# Asymptotic Analysis of Backlog Estimates for Dynamic Frame Aloha

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***Abstract*—In this paper a new analysis is presented that allows to investigate the asymptotic behavior of some backlog estimation procedures for Dynamic Frame Aloha (DFA) in Radio Frequency Identification (RFID) environment. Although efficiency $e^{-1}$ can theoretically be reached, none of the solution proposed in the literature has been shown to reach such value. Here we analyze first the Schoute's backlog estimate, which is very attractive for its simplicity, and formally show that its asymptotic efficiency is $0.311$ for any finite initial frame length. Since the analysis shows how the Schoute's estimate impairment can be avoided, we further propose the Asymptotic Efficient Estimate (AE$^2$), an improvement of the Schoute's one, that exploits the Frame Restart property of the standard and that is proved to asymptotically reach efficiency $e^{-1}$.**

***Index Terms*—RFID, DFA, DFSA, EPCglobal, Frame Aloha, Frame Restart, Tag Identification, Tag Estimate, Collision Resolution.**

## I. Introduction

Dynamic Frame Aloha (DFA) is a multiple access protocol proposed in the field of satellite communications by Schoute [1] in 1983. This protocol has been rediscovered about a decade ago for Radio Frequency Identification (RFID), an automatic identification system in which a reader interrogates a set of tags in order to identify each of them [2]. Upon being interrogated, concurrent tags responses may collide, and a collision resolution protocol is needed to arbitrate collisions. To this purpose DFA and its modified versions have become very popular, as demonstrated by the large body of literature on the topic, being also adopted in reference standards for UHF RFID systems [3].

In brief, DFA operates as follows: An initial number $N$ of users, also called *tags*, reply to a reader interrogation on a slotted time axis where slots are grouped into frames; a tag is allowed to transmit only one packet per frame in a randomly chosen slot. In the first frame all tags transmit, but only a part of them avoid collisions with other transmissions and get through. The remaining number of tags $n$, often referred to as the *backlog*, re-transmit in the following frames until all of them succeed. Outcomes of slots, *i.e.*, successfully used, not used, or collided, are continuously observed to derive an estimate of the backlog, $\hat{n}$, which is used to set the length $r$ of the next frame till all tags have been identified. The problem arises to get at each frame a suitable estimate $\hat{n}$, and to determine the most favorable frame length $r$. The RFID standard also introduces an additional capability, called *Frame Restart* (FR), that allows to restart a new frame at any slot even though the present frame is not finished.

Several works that appeared in the literature [4]–[11], some of them briefly discussed in the next section, have investigated backlog estimates $\hat{n}$ and optimal frame setting, including the method proposed by Schoute in his original paper, which is very attractive because of its simplicity and performance. The aim is at maximizing efficiency, defined as $\eta(N) = N/L(N)$, being $L(N)$ the average number of slots needed to identify all $N$ tags. In [12] we have proved that, when $N$ is known, the optimal frame setting is $r = n$, and the asymptotic efficiency, as $N \to \infty$, is $e^{-1}$, while in [13] we have shown that the FR procedure does not improve the DFA asymptotic efficiency. In practice $N$ is often unknown, and it turns out that the efficiency suffers from the mismatch between $N$ and the initial frame length $r_0$, especially when $N$ is unbounded, as it can be in future applications. Unfortunately, none of the cited works addresses high values of $N$.

Therefore, in recent years we set out to investigate the DFA limiting behavior of $\eta(N)$ as $N \to \infty$ under different estimates. In [14], we have analyzed many proposals both for DFA and DFA-FR assuming an initial population size which is Poisson distributed, as it happens in multiple access systems, or when a large population is subdivided in small groups. In [15] we have shown that, using DFA, a mechanism exists that provides efficiency $0.469$, very close to the best ever attained, $0.487$, reached with a sophisticated algorithm [16], [17] of the Tree Protocols family. In [18] we have analyzed the asymptotic efficiency of the first and simpler of the Schoute's method, and we have proved that its asymptotic efficiency is $0.311$, quite below the theoretical value $e^{-1}$, when the initial frame length is any finite value. In [13] we have also proposed a simple and efficient estimate, AE$^2$, when using the Frame Restart property.

In this paper we generalize the approach already exposed in [18] and, after deriving the asymptotic efficiency of the Schoute's estimate, equal to $0.311$, we use it to prove that estimate proposed in [13] is asymptotically efficient, namely is able to reach $e^{-1}$.

The paper is organized as follows. In Section II we show some preliminary results and review some of the literature. In Section III we produce the asymptotic analysis of the Schoute's protocol. Then, leveraging this analysis, in Section IV we introduce AE$^2$, an Asymptotically Efficient Estimate, and subsequently prove its asymptotic efficiency. Section V presents our conclusions.

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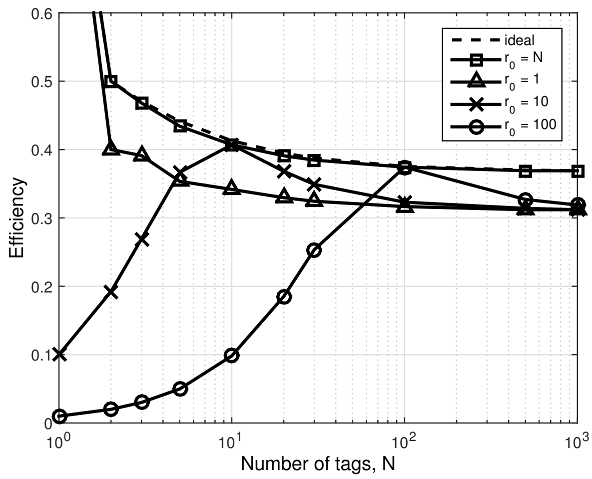


Figure 1. Efficiency versus the initial number of tags $N$ of Schoute's DFA mechanism for different values of the initial frame length $r_0$.

## II. Background and Previous Work

Since the average number of successful transmissions in a frame is maximized when $n = r$, Schoute's proposal is based on the idea that the mechanism should maintain the traffic, *i.e.*, the average number of transmissions per slot, equal to one. Therefore, it assumes that the number of tags transmitting in a slot can be approximated by a Poisson variate of average 1. Hence, the average number of terminals in a collided slot is

$$H = (1 - e^{-1})/(1 - 2e^{-1}) \approx 2.39,$$

and the estimate is $\hat{n} = \text{round}(Hc)$, where $c$ is the number of collided slot in the frame and $\text{round}(x)$ is the closest integer to $x$.

In Fig. 1 the efficiency for different values of $N$ with initial frame length $r_0 = N$, $r_0 = 1$, $r_0 = 10$ and $r_0 = 100$ is shown. Values up to $N = 30$ have been evaluated using the formula in [1], whereas values for $N = 500$ and $N = 1000$ have been obtained by simulating the algorithm. To allow comparisons we have also reported the performance with a perfect estimate $\hat{n} = n$ for each frame (dashed line), that represents a benchmark for all estimation mechanisms. We have also reported the case where only the estimate of the first frame is perfect, *i.e.*, when the first frame length is set to $N$. The comparison of the two latter cases suggests that the Schoute's mechanism is able to track the backlog exactly as in the perfect estimate case, asymptotically approaching the best possible efficiency $e^{-1}$; this result is, in fact, analytically proved in the next section. In all the other cases, the Schoute's estimate suffers from the mismatch between $N$ and the initial frame length $r_0$, and the efficiency degrades monotonically when $N$ increases beyond $r_0$, indicating the existence of a possible asymptote well below $e^{-1}$.

Among the first proposal in RFID, Vogt [4] introduces two estimation algorithms. One is the lower bound estimation $\hat{n} = 2c$, the other is the Minimum Distance Vector (MDV) based on Chebyshev's inequality theory. The proposed schemes are devised for a limited set of frame lengths and population size.

In [5], the authors consider two estimates, the collision ratio $c/r$ and $\hat{n} = 2.39c$, the same as Schoute's except for dropping the closest integer operator. Results on optimal frame length and collision probabilities are re-derived. The average identification periods of both proposals up to 900 tags appear practically equal.

In [7], the value of $n$ that maximizes the a posteriori probability $\Pr(n|s, c, e)$, having observed $s$ successes, $c$ collisions and $e$ empty slots, is assumed as estimate. In practice, this proposal uses a Maximum Likelihood method since no a priori distribution of $N$ is given. $\Pr(n|s, c, e)$ is obtained assuming independence among slots' outcomes. This estimate is shown to provide better performance than the previous ones, yielding an efficiency $\eta = 0.357$ for $N = 250$, which drops to $\eta = 0.277$ for $N = 50$.

A different class of estimates is given by the Bayesian estimate in [6], that evaluates the a posteriori probability distribution of the original population size $N$, conditioned to all the past observations, starting from the a priori distribution of the number of transmitting tags. Other proposals can be found in [8]–[10].

It appears that the proposed estimates can be roughly grouped into two categories, namely those derived from Schoute's and the ones that uses sophisticated estimation techniques such as Bayes or Maximum Likelihood. Because of the complexity of the latter, estimates of the first family seems more adequate when a large $N$ is considered, which is the reason why we started our asymptotic analysis from Schoute's method, as shown next.

## III. Asymptotic Analysis of Schoute's Estimate

The protocol analysis is subdivided into steps. In the remainder of the paper lowercase letters represent random variables, whereas calligraphic and upper cases represent averages.

**Step 1**. Here we derive recursive formulas for the backlog. We initially assume that the $i-$th frame size $r_i$, and the backlog $n_i$, are so large that the number of transmissions in a slot can be approximated by a Poisson variate with average $n_i/r_i$. This allows to evaluate the probability of an empty, successful, and collided slot respectively as

$$p_e = e^{-n_i/r_i}; \quad p_s = \frac{n_i}{r_i} e^{-n_i/r_i}; \quad p_c = 1 - p_e - p_s.$$

We note that relations above also hold when starting with small $r_0$, because in this case, being $N - i$ always very large, every slot is collided with probability one. In Appendix A we show that, in the conditions assumed, the ratio $k_i = n_i/r_i$ can be safely replaced by the ratio of the respective averages $\mathcal{K}_i = \mathcal{N}_i/\mathcal{R}_i$, which is the traffic per slot. With this substitution the probabilities above are denoted by $\mathcal{P}_e, \mathcal{P}_s, \mathcal{P}_c$. This means that the average number of collisions and the average backlog size can be expressed as

$$\mathcal{C}_i = \mathcal{R}_i \mathcal{P}_c, \qquad \mathcal{N}_{i+1} = \mathcal{N}_i(1 - \mathcal{P}_s). \tag{1}$$

The frame length evolves with law $r_{i+1} = \text{round}(Hc_i)$, so that

$$\mathcal{R}_{i+1} = \mathsf{E}\left\{\text{round}(Hc_i)\right\}, \tag{2}$$

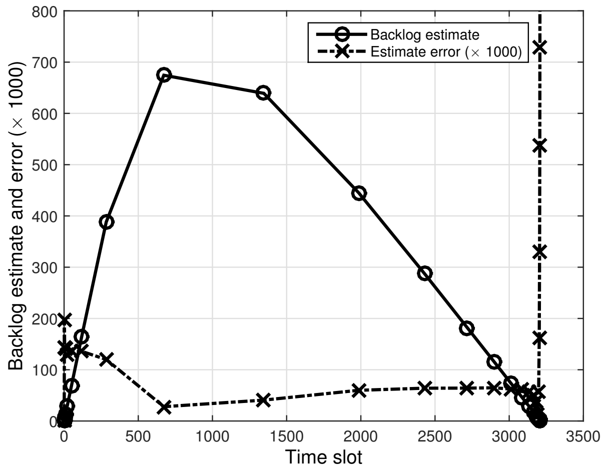


Figure 2. Average Schoute's backlog estimate $\hat{\mathcal{N}}_i$ at the end of the frames versus time slot ($N = 1000$, $r_0 = 1$). The dash-dotted line represents the relative error $\times 10^3$ with respect to the actual values $N_i$.

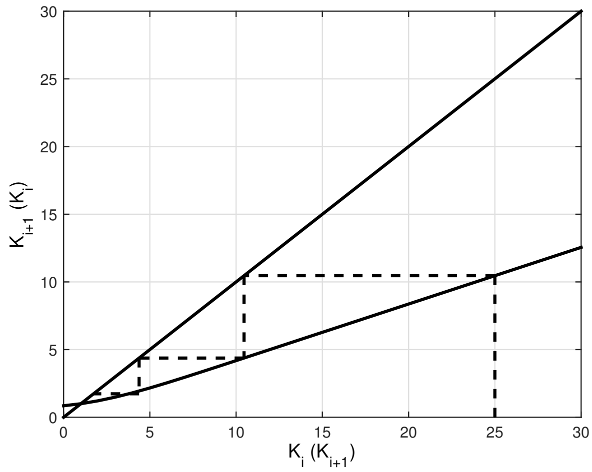


Figure 3. Representation of the trajectory of the sequence $\{K_i\}$. Solid lines: $K_{i+1} = K_i$ and Eq. (6).

where $\mathsf{E}\{\cdot\}$ is the expectation operator. Equations (1) and (2) form a recursion that provides sequences $\{\mathcal{R}_i\}$ and $\{\mathcal{N}_i\}$ that determine the efficiency. Unfortunately, the rounding operation in (2) makes their analysis practically unfeasible.

**Step 2**. When $c_i$ is large, by exploiting the limit $\lim_{x\to\infty} \mathrm{round}(x)/x = 1$, we can approximate the rounding operation $\mathrm{round}(Hc_i)$ in (2) with $Hc_i$, obtaining

$$\mathcal{R}_{i+1} = \mathsf{E}\{\mathrm{round}(Hc_i)\} \approx H\mathsf{E}\{c_i\} = H\mathcal{C}_i \triangleq R_{i+1}. \quad (3)$$

If we use (3) together with (1) we get the recursions, written with capital letters, that do not take into account the rounding operation:

$$R_{i+1} = HR_i \left(1 - K_i e^{-K_i} - e^{-K_i}\right), \quad (4)$$

$$N_{i+1} = N_i \left(1 - e^{-K_i}\right), \quad (5)$$

$$K_{i+1} = K_i \, \frac{1}{H} \frac{1 - e^{-K_i}}{1 - K_i e^{-K_i} - e^{-K_i}}. \quad (6)$$

Recursions (4)-(6) correspond to the actual sequences $\{\mathcal{R}_i\}$, $\{\mathcal{N}_i\}$, and $\{\mathcal{K}_i\}$, respectively. In Step 5 we show that replacing $(\{\mathcal{R}_i\}, \{\mathcal{N}_i\}, \{\mathcal{K}_i\})$ with $(\{R_i\}, \{N_i\}, \{K_i\})$ has no effect on the evaluation of the asymptotic performance. In Step 6 we show that this holds even for finite values of the initial frame size $r_0$. In practice, we find that sequence $\{R_i\}$ approximates fairly well sequence $\{\mathcal{R}_i\}$, even for moderate values of $N$, and this allows recurrence (6) to be used to evaluate the performance.

As an example, Fig. 2 shows sequence $\{\mathcal{N}_i\}$ derived by averaging $10^4$ simulation samples in the case $N = 10^3$ and $r_0 = 1$. We can clearly see a first phase where the estimate increases in order to converge to the true value $N = 10^3$; actually the estimate reaches a maximum value that is lower than the the true value because in the meantime some packets have been correctly transmitted. In the second phase, optimal conditions are met, collisions are solved and the backlog decreases steadily to reach zero at about the 25-th iteration. We prove in the next step that in the descending phase the rate of descent is $e^{-1}$, showing that Schoute's algorithm is capable to correctly track the backlog and to solve contentions in the most efficient way. Figure 2 also shows the relative error sequence $\{(\mathcal{N}_i - N_i)/\mathcal{N}_i\}$ multiplied by $10^3$ (dash-dotted line). The error is always very small except at the end of the process, where $\mathcal{N}_i$ becomes small and ignoring the rounding effect is no longer appropriate. However, this error has no effect on the efficiency since it occurs for a small period of time, negligible when compared to the entire collision resolution length.

**Step 3**. The evolution of the entire process is represented by recurrence (6) that depicts the evolution of the average traffic $K_i$. This is represented by the dashed trajectory in Fig. 3. This figure also shows that the evolution of the process is asymptotically stable since recurrence (6) leads to the fixed point in $K_i = 1$. This point is also a point of optimality because in here we attain the optimal condition $r_i = n_i$ that provides maximum throughput.

When the starting point in (6) is $K_0 = 1$, the protocol proceeds with a correct backlog estimate, yielding $K_i = 1$ for all subsequent $i$, and

$$R_{i+1} = (1 - e^{-1})R_i, \qquad i \geq 0. \quad (7)$$

The solution of recurrence (7) is

$$R_i = (1 - e^{-1})^i N, \qquad i \geq 0,$$

which shows that at each round the backlog reduces by the fraction $e^{-1}$. The total number of slot in this resolution phase is

$$L(N) = \sum_{i=0}^{\infty} R_i = Ne,$$

yielding an asymptotic throughput $N/L(N) = e^{-1}$.

When $K_0 = N/r_0 > 1$, the length of the entire procedure can be evaluated as

$$L(K_0) = \sum_{i=0}^{\infty} R_i = r_0 \sum_{i=0}^{\infty} a_i,$$

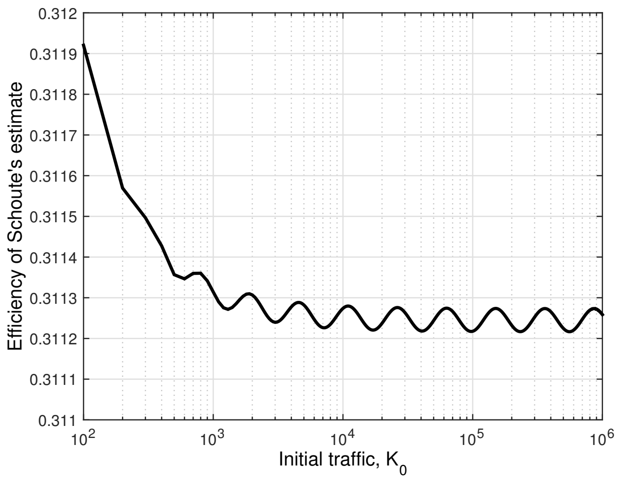


Figure 4. Efficiency of Schoute's backlog estimate versus initial traffic $K_0$.

where $r_0 = R_0$. The sequence $\{a_i = R_i/r_0\}$ just depends on $K_0$, whichever $r_0$ is, as it appears from (4). Therefore, the efficiency only depends on $K_0 = N/r_0$ and is evaluated as

$$\frac{N}{L} = \frac{K_0}{\sum_{i=0}^{\infty} a_i}.$$

**Step 4**. Here we show that for large values of the initial traffic $K_0$ the dependence of the efficiency on $K_0$ is negligible.

Since the protocol always starts with a finite $r_0$, large $N$ means large $K_0$, so we attain practically the same efficiency whichever the initial frame length $r_0$ is.

As an example, in Fig. 4 we have reported the efficiency $N/L(N)$, evaluated through (4) and (6), for different values of traffic $K_0$. Starting from $K_0 = 1$, the optimal case, not reported in the figure, the efficiency at first decreases as $K_0$ increases until about $K_0 = 500$ where it begins to oscillate without reaching an asymptote, around a mean value of 0.31125, with a period that increases geometrically with $H$.

To analyze the asymptotic behavior, during the solving process we consider three phases. The first phase, the *approaching phase*, starts at frame 0 with infinite traffic and ends at frame $u$, where $u$ is chosen in such a way that the traffic $K_u$ is finite and practically no successes occur up to frame $u$; as an example, we may arbitrarily assume $u$ such as $K_u \geq 10$. Although in this way $K_u$ and $u$ appear arbitrarily defined, we show below that this has no effect on the evaluation of the efficiency, as, in fact, the initial traffic $K_0$ has no effect. The assumed definition for $u$ assures that $u \to \infty$ as $N \to \infty$ and $R_u = N/K_u$.

The second phase, the *convergence phase*, starts at frame $u+1$ and ends at frame $u+v$ such that $K_{u+v} \approx 1$. At this point the third phase, the *tracking phase*, begins where tags are solved with efficiency $e^{-1}$. Denoting by $L', L''$, and $L'''$ the lengths of the three phases, respectively, the efficiency is evaluated as

$$\frac{N}{L(N)} = \frac{N}{L' + L'' + L'''}.$$

With high values of $K_0 = N/r_0$, in the first phase the frame length increases deterministically with law $R_i = r_0 H^i$, for $i \geq 0$. The average number of slots up to frame $u$ where the first phase ends is

$$L' = \sum_{i=0}^{u} R_i = r_0 \frac{H^{u+1} - 1}{H - 1} \approx \frac{H}{H-1} R_u.$$

Replacing $R_u = N/K_u$, the average length of the first phase becomes

$$L' = \frac{H}{H-1}\frac{N}{K_u} = NA(K_u),$$

where $A(K_u)$ is the proportionality constant, which expressly shows the dependence on $K_u$.

The second phase starts at frame $u+1$, when $K_u$ is such that the collision probability is practically one, and ends at frame $u+v$ when $K_{u+v} \approx 1$. Equation (4) can be used to evaluate the length of phase two by the following sum over a finite number of terms:

$$L'' = \sum_{j=1}^{v} R_{u+j} = R_u \sum_{j=1}^{v} \alpha_j = NB(K_u),$$

where terms $\alpha_j$ are all finite and, again, where $B(K_u)$ is the proportionality constant expressing the explicit dependence on $K_u$. The average backlog size at the end of the second phase can be evaluated by (5) as

$$N'' = N_{u+v} = N_u \prod_{j=1}^{v} \left(1 - e^{-K_{u+j}}\right) = NC,$$

where we have exploited the fact that $N_u = N$. The coefficient $C$ does not depend on $K_u$, since in frame $u+1$ we still observe all collisions ($e^{-K_{u+1}} \approx 0$).

The third phase presents efficiency $e^{-1}$ and its average length is

$$L''' = N''e = NCe.$$

The efficiency with very large $N$ is then

$$\frac{N}{L(N)} = \frac{N}{L' + L'' + N''e} = \frac{1}{A + B + Ce}. \tag{8}$$

We note that (8) does not depend on the choice of $v$, once the condition $K_{u+v} \approx 1$ is assured. If we replace $v$ by $v+1$, coefficient $A$ is not affected, and also term $B + Ce$ is not affected. In fact, $B$ is augmented by the term $R_{u+v+1}$ which, by (4) with $K_{u+v+1} \approx 1$, is equal to

$$R_{u+v+1} = N_{u+v}(1 - e^{-1}). \tag{9}$$

On the other side, term $Ce$ is diminished by

$$(N_{u+v} - N_{u+v+1})e = N_{u+v}(1 - e^{-1}),$$

that is equal to term (9). Nevertheless, efficiency (8) does depend on the choice of $K_u$, through coefficients $A$ and $B$. However, if we replace $K_u$, chosen as suggested above, with $K_u \cdot H$, efficiency (8) does not change because this only implies the shifting of term $R_u$ from term $A$ to term $B$. Therefore, the efficiency is periodic in a logarithmic scale and all the asymptotic amplitudes of the oscillations in Fig. 4 can be obtained by replacing $K_u$ with any value $K'$ in the range $(K_u, HK_u)$.

Table I
ANALYTICAL VALUES OF THE ASYMPTOTIC EFFICIENCY OF DFA WITH SCHOUTE'S ESTIMATE FOR DIFFERENT VALUES OF THE PARAMETER $K_u$.

| $K_u$ | 20 | 25 | 30 | 35 | 40 | 45 | 47.8 |
|---|---|---|---|---|---|---|---|
| $N/L(N)$ | 0.31125 | 0.31127 | 0.31125 | 0.31122 | 0.31122 | 0.31123 | 0.31125 |

Table I shows the efficiency attained by (8) for different values of $K_u$ chosen in the range $(20, 20H)$. As we can see, the values fit very well to those shown in Fig. 4. From what has been exposed above, we can conclude that the efficiency of Schoute's algorithm can be mathematically expressed as

$$\frac{N}{L(N)} = 0.311245 + \xi(\ln N) + \omega(N), \tag{10}$$

where $\xi(\ln N)$ is a periodic function of $\ln N$, such that $|\xi(\ln N)| < 0.0001$, and $\lim_{N\to\infty} \omega(N) = 0$. For all practical purposes, the asymptotic efficiency can be assumed equal to 0.311.

It is very interesting to note that expression (10) very closely resembles similar ones related to Tree Algorithms [19], which appears somehow originated by the geometric subdivision of the traffic operated by the protocol.

**Step 5**. Now we show that replacing $L', L''$, and $L'''$, in the limit $r_0 \to \infty$, with $\mathcal{L}', \mathcal{L}''$, and $\mathcal{L}'''$, in which the rounding operation is taken into account, does not change the results provided that the initial frame length is still $r_0$. In Appendix B we show that

$$\lim_{r_0\to\infty} \frac{\mathcal{L}'(N)}{L'(N)} = \frac{\sum_{i=0}^{\infty} \mathcal{R}_i}{\sum_{i=0}^{\infty} R_i} = 1.$$

We also have

$$\lim_{r_0\to\infty} \mathcal{L}''(N)/L''(N) = 1,$$

because the second phase is composed of a finite number $v$ of frames, each of them so large that the rounding effect is negligible. What shown also implies that at the end of the second phase we have $\lim_{r_0\to\infty} \mathcal{N}_i/N_i = 1$, and, therefore, since those tags are solved with efficiency $e^{-1}$, also for the length of the third phase we have

$$\lim_{r_0\to\infty} \mathcal{L}'''(N)/L'''(N) = 1.$$

**Step 6**. If $r_0$ is small and (3) can not be assumed, the first phase is split into two sub-phases in which the second sub-phase starts at frame-index $x$ such that, from this frame onward, the rounding operation in (2) can be disregarded. Index $x$ is finite and the length of the first sub-phase does not depend on $N$, whereas the length of the second sub-phase and of the other phases is proportional to $N$. Therefore, as $N \to \infty$, the length of the first sub-phase vanishes, with respect to the other phases, and the asymptotic efficiency remains approximately $0.311$ even with small $r_0$.

## IV. AE$^2$: AN ASYMPTOTICALLY EFFICIENT ESTIMATE

The analysis of Schoute's estimate of Sec. III has shown that the reduction of the asymptotic efficiency with respect to the theoretical value $e^{-1}$, when starting with a finite estimate, is not due to an intrinsic inefficiency of the estimate, but rather to the phase in which traffic $K$ converges to 1. This is the convergence phase composed of $L'$ and $L''$, whose length increases linearly with $N$. Specifically, the linear increase is because the frame length increases exponentially as $H^i$, and from the overhead point of view this is a complete waste of time, since in this phase almost no success occurs. On the other side, the frame increase is needed to reduce the traffic per slot and get locked to the optimal point $K = 1$. To get a good estimate of traffic $K$, we need not to explore the entire frame or, in another view, we need not to let all tags transmit in the frame; therefore, during the approaching phase toward $K = 1$ the frame can be shorter and provide a convergence phase with an average length $L' + L''$ such that

$$\lim_{N\to\infty} \frac{L' + L''}{N} = 0. \tag{11}$$

A way to reduce the number of tags transmitting in the frame, entirely respecting the EPC standard specifications, is to re-start a new frame before the exploration of the entire frame is completed. This has led to the proposal of AE$^2$ (Asymptotically Efficient Estimate), whose operation has been briefly anticipated in [13] and reads as follows.

Again, the traffic $n_i/r_i$ is determined by setting the frame length $r_i$; however, the exploration of frame $i$ of length $r_i$ is stopped at slot $z_i$, $z_i \le r_i$, in this way defining the *observed frame*, whose length is $z_i$. The estimate and frame setting are given by

$$\begin{aligned} \hat{n}_{i+1} &= \text{round}\left(H(\hat{n}_i) c_i \frac{r_i}{z_i}\right), \qquad c_i > 0, \\ \hat{n}_{i+1} &= \hat{n}_i - s_i, \qquad c_i = 0, \\ r_{i+1} &= \hat{n}_{i+1}, \end{aligned} \tag{12}$$

where $c_i$ and $s_i$ are respectively the number of collided and successful slots observed in frame $i$ up to slot $z_i$.

The meaning of the estimate in (12) is immediately apparent: $H(\hat{n}_i)c_i$ is the expected number of collided tags in the observed frame according to Schoute's method, which multiplied by $r_i/z_i$ extrapolates the figure to the entire frame. A similar estimate has been independently proposed in [11], but there $H(\hat{n}_i) = H$ has been used, exactly as in Schoute's. In Sec. IV-A we show that such a setting does not allow the estimate to converge to the actual backlog; the convergence requires, in fact, the following setting

$$H(\hat{n}_i) = \frac{1 - (\hat{n}_i/r_i)e^{-1}}{1 - 2e^{-1}}. \tag{13}$$

As for the increase in $z_i$, we asymptotically use the law

$$z_i = \min\{\text{round}((i+1)^b), r_i\}, \tag{14}$$

with $b > 0$. In (14), with large $N$ and with the exception of the first few slots, the observed frame size increases, at first, as $(i+1)^b$; later, when $r_i$ stabilizes and $i$ is such that $\text{round}((i+1)^b) > r_i$, the observed frame reaches the entire frame and the procedure becomes the classic DFA.

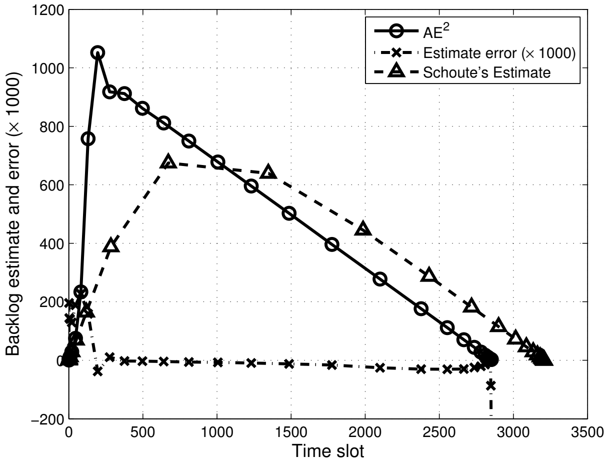


Figure 5. Average backlog estimate $\hat{N}_i$ at the end of frames versus time slot, for the AE$^2$ algorithm ($N = 1000$, $r_0 = 1$, $b = 2$). The red dash-dotted line represents the relative error $\times 10^3$ with respect to the analytical values $N_i$.

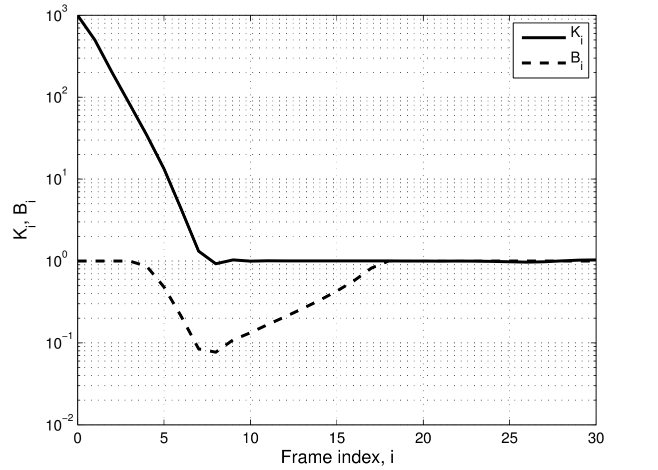


Figure 6. Sequences $\{K_i\}$ and $\{B_i\}$ versus the frame index $i$ ($N = 1000$, $r_0 = 1$, $b = 2$).

### A. Asymptotic Analysis of AE$^2$

The analysis here presented is much the same as the one presented in Sec. III. Therefore we limit our explanation to parts that differ. Adopting the same assumptions used in Sec. III we can write the recursions corresponding to (4)-(6) as

$$R_{i+1} = R_i \ H_i \left(1 - K_i e^{-K_i} - e^{-K_i}\right) \tag{15}$$

$$N_{i+1} = N_i \left(1 - \frac{Z_i}{R_i} e^{-K_i}\right)$$

$$K_{i+1} = K_i \ \frac{1}{H_i} \frac{1 - \frac{Z_i}{R_i} e^{-K_i}}{1 - K_i e^{-K_i} - e^{-K_i}}. \tag{16}$$

The key recursion (16) is different from (6) since now it also depends on $R_i$ which complicates the matter. Since, for an efficient estimation we want $K_i$ to converge to 1, sequence $\{H_i\}$ must be chosen as

$$H_i = \frac{1 - \frac{Z_i}{R_i} e^{-1}}{1 - 2e^{-1}}. \tag{17}$$

Recursion (16) is stable because it presents a unique fixed point in $K = 1$ and we have

$$-1 < \frac{\partial}{\partial K} \left\{ K \ \frac{1 - 2e^{-1}}{1 - Be^{-1}} \ \frac{1 - Be^{-K}}{1 - Ke^{-K} - e^{-K}} \right\} \Bigg|_{K=1} < 1,$$

for all $B \in (0, 1]$. Although values (17) could be evaluated a priori, in practice we can assume

$$H_i = \frac{1 - \frac{z_i}{r_i} e^{-1}}{1 - 2e^{-1}}.$$

Figure 5 validates the analysis carried out so far. In fact, it compares the results the analysis produces in terms of sequence $\{\mathcal{N}_i\}$ with exact values attained averaging $10^4$ simulation samples, in the case $N = 10^3$ and $r_0 = 1$. Again, the dash-dotted line represents the relative error multiplied by $10^3$, still very small. For comparison purposes we have also reported the curve in Fig. 2 that refers to Schoute's algorithm. We clearly see the advantage of AE$^2$: The estimate $\hat{\mathcal{N}}_i$ at first rises sharply reaching $N$ with some overshoot, higher and sooner with respect to Schoute's case. Right after the estimate begins a steady decline with rate $e^{-1}$.

What stated above is confirmed in Fig. 6 where we have reported sequences $\{K_i\}$ and $\{B_i\} = \{Z_i/R_i\}$ in the case $N = 10^3$, $r_0 = 1$ and $b = 2$. The former shows the convergence of the estimate in $K = 1$, while the latter reports the convergence of the observed frame $Z_i$ to the frame $R_i$. The protocol starts with $z_0 = r_0$, $e^{-K_i} \simeq 0$ and subsequently we have $H \simeq 2.39$ as in Schoute's, which yields $r_1 = z_1 = 2$. Condition $z_i = r_i$ is maintained up to $i = 3$ and then becomes $z_i < r_i$. $B_i$ decreases and when $z_i \ll r_i$ we have $H_i = H' \simeq 1/(1 - 2e^{-1}) \simeq 3.78$, and $r_{i+1} = H' r_i$, reducing the traffic more quickly than in Schoute's and speeding up the convergence phase, which is further reduced in time because the observed frame is shorter by far. $B_i$ reaches a minimum when $K_i$ reaches one. At this point the observed frame is so short that the collision solved are still very few. Beyond this point the protocol solves collisions with efficiency $e^{-1}$, $r_i$ decreases and $z_i$ increases until condition $z_i = r_i$ is reached again and never abandoned. From this point onward the backlog is solved exactly as in Schoute's algorithm.

It is worth noting that the recursion in $B$ does not get into its fixed point $B = 0$. In fact, once $K = 1$ is reached, by (15) we can write $B_{i+1}/B_i = Z_{i+1}/Z_i(1 - B_i e^{-1})^{-1} > 1$.

Now we prove that the efficiency of AE$^2$ equals $e^{-1}$. The efficiency can be evaluated by writing, as in Sec. III,

$$\eta = \lim_{N \to \infty} \frac{N}{L'(N) + L''(N) + N'' e}, \tag{18}$$

where $L^{'}(N)$ is the average number of slots of the first phase in which there are no successes, $L^{''}(N)$ is the average number of slots of the second phase in which the estimate of the backlog converges to the actual backlog. As it has been already observed, in the first phase we have $H_i = H'$, so that we have

$R_i = r_0(H')^i$, and assuming, as in the earlier analysis, that the first phase ends at frame $u$, where $R_u = N/K_u$, solving the expression of $R_u$ we get

$$u(N) = \log_{H'} \frac{N}{r_0 K_u}. \tag{19}$$

The length of this phase can be bounded as

$$\begin{aligned} L^{'}(N) &\leq \sum_{i=1}^{u(N)} i^{b+1} \leq \int_0^{u(N)+1} i^{b+1}\, di \\ &= \frac{(u(N)+1)^{b+2}}{b+2} \leq \frac{\left(1+\log_{H'} \frac{N}{r_0 K_u}\right)^{b+2}}{b+2}, \end{aligned} \tag{20}$$

where last inequality follows by (19), therefore we have $\lim_{N\to\infty} L^{'}(N)/N = 0$. The overhead of the second phase can be rewritten as

$$L^{''}(N) = (N - N^{''})(e - \epsilon), \qquad 0 < \epsilon < e, \tag{21}$$

where $N^{''}$ is the backlog size at the end of the second phase. At the end of the first phase we have

$$B_u \triangleq \frac{Z_u}{R_u} \approx \frac{K_u u^b}{N} = \frac{K_u(\log_{H'} N - \log_{H'} r_0 K_u)^b}{N},$$

which implies $\lim_{N\to\infty} B_u = 0$. In the second phase a few frames, $v$, are necessary to obtain $K = 1$, and we still have $\lim_{N\to\infty} B_{u+v} = 0$, which means that also the fraction of solved tags is asymptotically zero. Therefore, in (18) we have $\lim_{N\to\infty} N^{''}/N = 1$, and by (21) $\lim_{N\to\infty} L^{''}(N)/N = 0$, so that (18) yields $\eta = e^{-1}$.

## V. Conclusions

In this paper we have presented a new asymptotic analysis of the Dynamic Frame Aloha protocol for RFID systems applied to the family of Schoute's backlog estimates. The analysis shows that when the initial frame length is set to match the tag number $N$, the estimate is able to track the real value, providing the theoretical efficiency $e^{-1} \approx 0.367$. When a mismatch exists, such as when the initial frame length $r_0$ is finite and $N$ is much larger, the analysis shows that the asymptotic efficiency drops to $0.311$, far below the theoretical maximum. The analysis also shows that the low performance is due to the overhead caused by the geometric increase of the frame in the convergence phase, which is not needed to refine the estimate of $N$. Therefore, we introduce a new proposal, called AE$^2$, which exploits the Frame Restart property of the standard to reduce the increase of the frame in the convergence phase, making the overhead asymptotically vanish, thus reaching the theoretical efficiency $e^{-1}$.

## Appendix A

If $n_i$ and $r_i$ are both large, collided slots in frame $i$ become distributed according to a binomial with average $r_i p_c$ and variance $r_i p_c(1-p_c) \leq r_i$. Therefore, being, in the approaching phase and for large $r_0$, $r_{i+1} = Hc_i$ we have

$$\mathsf{Var}\{ r_{i+1} \mid n_i, r_i \} = H^2\, \mathsf{Var}\{ c_i \mid n_i, r_i \} \leq H^2 r_i. \tag{22}$$

Since the number of collisions can not be larger than $N/2$ it follows that

$$r_i \leq H\frac{N}{2},\ \forall i. \tag{23}$$

Substituting (23) into (22) yields

$$\mathsf{Var}\{ r_{i+1} \mid n_i, r_i \} \leq Nd,$$

where $d$ is a constant value. Using this bound with Chebyshev's inequality yields

$$\mathsf{P}\left(|r_{i+1} - \mathcal{R}_{i+1}| \geq \epsilon N | n_i, r_i\right) \leq \frac{d}{N\epsilon^2},$$

that can be reduced to

$$\mathsf{P}\left(|r_{i+1} - \mathcal{R}_{i+1}| \geq \epsilon N\right) \leq \frac{d}{N\epsilon^2}. \tag{24}$$

Relation (24) shows that, for $N \to \infty$, we have $r_i/N \to \mathcal{R}_i/N$, where the convergence is in probability.

Much in the same manner one can show that $n_i/N \to \mathcal{N}_i/N$, and, therefore, we have $n_i/r_i \to \mathcal{N}_i/\mathcal{R}_i$ in probability.

## Appendix B

Here we consider sequence $\{\mathcal{R}_i\}$ during the first phase, where all the slots are collided, *i.e.*, $\mathcal{C}_i = \mathcal{R}_i$ and relation (2) becomes

$$\mathcal{R}_{i+1} = \mathsf{E}\{Hc_i + \xi_i\} = H\mathcal{C}_i + \Xi_i = H\mathcal{R}_i + \Xi_i, \quad i \geq 0,$$

where $\xi_i$ is a random variable that accounts for the rounding operation, and is such that $|\xi_i| \leq 1/2$. On the other side we have

$$R_{i+1} = HR_i, \qquad i \geq 0,$$

with $\mathcal{R}_0 = R_0 = r_0$. Solving the recursions we get

$$\mathcal{R}_i = r_0 H^i + \sum_{k=0}^{i-1} H^{i-1-k}\Xi_k, \tag{25}$$

$$R_i = r_0 H^i, \tag{26}$$

for $i \geq 0$. Relation (25) can be rewritten as

$$\mathcal{R}_i = R_i + \sum_{k=0}^{i-1} H^{i-1-k}\Xi_k.$$

Since $|\Xi_k| \leq 0.5 < 1$, and being

$$\sum_{k=0}^{i-1} H^k = (H^i - 1)/(H-1),$$

we can write

$$R_i - \frac{H^i - 1}{H-1} < \mathcal{R}_i < R_i + \frac{H^i - 1}{H-1}, \qquad i \geq 0,$$

and

$$1 - \frac{f(H)}{r_0(H-1)} < \frac{\sum_{i=0}^{\infty} \mathcal{R}_i}{\sum_{i=0}^{\infty} R_i} < 1 + \frac{f(H)}{r_0(H-1)},$$

with

$$f(H) = \left(\sum_{i=0}^{\infty}(H^i - 1)\right) / \left(\sum_{i=0}^{\infty} H^i\right),$$

having exploited (26). Since it is

$$H^i - 1 < H^i, \qquad i \geq 0,$$

we also have $f(H) < 1$, and finally

$$\lim_{r_0 \to \infty} \left( \sum_{i=0}^{\infty} \mathcal{R}_i \right) / \left( \sum_{i=0}^{\infty} R_i \right) = 1.$$

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