q}}^T\dot{\boldsymbol{q}}$; now, by taking the upper–bounds of the followings terms,

$$\begin{aligned}-\epsilon\tilde{\boldsymbol{q}}^T C(\boldsymbol{q},\dot{\boldsymbol{q}})^T\dot{\boldsymbol{q}} &= -\epsilon\dot{\boldsymbol{q}}^T C(\boldsymbol{q},\dot{\boldsymbol{q}})\tilde{\boldsymbol{q}} \\ &\le |-\epsilon\dot{\boldsymbol{q}}^T C(\boldsymbol{q},\dot{\boldsymbol{q}})\tilde{\boldsymbol{q}}| \\ &\le \epsilon\|\dot{\boldsymbol{q}}\|\|C(\boldsymbol{q},\dot{\boldsymbol{q}})\tilde{\boldsymbol{q}}\| \\ &\le \epsilon k_{c_1}\|\dot{\boldsymbol{q}}\|^2\|\tilde{\boldsymbol{q}}\| \\ &\le \epsilon_0 k_{c_1}\|\dot{\boldsymbol{q}}\|^2\end{aligned} \tag{18}$$

where, by using the Property 2 and the definition of ϵ in (10).

$$\begin{aligned}-\dot{\epsilon}\tilde{\boldsymbol{q}}^T M(\boldsymbol{q})\dot{\boldsymbol{q}} &= -\frac{\epsilon_0}{\|\tilde{\boldsymbol{q}}\|[1+\|\tilde{\boldsymbol{q}}\|]^2}\tilde{\boldsymbol{q}}^T\dot{\boldsymbol{q}}\tilde{\boldsymbol{q}}^T M(\boldsymbol{q})\dot{\boldsymbol{q}} \\ &\le \left|-\frac{\epsilon_0}{\|\tilde{\boldsymbol{q}}\|[1+\|\tilde{\boldsymbol{q}}\|]^2}\tilde{\boldsymbol{q}}^T\dot{\boldsymbol{q}}\tilde{\boldsymbol{q}}^T M(\boldsymbol{q})\dot{\boldsymbol{q}}\right| \\ &\le \frac{\epsilon_0}{\|\tilde{\boldsymbol{q}}\|[1+\|\tilde{\boldsymbol{q}}\|]^2}\|\tilde{\boldsymbol{q}}\|\|\dot{\boldsymbol{q}}\|\|\tilde{\boldsymbol{q}}\|\|M(\boldsymbol{q})\dot{\boldsymbol{q}}\| \\ &\le \frac{\epsilon_0}{1+\|\tilde{\boldsymbol{q}}\|}\|\dot{\boldsymbol{q}}\|^2\lambda_{Max}\{M(\boldsymbol{q})\} \\ &\le \epsilon_0\|\dot{\boldsymbol{q}}\|^2\lambda_{Max}\{M(\boldsymbol{q})\}.\end{aligned} \tag{19}$$

By bounding the time derivative (15), the previous expression is reduced to

$$\begin{aligned}\dot{V}(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}}) \le& -[\lambda_{min}\{K_v(\dot{\boldsymbol{q}})\} + \lambda_{min}\{B\}- \\ & -\epsilon_0 k_{c_1} - 2\epsilon_0\lambda_{Max}\{M(\boldsymbol{q})\}]\|\dot{\boldsymbol{q}}\|^2- \\ & -\frac{\epsilon_0}{1+\|\tilde{\boldsymbol{q}}\|}\lambda_{min}\{K_p(\tilde{\boldsymbol{q}})\}\|\tilde{\boldsymbol{q}}\|^2+ \\ & +\epsilon_0[\lambda_{Max}\{K_v(\dot{\boldsymbol{q}})\}+ \\ & +\lambda_{Max}\{B\}]\|\tilde{\boldsymbol{q}}\|\|\dot{\boldsymbol{q}}\|\end{aligned} \tag{20}$$

which in turn may be written as

$$\dot{V}(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}}) \le -\begin{bmatrix}\|\tilde{\boldsymbol{q}}\|\\ \|\dot{\boldsymbol{q}}\|\end{bmatrix}^T Q \begin{bmatrix}\|\tilde{\boldsymbol{q}}\|\\ \|\dot{\boldsymbol{q}}\|\end{bmatrix} \tag{21}$$

where the entries of the matrix Q are defined by

$$\begin{aligned}Q(1,1) =& \frac{\epsilon_0}{1+\|\tilde{\boldsymbol{q}}\|}\lambda_{min}\{K_p(\tilde{\boldsymbol{q}})\} \\ Q(1,2) =& -\frac{\epsilon_0}{2}\lambda_{Max}\{K_v(\dot{\boldsymbol{q}})\} + \lambda_{Max}\{B\}] \\ Q(2,1) =& Q(1,2) \\ Q(2,2) =& \lambda_{min}\{K_v(\dot{\boldsymbol{q}})\} + \lambda_{min}\{B\} - \epsilon_0 k_{c_1}- \\ & -2\epsilon_0\lambda_{Max}\{M(\boldsymbol{q})\}\end{aligned} \tag{22}$$

the required conditions on ϵ_0 for $\dot{V}(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}})$ to be definite negative are given by,

$$\frac{4\lambda_{min}\{K_p(\tilde{\boldsymbol{q}})\}(\lambda_{min}\{K_v(\dot{\boldsymbol{q}})\} + \lambda_{min}\{B\})}{D_1 + D_2} > \epsilon_0 \ge \epsilon \tag{23}$$

where,

$$D_1 = [1+\|\tilde{\boldsymbol{q}}\|][\lambda_{Max}\{K_v(\dot{\boldsymbol{q}})\} + \lambda_{Max}\{B\}]^2 \tag{24}$$

$$D_2 = 4\lambda_{min}\{K_p(\tilde{\boldsymbol{q}})\}(k_{c_1} + 2\lambda_{Max}\{M(\boldsymbol{q})\}) \tag{25}$$

The knowledge of the actual numerical value of ϵ in the proposed controller is not required; it is just needed for stability purposes. In this way, we choose ϵ so as to satisfy simultaneously (11) and (23). Thus, by invoking Lyapunov's direct method (see [30, 31]), we conclude that the origin of the state space is a global, asymptotically stable equilibrium point of the closed–loop system (8).

5 POSITION ERROR BOUND

For more knowledge about the position error, consider the condition on ϵ defined by (23) then we have $\dot{V}(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}}) \le 0$ and in another hand the Proof 1 demonstrate that $V(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}}) > 0$, therefore the Lyapunov function candidate is decreasing function for $t \ge 0$ and it also has:

$$V(\tilde{\boldsymbol{q}}(0),\dot{\boldsymbol{q}}(0)) \ge V(\tilde{\boldsymbol{q}}(t),\dot{\boldsymbol{q}}(t)) > 0; \qquad \forall t \ge 0, \tag{26}$$

so,

$$\begin{aligned}V(\tilde{\boldsymbol{q}}(0),\dot{\boldsymbol{q}}(0)) =& \frac{1}{2}\dot{\boldsymbol{q}}(0)^T M(\boldsymbol{q}(0))\dot{\boldsymbol{q}}(0)+ \\ & +\int_0^{\tilde{\boldsymbol{q}}}\boldsymbol{\sigma}(0)^T K_p(\boldsymbol{\sigma}(0))d\boldsymbol{\sigma}- \\ & -\epsilon\tilde{\boldsymbol{q}}(0)^T M(\boldsymbol{q}(0))\dot{\boldsymbol{q}}(0)\end{aligned} \tag{27}$$

then taking the upper bounds evaluated in the initial condition, the previous expression may be written as,

$$\begin{aligned}V(\tilde{\boldsymbol{q}}(0),\dot{\boldsymbol{q}}(0)) \le& \frac{1}{2}\lambda_{Max}\{M(\boldsymbol{q}(0))\}\|\dot{\boldsymbol{q}}(0)\|^2+ \\ & +\frac{1}{2}k_{pu}\|\tilde{\boldsymbol{q}}(0)\|^2- \\ & -\epsilon_0\|\dot{\boldsymbol{q}}(0)\|^2\lambda_{Max}\{M(\boldsymbol{q}(0))\}\end{aligned} \tag{28}$$

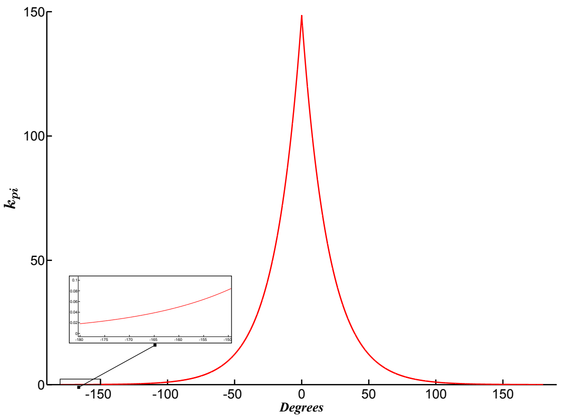

Fig. 1. i-th element of matrix K_p

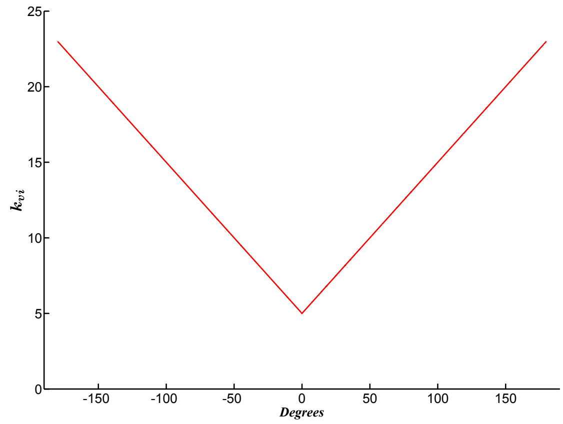

Fig. 2. i-th element of matrix K_v

replacing (13), (28) in (26) and after some algebra, the norm of position error can be expressed as,

$$\|\tilde{\boldsymbol{q}}(t)\| \leq \sqrt{\frac{\frac{1}{2}k_{pu}\|\tilde{\boldsymbol{q}}(0)\|^2}{\frac{1}{2}k_{pl}}} \tag{29}$$

It is clear that the position error is bounded and independent of the model uncertainties, so we can conclude the robustness of the controllers against to this type of uncertainty.

6 DESIGN OF VARIABLE-GAINS MATRIX

For the stability of the closed–loop systems described in eqs. (5) and (8), the proportional $K_p(\tilde{\boldsymbol{q}})$ and derivative K_v $n \times n$ matrices need to be diagonal; the entries of $K_p(\tilde{\boldsymbol{q}})$ and K_v are, by design, nonlinear, positive even functions.

6.1 Proportional matrix $K_p(\tilde{\boldsymbol{q}})$

In this case we propose, as a tuning rule for the entries of the proportional matrix, continuous functions inversely proportional to the absolute value of the position error. In this way, saturation of the maximum torque of the robot's actuators is avoided by choosing a small constant for large position errors, and a higher constant for small positioning errors.

The following function is proposed, for each entry of the matrix,

$$k_{pi}(\tilde{q}_i) = e^{-a_i|\tilde{q}_i|+b_i}; \tag{30}$$

where a_i and b_i are positive constants. Fig. 1 presents a graph of (30).

The value of b_i can be used as upper bound, chosen as follows,

$$b_i = \log(k_{pu}) \tag{31}$$

where k_{pu} is the maximum desired value of $k_{pi}(\tilde{q}_i)$.

The value of a_i is chosen as a function of the desired position and maximum torque of servoactuators.

$$a_i = \frac{b_i - \log\left(\frac{\tau_i^{max}}{|f(q_{di})|}\right)}{|f(q_{di})|} \tag{32}$$

where $f(q_{di})$ is the value of the desired position evaluated in the controller's function while τ_i^{max} is the maximum torque. In order to avoid saturation of the motors, they operate in the linear range, for example, at a value of 80% of the maximum torque.

6.2 Derivative matrix $K_v(\dot{\boldsymbol{q}})$

The entries for the derivative matrix were chosen as continuous functions, proportional to the absolute value of the joint velocities plus an offset.

The following functions

$$k_{vi}(\dot{q}_i) = c_i|\dot{q}_i| + d_i, \tag{33}$$

represent our approach for the entries of each element of the derivative matrix. Fig. 2 presents the graph of (33).

The values of constants $c_1 = 0.1$, $c_2 = 0.1$ and $c_3 = 0.01$ correspond to the base, shoulder and elbow, respectively. The offset values are $d_1 = 15$, $d_2 = 60$ and $d_3 = 1.25$ for the base, shoulder and elbow respectively.

7 EXPERIMENTAL RESULTS

The experimental results were obtained by using a three-degree-of-freedom, direct-drive, antrophomorfic

Fig. 3. Experimental robot manipulator "Rotradi"

robot manipulator named "Rotradi", which is shown in Fig. 3. This device consists of three 6061 aluminum links, actuated with three brushless, direct-drive servoactuators, DM–1050A, DM–1150A and DM–1015B models from Parker Compumotor, for the base, the shoulder and elbow joints, respectively [32]. The robot was designed and built at the Robotics Laboratory of Benemérita Universidad Autónoma de Puebla (BUAP). The servoactuators are operated in torque mode, which means that the motor acts as a torque source where an analogue voltage is provided as a reference for the torque signal. The servoactuators features are shown in Table 1. The robot system has a device designed for reading the encoders and generate reference voltages, which is a motion–control board of Precision MicroDynamics Inc. The system runs in real time in a PC host computer Pentium–1 at 166MHz with a 2.5 ms sample rate and the control programs are written in C code.

Table 1. Robot arm servo actuators

Joint	Model	Max. Torque	Resolution
Base	DM–1050	50 [Nm]	1,024,000 [cpr]
Shoulder	DM–1150A	150 [Nm]	1,024,000 [cpr]
Elbow	DM–1015B	15 [Nm]	1,024,000 [cpr]

For the PD controller, the initial value of the proportional gains are $k_{p1}(\tilde{q}_1(t_0)) = 25.33$, $k_{p2}(\tilde{q}_2(t_0)) = 151.92$ and $k_{p3}(\tilde{q}_3(t_0)) = 15.20$; the final values are $k_{p1}(\tilde{q}_1(t_f)) = 197.6$, $k_{p2}(\tilde{q}_2(t_f)) = 592.8$ and $k_{p3}(\tilde{q}_3(t_f)) = 198.1$. For the derivative gains, the maximum values are $k_{v1} = 24$, $k_{v2} = 70.8$ and $k_{v3} = 3.9$.

The position errors for the PD controller with variable gains are shown in the Fig. 4 and, for the SINH controller with variable gains, the errors are shown in Fig. 8. The performance of both controllers is similar and validate the functionality of the proposed variable-gains scheme.

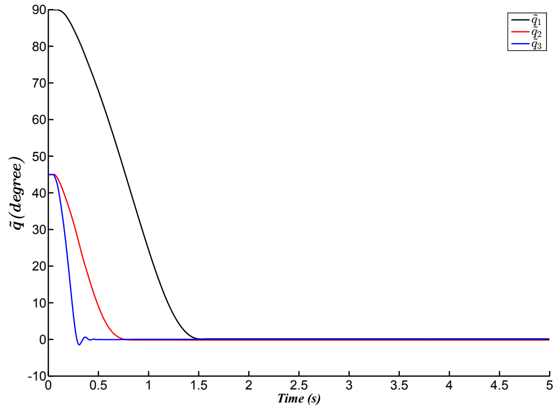

Fig. 4. Position errors for PD controller with variable gains

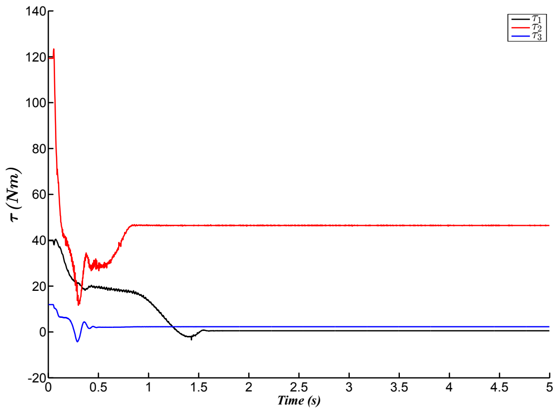

Fig. 5. Applied torque for PD controller with variable gains

Figures 5 and 9 show the applied torque, which do not exceed the maximum torque of the servoactuators.

Finally, the change of the proportional gains are shown in Fig. 6 and Fig. 10, for the PD and SINH controllers, respectively. In the case of the derivative gains, the change of the gains are shown in Fig. 7 for the PD controller, and in Fig. 11 for the SINH controller.

For the SINH controller, the initial value of the proportional gains are $k_{p1}(\tilde{q}_1(t_0)) = 17.37$, $k_{p2}(\tilde{q}_2(t_0)) = 137.18$ and $k_{p3}(\tilde{q}_3(t_0)) = 13.73$. The final values are $k_{p1}(\tilde{q}_1(t_f)) = 196.2$, $k_{p2}(\tilde{q}_2(t_f)) = 593.7$ and $k_{p3}(\tilde{q}_3(t_f)) = 198.3$. For the derivative gains, the maximum values are $k_{v1} = 23.5$, $k_{v2} = 70.6$ and $k_{v3} = 3.9$.

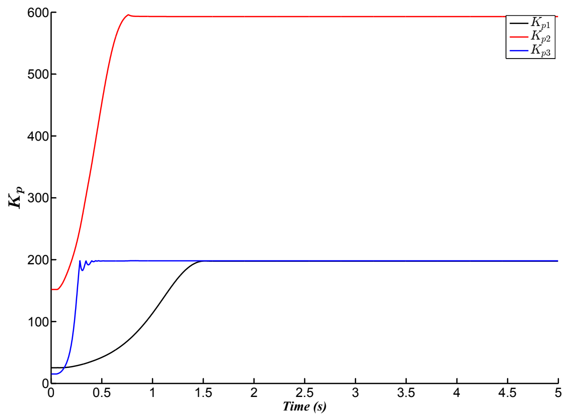

Fig. 6. Proportional gains for PD controller

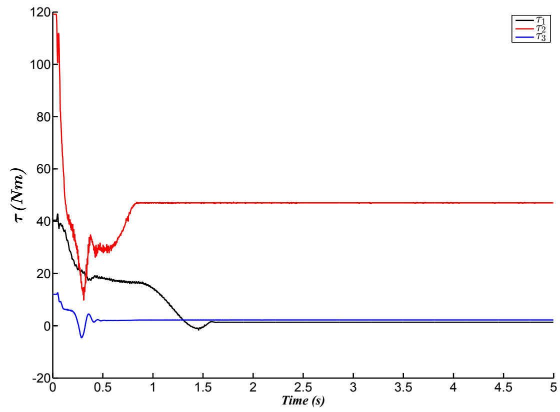

Fig. 9. Applied torque for SINH controller with variable gains

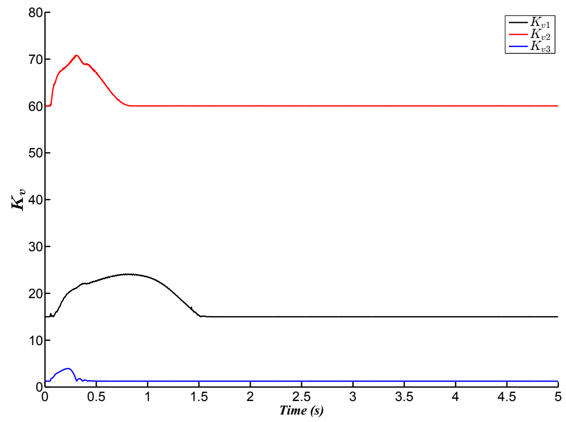

Fig. 7. Derivative gains for PD controller

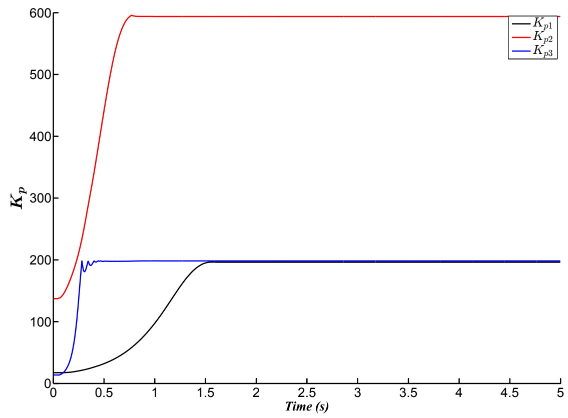

Fig. 10. Proportional gains for SINH controller

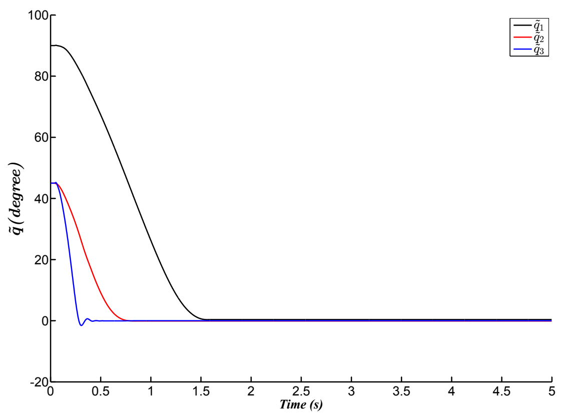

Fig. 8. Position errors for SINH controller with variable gains

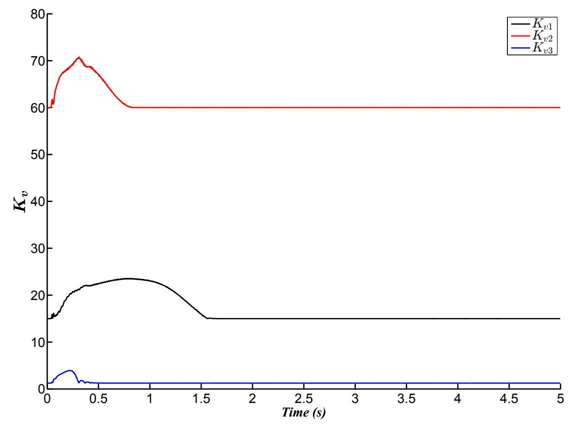

Fig. 11. Derivative gains for SINH controller

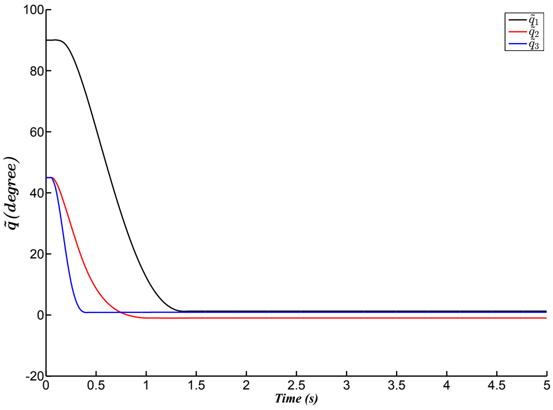

Fig. 12. Position errors for a manually-tuned PD controller

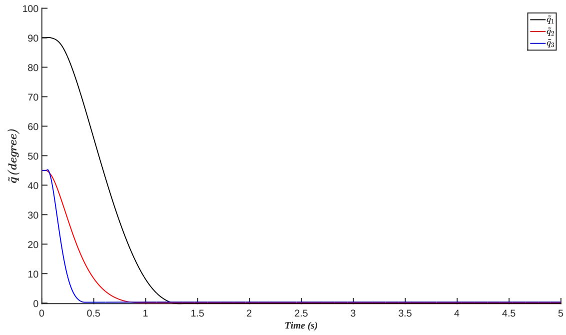

Fig. 13. Position errors for a manually-tuned PID controller

The PID control with constant gains is an unbounded scheme given by:

$$\boldsymbol{\tau} = K_p\tilde{\boldsymbol{q}} + K_i\int_0^t \tilde{q}(\sigma)d\sigma - K_v\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) \quad (34)$$

while that the hyperbolic tangent control (Tanh) with constant gains is a bounded scheme with the following form:

$$\boldsymbol{\tau} = K_p\tanh(\tilde{\boldsymbol{q}}) - K_v\tanh(\dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) \quad (35)$$

where K_p, K_i, K_v are 3×3 proportional, integral and derivative constant gains, respectively.

The L^2-norm is a method for measuring the performance of controllers. Manually tuning methods for the constant gains were made for PD, PID and hyperbolic tangent (Tanh) control schemes; for comparison reasons among these gains, their profiles are presented in Fig. 12, 13 and 14 respectively.

In Fig. 15, the L^2-norm for the transient state is shown, where PD, PID and Tanh are a manually-tuned controllers for reference, PDv and SINHv is a PD an SINH controllers with variable gains, respectively.

In transient state, the controllers' performance is similar and competitive because it was scheduled to smoothly arrive at the desired point by setting the settling speed, except Tanh controller, that was the slowest shown here.

With the L^2-norm for the steady-state, shown in Fig. 16, we can show the performance of the controllers with variable gains, where the errors of the PD controller with variable gains is 2.1%, 16.3% and 11.6% of the errors for the manually-tuned PD controller in the elbow, shoulder and base, respectively and competing with PID and Tanh controllers, which have proven a good performance in a steady-state. In the case of the SINH controller with

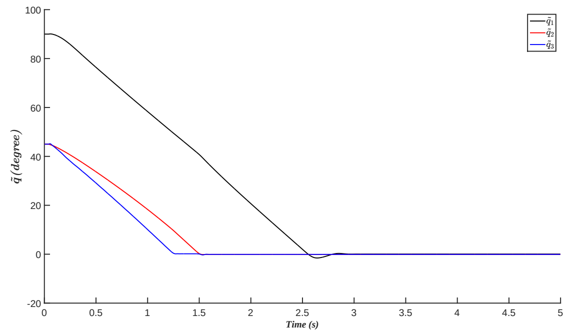

Fig. 14. Position errors for a manually-tuned Tanh controller

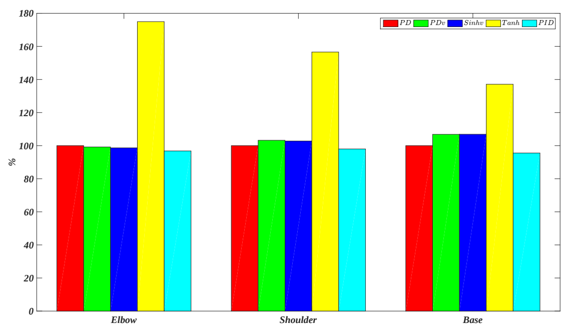

Fig. 15. L^2-norm during the transient state.

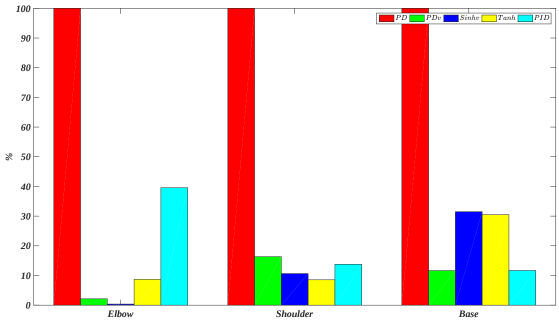

Fig. 16. L^2−norm in steady state.

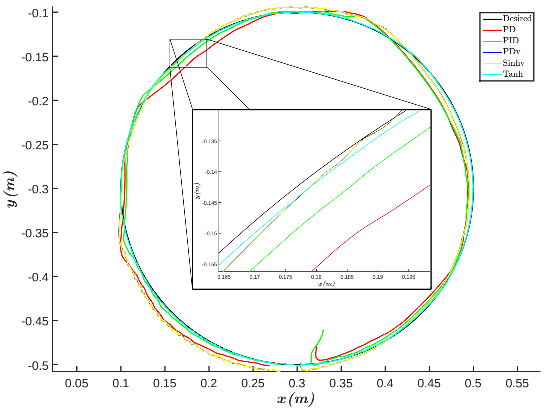

Fig. 17. Planned trajectory for PD controller

variable gains, the errors were 0.3%, 10.6% and 31.5% of the error for the manually-tuned PD controller in the elbow, shoulder and base, respectively.

The presented control schemes were designed for regulation, i.e., position control considering a constant reference q_d. Greater demands are placed on point-to-point control, which is why a planned trajectory consisting of a circle with a radius of 20 cm, with center in $(0.3, -0.3)$ m, that is to be completed in 20 seconds, was implemented and is presented in Fig. 17, where PDv is a PD controller with variable gains, SINH is our proposed controller with variable gains, while PD, PID and Tanh are manually-tuned controllers. It is easy to observe that the performance of the manually-tuned PD controllers is very poor, unlike the controllers with variable gains (SINH and PD), which have a better performance.

8 CONCLUSION

In this paper a new control scheme based on hyperbolic sine (SINH) with variable gains, has been presented, where the equilibrium point of the closed-loop equation is asymptotically stable according to Lyapunov theory. The tuning procedure of the variable gains is based on continuous functions in order to define the profile of these gains; this method can be extended to other schemes; this is the case of PD controllers whose performance can be improved, as compared to the same scheme with constant gains.

The usefulness of the proposed SINH scheme was validated experimentally by using a direct-drive robot manipulator involving constant gains controllers. It was shown that the position error is bounded and it does not depend on the robot manipulator dynamics evidencing robustness to parametric uncertainty.

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