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Half step position sensorless control of a Linear Switched Reluctance Motor based on back EMF

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Original scientific paper

This paper presents a position sensorless closed loop control of a switched reluctance linear motor. The aim of the proposed control is to damp the position of the studied motor. Indeed, the position oscillations can harm some applications requiring high position precision. Moreover, they can induce the linear switched reluctance motor to an erratic working. The proposed control solution is based on back Electromotive Forces which give information about the oscillatory behaviour of the studied motor and avoid the use of a cumbersome and expensive position linear sensor. The determination of the designed control law parameters was based on the singular perturbation theory. The efficiency of the proposed control solution was proven by simulations and experimental tests.

Key words: Linear switched reluctance motor, back EMF, position sensorless control, damping, large scale system, singular perturbation, order reduction

Polukoračno upravljanje pozicijom linearnog prekidačko-reluktantnog motora bez senzora bazirano na stražnjim elektromagnetskim silama. Ovaj rad predstavlja upravljanje linearnim prekidačko-reluktantnim motorom u zatvorenoj petlji bez senzora. Cilj predloženog upravljanja je prigušiti oscilacije pozicije navedenog motora. Oscilacije pozicije mogu štetno utjecati na aplikacije koje zahtijevaju visoku preciznost pozicije. Također oscilacije mogu dovesti do nepravilnog rada motora. Predloženo rješenje upravljanja zasniva se na stražnjim elektromotornim silama koje daju informaciju o oscilatornom vladanju navedenog motora čime se izbjegava potreba za skupim i nezgrapnim linearnim senzorom pozicije. Određivanje parametara upravljanja je bazirano na singularnoj teoriji perturbacije. Efikasnost predloženog sustava upravljanja dokazana je pomoću simulacijskih i eksperimentalnih testiranja.

Ključne riječi: Linearni prekidačko-reluktantni motor, stražnji EMF, pozicijsko upravljanje bez senzora, prigušenje, sustav velikih razmjera, singularna perturbacija, reduciranje reda

1 INTRODUCTION

A Linear Switched Reluctance Motor (LSRM) is an excellent solution for positioning applications that need rapid acceleration and high-speed. A LSRM is characterized by its mechanical simplicity and simple open-loop operation. It represents the linear counterpart of the rotary switched reluctance motor. The linear motor is an alternative to conventional rotary to linear conversion devices such as lead-screws or belt drives which give to the motor some velocity and acceleration limitations. With a linear motor, the force generated by the machine is directly applied to the load [1-3].

A LSRM can have several mechanical structures which can be double sided, flatted or tubular. The studied actuator is a LSRM characterized by four phases and having a tubular structure. Its cylindrical mechanical geometry allows simplicity of construction. In addition, it is simple to use and it is characterized by its accuracy of positioning in open-loop operation. It offers solutions to a variety of applications requiring high linear positioning precision. Nowadays, it is widely used in automobile industry, machine tools, robotics domains and electronic industry [4,5].

Open loop position control can be applied easily to a LSRM so that it acts as a stepper motor. However, the position response of the LSRM is generally very oscillatory. In application requiring accurate positioning, this poorly damped response can be a great disadvantage [4,5]. If the inertia of the motor raises, overshoot and oscillation increase. These characteristics disturb the normal operation of this kind of motors. At some speeds, the magnitude of oscillatory response increases with time. As a result, the motor can lose synchronism inducing dynamic instability and erratic working [3].

There are numerous solutions allowing the reduction of the LSRM oscillations which can be classified into mechanical solutions and control solutions [3,4].

As a mechanical solution, it is possible to introduce additional viscous friction. However, this solution gives to the motor some speed and acceleration limitations and reduces the linear motor nominal force [3,4].

The application of a closed loop control needs the use of some sensors. For the case of a LSRM, the position sensor is expensive and cumbersome [6-8].

One solution consists into a position sensorless control. Several research works proposed the use of back EMF as feedback information for rotary motors control to avoid the use of a position or a speed sensor. In [9], a position sensorless closed loop control of brushless DC motors with low inductance and weak back EMF is proposed. In [10], a position observer based on back EMF estimation is developed for permanent synchronous rotary motors.

Sensorless control solutions based on back EMF were also applied for linear motors and especially for permanent magnet linear motors. In [11] and in [12], the authors designed a control strategy using back EMF for linear hybrid stepper motors speed control. In [13], a sensorless control method of a tubular linear permanent magnet motor based on a linear observer is developed.

As in [14-17], several position estimation methods based on back EMF have been reported for rotary switched reluctance motors. But for LSRMs, there are few literatures referring to position estimation based on back EMF so far. Rotary and linear switched reluctance motors are similar in term of principle of operation. But they have different mechanical structures. Thus, an accurate estimation of LSRM position is a difficult challenge because of this kind of linear machines specificities.

We propose, in this paper, a specific position sensorless closed loop half step control solution for a LSRM based on the motor back EMF. The main contribution of the proposed control solution is that it increases the friction to reduce the plunger oscillations without the estimation of the plunger position or speed. An analytical study based on singular perturbation theory and allowing the control law parameters determination, is proposed.

This paper is organized as follows; the studied motor structure and mathematical model are presented in section 2. Section 3 is reserved to the problem statement. The proposed control law is explained in section 4 and its efficiency is proven, by simulation and experimental tests, in section 5.

2 STUDIED MACHINE MATHEMATICAL MODEL

The studied LSRM, presented in Fig.1, has four electrically identical phases, denoted by A, B, C and D, separated by a non-magnetic material. The linear motor moving element is called forcer or plunger, and the stationary one is called stator. The stator and the plunger are regularly toothed. A LSRM operates on the same electromagnetic principles as a rotary switched reluctance machine. When a current passes through one phase winding the plunger tooth tends to align the stator tooth; that is, it produces a force that tends to move the plunger to a minimum reluctance position.

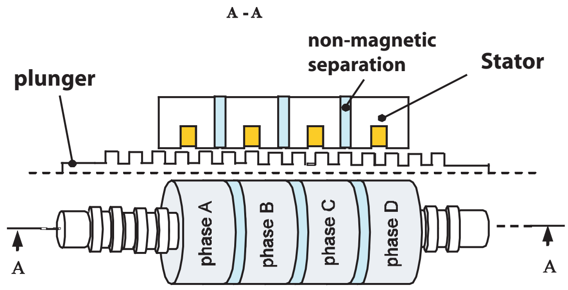


Fig. 1. Studied Linear stepping motor structure

The mathematical model the LTSRSM was elaborated by neglecting the mutual inductances because of the non-magnetic separation between the four phases and by supposing that the four phase inductances and the generated forces are sinusoidal. Thus, it is given by the following nonlinear differential equations [18,19]:

$$u_A = Ri_A + L_A(x)\frac{di_A}{dt} + e_{mA} \tag{1}$$

$$u_B = Ri_B + L_B(x)\frac{di_B}{dt} + e_{mB} \tag{2}$$

$$u_C = Ri_C + L_C(x)\frac{di_C}{dt} + e_{mC} \tag{3}$$

$$u_D = Ri_D + L_D(x)\frac{di_D}{dt} + e_{mD} \tag{4}$$

$$\begin{aligned} m\tfrac{dv}{dt} &= F_A(x) + F_B(x) + F_C(x) + F_D(x) \\ &\quad -F_0 sgn(v) - \xi v - F_c \end{aligned} \tag{5}$$

$$\frac{dx}{dt} = v \tag{6}$$

i_A, i_B, i_C and i_D are the phase currents, u_A, u_B, u_C and u_D are the phase voltages. x is the plunger position. v is the plunger speed. F_c is the load force. e_{mA}, e_{mB} , e_{mC} and e_{mD} are the back EMF of the phases A, B, C and D defined as follows:

$$e_{mA} = \frac{2\pi}{\lambda}L_1 \sin(\frac{2\pi x}{\lambda})vi_A \tag{7}$$

$$e_{mB} = \frac{2\pi}{\lambda}L_1 \sin(\frac{2\pi x}{\lambda} - \frac{\pi}{2})vi_B \tag{8}$$

$$e_{mC} = \frac{2\pi}{\lambda} L_1 \sin(\frac{2\pi x}{\lambda} - \pi) v i_C \quad (9)$$

$$e_{mD} = \frac{2\pi}{\lambda} L_1 \sin(\frac{2\pi x}{\lambda} - \frac{3\pi}{2}) v i_D \quad (10)$$

$F_A(x)$, $F_B(x)$, $F_C(x)$ and $F_D(x)$ are the forces generated by excitation of the phases A, B, C and D respectively. They are expressed as follows:

$$F_A(x) = -\frac{\pi i_A^2 L_1}{\lambda} \sin\left(\frac{2\pi x}{\lambda}\right) \quad (11)$$

$$F_B(x) = -\frac{\pi i_B^2 L_1}{\lambda} \sin\left(\frac{2\pi x}{\lambda} - \frac{\pi}{2}\right) \quad (12)$$

$$F_C(x) = -\frac{\pi i_C^2 L_1}{\lambda} \sin\left(\frac{2\pi x}{\lambda} - \pi\right) \quad (13)$$

$$F_D(x) = -\frac{\pi i_D^2 L_1}{\lambda} \sin\left(\frac{2\pi x}{\lambda} - \frac{3\pi}{2}\right) \quad (14)$$

$L_A(x)$, $L_B(x)$, $L_C(x)$ and $L_D(x)$ are the inductances of the phases A, B, C and D, respectively. They are given by the following expressions:

$$L_A(x) = L_0 + L_1 \cos(\frac{2\pi x}{\lambda}) \quad (15)$$

$$L_B(x) = L_0 + L_1 \cos(\frac{2\pi x}{\lambda} - \frac{\pi}{2}) \quad (16)$$

$$L_C(x) = L_0 + L_1 \cos(\frac{2\pi x}{\lambda} - \pi) \quad (17)$$

$$L_D(x) = L_0 + L_1 \cos(\frac{2\pi x}{\lambda} - \frac{3\pi}{2}) \quad (18)$$

sgn is the signum function defined as follows:

$$sgn(x) = \begin{cases} 1 & if\ x > 0 \\ 0 & if\ x = 0 \\ -1 & if\ x < 0 \end{cases} \quad (19)$$

The mathematical model described by the differential equations (1), (2), (3), (4), (5) and (6) was simulated by using Matlab software and resolved by the Runge Kutta algorithm.

The electrical and mechanical characteristics of the studied LSRM are given in Table 1 [20-22].

Table 1. Studied machine characteristics

Nominal voltage	U_n=*18V*
Phase resistance	$R = 18\,\Omega$
Phase inductance	$L_0 = 225mH$
Amplitude of the phase inductance	$L_1 = 50mH$
Nominal current	*1A*
Maximum current	*1,5A*
Air gap	*1mm*
Total course length	*100mm*
Plunger weight	$m = 5Kg$
Maximum force	15.5N
Tooth size	$\lambda = 10,16mm$
Maximal speed	*1 m/s*
Dynamic viscosity coefficient	$\xi = 65N.m/s$
Dry friction force	$F_0 = 0.1N$

3 PROBLEM STATEMENT

Half step position control mode is a classical open loop control that improves the resolution of stepper motors. It consists into the simultaneous excitation of two phases followed by the excitation of one phase. We propose in the following to apply this control technique to the studied LSRM.

For the case of the studied actuator, half step control voltage sequence is presented in Fig. 2. Let us consider that initially the phase A is excited, and that the initial equilibrium position is $x = 0\ mm$, Fig. 3. The excitation of the phases A and B drives the plunger to an intermediate equilibrium position resulting from the equality of the two opposite forces $\overrightarrow{F}_1$and $\overrightarrow{F}_2$ generated by the phases A and B, respectively. Thus, the obtained equilibrium position is $x = \frac{\lambda}{8} = 1,27\,mm$, Fig.4. The excitation of the phase B drags the plunger to the natural equilibrium position $x = \frac{\lambda}{4} = 2,54\,mm$ obtained by the alignment of the plunger and the stator teeth, Fig.5.

Figure 6 presents the simulated position evolution obtained by application of the half step voltage sequence. For both full step and half step positions, we can observe some oscillations and overshoots. Moreover, half step position responses present more oscillations and a larger overshoot than those obtained for natural equilibrium positions.

We propose in the following section a closed loop control to damp the half step position responses of the studied LSRM.

4 PROPOSED CONTROL LAW

4.1 Basic Idea of the proposed control law

Our aim is to design a closed loop control law allowing the elimination of the studied actuator position oscillations without the use of a position sensor. Thus, we propose to use the back EMF as feedback information of the oscillatory behaviour of the studied motor.

Let us consider the case of a half step position evolution. Figure 7 and Figure 8 present the position evolution and the back EMF of the phase B respectively. We can notice that back EMF gives information about the oscillatory behaviour of the studied actuator. Indeed, when the

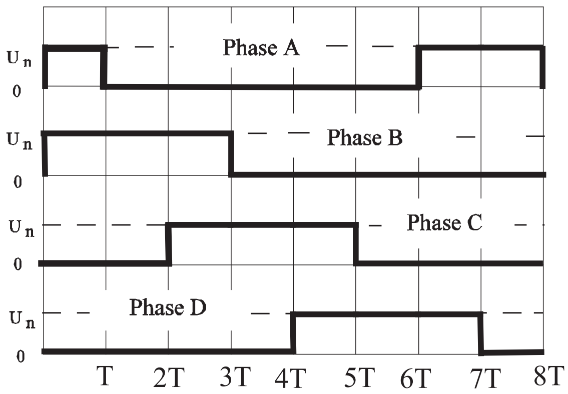


Fig. 2. Half step voltage sequence

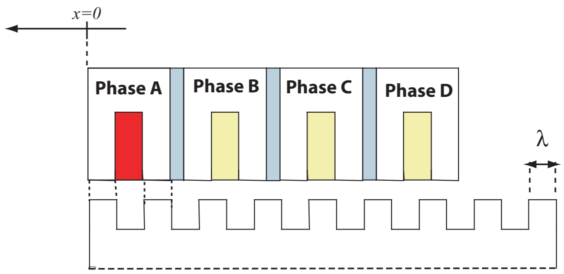


Fig. 3. Position of the LTSRM by excitation of the phase A

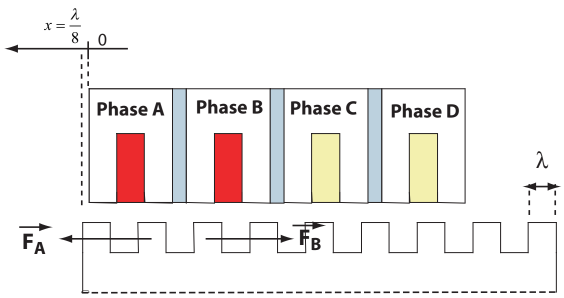


Fig. 4. Position of the LTSRM by excitation of the phase B

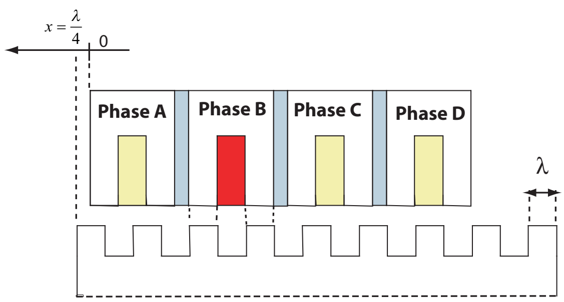


Fig. 5. Position of the Position of the LTSRM by excitation of the phase C

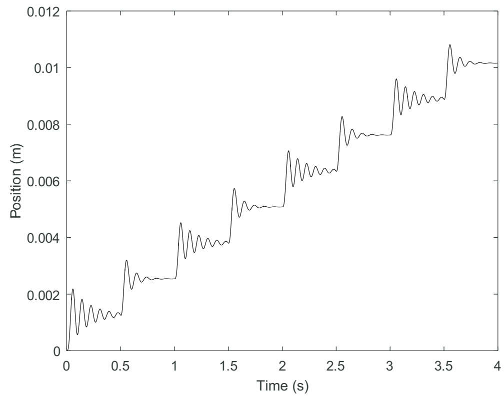


Fig. 6. Open loop half Step Position evolution

plunger oscillates the back EMF oscillates as well. When the plunger attends the equilibrium position, back EMF becomes null [19-21].

Our objective is to increase the motor friction by the application of a specific control law based on the half step control technique described in section 3.

For the case of a LSRM, the force developed when a phase is proportional to the square of the current. Thus, we propose to impose in phases A and B the currents $i_A(v)$ and $i_B(v)$ having the following expressions:

$$i_A(v) = \sqrt{\left(\frac{U_n}{R}\right)^2 + K_m \frac{e_{mA}}{i_A}} \tag{20}$$

$$i_B(v) = \sqrt{\left(\frac{U_n}{R}\right)^2 + K_m \frac{e_{mB}}{i_B}} \tag{21}$$

which can be rewritten as follows:

$$i_A(v) = \sqrt{\left(\frac{U_n}{R}\right)^2 + K_m \frac{2\pi}{\lambda} L_1 \sin(\frac{2\pi x}{\lambda}) v} \tag{22}$$

$$i_B(v) = \sqrt{\left(\frac{U_n}{R}\right)^2 + K_m \frac{2\pi}{\lambda} L_1 \sin(\frac{2\pi x}{\lambda} - \frac{\pi}{2}) v} \tag{23}$$

By replacing the current expressions (22) and (23) in (5) we obtain:

$$\begin{aligned} m\tfrac{dv}{dt} = & -\tfrac{\pi L_1}{\lambda}\Big[\Big(\big(\tfrac{U_n}{R}\big)^2 + K_m \tfrac{2\pi}{\lambda} L_1 \sin(\tfrac{2\pi x}{\lambda}) v\Big)\sin(\tfrac{2\pi x}{\lambda}) \\ & + \Big(\big(\tfrac{U_n}{R}\big)^2 + K_m \tfrac{2\pi}{\lambda} L_1 \sin(\tfrac{2\pi x}{\lambda} - \tfrac{\pi}{2}) v\Big)\sin(\tfrac{2\pi x}{\lambda} - \tfrac{\pi}{2})\Big] \\ & - F_0 signe(v) - \xi v - F_C \end{aligned} \tag{24}$$

Equation (24) can be rewritten as follows:

$$\begin{aligned} m\tfrac{dv}{dt} = & F_m(x) - 2\ \left(\tfrac{\pi}{\lambda} L_1\right)^2 K_m\, v - F_0 signe(v) \\ & -\xi v - F_C \end{aligned} \tag{25}$$

where $F_m(x)$ is the force developed by excitation of the phases B and A. It is defined as follows:

$$F_m(x) = -\frac{\pi L_1 I_n^2}{\lambda}\left(\sin(\frac{2\pi x}{\lambda}) + \sin(\frac{2\pi x}{\lambda} - \frac{\pi}{2})\right) \quad (26)$$

We notice from (25) that the friction is increased by the term $2\left(\frac{\pi}{\lambda}L_1\right)^2 K_m$.

Thus, it is possible to increase the friction by generating the currents in the motor phases having the form described by (20) and (21).

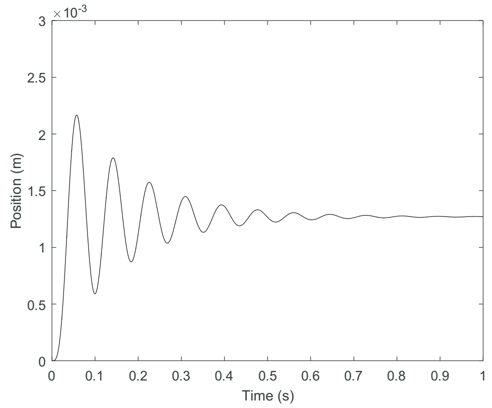


Fig. 7. Half step position evolution

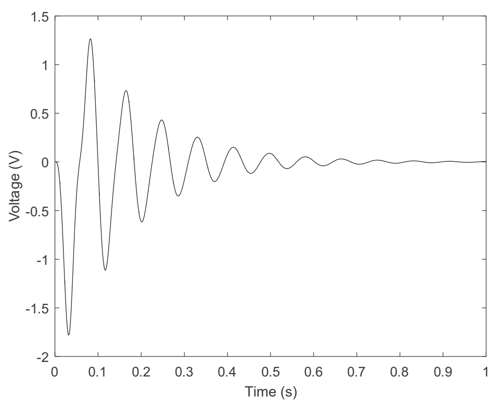


Fig. 8. Phase B Back EMF evolution

4.2 Principle of the proposed control law

The proposed control law is an extension of the classical half step control. In fact, two phases are excited for both full step and half step equilibrium positions. During the transient regime, the first phase is used to drag the plunger to the equilibrium position. The second one is the braking phase. It is used to absorb the kinetic energy developed when the pull phase is excited. The pull phase and the braking one are defined in Table 2 for one electrical cycle.

Table 2. Pull and braking phases definition

Step order	Pull phase	Braking phase
1^{st}half step	B	A
1^{st} step	B	A
2^{nd} half step	C	B
2^{nd} step	C	B
3^{rd} half step	D	C
3^{rd} step	D	C
4^{th} half step	A	D
4^{th}step	A	D

For a half step position, the pull and the braking phases must be excited with the voltages u_j^{ref}, where j denotes the corresponding phase. To obtain a full step position, the pull phase must be excited with the voltage u_j^{ref} and the braking one with the voltage v_j^{ref}.

u_j^{ref} and v_j^{ref} are defined as follows:

$$u_j^{ref} = (i_{1j}(v) - i_j)\,K_i + U_n \quad (27)$$

$$v_j^{ref} = (i_{2j}(v) - i_j)\,K_i \quad (28)$$

$i_{1j}(v)$ and $i_{2j}(v)$are the currents that we have to impose and to regulate in the motor phases. They are expressed as follows:

$$i_{1j}(v) = \sqrt{\left(\frac{U_n}{R}\right)^2 + K_m\,\frac{e_{mj}}{i_j}} \quad (29)$$

$$i_{2j}(v) = \sqrt{K_m\,\frac{e_{mj}}{i_j}} \quad (30)$$

$i_{1j}(v)$ and $i_{2j}(v)$ oscillate around zero during the position transient regime. However, when the plunger attends its equilibrium position ($v = 0$), we have $i_{1j}(v) = \frac{U_n}{R} = I_n$ and $i_{2j}(v) = 0$.

Figure 9 and Figure 10 present the closed loop regulation principle of the currents $i_{1j}(v)$ and $i_{2j}(v)$ respectively.

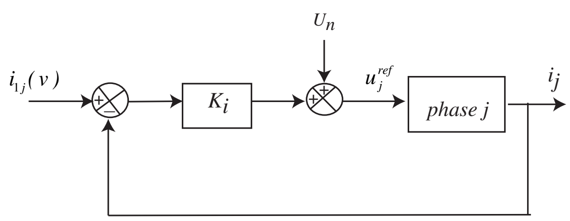


Fig. 9. Current $i_{1j}(v)$ regulation loop

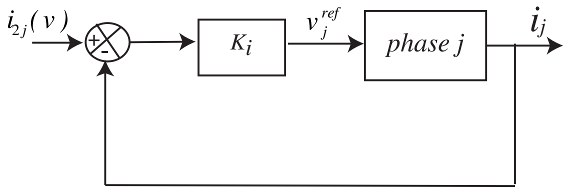


Fig. 10. Current $i_{2j}(v)$ regulation loop

Table 3. Phase order excitation

Step order	Equilibrium position	$\mathbf{u_A^C}$	$\mathbf{u_B^C}$	$\mathbf{u_C^C}$	$\mathbf{u_D^C}$
1^{st}half step	$\frac{\lambda}{8}$	u_A^{ref}	u_B^{ref}	0	0
1^{st} step	$\frac{\lambda}{4}$	v_A^{ref}	u_B^{ref}	0	0
2^{nd} half step	$\frac{3\lambda}{8}$	0	u_B^{ref}	u_C^{ref}	0
2^{nd} step	$\frac{\lambda}{2}$	0	v_B^{ref}	u_C^{ref}	0
3^{rd} half step	$\frac{5\lambda}{8}$	0	0	u_C^{ref}	u_D^{ref}
3^{rd} step	$\frac{3\lambda}{4}$	0	0	v_C^{ref}	u_D^{ref}
4^{th} half step	$\frac{7\lambda}{8}$	u_A^{ref}	0	0	u_D^{ref}
4^{th}step	λ	u_A^{ref}	0	0	v_D^{ref}

Table 3 defines, for one electrical cycle, the control voltages for each equilibrium position. The proposed closed loop control is presented in Fig.11. It consists into measuring the voltages and the currents of the studied actuator phases to estimate the generated back EMF. Once the control voltages calculated, they are transformed into PWM signals with duty cycles τ_j defined as follows:

$$\tau_j = \frac{u_j^C}{V_{in}} \tag{31}$$

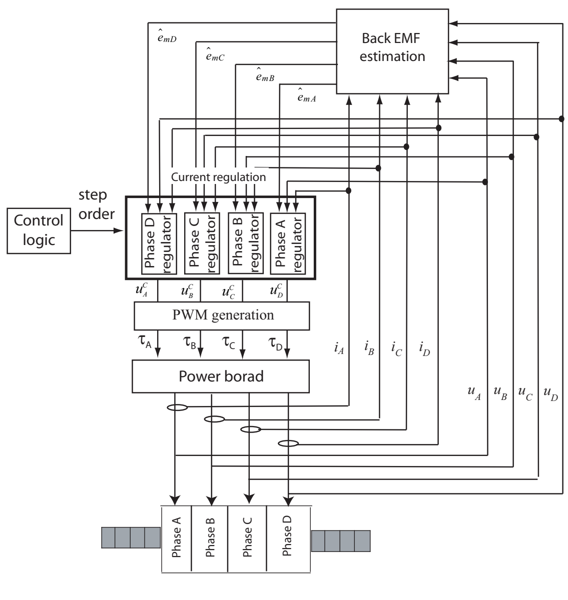


Fig. 11. Block diagram of the proposed control

In the following, we will propose a method allowing the determination of the parameters K_m and K_i.

4.3 Determination of the parameters K_m and K_i

The proposed control method depends on the two gains K_m and K_i. The gain K_i involved in the current regulation loops. The gain K_m affects the viscosity friction of the studied motor.

The studied motor model is nonlinear. So to determine the control parameters, we propose, as a first simplification, to linearise the controlled model. Let us consider the case of a half step position evolution obtained by the simultaneous excitation of the phases A and B respectively by application of the voltages u_A^C and u_B^C defined in the table 2 and by (27) and (29). For this case, the equilibrium position, in steady regime, is defined by $i_A = 1\,A$, $i_B = 1\,A$, $v = 0\ m.s^{-1}$and $x = \frac{\lambda}{8}$. By considering the state space vector $X = \begin{pmatrix} i_A & i_B & v & x \end{pmatrix}^T$, the linearization of the studied motor model around the equilibrium position $X_0 = \begin{pmatrix} 1 & 1 & 0 & \frac{\lambda}{8} \end{pmatrix}^T$ leads to the following state space representation:

$$\overset{\bullet}{\overline{X}} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \overline{X} \tag{32}$$

where $\overline{X}$, A_{11}, A_{12}, A_{21} and A_{22} are defined as follows: $\overline{X} = \begin{pmatrix} \overline{i}_A & \overline{i}_B & \overline{x} & \overline{v} \end{pmatrix}^T = X - X_0$

$$A_{11} = \begin{pmatrix} -\frac{R+K_i}{L_0+\frac{\sqrt{2}}{2}L_1} & 0 \\ 0 & -\frac{R+K_i}{L_0+\frac{\sqrt{2}}{2}L_1} \end{pmatrix}$$

$$A_{12} = \begin{pmatrix} \frac{\frac{\sqrt{2}\pi L_1}{\lambda}\left(\frac{K_i K_m}{2}-1\right)}{L_0+\frac{\sqrt{2}}{2}L_1} & 0 \\ -\frac{\frac{\sqrt{2}\pi L_1}{\lambda}\left(\frac{K_i K_m}{2}-1\right)}{L_0+\frac{\sqrt{2}}{2}L_1} & 0 \end{pmatrix}$$

$$A_{21} = \begin{pmatrix} -\frac{\sqrt{2}\pi L_1}{2m\lambda} & \frac{\sqrt{2}\pi L_1}{2m\lambda} \\ 0 & 0 \end{pmatrix}$$

$$A_{22} = \begin{pmatrix} -\frac{\xi}{m} & -2\sqrt{2}\frac{L_1}{m}\left(\frac{\pi}{\lambda}\right)^2 \\ 1 & 0 \end{pmatrix}$$

As a second simplification, we propose to reduce the order of the studied system by considering only the mechanical state variables x and v.

As an electromechanical system, the studied actuator is a two time scale system characterized by a fast subsystem and a slow one [23]. For the case of an electrical motor, the currents can be considered as the fast variables the position and the speed can be regarded as the slow ones.

We propose in the following to apply the singular perturbation theory to separate the fast and the slow dynamics of the studied motor in order to determine the parameters K_i and K_m.

Singular perturbation technique allows the order reduction of a large scale system to a smaller one. It lowers the model order by first neglecting the fast phenomenon [23-26].

A_{11} is associated to the currents $\bar{i}_A$ and $\bar{i}_B$. A_{22} is related to $\overline{x}$ and $\overline{v}$. Block diagonal matrix, A_{11} and A_{22} have an influence on the fast and slow dynamics respectively. Indeed, the fast eigenvalues of $A_{11}$and A_{22} approach the fast and the slow eigenvalues of the large system, respectively [26]. However, we cannot reduce the scale large system by neglecting the fast dynamic and by considering A_{22} as the reduced model state matrix.

To reduce the order of the studied system, some conditions must be fulfilled. Indeed, the system described by (32) is singularly perturbed, if the following condition is satisfied [23,26]:

$$\left\|A_{11}^{-1}\right\|\left(\left\|A_{22}-A_{21}A_{11}^{-1}A_{12}\right\|+\left\|A_{21}A_{11}^{-1}\right\|\left\|A_{12}\right\|\right)<\frac{1}{3} \tag{33}$$

where $\|.\|$ denotes a matrix norm.

For such case, the slow dynamics can be studied separately from the fast one and it is given by the following relation [26]:

$$\dot{x}_{2l}=\left(A_{22}-A_{21}A_{11}^{-1}A_{12}\right)x_{2l} \tag{34}$$

where $x_{2l}=\begin{pmatrix}\widetilde{v} & \widetilde{x}\end{pmatrix}^T$with $\widetilde{x}$ and $\widetilde{v}$ are estimates of $\overline{x}$ and $\overline{v}$.

For the studied LSRM motor, if K_i and K_m satisfy the condition (33), the slow system dynamic can be approximated by the following second order differential equation:

$$\begin{aligned}&\frac{d^2\widetilde{x}_l}{dt^2}+\frac{1}{m}\left(\xi+\frac{4\pi^2L_1^2}{\lambda^2}\frac{\left(\frac{K_iK_m}{2}-1\right)}{R+K_i}\right)\frac{d\widetilde{x}_l}{dt}\\&=-\frac{2\pi^2}{\lambda^2}\frac{\sqrt{2}L_1}{m}\widetilde{x}_l\end{aligned} \tag{35}$$

From (35) we can identify the undamped natural frequency ω_n and the damping ratio $z$of the reduced system which are defined respectively as follows:

$$\omega_n=\frac{\pi}{\lambda}\sqrt{\frac{2\sqrt{2}L_1}{m}} \tag{36}$$

$$z=\frac{\xi}{2m\omega_n}+\frac{\omega_nL_1}{\sqrt{2}}\frac{\left(\frac{K_iK_m}{2}-1\right)}{R+K_i} \tag{37}$$

From (37) we can notice that the two parameters have an influence on the damping ratio of the reduced system. To get a damped response, we can impose $z=1$. Thus, the conditions that allow the reduction of the system order and the damping of the position response of the studied motor are given by the following system:

$$\begin{cases}z=1\\\left\|A_{11}^{-1}\right\|\left(\left\|A_{22}-A_{21}A_{11}^{-1}A_{12}\right\|+\left\|A_{21}A_{11}^{-1}\right\|\left\|A_{12}\right\|\right)<\frac{1}{3}\end{cases} \tag{38}$$

The condition $z=1$ leads to the following expression:

$$\frac{K_mK_i-1}{K_i+R}=\frac{\sqrt{2}}{L_1\omega_n}\left(1-\frac{\xi}{2m\omega_n}\right) \tag{39}$$

By using the infinity norm, we can establish that:

$$\begin{aligned}&\left\|A_{11}^{-1}\right\|_\infty\cdot\left(\left\|A_{22}-A_{21}A_{11}^{-1}A_{12}\right\|_\infty+\left\|A_{21}A_{11}^{-1}\right\|_\infty\left\|A_{12}\right\|_\infty\right)\\&=\frac{L_0+\frac{\sqrt{2}}{2}L_1}{K_i+R}\left(\frac{\xi}{m}+\omega_n^2\left(1+2\sqrt{2}L_1\frac{\frac{K_mK_i}{2}-1}{K_i+R}\right)\right)<\frac{1}{3}\end{aligned} \tag{40}$$

By using (39), the inequality (40) can be expressed as follows:

$$3\left(L_0+\frac{\sqrt{2}}{2}L_1\right)\left(4\omega_n-\frac{\xi}{m}+\omega_n^2\right)-R<K_i \tag{41}$$

To determine the parameters K_m and K_i the nonlinear model of the studied actuator was linearized and reduced. The reduced model presenting the slow dynamic of the LSRM is described by (34). If K_m and K_i satisfy the conditions (38), the slow dynamic of the motor, represented by the position and the speed, can be separated from the fast one, represented by the currents. For such case the position responses obtained from the reduced model, represented by (34), and the nonlinear large scale model, represented by (1), (2), (3) and (4), must be comparable.

Figure 12 presents the simulated position responses obtained from the nonlinear and entire studied model and the reduced one by application of the proposed control law for $K_m=0.95$and $K_i=2500$. For such case, K_m and K_i satisfy the conditions (34). We can observe from Fig.12 that the two curves have nearly the same rate and appearance during the transient regime.

However, if K_m and $K_i$not satisfy the separability condition ($K_m=0.95$ and $K_i=1000$) the obtained simulation results presented in Fig.13 show that, during the transient regime, the position responses dynamics are different. This result is quite normal because the reduced model is not valid.

4.4 Back EMF voltage estimation

Back EMF voltage is defined as follows:

$$e_{mj}=u_j-Ri_j-L(x)\frac{di_j}{dt} \tag{42}$$

The inductances of the LSRM motor, expressed by (15), (16), (17) and (18), depend on the motor position. Such characteristic makes the determination of e_{mj} difficult.

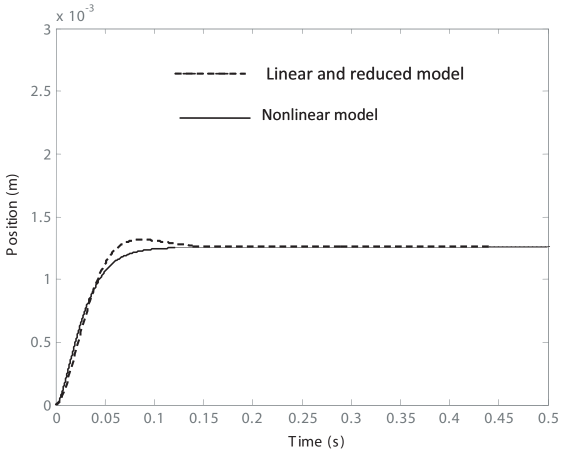


Fig. 12. Half step position evolutions $(K_m = 0.95, K_i = 2500)$

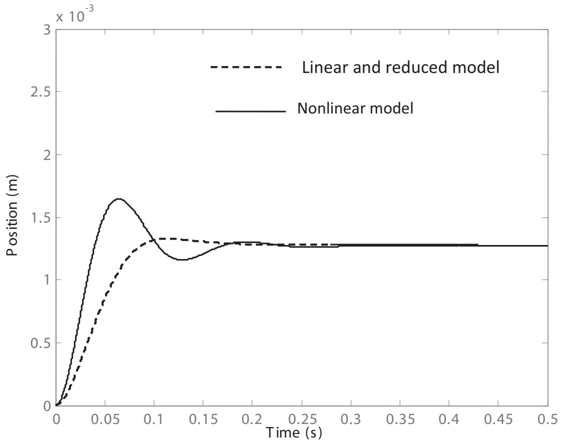


Fig. 13. Half step position evolutions $(K_m = 0.95, K_i = 1000)$

As a solution, we suppose that the motor inductance is constant by neglecting the terms $L_1 \cos(\frac{2\pi x}{\lambda} - ...)$ in (7), (8), (9) and (10). The estimation of the back EMF can be performed, in practice, by measuring the currents and the voltages of the motor phases and estimated as follows:

$$\widehat{e}_{mj} = u_j - Ri_j - L_0 \frac{di_j}{dt} \tag{43}$$

Figure 14 presents the phase B back EMF and estimated back EMF. By considering the assumption that the inductance is constant the estimated back EMF, obtained from (27) is close to the real one, obtained from (28).

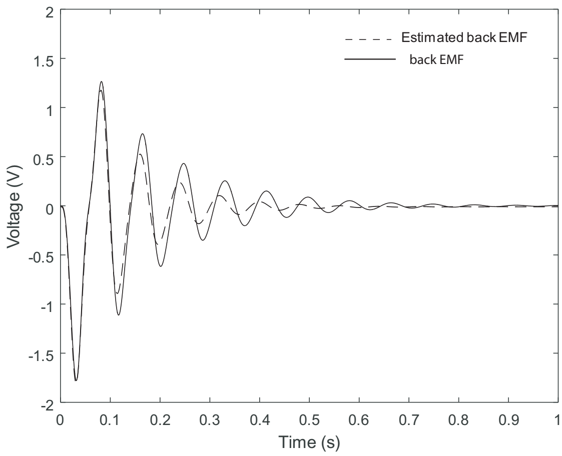


Fig. 14. Phase B back EMF and estimated back EMF evolutions

5 SIMULATION AND EXPERIMENTAL RESULTS

5.1 Simulation results

In the following, all the simulations were performed by using the nonlinear motor model represented by (1), (2), (3), (4), (5) and (6).

Figure 15 presents the open loop half step position evolution obtained by excitation of the phases A and B. We can notice that the half step position evolution is poorly damped.

Figures 16 present the currents of the phases A and B and the half step position by application of the proposed control law. For this simulation, the control law was applied with $K_m = 0.95$ and $K_i = 2500$.

The plunger moved with no oscillation and no overshoot was observed. As the phase B is the pull phase and the phase A is the braking one i_A increases and i_B decreases during the position response transient regime.

In order to generalize the proposed control law to all the motor phases, it was applied by simulation for an entire one electrical cycle. Figure 17 presents the position evolution. We can notice that the obtained half step positions are damped.

5.2 Experimental tests

5.2.1 Test bench presentation

The proposed control law was validated experimentally. The test bench structure is presented in Fig. 18. It is built essentially around:

- a power supply unit,

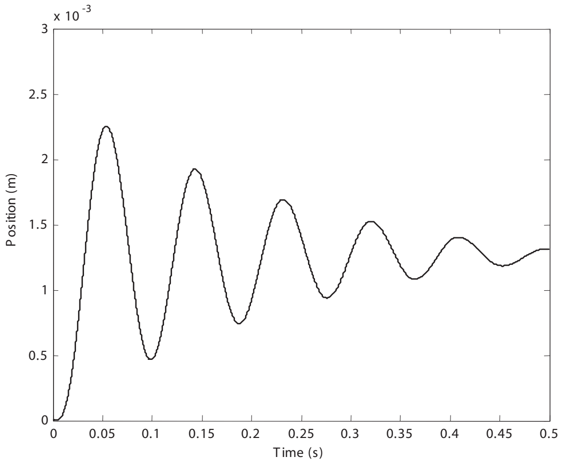


Fig. 15. Open loop half step position evolution

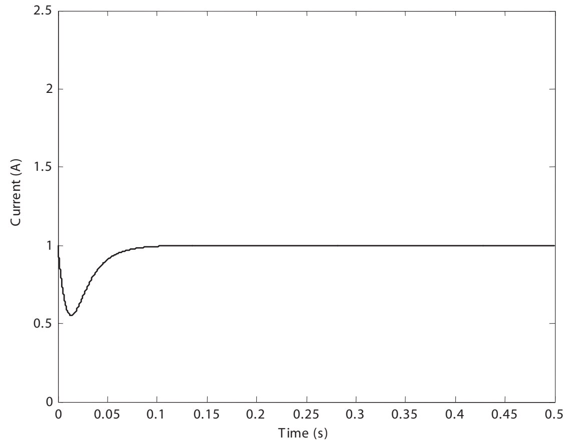
- a compact RIO controller,
- a power board connected to the four motor phases,
- the studied LTSRSM,
- a LVDT position linear sensor.

The test bench photo is given in Fig.19. The LVDT sensor is used to monitor the studied linear motor position evolution.

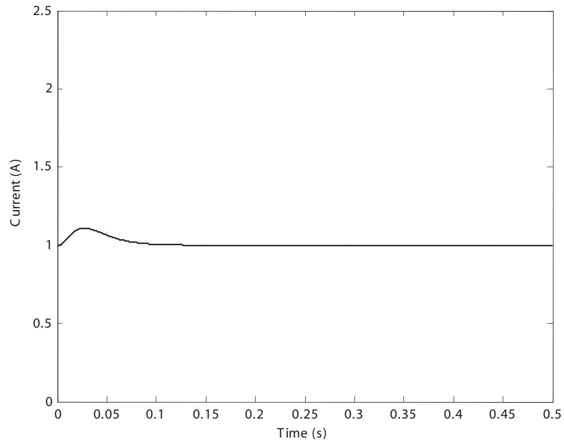
A compactRIO (cRIO) is a reconfigurable embedded control and acquisition system. It includes analog and digital I/O modules, a reconfigurable FPGA and a processor. This real-time controller was programmed with Labview graphical language. For the proposed control law, we used 4 digital outputs for the PWM control signals and two analog modules to convert the motor phase voltages and currents. Each module contains four independent analog to digital converters.

As illustrated in Fig. 20, the estimation of the four back EMF is performed by the FPGA. The control voltages and the duty cycles are computed by the processor.

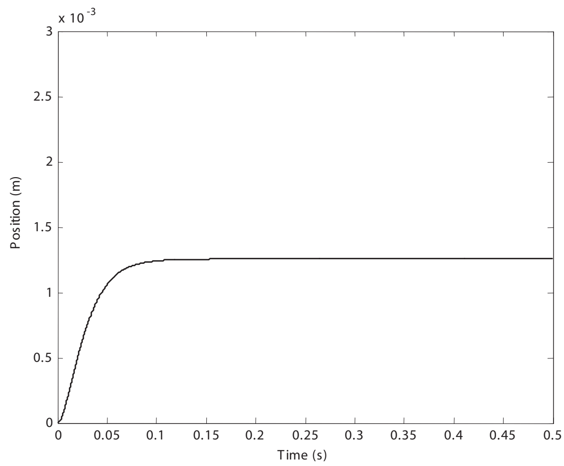
The structure of the power board is represented in Fig 21. It is composed essentially by four transistors controlled in ON OFF mode through four 10 kHz frequency PWM signals generated by the cRIO controller. The four control signals were applied to the control board through optocouplers. The currents are measured, from each phase of the motor, by Hall effect sensors. The voltages were adapted and converted by the cRIO ADC modules. V_{in} is delivered by the supply voltage. For the experiments, we used $V_{in} = 22\,V$.



(a) Current i_B



(b) current i_A



(c) Position x

Fig. 16. Application of the proposed control for a half step position evolution

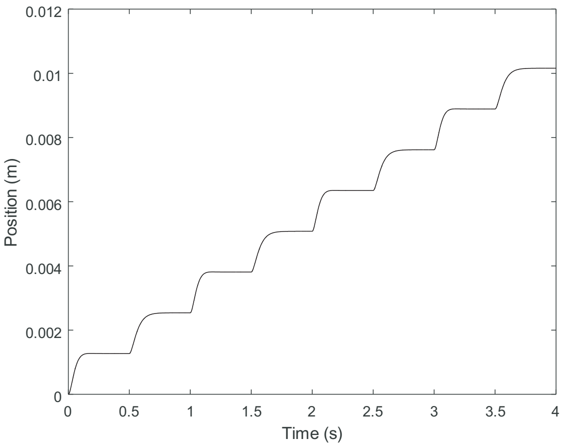


Fig. 17. Application of the proposed control law for an electrical cycle

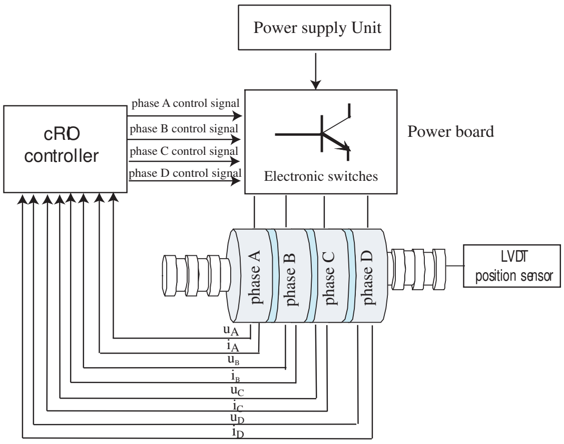


Fig. 18. Test bench structure

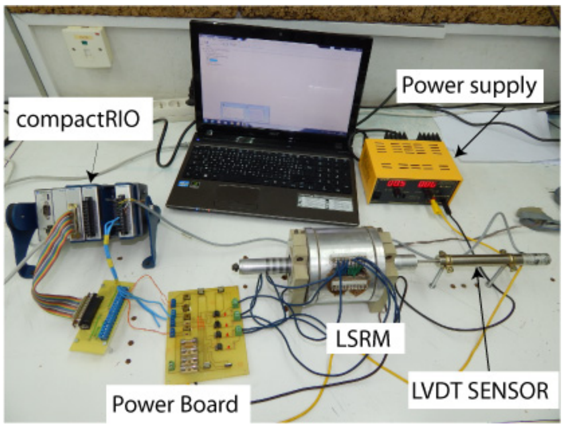


Fig. 19. Photo of the test bench

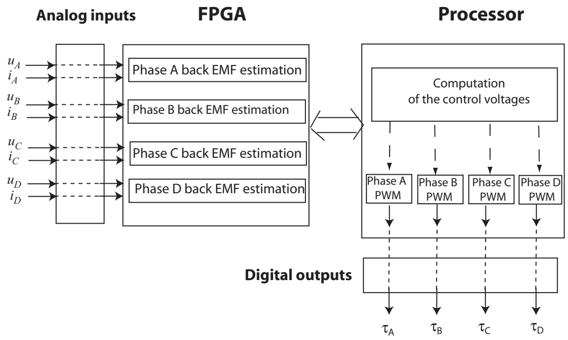


Fig. 20. Implementation of the control on the cRIO FPGA and Processor

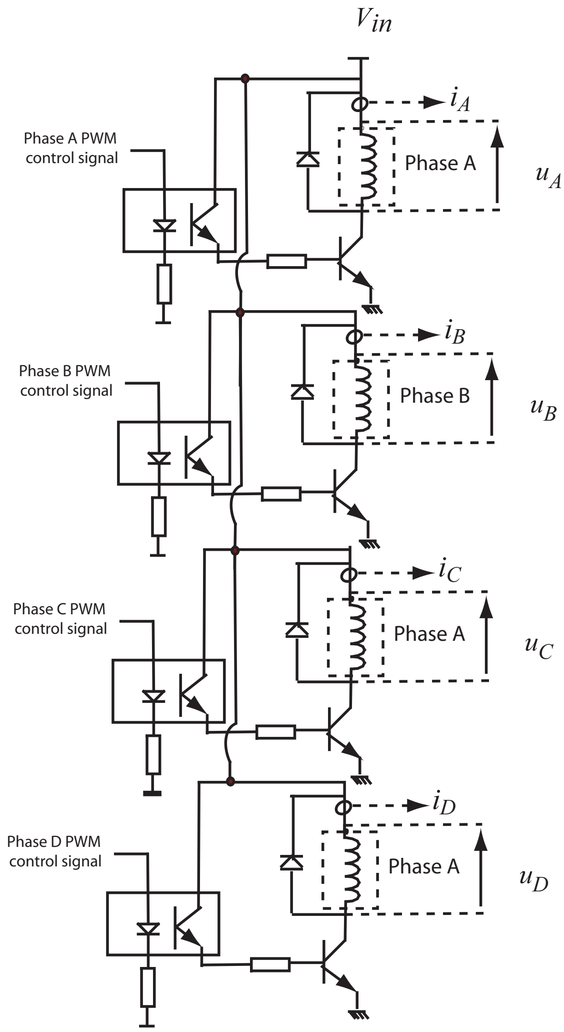


Fig. 21. Power board structure

5.2.2 Experimental results

Figure 22 presents the experimental half step open loop position evolution obtained for one electrical cycle. We can notice that the position response is poorly damped. Moreover, the half steps are different. This can be explained by the specificities of the studied LSRM. Indeed, a linear motor phases are not identical because of its mechanical geometry.

Figure 23 presents the experimental position evolution obtained by the application of the proposed control law. We can observe that the oscillations and the overshoot were eliminated from the half steps position responses.

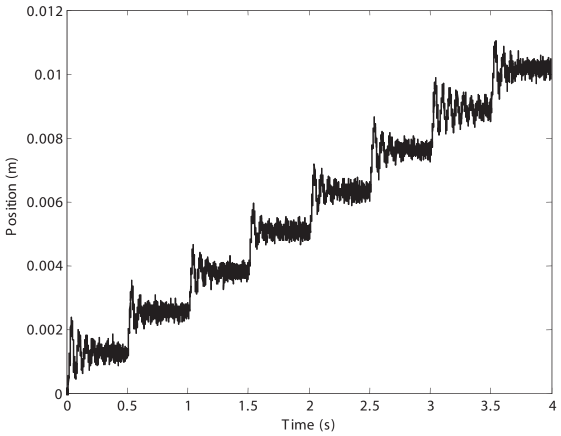


Fig. 22. Open loop half step position evolution for one electrical cycle

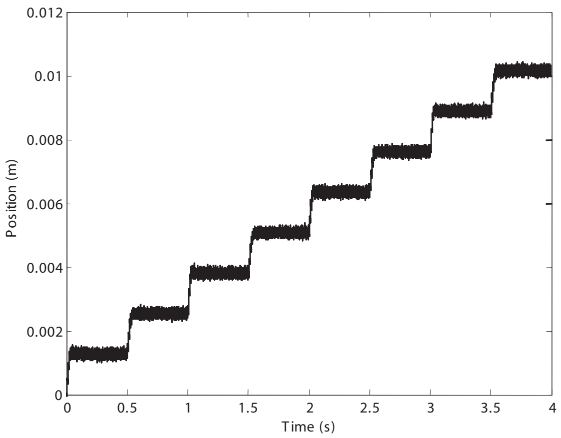


Fig. 23. Application of the proposed control law for one electrical cycle

6 CONCLUSION

In this paper, we presented a closed loop control based on back EMF. The aim of the proposed control is to damp the studied LSRM position evolution. It uses the motor back EMF to avoid the use of a linear position sensor which is cumbersome and expensive. Indeed, back EMF gives information about the motor plunger oscillations.

For the proposed control, we have no need to estimate or to observe the position and the speed of the motor. Back EMF is used to extract the information about the oscillatory behaviour of the plunger. The control strategy uses this information to increase the friction in order to eliminate the oscillations and to damp the overshoot. The proposed control law is an extension of the classical half step control of stepper motors.

To determine the proposed control parameters, the singular perturbation technique was applied to reduce the order of the studied system after the linearization of the nonlinear studied motor mathematical model. Two analytical conditions were determined. The first condition is relative to the separation between the slow and fast motor dynamics. The second one is relative to the damping ratio.

For the case of speed control of the LSRM, the oscillations can induce erratic working. The proposed control law can be easily applied especially during the acceleration phase from a starting speed to a desired target speed.

REFERENCES

[1] R. Krishnan, *Switched reluctance motor drives*, CRC Press, New York, 2001.

[2] M. Sakamoto, "PM type 3 phases stepping motors and their development to the linear stepping motors," in *Proceedings book of the* 4^{th} *International Symposium on Linear Drives for Industry Application,* (Birmingham, UK), pp. 435-438, 2003.

[3] P. Acarnley, *Stepping motors: A guide to theory and practice*. IEE Control Engineering, series 63, 4^{th} edition 2002.

[4] M. Jufer, *Electromécanique*. Lausanne, Suisse, Presses polytechniques et universitaires romandes, 1995.

[5] J.F. Gieras, "Status of the linear motors in the United States," in *Proceedings book of the* 4^{th} *International Symposium on Linear Drives for Industry Application*, LDIA 2003, Birmingham, pp. 169-176.

[6] J. F. Pan, Y. Zou, G. Cao, "An asymmetric linear switched reluctance motor," *IEEE Transaction on Energy Conversion*, vol. 28, no. 2, June 2013.

[7] J.F. Pan, Y. Zou , G. Cao, "Adaptive controller for the double-sided linear switched reluctance motor based on the nonlinear inductance modeling," IET Electric Power Applications, volume 7, issue 1, January 2013, pp. 1 – 15.

[8] C.G. Chen, M.T. Lin, "Implementation of sensorless techniques for switched reluctance motor drive systems," in *Proceedings book of the IEEE International Symposium on Industrial Electronics*, (Bari, Italy), pp. 1321 – 1326, 4-7 July 2010.

[9] W. Li, J. Fang, H. Li, J. Tang, "Position sensorless control without phase shifter for high-speed BLDC motors with low Inductance and nonideal Back EMF," IEEE Transactions on Power Electronics, vol.28 , no.2, February 2016.

[10] K. Urbanski, K. Zawirski, "Improved Method for position Estimation Using a Two-Dimensional Scheduling Array," *Automatika*, vol 56, no 3 pp. 331–340, 2015.

[11] I. A. Viorel, L. Szabo *Hybrid Linear Stepper Motors*, Mediamira, Cluj-Napoca, Romania, 1998.

[12] L. Szabo, I. A. Viorel, Z. Kovács, "E.M.F Sensing Controlled Variable Speed Drive System of a Linear Stepper-Motor", in *Proceedings book of the Power Electronics, Motion Control Conference* (Warsaw, Poland), pp. 366-371, 1994.

[13] H. A. Hussain, A Hamid, "Back-EMF Based Sensorless Vector Control of Tubular PM Linear Motors," in *Proceedings book of the IEEE International Electric Machines & Drives Conference* (Coeur d'Alene, France), pp. 878 – 883, 10-13 May 2015.

[14] K. Urbanski, K. Zawirski, "Sensorless control of SRM using position observer," in *Proceedings book of the European Conference on Power Electronics and Applications*, (Aalborg, Denmark), 2-5 Sept. 2007, pp.1-7

[15] S. Saha, K. Ochiai, T. Kosaka, N. Matsui, Y. Takeda, "Developing a Sensorless Approach for Switched Reluctance Motors from a New Analytical Model," in *Proceedings book of the* of 34^{th} Industry application conference, (Phoenix , USA), vol. 1, pp. 525-532, 03 -07 October 1999.

[16] B. Fahimi, A. Emadi, "Robust Position Sensorless Control of Switched Reluctance Motor Drives over the Entire Speed Range," in *Proceedings book of the Power Electronics Specialists Conference*, (Cairns, Autralia), vol.1, pp. 282 – 288, 23-27 June 2002.

[17] C.S. Dragu, R. Belmans, "Sensorless control of switched reluctance motor," in *Proceedings book of the 15th International conference on electrical machines*, (Brugge, Belgium), 25-28 August 2002.

[18] L. El Amraoui, B. Ben Salah, M. Benrejeb, F. Gillon, P. Brochet, "Dynamic modelling of a switched linear step motor, " in *Proceedings book of the 4^{th} International Symposium on Linear Drives for Industry Applications*, (Birmingham, UK), pp. 461-464, 2003.

[19] K. Ben Saad, A. Mbarek, M. Benrejeb, "*Determination of the switching times of the bang-bang control for a linear stepping motor by Lyapunov functions,*" Studies in Informatics and Control 2008; 17: pp. 393-406, 2008.

[20] K. Ben Saad, *Modélisation et commande d'un moteur pas à pas tubulaire à réluctance variable et à quatre phases : Approches conventionnelles, par logique floue et par réseaux de neurones artificiels*, PhD Thesis, Ecole Centrale de Lille, 2005.

[21] K. Ben Saad, M. Benrejeb, P. Brochet, "Linear Tubular Switched Reluctance Stepping Motor Modeling," *International Review of Electrical Engineering*, Vol. 2, no 2, pp. 273-281, 2007.

[22] A Ben Amor, "Experimental identification of a linear tubular four phases stepping motor," in *Proceedings book of the IEEE Systems, Man and Cybernetics Conference*, (Hammamet, Tunisia), 6-9 October 2002,.

[23] P.V. Kokotovic, R. Yackel, "Singular perturbation of linear regulators: Basic theorems," IEEE transactions on automatic control , vol. 17, no. 1, pp. 29 – 37, 1972.

[24] H. K. Khalil, P. V. Kokotovic, "Control of linear systems with multiparameter singular perturbations", Automatica, vol. 15, no. 2, pp. 197–207, 1976.

[25] P. V. Kokotovic, R. E. O'Malley, P. Sannuti, "Singular perturbations and order reduction in control theory an overview", Automatica, vol. 12, no. 2, pp. 123–132, 1976.

[26] H. Khalil,"Output feedback control of linear two time scale systems," *IEEE transactions on automatic control* , vol. 32, no. 9, pp. 784 – 792, 1987.

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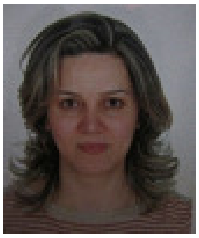

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