Impact of Uncertain Channel Estimation and Outdated Feedback on the Adaptive M-PSK Modulation

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Abstract—**This paper investigates an adaptive M-ary phase-shift keying (M-PSK) modulation scheme over Rayleigh flat fading channels. The data rate is adapted according to the channel state. At the receiver, the fading is estimated using pilot symbols. To cancel the channel impact, we correct the received signal by dividing it by the estimated value of the fading. So, we propose to adjust the modulation level by examining the statistics of the corrected signal. In contrast to the previous works on the adaptive M-PSK modulation techniques, our modulation switching protocol takes into account the channel estimation error variance. Moreover, we derive a new closed-form expression for the average bit error rate of the considered system.**

Index Terms—**Adaptive modulation, M-PSK, Channel estimation, Rayleigh channels**

I. INTRODUCTION

The growing demand for wireless systems with high data rates and quality of service requires spectrally efficient transmission techniques. Classical systems with a robust nonadaptive modulation are generally designed to maintain an acceptable performance in deep fades [1]. In fact, these systems are implemented to take account of the poorest channel conditions. Furthermore, to ensure the required quality of service, robust modulation schemes decrease the system throughput. Adaptive modulation has been proposed as a powerful method to maintain the desired quality of service and to maximize the transmission throughput given channel conditions [2], [3]. The basic idea of this technique is to switch between different modulation constellation sizes depending on the channel state. For a deep fade, a modulation with a small size constellation is chosen to reduce the error probability and maintain the target bit error rate (BER). However, if the channel conditions are considered to be good, the throughput is increased by a dense constellation modulation. Therefore, the transmitter needs the knowledge of the channel fading state to adjust the modulation level. For this purpose, the receiver estimates the received signal power and sends the monitored channel fading information to the transmitter over a reserve channel.

The choice of the modulation and the setting of the modulation switching levels are major parameters to design adaptive modulation systems. Thus, an accurate channel prediction and a reliable feedback link between the receiver and the transmitter are required to achieve good performances. Adaptive

modulation has been investigated by several researchers [1]–[5]. An exhaustive analysis of adaptive multilevel quadrature amplitude modulation (M-QAM) for Rayleigh fading channels has been examined in [1]. The effect of the imperfect channel prediction and the impact of the time delay on the performance of an adaptive M-QAM modulation have been discussed in the literature [5]. A variable rate QAM for data transmission over selective channels is given in [6]. An adaptive trellis-coded M-PSK modulation system for Rayleigh fading channels is proposed in [7]. A non coherent M-ary frequency shift keying (NC-MFSK) modulation scheme for Nakagami fading channels has been studied in [8]. An adaptive M-PSK modulation without channel estimation has been introduced in [9]. Recently, the authors in [10] have proposed an adaptive transmission technique for free space optical systems with sub-carrier phase shift keying (S-PSK) intensity modulation. The adaptive modulation approach has been extended to multiple-input-multiple-output (MIMO) systems in [11]–[14].

In this paper, we study an adaptive M-PSK modulation technique with a coherent detection over Rayleigh fading channels. To reduce the channel impact, fading estimates are used to correct the received signal before the coherent detection. This is done by dividing the received symbol by the estimated value of the complex fading gain. This process is called automatic gain control (AGC) [15]. Thus, we propose a modulation switching protocol based on the power of the corrected signal instead of that of the received one. In fact, if the channel estimation is imperfect, the AGC improperly scales the received signal and the demodulator can perform incorrectly [15]. In this paper, we study the effect of this demodulation on the adaptive modulation behavior. To the best of our knowledge, the investigation of the impact of channel estimation error on the performance of the adaptive M-PSK modulation schemes has not been considered before. The switching strategy and the closed-form expression of the average BER derived in this paper are novel.

The outline of this paper is as follows. In section II, the system and channel models are described. In section III, channel estimation and prediction techniques based on pilot symbols are presented. Section IV gives an analytic expression of the BER for M-PSK system with imperfect channel estimation. The impact of the channel estimation error is also presented in this section. Section V discusses the adaptive modulation procedure and the modulation switching protocol. The effect of the channel estimation and prediction error is illustrated in

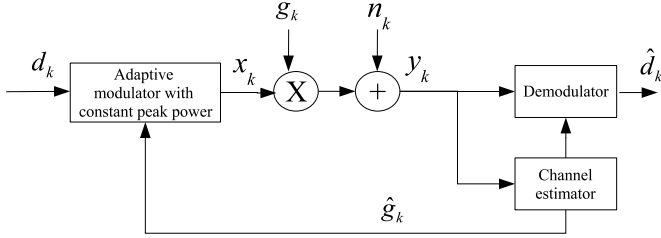section VI. A conclusion is given in section VII.

II. SYSTEM MODEL

Figure 1. Transmission scheme.

In this paper, we consider the discrete baseband system model shown in Fig. 1. The scheme is summarized as follows. The receiver sends the monitored channel fading information to the transmitter on the reverse channel. According to the channel conditions, the transmitter chooses a proper modulation scheme from a set of M-PSK modulations with different constellation sizes, a constant transmission power E_s and a fixed symbol rate T_s. The setting of the modulation switching levels will be discussed in the next section. The symbol rate and the carrier frequency remain constant. Hence, the spectrum usage is unmodified by the approach.

The M-PSK modulated sequence is transmitted over a correlated Rayleigh flat fading channel with an additive white Gaussian noise (AWGN). Let us denote x_n the transmitted signal and g_n the multiplicative distortion of the flat fading channel. The received signal at time nT_s is then

$$y_n = g_n x_n + w_n \qquad (1)$$

where w_n is a zero-mean AWGN with variance $\sigma_w^2 = N_0/2$.

The Rayleigh fading process is generated according to Jakes' model [16]. So, g_n is a correlated complex Gaussian process with zero mean and variance σ_g^2. The fading autocorrelation function is determined by the maximum Doppler spread f_d as [16]

$$\rho_m = E\{g_n g_{n-m}^*\} = \sigma_g^2 J_0(2\pi f_d T_s m) \qquad (2)$$

where $J_0(.)$ is the zeroth-order Bessel function of the first kind and $*$ denotes the complex conjugate. The real and imaginary parts of g_n are supposed to be mutually uncorrelated.

At the receiver, the signal y_n is used to estimate the fading multiplicative distortion g_n. Having the channel estimate, we can compensate the impact of the fading gain by dividing the received signal by the fading estimate $\hat{g}_n$ as [17]

$$z_n = y_n/\hat{g}_n \qquad (3)$$

The corrected signal z_n is then fed to the decision device to detect the demodulated data bits.

III. CHANNEL ESTIMATION AND PREDICTION

The estimation of Rayleigh flat fading channels has been widely investigated in the literature. In this paper, we use the well known pilot symbol assisted modulation (PASM) technique [18]. For this method, known pilot symbols are periodically inserted into the data sequence. At the receiver, pilot symbols are used to estimate the channel fading. Let us assume that the pilot sequence is of length N_p. Thus for known symbols x_n (i.e. $n \le N_p$), a scaled received sample is introduced as [19]

$$v_n = y_n/x_n \qquad (4)$$

To estimate the channel gain, scaled samples of equation (4) are used in order to minimize the following mean square error (MSE)

$$E\left[|\hat{g}_n - g_n|^2\right] \qquad (5)$$

where $\hat{g}_n$ denotes the estimate of g_n. When scaled samples are known, the minimum mean square solution is given by

$$\hat{g}_n = \boldsymbol{W}' \boldsymbol{v}_n \qquad (6)$$

where the apostrophe denotes the transpose operator, $\boldsymbol{v}_n = [v_{n-1}, \cdots, v_{n+N_e}]'$ and $\boldsymbol{W} = [w_1, \cdots, w_{N_e}]'$ is the set of N_e filter coefficients obtained by solving the Wiener-Hopf equations [20]. Indeed, given the scaled received sequence $\{v_n\}$, the fading process has a Gaussian distribution $f_{\boldsymbol{g}|\boldsymbol{v},\mathbf{x}}(g_n|v_n, x_n)$ with a conditional mean [20]

$$\mu_{\boldsymbol{g}|\boldsymbol{v},\mathbf{x}} = E\{g_n|v_n, x_n\} = \boldsymbol{R}_{gv}\boldsymbol{R}_{vv}^{-1}\boldsymbol{v}_n \qquad (7)$$

and with a conditional covariance $\boldsymbol{R}_{\boldsymbol{g}|\boldsymbol{v},\mathbf{x}}$ given by [20]

$$\boldsymbol{R}_{\boldsymbol{g}|\boldsymbol{v},\mathbf{x}} = \boldsymbol{R}_g - \boldsymbol{R}_{gv}\boldsymbol{R}_{vv}^{-1}\boldsymbol{R}_{gv}' \qquad (8)$$

where $\boldsymbol{R}_{gv} = E\{g_n \boldsymbol{v}_n^H\}$ and $\boldsymbol{R}_{vv} = E\{\boldsymbol{v}_n \boldsymbol{v}_n^H\}$ an $N_e \times N_e$ Toeplitz matrix, and superscript H is the transpose and complex conjugate operator. Therefore, the estimate $\hat{g}_n$ is the conditional mean given by (7) and the Wiener filter $\boldsymbol{W}$ can be expressed as in [20]

$$\boldsymbol{W}' = \boldsymbol{R}_{gv}\boldsymbol{R}_{vv}^{-1} \qquad (9)$$

For $n > N_p$, data symbols are unknown at the receiver. In this case, the channel estimation can be performed using past decisions $\hat{x}_n$ to scale the received signal instead of x_n

$$v_n = y_n/\hat{x}_n \qquad (10)$$

The channel estimate given by (6) can be expressed as

$$\hat{g}_n = g_n + e_n \qquad (11)$$

where e_n denotes the channel estimation error. It is easily proven that e_n is a complex Gaussian random variable with $E\{e_n\} = 0$, which means that the channel estimator is unbiased. The channel estimation error variance is [20]

$$\sigma_e^2 = E\left\{|\hat{g}_n - g_n|^2\right\} = \sigma_g^2 - \boldsymbol{R}_{gv}\boldsymbol{R}_{vv}^{-1}\boldsymbol{R}_{gv}' \qquad (12)$$

Due to the feedback link delay, the channel estimate obtained at time n is available at the transmitter at time $n + K$. Therefore, to adjust the modulation size properly, the fading prediction is needed [5]. The channel gain at time

n can be predicted by an unbiased finite-impulse response (FIR) filter based on a finite number (L) of past estimates $\hat{\boldsymbol{g}}_{n-L} = [\hat{g}_{n-K}, \hat{g}_{n-K-1}, \cdots, \hat{g}_{n-K-(L-1)}]'$ [5]. The unbiased predicted channel gain is given in [5] by

$$\tilde{g}_n = \boldsymbol{\Phi}' \hat{\boldsymbol{g}}_{n-L} \tag{13}$$

where $\boldsymbol{\Phi}$ is the optimal linear predictor in the MSE sense given by [5]

$$\boldsymbol{\Phi}' = \boldsymbol{R}_{g\hat{g}} \boldsymbol{R}_{\hat{g}\hat{g}}^{-1} \tag{14}$$

where $\boldsymbol{R}_{g\hat{g}} = \mathrm{E}\left\{ g_n \hat{\boldsymbol{g}}_{n-L}^H \right\}$ and $\boldsymbol{R}_{\hat{g}\hat{g}} = \mathrm{E}\left\{ \hat{\boldsymbol{g}}_{n-L} \hat{\boldsymbol{g}}_{n-L}^H \right\}$.

The prediction error variance is given by [5]

$$\sigma_p^2 = \mathrm{E}\left\{ |\tilde{g}_n - g_n|^2 \right\} = \sigma_g^2 - \boldsymbol{R}_{g\hat{g}} \boldsymbol{R}_{\hat{g}\hat{g}}^{-1} \boldsymbol{R}_{g\hat{g}}' \tag{15}$$

IV. ERROR PROBABILITY FOR M-PSK

Once the channel estimate $\hat{g}_n$ is evaluated, it is passed to the demodulator and sent to the transmitter. To compensate the channel effect, we divide the received signal y_n by $\hat{g}_n$. Thus, the demodulation is performed using the following decision variable

$$z_n = y_n / \hat{g}_n \tag{16}$$

Unlike previous works, we propose in this paper a modulation switching strategy based on the power of the decision variable z_n instead of that of the received signal y_n. In fact, demodulation decision regions must correspond to the transmitted M-PSK constellation. In this work, we assume that the channel does not vary significantly over two symbol blocks.

A. Decision variable statistics

The decision variable of equation (16) depends on the channel estimate. It can be written as [17]

$$z_n = \frac{y_n}{\hat{g}_n} = x_n + \frac{w_n - e_n x_n}{\hat{g}_n} = x_n + m_n \tag{17}$$

Let us derive the probability distribution function (PDF) of the "final-noise" m_n conditioned on x_n and $\hat{g}_n$. It is known that the channel estimate $\hat{g}_n$ is a zero mean complex Gaussian random variable. Since channel estimation error e_n and $\hat{g}_n$ are uncorrelated, it is easily shown that the channel estimator variance is

$$\mathrm{var}(\hat{g}_n) = \sigma_{\hat{g}}^2 = \sigma_g^2 - \sigma_e^2 \tag{18}$$

The Gaussian real and imaginary parts of w_n and $e_n x_n$ are independent. Thus, both w_n and $e_n x_n$ are circularly symmetric. This yields to the circular symmetry of $\hat{g}_n$ [17]. So, the phase of the channel estimate $\arg(\hat{g}_n)$ can be ignored, and a new random variable can be used [17]

$$m_n' = m_n \arg(\hat{g}_n) = \frac{w_n - e_n x_n}{|\hat{g}_n|} = \alpha w_n' \tag{19}$$

where $w_n' = w_n - e_n x_n$ and $\alpha = 1/|\hat{g}_n|$.

Given the transmitted symbol x_n, the noise w_n' is the sum of two independent Gaussian variables with independent real and imaginary parts. So, the conditional PDF of w_n' is

$$p(w_n'|x_n) = p\left(\Re[w_n'], \Im[w_n'] \right) =$$
$$\frac{1}{2\pi \left(\sigma_w^2 + |x_n|^2 \sigma_e^2 \right)} \exp\left[-\frac{|w_n'|^2}{2\left(\sigma_w^2 + |x_n|^2 \sigma_e^2 \right)} \right] \tag{20}$$

where $\Re[\cdot]$ and $\Im[\cdot]$ denote the real and the imaginary parts.

For M-PSK modulation, $|x_n|^2 = E_s$ for all n. So, the conditional PDF $p(w_n'|x_n)$ does not depend on x_n

$$p(w_n'|x_n) = p(w_n') = \frac{1}{2\pi\sigma_T^2} \exp\left(-\frac{|w_n'|^2}{2\sigma_T^2} \right) \tag{21}$$

where $\sigma_T^2 = \left(\sigma_w^2 + E_s \sigma_e^2 \right)$.

B. Instantaneous BER

For adaptive modulation systems, the constellation size is adjusted based on the instantaneous SNR. Given the channel gain, the "final noise" is Gaussian with the conditional PDF

$$p\left(m_n|x_n, \hat{g}_n \right) = p\left(m_n'|x_n, \hat{g}_n \right)$$
$$= \frac{|\hat{g}_n|^2}{2\pi\sigma_T^2} \exp\left(-\frac{|w_n'|^2 |\hat{g}_n|^2}{2\sigma_T^2} \right) \tag{22}$$

It is noted that the PDF $p\left(m_n|x_n \right)$ is not Gaussian. The instantaneous estimated SNR is then

$$\hat{\gamma}_n = \frac{E_s |\hat{g}_n|^2}{\sigma_T^2} \tag{23}$$

The relationship between the true instantaneous SNR and the estimated instantaneous SNR is given by

$$\hat{\gamma}_n = \frac{E_s |\hat{g}_n|^2}{\sigma_w^2 + E_s \sigma_e^2} \approx \frac{E_s |g_n|^2 + E_s \sigma_e^2}{\sigma_w^2 + E_s \sigma_e^2} = \frac{\gamma_n + E_s \sigma_e^2/\sigma_w^2}{1 + E_s \sigma_e^2/\sigma_w^2}$$
$$\approx \frac{\gamma_n}{1 + E_s \sigma_e^2/\sigma_w^2} \tag{24}$$

For a given SNR γ, the BER of the M-PSK modulation with a Gray mapping in AWGN can be approximated by

$$P_b(M, \gamma) = \frac{2}{k} Q\left[\sqrt{2\gamma} \sin\left(\frac{\pi}{2^k} \right) \right] \tag{25}$$

where $k = \log_2(M)$ is the number of bits per symbol. This BER expression will be used by the transmitter to adjust the modulation size. Unfortunately, it is not invertible in its arguments M and $\bar{\gamma}$. It was suggested in [21] to approximate the BER expression by the following generic form

$$P_b(M, \gamma) = c_1 \exp\left[-\frac{c_2 \gamma}{2^{kc_3} - c_4} \right] \tag{26}$$

where c_1, c_2, c_3 and c_4 are real constants. The determination of these constants is a nonlinear curve-fitting problem. It can be solved by minimizing the least-squares criterion. Three models for BER approximation with different values of $\{c_i\}$ are given in [21].

C. Average BER for fixed M-PSK modulation

Using the Rayleigh distribution of $|\hat{g}_n|$, it was shown in [17] that the PDF of $\alpha = 1/|\hat{g}_n|$ is

$$p(\alpha) = \frac{2}{\tilde{\sigma}_g^2 \alpha^3} \exp\left(-\frac{\alpha^{-2}}{\tilde{\sigma}_g^2}\right) \ ; \ \alpha > 0 \qquad (27)$$

The PDF of m_n can be obtained from (21) and (27) [17]

$$
\begin{aligned}
p(m_n) & = p\left(m_n'\right) = \int_0^\infty p\left(m_n'|\alpha\right) d\alpha \\
& = \int_0^\infty p\left(m_n'|\alpha\right) p(\alpha) d\alpha \\
& = \frac{\bar{\gamma}_e}{\pi \left(1 + \bar{\gamma}_e |m_n|^2\right)^2} \qquad (28)
\end{aligned}
$$

where $\bar{\gamma}_e$ is defined as [17]

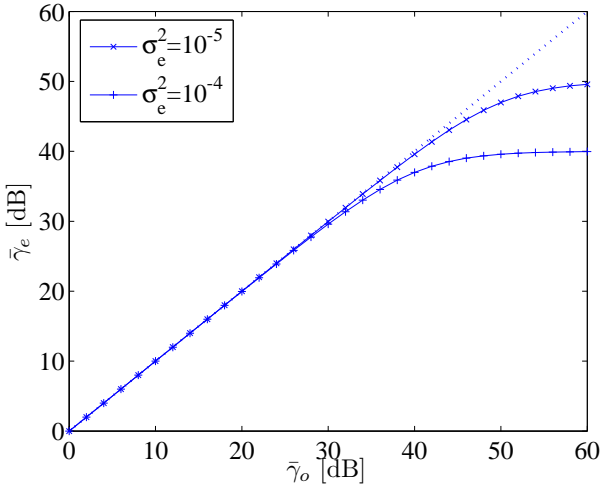$$\bar{\gamma}_e = \frac{\tilde{\sigma}_g^2}{\sigma_w^2 + E_s \sigma_e^2} \qquad (29)$$

Figure 2. $\bar{\gamma}_e$ vs $\bar{\gamma}_0$ for $\sigma_e^2 = 10^{-5}$ and $\sigma_e^2 = 10^{-4}$.

Figure 2 illustrates the impact of the channel gain estimation error on the system SNR. For smaller SNRs, the estimation error σ_e^2 has a negligible effect. However, for higher SNRs, the discrepancy between $\bar{\gamma}_e$ and $\bar{\gamma}_0$ is important. For $\sigma_e^2 = 10^{-5}$, a saturation floor is observed at 50 dB.

For systems with a perfect channel estimation ($\sigma_e^2 = 0$), the "final noise" PDF is [17]

$$p(m_n) = \frac{\bar{\gamma}_0}{\pi \left(1 + \bar{\gamma}_0 |m_n|^2\right)^2} \qquad (30)$$

where $\bar{\gamma}_0$ is the average free noise signal-to-noise ratio

$$\bar{\gamma}_0 = \frac{\sigma_g^2}{\sigma_w^2} \qquad (31)$$

It was shown in [17] that the average symbol error rate (SER) with a perfect channel estimation is

$$P_{s,0} = \frac{M-1}{M} - \zeta_M \left[\frac{1}{2} + \frac{1}{\pi} \tan^{-1}\left(\zeta_M \cot \frac{\pi}{M}\right)\right] \qquad (32)$$

where

$$\zeta_M = \sqrt{\frac{\bar{\gamma}_0 \sin^2\left(\frac{\pi}{M}\right)}{1 + \bar{\gamma}_0 \sin^2\left(\frac{\pi}{M}\right)}} \qquad (33)$$

The BER can be approximated by

$$P_{b,0} = \frac{P_{s,0}}{\log_2(M)} \qquad (34)$$

For an imperfect channel gain estimation, the BER of the M-PSK system is obtained by replacing $\bar{\gamma}_0$ by $\bar{\gamma}_e$ [17]. In fact, it can be seen that the "final noise" PDF has the same form for both perfect estimation (30) and imperfect estimation (28) cases.

The BER degradation due to the channel gain estimation error is plotted in Fig. 3. Curves correspond to $M \in \{4, 16, 64, 256\}$, Wiener filter length $N_e = 20$ and normalized Doppler spread $f_d T_s = 10^{-2}$. It is shown that, for higher SNRs, the BER degradation is not negligible. The channel estimation error can lead to a 2 dB performance loss. Hence, to set the modulation size switching levels, it is important to take into account the channel estimation error.
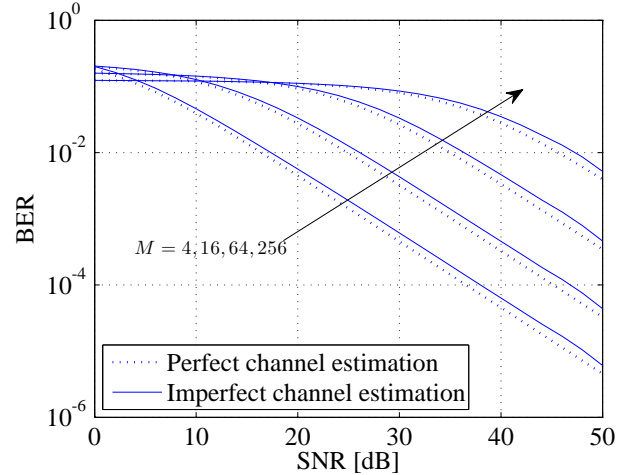

Figure 3. Average BER degradation due to channel gain estimation error. $M \in \{4, 16, 64, 256\}$, $N_e = 20$ and $f_d T_s = 10^{-2}$.

V. ADAPTIVE MODULATION

Wireless radios are used over a wide range of link conditions. Furthermore, a small error probability and a high transmission throughput are required. Adaptive modulation is a powerful method to maintain the desired BER and to maximize the transmission throughput given channel conditions [1]–[5]. The basic idea of this technique is to switch between different modulation constellation sizes depending on the channel state. So, adaptive modulation improves the use of the channel capacity [1]. The choice of the modulation mode and the determination of the switching metric are major parameters to design an adaptive modulation. In this context, we propose a fixed power M-PSK scheme with a variable discrete rate of transmission. All uncertainly about the the channel gain (estimation error and prediction error) are considered.

A. Modulation switching protocol

An important issue in an adaptive modulation system is the strategy for the choice of the modulation level. The transmitter adjusts the modulation size based on the required BER and the predicted instantaneous SNR

$$\tilde{\gamma}_n = \frac{E_s |\tilde{g}_n|^2}{\sigma_w^2} \tag{35}$$

Let us consider an adaptive M-PSK scheme with N different modulation sizes $(M_1, M_2, \cdots, M_N)$ varying from the lower constellation size to the higher constellation size with an increasing order. For adaptive discrete rate modulation systems, the SNR range is divided to N regions given by $\mathcal{R}_l = [\gamma_l^s, \gamma_{l+1}^s)$, where $(\gamma_0^s, \gamma_1^s, \cdots, \gamma_N^s)$ are the switching levels and $\gamma_N^s = \infty$. If the predicted SNR $\tilde{\gamma}_n$ is in the region $\mathcal{R}_l$, the associated modulation level $M_l(\tilde{\gamma}_n) = 2^{k_l}$ is selected and transmitted. When the SNR $\tilde{\gamma}_n$ is less than γ_0^s, there is no transmission. So, γ_0^s is the channel cutoff SNR below which transmission is stopped.

For a Gray mapping and a constellation size $M_l(\tilde{\gamma}_n) = 2^{k_l}$, the instantaneous BER can be expressed from (24) and (26) as

$$P_b(\tilde{\gamma}_n, \gamma_n, \sigma_e^2) = c_1 \exp\left[-\frac{c_2' \gamma_n}{2^{c_3 k_l} - c_4}\right] \tag{36}$$

where

$$c_2' = \frac{c_2}{1 + E_s \sigma_e^2 / \sigma_w^2} \tag{37}$$

This BER expression depends on the channel gain and the estimation error variance σ_e^2. So, the transmitter can use equation (12) to evaluate σ_e^2.

B. Average BER

The average instantaneous BER is given by averaging (36) over the true SNR γ_n [5]

$$\bar{P}_b(\tilde{\gamma}_n, \sigma_e^2, \sigma_p^2) = \int_0^\infty P_b(\tilde{\gamma}_n, \gamma_n, \sigma_e^2) p(\gamma_n | \tilde{\gamma}_n) d\gamma \tag{38}$$

where the conditioned PDF $p(\gamma_n | \tilde{\gamma}_n)$ is given in [5]

$$p(\gamma_n | \tilde{\gamma}_n) = \frac{U(\gamma) U(\tilde{\gamma} - \bar{\gamma}_p)}{\bar{\gamma}_p} \exp\left[-\frac{\tilde{\gamma} + \gamma - \bar{\gamma}_p}{\bar{\gamma}_p}\right] \times I_0\left(\frac{2}{\bar{\gamma}_p}\sqrt{\gamma(\tilde{\gamma} - \bar{\gamma}_p)}\right) \tag{39}$$

where $U(\cdot)$ is the Heaviside's step function, $I_0(\cdot)$ is the zeroth order modified Bessel function and

$$\bar{\gamma}_p = \frac{\bar{\gamma}_0 \sigma_p^2}{\sigma_g^2} \tag{40}$$

where σ_p^2 is the prediction error variance given by (15). Following results presented in [5], the average instantaneous BER is

$$\bar{P}_b(\tilde{\gamma}_n, \sigma_e^2, \sigma_p^2) = \frac{c_1 \beta_l}{\beta_l + c_2' \bar{\gamma}_p} \exp\left[\frac{c_2'(\bar{\gamma}_p - \tilde{\gamma}_n)}{\beta_l + c_2' \bar{\gamma}_p}\right] \tag{41}$$

where
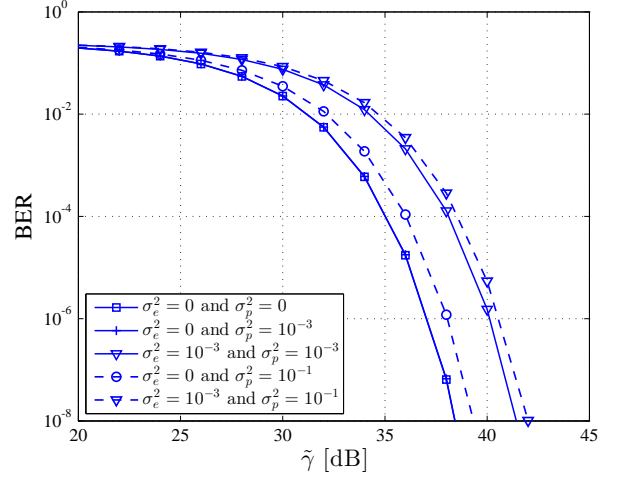
$$\beta_l = \left(2^{c_3 k_l} - c_4\right) \tag{42}$$

Figure 4. Impact of channel estimation and prediction error variances on the instantaneous BER of the 64-PSK modulation. The BER is evaluated for $\bar{\gamma}_0 = 30$ dB using the model 3 of [21] and equation (41).

The impact of both channel estimation and prediction error variances in the BER is shown in Fig. 4. In this figure, we have plotted the instantaneous BER of the 16-MPSK modulation versus the predicted SNR $\tilde{\gamma}_n$ for $\bar{\gamma}_0 = 30$ dB and different values of σ_e^2 and σ_p^2. It is shown that a small prediction error variance $\sigma_p^2 = 10^{-3}$ has a negligible impact. However, a loss of 1 dB is observed for $\sigma_p^2 = 10^{-1}$. The channel estimation error variance has a more noticeable effect. At a BER of 10^{-6}, the performance loss due to $\sigma_e^2 = 10^{-3}$ is about 3 dB.

The average BER of an adaptive M-PSK can be obtained using definition in [21]

$$\overline{\text{BER}}(\sigma_e^2, \sigma_p^2) = \frac{\sum_{l=0}^{N-1} k_l \int_{\gamma_l^s}^{\gamma_{l+1}^s} \bar{P}_b(\tilde{\gamma}_n, \sigma_e^2, \sigma_p^2) p(\tilde{\gamma}_n) d\tilde{\gamma}_n}{\sum_{l=0}^{N-1} k_l \int_{\gamma_l^s}^{\gamma_{l+1}^s} p(\tilde{\gamma}_n) d\tilde{\gamma}_n} \tag{43}$$

VI. SPECTRAL EFFICIENCY

The spectral efficiency of any modulation scheme is defined as the average data rate per unit bandwidth. When a M-PSK modulation of size $M_l = 2^{k_l}$ is used, the instantaneous throughput is k_l/T_s (bps). Assuming Nyquist data pulses (the transmitted signal bandwidth is $B = 1/T_s$), the spectral efficiency is then

$$\eta_B = \frac{R}{B} = \sum_{l=0}^{N-1} k_l \int_{\gamma_l^s}^{\gamma_{l+1}^s} p(\tilde{\gamma}) d\tilde{\gamma} \tag{44}$$

where R is the data rate.

Besides spectral efficiency, the system outage probability is an important metric for the performance analysis. The outage probability is defined as the probability that the received SNR is bellow the cutoff SNR γ_0^s. This latter corresponds to the minimum acceptable SNR which ensures the required BER.

In this section, we consider the maximization of the spectral efficiency with a constant transmit power and an instantaneous

BER constraint

$$\bar{P}_b(\tilde{\gamma}_n, \sigma_e^2, \sigma_p^2) \leq \overline{\text{BER}}_d \qquad (45)$$

This condition must be satisfied for all SNR in the corresponding region $\mathcal{R}_l = [\gamma_l^s, \gamma_{l+1}^s)$ [5]

$$\bar{P}_b(\tilde{\gamma}_n, \sigma_e^2, \sigma_p^2) \leq \bar{P}_b(\gamma_l^s, \sigma_e^2, \sigma_p^2) = \overline{\text{BER}}_d, \ \forall \tilde{\gamma} \in \mathcal{R}_l \quad (46)$$

Therefore, the optimal rate region boundaries can be obtained from (41) and (46)

$$\gamma_l^s = \bar{\gamma}_p - \frac{\beta_l + c_2'\bar{\gamma}_p}{c_2'} \ln\left[\frac{\left(\beta_l + c_2'\bar{\gamma}_p\right)}{c_1\beta_l}\overline{\text{BER}}_d\right] \qquad (47)$$

Figure 5 gives the optimal rate adaptation based on (47) for different values of σ_e^2 and σ_p^2. Dotted curves correspond to the ideal scenario with a perfect channel estimation and prediction which serves as a benchmark. The instantaneous number of bits per symbol $k(\tilde{\gamma}_n)$ increases as the instantaneous predicted SNR $\tilde{\gamma}_n$ increases. Once again, it can be seen that the channel estimation error has more impact than the prediction error. For a given $\overline{\text{BER}}_d$, σ_e^2 and σ_p^2, the optimum switching levels $\{\gamma_0^s, \gamma_1^s, \cdots, \gamma_N^s\}$ can be easily obtained from Fig. 5.
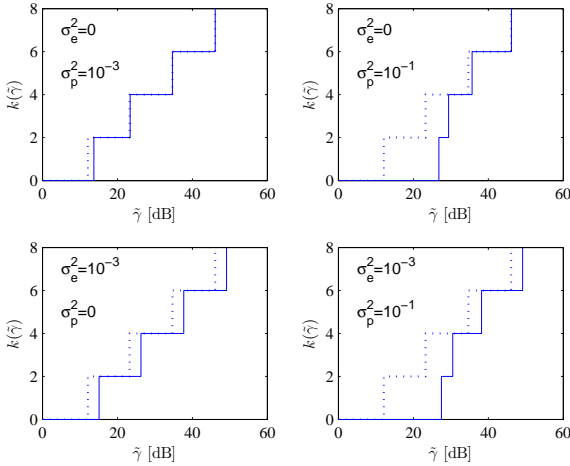

Figure 5. Optimal rate adaptation for $\overline{\text{BER}}_d = 10^{-4}$ and $\bar{\gamma}_0 = 30$.

The variation of the spectral efficiency as a function of $\bar{\gamma}_0$ for different values of σ_e^2 and σ_p^2 is depicted in Fig 6. Solid curves and dashed curves correspond respectively to target $\overline{\text{BER}}_d = 10^{-3}$ and $\overline{\text{BER}}_d = 10^{-6}$. At an SNR $\bar{\gamma}_0 = 35$ dB and for a $\overline{\text{BER}}_d = 10^{-3}$, an estimation error variance of $\sigma_e^2 = 10^{-3}$ reduces the spectral efficiency from 4 bps/Hz to 3 bps/Hz. The impact predication error $\sigma_p^2 = 10^{-3}$ is insignificant. Nevertheless, this estimation error variance has a noticeable impact on the outage probability specially for higher SNRs as it is illustrated in Fig 7. In fact, this value of σ_e^2 reduces the performance of the 4-PSK modulation and has a negligible impact on modulations with $M > 4$. This explains the degradation of the outage probability with a small estimation variance.
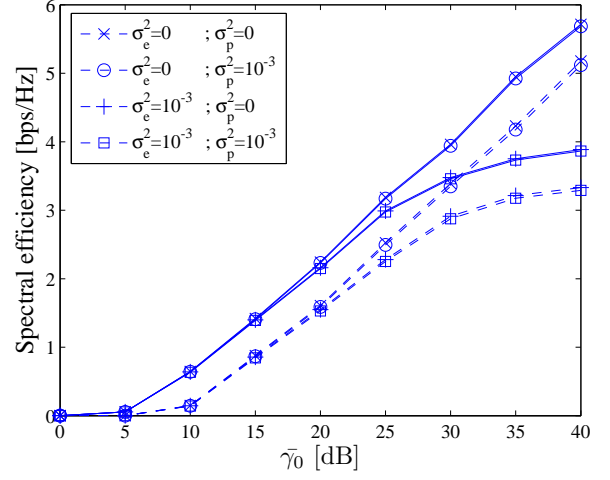

Figure 6. Spectral efficiency vs $\bar{\gamma}_0$ for $\overline{\text{BER}}_d = 10^{-3}$ (solid curves) and $\overline{\text{BER}}_d = 10^{-6}$ (dashed curves).
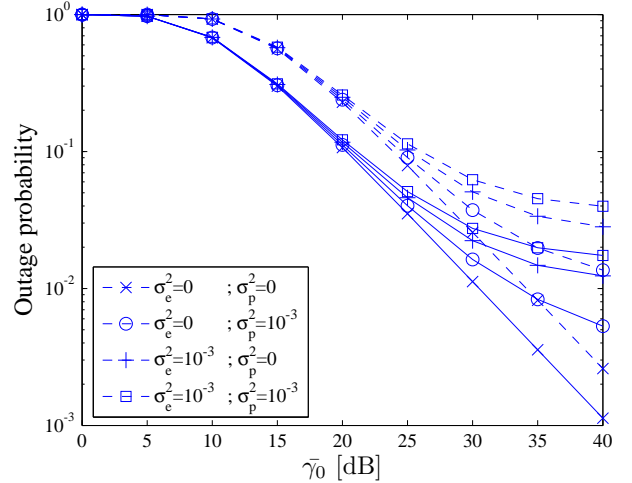

Figure 7. Outage probability vs $\bar{\gamma}_0$ for $\overline{\text{BER}}_d = 10^{-3}$ (solid curves) and $\overline{\text{BER}}_d = 10^{-6}$ $\overline{\text{BER}}_d = 10^{-6}$ (dashed curves).

VII. CONCLUSION

A commonly used coherent detection technique in a Rayleigh flat fading channel consists in dividing the received signal by the estimated fading in order to compensate for amplitude and phase deviations. In this paper, we investigated an adaptive M-PSK modulation scheme for Rayleigh flat fading channels when the received signal is corrected with imperfect channel estimates. The combined impact of channel estimation and channel prediction errors is evaluated.

The evaluation of the BER conditioned on the estimated and predicted values of the channel gain has been presented. The BER expression takes into account the channel estimation error variance. The receiver uses this BER expression to adjust the modulation size based on the required BER.

Unlike previous works, the modulation switching strategy that we propose is based on the power and statistics of the corrected signal. The influence of the imperfect AGC is

analyzed. In this paper, we have considered an adaptive rate system with a constant power. The presented analysis can be extended to variable power systems following [5], [21].

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