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Dynamic Management of Electrical Power Distribution Networks

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Original scientific paper

This paper considers the problem of optimal dynamic management of electrical power distribution networks with distributed generation and storage. Initially, analysis is performed of the Optimal Power Flow (OPF) problem – a paramount optimization problem that needs to be solved to ensure optimal steady-state power network operation. In the rest of the paper we present a hierarchical control structure for solving the considered optimal control problem in a dynamical framework. At the upper level a dynamic OPF solver computes the optimal power references for distributed generators and storages at slow rate. These references are then transmitted to the intermediate level, where a faster Model Predictive Control algorithm computes small deviations from power references given by the OPF solver to take into account the variability of load profiles that was neglected at the upper layer. Finally, the power references are forwarded to the primary level where local controllers track these power reference values. A realistic simulation case study of a Croatian power distribution grid is used for testing purposes and to demonstrate the applicability and usefulness of the proposed control strategy.

Key words: model predictive control, power distribution system, optimal power flow, convex relaxation, hierarchical control

Dinamičko upravljanje distribucijskim elektroenergetskim sustavima. U ovom članku razmatramo problem optimalnog dinamičkog upravljanja elektroenergetskim distribucijskim sustavom sa distribuiranom proizvodnjom i pohranom energije. Analizirali smo problem optimalnih tokova snage (OPF), koji je od najveće važnosti za razmatrani upravljački problem. U nastavku članka smo opisali hijerarhijsku upravljačku strukturu za rješavanje razmatranog problema optimalnog upravljanja. Na najvišoj razini (dinamički) OPF algoritam izračunava optimalne reference snage za distribuirane izvore i spremnike energije na sporoj vremenskoj skali. Te se reference zatim šalju srednjoj razini, gdje brži algoritam, temeljen na modelskom prediktivnom upravljanju, izračunava male devijacije od referenci koje je dobio od nadređene razine, kako bi uzeo u obzir brže varijacije profila potrošnje koje su zanemarene u nadređenoj razini. Konačno, reference se prosljeđuju najnižoj razini gdje se nalaze lokalni regulatori koji su zaduženi za njihovo praćenje. Realističan simulacijski ispitni primjer hrvatske elektroenergetske distribucijske mreže korišten je za ispitivanje i demonstraciju primjenjivosti i korisnosti predložene upravljačke strategije.

Ključne riječi: modelsko prediktivno upravljanje, distribucijski elektroenergetski sustav, optimalni tokovi snage, konveksna relaksacija, hijerarhijsko upravljanje

1 INTRODUCTION

The ever increasing demands for electrical energy, limited conventional fuel reserves, climate change, the desire for energy independence and diversification of energy sources, put in focus the distributed production of electrical energy from renewable sources as a key element in achieving a sustainable development. Since most of the electricity generated in developed countries is consumed in homes, buildings, and industry, the idea is to bring the distributed energy production closer to the end-consumers and that means to the power distribution level of the overall electrical power system. Hence, the power distribution system ceases to be a passive part of the electrical power system and starts to be actively involved in the production of electrical energy.

Despite all the advantages of distributed production of electrical energy, the rapidly growing penetration of intermittent renewable energy sources and other distributed sources poses vast challenges for electricity distribution systems. The challenges relate first of all to maintaining grid stability while adhering to the grid codes in order to ensure reliable and efficient power supply to all consumption entities spatially distributed over the distribution grid. Moreover, the dynamic interaction of locally managed components gives rise to complex dynamic behavior of the overall power distribution system and can lead to large-scale disruptions, i.e. black-outs in the electric grid, so the optimal operation of the system requires the designer to take this dynamic behavior into account.

With the lack of reliable and efficient inter-operation concepts that take this complex dynamic behavior into account, the distribution system operator (DSO) in current practice acts very conservatively and imposes additional costs to the potential investors for connection of units with production capacity to certain grid points in situations when worst-case simulations show that grid codes violations might occur. These costs can be too high and consequently the investor often gives up on building the unit. This significantly hinders the penetration of distributed (renewable) energy sources and is a standing obstacle to achieve the promised benefits of smart grids -– reduction of electricity losses, integration of renewable generation and storage, reduced use of fossil fuels, and improved grid reliability. The problem considered in this paper is, therefore, the problem of optimal dynamic management of electrical power distribution system with distributed generation and storage that will result in an efficient and reliable inter-operation of the power distribution system and all its components.

The optimal power flow (OPF) problem deals with finding the optimal electrical power network operating point, where the optimization objective is usually designated as the minimization of transmission losses, generation cost, etc. [1]. Mathematically, the exact solution to the OPF problem is very difficult to obtain for general AC networks due to its non-convex constraints. Recently, there have been considerable advances in convex relaxation approaches to solving the OPF problem that, under some assumptions, can guarantee a globally optimal solution. Semidefinite relaxations of the OPF problem are first proposed in [2] and [3]. Whether and when these convex relaxations are exact (i.e. when a globally optimal solution of the non-convex OPF problem can be recovered from the optimal solution of the convex relaxation of the OPF problem) is first studied in [4]. In [5], the OPF problem is formulated as a convex quadratic program (QP), although the results hold only for lossless radial networks.

One obvious drawback of the conventional OPF problem is the fact that it is essentially a static optimization problem, i.e. a steady-state operation of the power network is assumed. One reason for this is certainly a great computational complexity of the optimization procedure. In [6] the static OPF formulation is extended to include a slow charge/discharge dynamics of the storage units distributed in the network. The obtained optimization problem is formulated over a finite time horizon and a general procedure for solving this problem follows the convex relaxation approach from [4]. However, the dynamic storage model used in [6] is unrealistic as it neglects different efficiencies of the storage unit during charging and discharging.

Since the (dynamic) OPF problem cannot be solved on a time-scale where dynamics of generators are relevant, we propose a hierarchical three-layer control structure, similar to the control structure in [7], where such structure was used to control the voltage profiles in medium voltage networks with distributed generation. At the upper level of the control structure in [7], a static OPF problem is solved and the voltage and reactive power references at the nodes of the network are computed. Based on references from the upper layer, the intermediate level centralized controller computes the references for the local power factors of the distributed generators where standard local controllers track these references. The intermediate layer in [7] is designed as a Model Predictive Control (MPC) algorithm based on an impulse response model of the power network.

In the proposed hierarchical control structure a dynamic OPF algorithm is used at the upper level. The dynamic OPF formulation from [2] is extended to include a realistic dynamic model of storage units. We show that, despite the piece-wise affine (PWA) nature of the realistic storage model, it is possible to avoid the introduction of mixed-integer variables to the overall optimization problem. The objective of the optimization is to minimize the cumulative active power losses. The prediction of variable future load profiles is assumed to be available to the OPF controller, but at the slow time-scale ΔT_{opf} used in this level, i.e. the load profiles are assumed fixed between the two samples of the OPF controller. The secondary controller is designed as an MPC algorithm based on a state-space model of the power network linearized around the operating point computed by the OPF. The MPC algorithm is designed on a faster time-scale $\Delta T_{\text{mpc}} < \Delta T_{\text{opf}}$ to capture the dynamics of distributed generators. The MPC computes optimal power reference deviations from the references computed by the OPF to take into account the variability of the load profiles on a faster time-scale. Finally, active and reactive power references are sent to the low-level PI-type controllers of the individual generators and storage units. The main assumption here is that the DSO acts as an independent aggregator, dispatching flexibility providers in order to optimally utilize all network resources and comply with the technical constraints of the grid. Incentives/costs of this services are not the topic of the paper and they are assumed to be resolved through long terms contracts between DSO and service providers.

This paper is organized as follows. The conventional OPF problem, with the notation used in the rest of the paper, is presented in Section 2. Section 3 describes the proposed hierarchical control structure with focus on the design of the two upper layers. Finally, in Section 4, a realistic simulation case study based on a real-life power distribution grid located in the city of Koprivnica, Croatia, is used to illustrate the control method described in the paper.

2 PROBLEM SETUP

Consider a power network represented by the graph $G = (\mathcal{V}, \mathcal{E})$ and the set of generator buses $\mathcal{G} \subseteq \mathcal{V}$, where $\mathcal{V} := \{1, 2, \ldots, n\}$ is the set of nodes and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of flow lines (i, j), where $i, j \in \mathcal{V}$ and $i \neq j$. Let $N(i)$ denote a set of all nodes adjacent to node i, i.e. $N(i) := \{j \mid (i, j) \in \mathcal{E}\}$. Node 1 is designated as the root of the network and represents the substation node, e.g. the node that connects the distribution network to the rest of the power system. This node is also used to balance the active and reactive power in the network (in the literature known as the "slack bus"). We define the following variables and parameters of the system model:

- P_i^{D} and Q_i^{D}, the active and reactive power of the load connected to node $i \in \mathcal{V}$ ($P_i^{\mathrm{D}} = Q_i^{\mathrm{D}} = 0$ when there is no load at node i).
- P_i^{G} and Q_i^{G}, the active and reactive power of the generator connected to node $i \in \mathcal{V}$ ($P_i^{\mathrm{G}} = Q_i^{\mathrm{G}} = 0$ when $i \in \mathcal{V} \backslash \mathcal{G}$).
- V_i, the voltage magnitude at node $i \in \mathcal{V}$.
- θ_i, voltage angle at node i.
- θ_{ij}, voltage angle difference between nodes i and j, $(i, j) \in \mathcal{E}$, $\theta_{ij} = \theta_i - \theta_j$.
- P_{ij} and Q_{ij}, the active and reactive power transferred from node $i \in \mathcal{V}$ to the rest of the network through line $(i, j) \in \mathcal{E}$.

The circuit model of the power network can be derived by replacing every transmission line and transformer with their equivalent Π-models, see [8] for details. In this circuit model, for each line $(i, j) \in \mathcal{E}$, let z_{ij} denote its impedance (with $r_{ij} = \Re\{z_{ij}\}$ and $x_{ij} = \Im\{z_{ij}\}$), and $y_{ij} = z_{ij}^{-1}$ its admittance (with $g_{ij} = \Re\{y_{ij}\}$ and $b_{ij} = \Im\{y_{ij}\}$). In the rest of this chapter, in general, a variable without subscript denotes a vector with appropriate components, e.g. $P^{\mathrm{D}} := \left[P_1^{\mathrm{D}}, \ldots, P_n^{\mathrm{D}}\right]^{\mathrm{T}}$.

For every node $i \in \mathcal{V}$ of the network, the following power flow constraints on active and reactive power injection (P_i^{I} and Q_i^{I}, respectively) must be ensured:

$$P_i^{\mathrm{I}} = P_i^{\mathrm{G}} - P_i^{\mathrm{D}}, \ \forall i \in \mathcal{V}, \tag{1a}$$

$$Q_i^{\mathrm{I}} = Q_i^{\mathrm{G}} - Q_i^{\mathrm{D}}, \ \forall i \in \mathcal{V}, \tag{1b}$$

where

$$P_i^{\mathrm{I}} = \sum_{j \in N(i)} \left[g_{ij} V_i^2 - V_i V_j \left(g_{ij} \cos\theta_{ij} + b_{ij} \sin\theta_{ij}\right)\right], \tag{2a}$$

$$Q_i^{\mathrm{I}} = \sum_{j \in N(i)} \left[-b_{ij} V_i^2 + V_i V_j \left(b_{ij} \cos\theta_{ij} - g_{ij} \sin\theta_{ij}\right)\right]. \tag{2b}$$

The active and reactive power generation is bounded as:

$$\underline{P}_i \leq P_i^{\mathrm{G}} \leq \overline{P}_i, \ \forall i \in \mathcal{V}, \tag{3a}$$

$$\underline{Q}_i \leq Q_i^{\mathrm{G}} \leq \overline{Q}_i, \ \forall i \in \mathcal{V}, \tag{3b}$$

where, by definition, $\underline{P}_i = \overline{P}_i = \underline{Q}_i = \overline{Q}_i = 0$, $\forall i \in \mathcal{V} \backslash \mathcal{G}$.

The voltage magnitude at node $i \in \mathcal{V}$ lies within predefined lower and upper bounds $\underline{V}$ and $\overline{V}$, respectively:

$$\underline{V} \leq V_i \leq \overline{V}, \ \forall i \in \mathcal{V}. \tag{4}$$

For example, if voltages must not deviate by more than 5% from their nominal values, then $0.95 \leq V_i \leq 1.05$ p.u.

The network active power losses are equal to the difference between the total system active power generation and the total system active power consumption. Consequently, the network active power losses can be computed as a sum of active power injections at all nodes:

$$P_{\text{loss}} = \sum_{i \in \mathcal{V}} P_i^{\mathrm{I}} \tag{5}$$

The optimal power flow (OPF) problem can now be stated as follows:

$$\begin{aligned} \min_{V, \theta, P^{\mathrm{G}}, Q^{\mathrm{G}}} \quad & \sum_{i \in \mathcal{V}} P_i^{\mathrm{I}}, && (6a)\\ \text{s.t.} \quad & P_i^{\mathrm{I}} = P_i^{\mathrm{G}} - P_i^{\mathrm{D}}, \ \forall i \in \mathcal{V}, && (6b)\\ & Q_i^{\mathrm{I}} = Q_i^{\mathrm{G}} - Q_i^{\mathrm{D}}, \ \forall i \in \mathcal{V}, && (6c)\\ & \underline{P}_i \leq P_i^{\mathrm{G}} \leq \overline{P}_i, \ \forall i \in \mathcal{V}, && (6d)\\ & \underline{Q}_i \leq Q_i^{\mathrm{G}} \leq \overline{Q}_i, \ \forall i \in \mathcal{V}, && (6e)\\ & \underline{V} \leq V_i \leq \overline{V}, \ \forall i \in \mathcal{V}. && (6f) \end{aligned}$$

2.1 Convex relaxation of the OPF problem

The objective of the optimization problem (6) is to minimize the active power losses while satisfying the operating and physical constraints. In other words, it aims to find the optimal active and reactive power references for all generators which will ensure the supply of energy to all consumers with minimum losses in the network and all voltages kept in a predefined safe interval. Note that additional constraints on maximum power flows through different lines can be easily added but are omitted here for brevity. The optimization problem (6) is nonconvex but a convex relaxation which recovers the exact solution at optimality can be obtained by reformulating the problem as a convex second-order cone program (SOCP) [2]. The following new variables need to be defined:

$$v_i = \frac{V_i^2}{\sqrt{2}},\ \forall i \in \mathcal{V}, \tag{7a}$$

$$R_{ij} = V_i V_j \cos\theta_{ij},\ \forall (i,j) \in \mathcal{E}, \tag{7b}$$

$$T_{ij} = V_i V_j \sin\theta_{ij},\ \forall (i,j) \in \mathcal{E}. \tag{7c}$$

In terms of these new variables, (6) becomes:

$$\min_{v,R,T,P^{\mathrm{G}},Q^{\mathrm{G}}} \sum_{i\in\mathcal{V}} P_i^{\mathrm{I}}, \tag{8a}$$

$$\text{s.t.} \quad P_i^{\mathrm{I}} = P_i^{\mathrm{G}} - P_i^{\mathrm{D}},\ \forall i \in \mathcal{V}, \tag{8b}$$

$$Q_i^{\mathrm{I}} = Q_i^{\mathrm{G}} - Q_i^{\mathrm{D}},\ \forall i \in \mathcal{V}, \tag{8c}$$

$$2v_i v_j \geq R_{ij}^2 + T_{ij}^2,\ \forall (i,j) \in \mathcal{E}, \tag{8d}$$

$$R_{ij} \geq 0,\ \forall (i,j) \in \mathcal{E}, \tag{8e}$$

$$\underline{P}_i \leq P_i^{\mathrm{G}} \leq \overline{P}_i,\ \forall i \in \mathcal{V}, \tag{8f}$$

$$\underline{Q}_i \leq Q_i^{\mathrm{G}} \leq \overline{Q}_i,\ \forall i \in \mathcal{V}, \tag{8g}$$

$$\frac{\underline{V}^2}{\sqrt{2}} \leq v_i \leq \frac{\overline{V}^2}{\sqrt{2}},\ \forall i \in \mathcal{V}, \tag{8h}$$

where

$$P_i^{\mathrm{I}} = \sum_{j\in N(i)} \left(\sqrt{2} g_{ij} v_i - g_{ij} R_{ij} - b_{ij} T_{ij}\right), \tag{9a}$$

$$Q_i^{\mathrm{I}} = \sum_{j\in N(i)} \left(-\sqrt{2} b_{ij} v_i + b_{ij} R_{ij} - g_{ij} T_{ij}\right). \tag{9b}$$

Note that the convex relaxation (8) is exact if and only if the cone constraint (8d) is binding at the optimal point. The condition can be easily checked after the optimization is finished. The inequality (8e) follows from (7b) and the observation that the angle difference θ_{ij} between adjacent nodes is not expected to be greater than $\pi/2$ in absolute value for real-life power distribution networks. Furthermore, note that the objective (8a) will force values of R_{ij} to increase (beacuse $g_{ij} > 0$ in (8b)) and this should result in the cone constraint (8d) being active at the optimal point. The OPF formulation (8) is static but it can be extended to include dynamics [9].

3 CONTROL STRUCTURE

As can be seen from (6), the conventional OPF problem is assuming the steady-state operation of the network, i.e. (6) is a static optimization problem. The dynamics of generators are neglected while the load profiles are assumed constant. In reality, load profiles vary through the day considerably, so the power generation references computed by the OPF (6) (with the assumption of fixed loads) cannot be considered optimal when a change in load profiles occurs.

Therefore, to ensure optimality, one should solve the OPF problem whenever loads change, taking into account dynamics of generators on a prediction horizon to ensure the compliance with all operating constraints (e.g. voltage magnitude constraints). This would generate an optimization problem that needs to be solved at every time sample, which is intractable for large-scale networks.

To circumvent this difficulty, we propose a hierarchical three-layer control structure (see Fig. 1), similar to the control structure in [7]. At the upper layer (tertiary control) slow OPF-based controller computes optimal power references while at the intermediate layer (secondary control) a faster centralized model-based predictive controller computes optimal deviations from power references given by the OPF controller to take into account the variability of load profiles between the two time samples of the OPF controller. Finally, at the lower layer, these power references are forwarded to local regulators of active and reactive power at each distributed generator. Since the design of the lower control layer is quite standard (simple PI-type regulators) [8], we focus on the design of two upper layers.

3.1 Tertiary control layer

The non-convexity of constraints in (2) make the OPF problem in (6) non-convex and thus very hard to solve. Recently there have been considerable advances in convex relaxation approaches to solving this problem. These methods, under some conditions, can guarantee a global optimality of the recovered solution. Our tertiary control layer is designed by convexification of (6), as already outlined in Subsection 2.1. It was shown in [10] that under some mild technical conditions, the OPF problem for radial networks can be solved exactly via a second-order cone program (SOCP), a convex optimization problem which can be solved to a global optimum reliably by using openly available solvers (e.g. SDPT3 or SeDuMi) , cf. [4].

The objective of the tertiary control layer is to minimize the cumulative active power losses in the network. The underlying OPF problem is solved periodically with a relatively large time step ΔT_{opf} (e.g $\Delta T_{\mathrm{opf}} = 60$ min), to provide enough time for solving it. Predictions of the future load profiles are assumed to be available to tertiary controller. Since the OPF formulation assumes fixed loads, the mean load profiles over next ΔT_{opf} minutes are used in computations. The solution of the optimization, namely

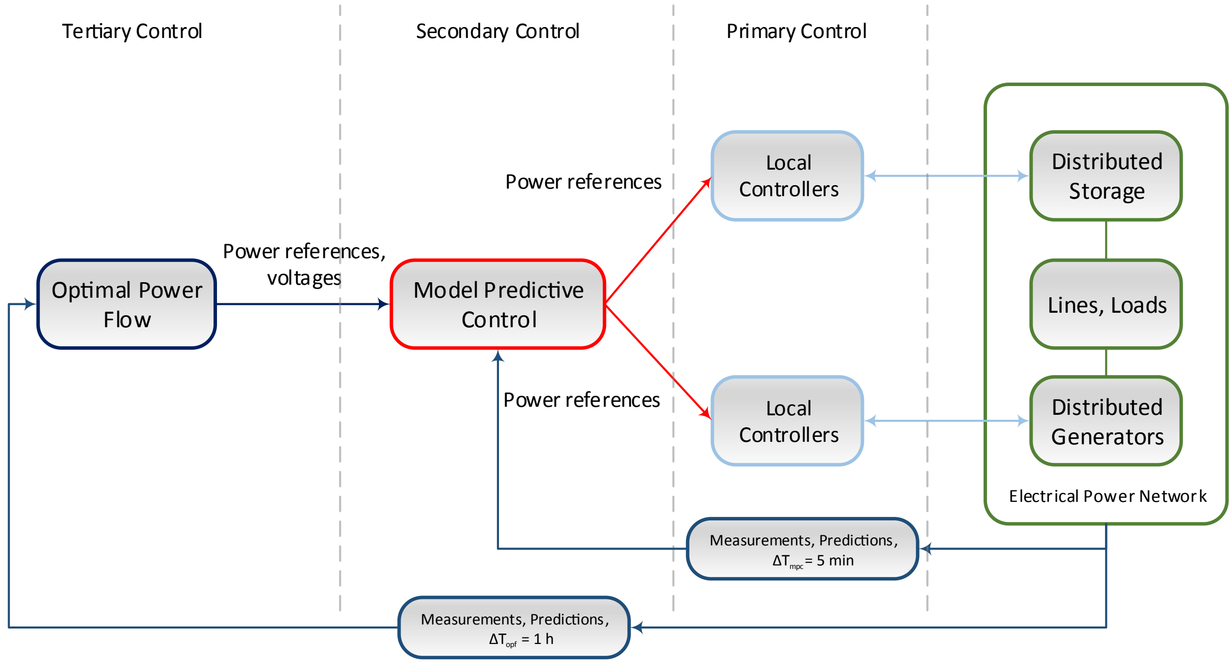


Fig. 1. Centralized hierarchical control structure.

optimal active and reactive power references for distributed generators, are forwarded to the secondary control layer.

3.2 Extension of tertiary control layer with slow dynamics

Next, we extend the baseline tertiary controller described in Subsection 3.1 to include slow dynamics of distributed storages. We consider the following discrete time model of a storage unit connected to bus $i \in \mathcal{B}$:

$$x_i^{\mathrm{b}}(t+1) = x_i^{\mathrm{b}}(t) - \eta_i r_i(t), \tag{10a}$$

$$\eta_i = \begin{cases} \eta_i^{\mathrm{c}}, & \text{if } r_i(t) < 0 \text{ (charging)}, \\ 1/\eta_i^{\mathrm{d}}, & \text{otherwise (discharging)}, \end{cases} \tag{10b}$$

$$-\overline{r}_{\mathrm{c}} \le r_i(t) \le \overline{r}_{\mathrm{d}}, \tag{10c}$$

$$\underline{q} \le q_i^{\mathrm{b}}(t) \le \overline{q}, \tag{10d}$$

$$0 \le x_i^{\mathrm{b}}(t) \le \overline{B}_i, \tag{10e}$$

where $\mathcal{B} \subseteq \mathcal{N}$ denotes a set of nodes where a storage unit is connected, $x_i^{\mathrm{b}}(t)$ denotes the level of the energy stored at time t, $r_i(t)$ is the rate of charge and discharge, η_i accounts for losses during charging and discharging ($\eta_i^{\mathrm{c}}, \eta_i^{\mathrm{d}} \le 1$), $\overline{r}_{\mathrm{c}}, \overline{r}_{\mathrm{d}} > 0$ are maximum rates of charge and discharge, q_i^{b} is the reactive storage power inflow/outflow which is bounded between $\underline{q}$ and $\overline{q}$, and $\overline{B}$ is the maximum storage level.

The OPF formulation from [4] is extended in [6] to integrate simple charge and discharge dynamics for energy storage distributed over network. A similar storage model to (10) is used in [6] but with one important difference: the losses during charging and discharging are neglected [6], i.e. they use $\eta_i = 1$. In the remainder of this subsection we extend the OPF formulation (8) to include a realistic storage model (10).

A similar storage model to (10) was also used in [11] to implement a model predictive control approach for microgrid operation optimization. However, in [11] the optimal control problem is formulated by following a standard procedure from [12] which leads to a mixed-integer optimization problem. One can avoid introduction of integer optimization variables to the optimization problem with introduction of an equivalent description of storage dynamics to (10):

$$x_i^{\mathrm{b}}(t+1) = x_i^{\mathrm{b}}(t) - \eta_i^{\mathrm{c}} r_i^{\mathrm{c}}(t) - \frac{1}{\eta_i^{\mathrm{d}}} r_i^{\mathrm{d}}(t), \tag{11a}$$

$$-\overline{r}_{\mathrm{c}} \le r_i^{\mathrm{c}}(t) \le 0, \tag{11b}$$

$$0 \le r_i^{\mathrm{d}}(t) \le \overline{r}_{\mathrm{d}}, \tag{11c}$$

$$\underline{q} \le q_i^{\mathrm{b}}(t) \le \overline{q}, \tag{11d}$$

$$0 \le x_i^{\mathrm{b}}(t) \le \overline{B}_i, \tag{11e}$$

where r_i^{c} is the rate of charge and r_i^{d} is the rate discharge. Note that the equivalence with the mixed-integer model (as in [11]) is ensured if only one of these rates is non-zero at the optimum. This condition will be ensured, e.g. if the storage state x_i^{b} does not appear in the overall optimization objective function. However, it should be pointed out that, in practical application, any scenario where both $r_i^{\mathrm{c}}(t)$ and $r_i^{\mathrm{d}}(t)$ are non-zero at the optimal solution (for some t and any i), would indicate unrealistic objective function, since this implies that it pays out to charge and discharge the storage at the same time!

The inclusion of storage dynamics in the baseline static OPF formulation leads to a finite-horizon optimal control problem that enables optimization over time. The objective function is stated as:

$$\sum_{t=0}^{N-1} \sum_{i \in \mathcal{V}} P_i^{\mathrm{I}}(t), \tag{12}$$

where N is the length of the prediction horizon.

For brevity we omit derivation of the extended (dynamic) OPF problem formulation and solution procedure. The extension is rather straightforward. The optimization problem (8) is defined on a prediction horizon and storage model constraints (11) are added to its list of constraints. Finally, the active and reactive power injections of battery storage are added to the power balance constraints (8b) and (8c). The overall structure of the optimization problem as well as all the results following from the relaxation procedure stay the same as before.

3.3 Secondary control layer

The secondary control layer is designed as a Model Predictive Control (MPC) algorithm based on a linearized state-space model of distributed generators interconnected by the electrical power network. For simplicity, all distributed generators are modeled as synchronous machines driven by steam turbines (e.g. a biomass power generation facility). Since the modeling of these components and the derivation of the overall linearized model of multimachine system is well known in the literature, the details are omitted here and the interested reader is referred to [8]. The final discrete time model of each distributed generator at bus $i \in \mathcal{G}$ is of the form:

$$\Delta x_i(t+1) = A_i \Delta x_i(t) + B_i \Delta u_i(t), \tag{13a}$$

$$\Delta y_i(t) = C_i \Delta x_i(t), \tag{13b}$$

where Δx_i is the state vector, Δu_i the input vector, $\Delta y_i = \left[\Delta P_i^{\mathrm{G}}, \Delta Q_i^{\mathrm{G}}\right]^T$ the output vector, and Δ indicates that the model is linearized around the operating point computed by the OPF solver. The time sample used for the discretization is ΔT_{mpc}. Inputs to the model are active and reactive power references, while the outputs are actual active and reactive power injections to the network.

In addition to the standard constraints in the MPC formulation that the state vector and the input vector must lie in some predefined sets, e.g. $x_i \in \mathcal{X}_i$ and $u_i \in \mathcal{U}_i$, additional constraints imposed by the electrical power network must also be considered, i.e. power flow and voltage constraints. The power flow constraints are non-convex and including them in the MPC formulation would generate a non-convex optimization problem that needs to be solved at every time sample. Theoretically, the MPC formulation could be solved via convex relaxation approach discussed in Subsection 3.1. However, for real-life power networks, with large numbers of nodes and generators, this optimization problem cannot be solved at the fast sample rate of the secondary control layer.

To make the MPC algorithm more tractable, we linearize the following nonconvex constraint around the operating point (computed by the tertiary control):

$$2v_i v_j = R_{ij}^2 + T_{ij}^2. \tag{14}$$

By applying small perturbation and neglecting second-order terms involving perturbed values one can obtain the following expression:

$$v_{j0}\Delta v_i + v_{i0}\Delta v_j = R_{ij0}\Delta R_{ij} + T_{ij0}\Delta T_{ij}, \tag{15}$$

where the variables with zero subscript denote values of respective variables at the operating point around which the linearization is performed.

We are now ready to formulate the MPC optimization problem on a given prediction horizon N. By defining objective functions at fixed time sample t as follows:

$$J_1(t) = \sum_{i \in \mathcal{V}} \Delta P_i^{\mathrm{I}}(t), \tag{16a}$$

$$J_2(t) = \sum_{i \in \mathcal{G}} \left\| C_i \Delta x_i(t) \right\|_1, \tag{16b}$$

where function $J_1(t)$ denotes the total losses in network with regard to the operating point computed by the tertiary control layer, i.e. $\Delta P_i^I(t)$ is the deviation of active power injection at bus i in time t from the reference $P_{i0}^{\mathrm{I}}(t)$ computed by the OPF solver. Similarly, $\Delta P_i^{\mathrm{D}}(t)$ is the deviation of load profile at bus i in time t from the fixed load profile P_{i0}^{D} assumed by the OPF solver. Function $J_2(t)$ is used to penalize too large deviations of active and reactive power injections at generator nodes from the power references computed by the OPF.

The following optimization problem represents the MPC formulation of our secondary control layer:

$$\min \sum_{t=1}^{N} \alpha J_1(t) + \beta J_2(t), \tag{17a}$$

$$\text{s.t.} \quad \Delta x_i(t+1) = A_i \Delta x_i(t) + B_i \Delta u_i(t), \tag{17b}$$

$$\Delta x_i(t) \in \mathcal{X}_i,\ \Delta u_i(t) \in \mathcal{U}_i,\ \forall i \in \mathcal{G}, \tag{17c}$$

$$\Delta P_i^{\mathrm{I}}(t) = \Delta P_i^{\mathrm{G}}(t) - \Delta P_i^{\mathrm{D}}(t), \forall i \in \mathcal{V}, \tag{17d}$$

$$\Delta Q_i^{\mathrm{I}}(t) = \Delta Q_i^{\mathrm{G}}(t) - \Delta Q_i^{\mathrm{D}}(t), \forall i \in \mathcal{V}, \tag{17e}$$

$$V_{ij0}^{\mathrm{T}} \Delta V_{ij}(t) = S_{ij0}^{\mathrm{T}} S_{ij}(t),\ \forall (i,j) \in \mathcal{E}, \tag{17f}$$

$$\Delta R_{ij}(t) \geq 0,\ \forall (i,j) \in \mathcal{E}, \tag{17g}$$

$$\underline{P}_i^{\mathrm{G}}(t) \leq P_{i0}^{\mathrm{G}} + \Delta P_i^{\mathrm{G}}(t) \leq \overline{P}_i^{\mathrm{G}}(t),\ \forall i \in \mathcal{V} \tag{17h}$$

$$\underline{Q}_i^{\mathrm{G}}(t) \leq Q_{i0}^{\mathrm{G}} + \Delta Q_i^{\mathrm{G}}(t) \leq \overline{Q}_i^{\mathrm{G}}(t),\ \forall i \in \mathcal{V} \tag{17i}$$

$$\frac{\underline{V}^2}{\sqrt{2}} \leq v_{i0} + \Delta v_i(t) \leq \frac{\overline{V}^2}{\sqrt{2}},\ \forall i \in \mathcal{V}, \tag{17j}$$

where

$$\Delta P_i^{\mathrm{I}} = \sum_{j \in N(i)} \sqrt{2} g_{ij} \Delta v_i - g_{ij} \Delta R_{ij} - b_{ij} \Delta T_{ij}, \tag{18a}$$

$$\Delta Q_i^{\mathrm{I}} = \sum_{j \in N(i)} -\sqrt{2} b_{ij} \Delta v_i + b_{ij} \Delta R_{ij} - g_{ij} \Delta T_{ij}, \tag{18b}$$

$$V_{ij0} = [v_{j0},\ v_{i0}]^{\mathrm{T}},\ \Delta V_{ij} = [\Delta v_i,\ \Delta v_j]^{\mathrm{T}}, \tag{18c}$$

$$S_{ij0} = [R_{ij0},\ T_{ij0}]^{\mathrm{T}},\ \Delta S_{ij} = [\Delta R_{ij},\ \Delta T_{ij}]^{\mathrm{T}}. \tag{18d}$$

The optimization variables in (17) are $\Delta P_i^{\mathrm{G}}(t)$, $\Delta Q_i^{\mathrm{G}}(t)$, $\Delta x_i(t)$, $\Delta u_i(t)$, $\Delta v_i(t)$, $\Delta R_{ij}(t)$, and $\Delta T_{ij}(t)$. Real scalars α and β are used to fine-tune the optimization. Sets $\mathcal{X}_i$ and $\mathcal{U}_i$ are defined as simple polytopic constraints.

4 SIMULATION CASE STUDY

This section illustrates the proposed control method for dynamic management of electrical power networks with distributed generation and storage which are described in Section 3, on a simulation example shown in Fig. 2.

The considered network is a representation of a part of an actual power distribution grid from the city of Koprivnica, Croatia. It comprises thirteen nodes, where node 1 is the slack node, i.e. it provides the balance of power in the grid. All nodes are in the medium voltage level (110 kV, 35 kV and 10 kV). Data for all nodes are given in Table 1.

There are three transformers in the grid. Table 2 summarizes their parameters: series resistance, series inductance, nominal voltage at winding 1, nominal voltage at winding 2, and nominal power rating. A 110/35 transformer is located between nodes 1 and 2. It connects the considered power distribution network to the external grid (i.e. the transmission level of the power system). The external grid is modeled as a voltage source of constant voltage and constant frequency (typically referred to as an *infinite bus*, [8]).

Transmission line data are given in Table 3. Each line is characterized by its series resistance, series reactance, and length. The topology of the network is radial, which is typical for power distribution systems.

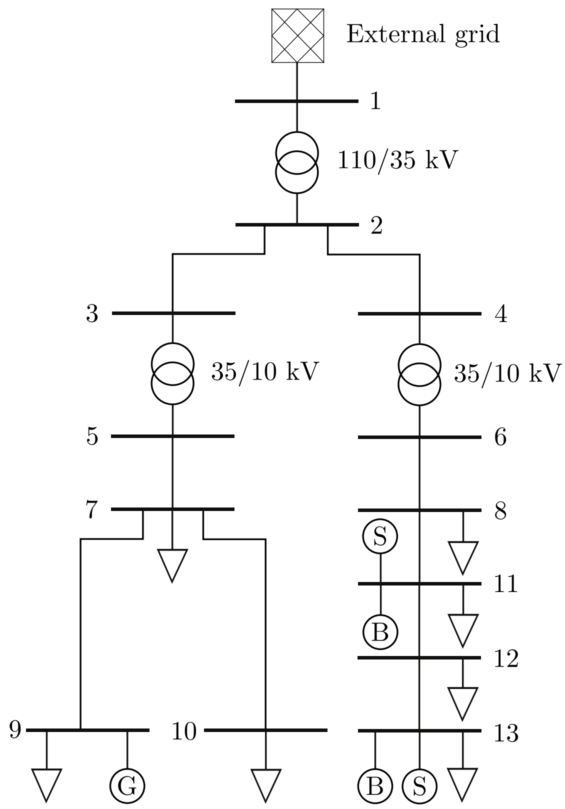


Fig. 2. Benchmark grid topology.

There are three controllable distributed generators in the grid: one combustion turbine generator at node 9 (denoted by "G") and two photovoltaic generators at nodes 11 and 13 (denoted by "S"). For simplicity, a second-order model of a combustion turbine generator is used, neglecting faster electro-magnetic dynamics and thus focusing on slower mechanical dynamics that are relevant at the secondary control layer. In particular, we use simple model from [13], discretized at sampling time of 5 [min]. The maximum active and reactive power of the turbine generator is set to 0.8 [MVA]. The rated power of both photovoltaic generators is 100 [kW]. Additionally, there are also two battery storage systems at nodes 11 and 13. Both battery storage systems have maximum capacity of 1 [MWh] with maximum rate of charge/discharge of 250 [kW]. For simplicity, we use $\eta_i^{\mathrm{c}} = \eta_i^{\mathrm{d}} = 0.9$.

The control problem considered here aims to minimize the cumulative active power losses in the network. Control variables are active and reactive power references for distributed generator (labeled with "G" in Fig. 2) and distributed storage units (labeled with "B" in Fig. 2). The predictions of future load profiles, depicted in Fig. 3 and future photovoltaic power generation profiles, depicted in Fig. 4, are assumed to be available. The profiles in Fig. 3

Table 1. Node data: peak active and reactive power demand, nominal voltage magnitude, and voltage constraints (maximum and minimum deviation from the nominal value).

Node num.	P^{D} [MW]	Q^{D} [MVar]	V_{nom} [kV]	$\overline{V}$ [p.u.]	$\underline{V}$ [p.u.]
1	-	-	110	1.05	0.95
2	-	-	35	1.05	0.95
3	-	-	35	1.05	0.95
4	-	-	35	1.05	0.95
5	-	-	10	1.05	0.95
6	-	-	10	1.05	0.95
7	0.1277	0.0372	10	1.05	0.95
8	0.1219	0.0363	10	1.05	0.95
9	0.1302	0.0380	10	1.05	0.95
10	0.0757	0.0221	10	1.05	0.95
11	0.0683	0.0199	10	1.05	0.95
12	0.1065	0.0225	10	1.05	0.95
13	0.1020	0.0227	10	1.05	0.95

Table 2. Transformer data: IDs of endpoint nodes, series resistance and inductance, nominal voltage at winding 1, nominal voltage at winding 2, and nominal power rating.

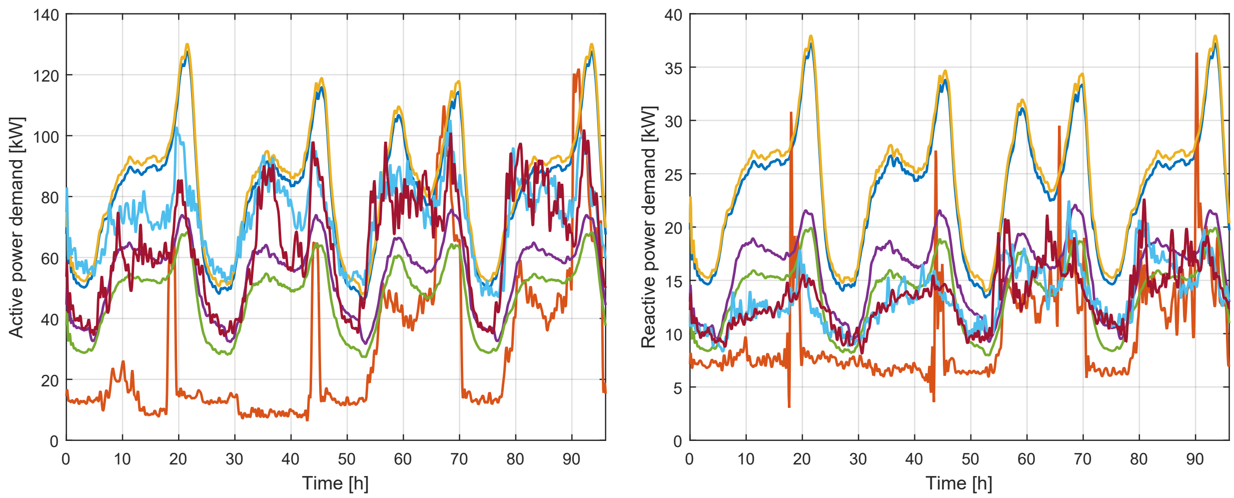
From node	To node	R [Ω]	L [H]	U_{n_1} [kV]	U_{n_2} [kV]	S_{nom} [MVA]
1	2	1.3000	0.1040	110	35	40
3	5	1.0311	0.0339	35	10	8
4	6	1.0311	0.0339	35	10	8

Table 3. Line data: IDs of endpoint nodes, series resistance and reactance per unit length, and length.

From node	To node	R [Ω/km]	X [Ω/km]	L [km]
2	3	0.0870	0.1160	0.3224
2	4	0.1380	0.1160	0.1360
5	7	0.2060	0.1220	1.3500
7	9	0.1930	0.1000	0.4000
7	10	0.2060	0.1220	0.4150
6	8	0.2060	0.1220	1.0420
8	11	0.3200	0.1000	0.1610
11	12	0.1930	0.1000	0.2870
12	13	0.3200	0.1000	0.0870

and Fig. 4 represent load predictions with a 5 minutes sampling rate that are available to the MPC controller. At the OPF control layer one hour averaged predictions are used. The sampling rates used are accordingly $\Delta T_{\mathrm{opf}} = 60$ minutes and $\Delta T_{\mathrm{mpc}} = 5$ minutes. The profiles depicted in Figs. 3 and 4 were provided by the Koprivnica distribution system operator (HEP-ODS, Elektra Koprivnica) and represent real-life measurements during four consecutive days.

The dynamic model of the case study is implemented in Matlab. Furthermore, Matlab toolbox YALMIP [14] and CPLEX solver (version 12.6.0.0) were used for setting up and solving the optimization problems discussed in this paper. Two simulations were run and results compared. In one simulation the MPC control layer was not used (this scenario is denoted with shortcut OPF) while in the other the complete hierarchical control structure was used to control the power network (this scenario is denoted with shortcut OPF+MPC). Fig. 5 shows the results. Nodal voltages during the simulation are shown in Fig. 5(a) (for brevity and clarity, nodal voltages are shown only for the OPF+MPC scenario). Fig. 5(b) shows cumulative active power losses in the network resulting from both simulation scenarios. The inclusion of intermediate MPC layer in the overall control structure proved to be beneficial because it succeeds to generate less (on average) active power losses than the baseline OPF controller. The total losses over the entire simulation in the OPF scenario were 67.12 [kWh], while in the OPF+MPC scenario these losses were reduced to 60.85 [kWh]. Thus, by utilizing the entire hierarchical control structure the cumulative active power losses were reduced by 9.34%.



(a) Active power demand. (b) Reactive power demand.

Fig. 3. Typical nodal power demand profiles in the city of Koprivnica, Croatia, during four consecutive days.

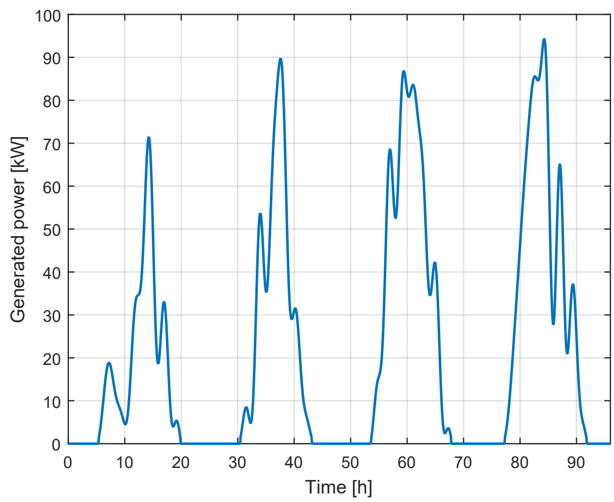


Fig. 4. Typical photovoltaic power generation profile during four consecutive days.

5 CONCLUSION

A hierarchical control structure for optimal dynamic management of electrical power networks with distributed generation and storage is presented. It comprises optimal power flow (OPF) computation at the highest level, model predictive control (MPC) at the intermediate level, and fixed local network controllers at the bottom level. After analysis of the conventional steady-state OPF problem, we extended it to a dynamic optimization framework by considering realistic dynamic models of storage units. It was shown that the overall optimization problem can be formulated without introduction of integer variables even in the case of a piece-wise affine model of the storage dynamics. At the intermediate level of the control structure we define and solve an MPC problem based on a state-space model of the system linearized around the operating point computed by the upper level OPF controller. The proposed control approach was tested on a realistic case study of the power network of the city of Koprivnica. Obtained results illustrate that, with the inclusion of the dynamics of the electrical power system, one can significantly improve the overall system performance.

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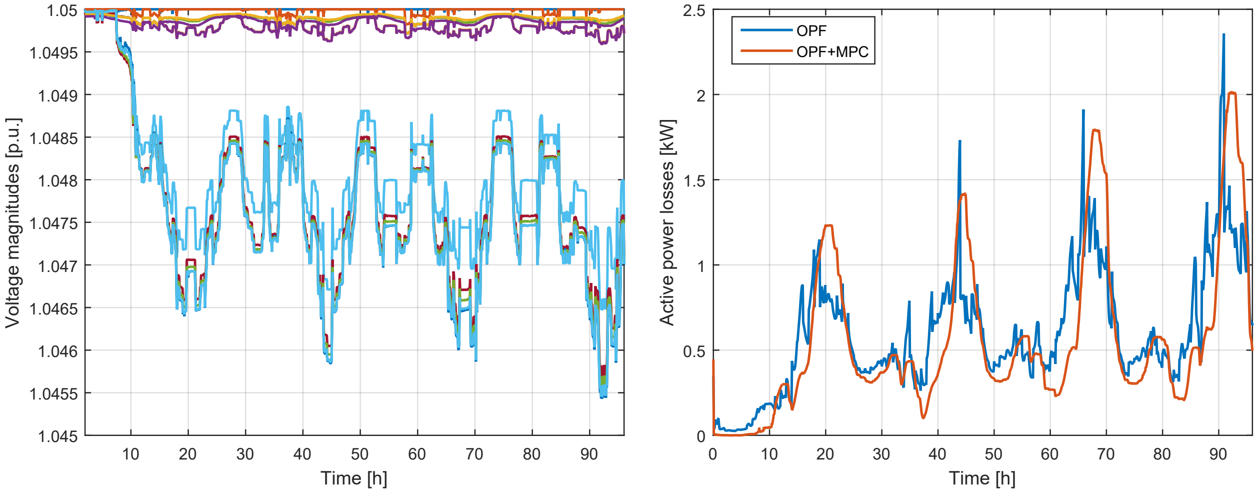

(a) Nodal voltages during the simulation. (b) Total grid active power losses during the simulation.

Fig. 5. Simulation results: (a) nodal voltages are always kept within predefined limits, (b) total grid active power losses in two simulation scenarios (one where only OPF control layer is used to control the grid and the other where the entire hierarchical control structure is utilized, i.e. both OPF and MPC).

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