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# Enhancing growth and welfare through debt-financed education

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## ABSTRACT

Using an overlapping generations (OLG) model, we show how relatively small open economies can enhance their growth through educational subsidies financed via public debt and reduce their fertility rate. We show that subsidising education through public debt leads to an A-Pareto improvement of all generations. Even if a country is a net borrower in the international capital market, we show that this subsidy policy can help, under certain conditions, to improve its net borrowing position. This has strong implications for the calculation of the 3% deficit to Gross Domestic Product ratio set by the European Union because the analysis implies that public expenditures for education should be subtracted from the government deficit before applying the deficit criterion.

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## 1. Introduction

In the presence of the austerity policy applied in the European Union (EU) and the restrictions set by the stability and growth pact (SGP) for European Union member states, it should be considered if the public expenditures for education should be exempted from calculation of 3% government deficit to gross domestic product (GDP) limit. In 2011, the share of public expenditures for education as a percentage of GDP in the EU differed between 3.1% (Romania) and 8.8% (Denmark).<sup>1</sup> On the other hand, in the same period, the private expenditures for education in the EU were relatively low – between 0.1% (Romania) and 1.75% (Cyprus).<sup>2</sup> Since public expenditure for education is accounted for as government consumption in the national accounts and not investment expenditure, the education expenditure is exposed to austerity measures.<sup>3</sup>

It is well-known in the economic literature that human capital accumulation creates positive externalities for future generations (cf. Acemoglu & Angrist, 2000). This externality is due to the fact that parents who invest in the human capital of their children do not internalise the increase of the overall efficiency of the human capital accumulation

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process. Therefore, many economists (cf. Becker & Murphy, 1988; Boldrin & Montes, 2005; Kaganovich & Meier, 2012; Peters, 1995; Rangel, 2003) interpret a pay-as-you-go (PAYG) pension system as a mechanism through which the intergenerational externality can be internalised. The basic idea in these models is that in a PAYG pension system, the pensions depend on the labour income of the working generation. The labour income depends on the human capital stock of the young generation and therefore on the investments of their parents in education. Consequently, the parents benefit from it through higher pensions.

However, the majority of papers which focus on the question of how to internalise the externality created by human capital building assume a constant population or a constant fertility rate. In contrast to these models, we endogenise the fertility behaviour and instead of analysing a PAYG pension system, we propose an education subsidy which is financed by a public debt to internalise the intertemporal externality.

Even though a public debt is similar to a PAYG pension system, there are some differences which make an investigation worthwhile. The difference between a pension system and a public debt is that the former immediately redistributes income from the working population to the older non-working generation and therefore reduces the private savings. If the government sells bonds on the international capital market or borrow alternatively from an institution like the World Bank or donor country in the case of a less-developed country (LDC), the country will experience an inflow of income from abroad. Additionally, in the case of LDCs, the introduction of a PAYG pension system seems to be unrealistic because of the existing public budget constraints in these countries. Moreover, even though the argument that parents benefit from educational investments in the presence of a PAYG pension system is justifiable from an economic point of view, it is not clear if this coherence is recognised by parents in reality. In contrast, as noted by Boldrin and Montes (2004, p. 22), a subsidy seems to be a *more compelling* option.

The literature on endogenous fertility and human capital in growth model goes back to Becker, Murphy, and Tamura (1990) and Becker and Barro (1988). Using an intertemporal utility function, they assume that a higher fertility rate of the present generation raises the discount factor on future per capita consumption. Consequently, a higher fertility rate discourages investments in human and physical capital. On the other hand, higher stocks of physical and human capital (education) reduce the number of children because of the high cost of raising and caring for children.

One strand of the literature is based on the work of Kolmar (1997) who integrated only endogenous fertility behaviour in a standard OLG framework. His idea was further extended by Fenge and Meier (2005, 2009) and van Groezen, Leers, and Meijdam (2003). However, these scholars did not concentrate on public debts but on PAYG pension systems. They show that for a small country, under certain conditions, it is possible to increase the fertility rate and the welfare simultaneously. They assumed that the pensions depend on the number of children and that the growth in population attracts higher capital inflows so that all individuals are better off, are results which hold only for small open economies. On the contrary, Stauvermann, Ky, and Nam (2013) argue that, except for increase in fertility rates, these results do not hold in an OLG-model with endogenous growth because reduced savings tend to reduce the per-capita growth rates and hence the welfare of future generations. Further, they show that an increase in fertility rate is accompanied by high costs for future generations, which further decrease the rates of capital accumulation, per capita growth, per capita capital and the labour income.

Another strand of literature goes back to Bental (1989), Raut (1992), Cigno (1993), Stauvermann (1996), Zhang and Nishimura (1993), Zhang and Zhang (1995) and Zhang (1995). These authors interpret children as insurance for the old age. In their models, the introduction of a PAYG pension scheme decreases the importance of children and therefore the number of them declines. As a consequence, the per capita income growth rises and the fertility-reducing effect of a PAYG pension system offset the savings-reducing effect.

Moreover, Zhang (1997, 2003, 2006) and Li and Zhang (2008) provide another dimension on fertility and human capital accumulation. Their main assumptions are that individuals are perfectly altruistic in the sense of Barro (1974) with respect to their offspring and that the human capital accumulation is associated with external economies of scale. The latter idea introduced by Lucas (1988) shows that the private rate of return of human capital building is lower than the social rate of return. Subsequently, a government subsidy as an incentive to invest in human capital is able to equalise the private and social rate of return.

We acknowledge the role of external economies of scale. However we develop our model and present arguments even without explicitly factoring the role of external economies of scale. Notably, all models which include human capital accumulation but not perfect altruism in the sense of Barro (1974) exhibit intergenerational externalities because parents who invest in the human capital of their offspring do not take into account the positive effects of investments for their grandchildren and great-grandchildren.

Another approach was developed by Wigger (2001), who argues that an education subsidy financed by a lump-sum tax can be welfare-enhancing in a closed economy with a constant population, provided the cross derivative of the production function between human capital and physical capital is positive and sufficiently large. Additional empirical studies on the importance of education for economic growth can be found in De Fuentes (2003), Hanushek and Woessmann (2010) and the references therein.

As many others in the literature (cf. Peters, 1995; de la Croix & Doepke, 2003; Boldrin & Montes, 2005), we assume that parents derive a benefit from investing in the number of children and from investments in their children's education. In such a framework we show that it is possible to internalise the intergenerational externality in a small open economy with the help of a public debt without harming any generation. We argue that the generation that gains from a human capital investment (children) should pay for it instead of putting the entire burden of responsibility on the current generation (parents). Under these conditions we show that it is possible to increase the growth of the economy by decreasing the number of children and increasing the growth rate of human capital accumulation. We argue that by subsidising education of children and not the number of them (a quality and quantity trade-off) will have a growth- and welfare-enhancing effect and a negative effect on fertility if the preference for the quantity of children exceeds the preference for the quality for children. Our main intention is to give a normative theoretical reasoning that public education and other educational subsidies should be financed by a government debt in the presence of a quality and quantity trade-off.

The rest of the article is outlined as follows. In Section 2, we introduce the basic model, which to a large extent is similar to Stauvermann and Kumar (2016). In Section 3, we derive the short-run effects caused by the introduction of an education subsidy followed by an analysis of the long-run effects. Finally, in Section 5, we conclude.

## 2. The model

To model the production side of a small open economy, we use the approach of Lucas (1988) and Uzawa (1965). The production depends on physical and human capital and the production function has the following form:

$$Y_t = F(K_t, H_t) = F(K_t, L_t h_t). \quad (1)$$

Here  $Y_t$  is the production,  $K_t$  is the capital stock, and  $H_t = h_t L_t$ , is the human capital stock which results from the product of the average human capital per head ( $h_t$ ) times the aggregate labour time ( $L_t$ ). The production function exhibits the usual diminishing marginal productivities in each input factor, fulfils the Inada conditions, and is linear homogenous.<sup>4</sup> The subscript  $t$  indicates the period of time.

Regarding the creation of human capital, for our purposes a very simple formulation of the human capital building process is sufficient because we assume that all individuals are identical:

$$h_t = h_{t-1}(1 + \varphi u_t). \quad (2)$$

Therefore, the average human capital stock equals the individual human capital stock. This means the average human capital in time  $t$  depends on the parental average human capital stock, the time variant parental investments in education  $u_t$  like schooling, and the effectiveness of education  $\varphi > 0$ . The investments can be measured in monetary units,<sup>5</sup> or in time units like parental time of home-schooling. However, we measure the human capital investments without loss of generality in time units the parents spend for teaching their children. These investments can be expressed in monetary terms if we use the wage rate as the value of time at the margin. For simplicity, we define the total available time of an adult to be equal to 1. The efforts of the parents or investments in education are represented by  $u_t$ , where  $0 \leq u_t < 1$ . It is easy to see that the human capital production function is a specification of the more general function of Azariadis and Drazen (1990) or de la Croix and Doepke (2003). In any case, the resulting growth rate of human capital  $g_t^h$  becomes:

$$g_t^h = \varphi u_t. \quad (3)$$

We accept that there is a positive externality caused by human capital building because subsequent generations benefit from educational investments of former generations.

The wage rate per human capital unit  $\tilde{w}_t$  and the interest factor  $R_t$  are determined on the world capital market, since we consider a reasonably small country. The capital stock used in this economy adjusts according to the factor prices on the world market. Assuming that the physical capital is totally depreciated within one period, we get the following equations:

$$\tilde{w}_t = f(\tilde{k}_t) - f'(\tilde{k}_t)\tilde{k}_t \quad (4)$$

$$R_t = f'(\tilde{k}_t) \quad (5)$$

The function  $f(\tilde{k}_t = F\left(\frac{\tilde{k}_t}{H_t}, 1\right)$  represents the production function per human capital unit. Then the resulting wage rate per capita is given by  $w = \tilde{w}_t h_t$ . For the rest of the article

we assume that world economy is in the long-run equilibrium and therefore,  $\tilde{w}_t = \tilde{w}$ ,  $\forall t$  and  $R_t = R, \forall t$ .

To model the consumption side of the economy, we use a three-period overlapping generation (OLG) model of Allais (1947), Samuelson (1958) and Diamond (1965). This approach is extended by the introduction of human capital and endogenous fertility. Even though the fertility behaviour is based on similar considerations as in Becker (1960) and Becker and Lewis (1973)<sup>6</sup> who introduced the quantity–quality trade-off of children, we use a different definition of a child's quality. While Fanti and Gori (2008a, 2008b) use the definition of Becker (1960), where quality is defined as all expenditures for children, we define quality as time spent for the education of a child similar to Galor and Weil (1999) or de la Croix and Doepke (2003), and the results of the model is dependent on this definition. The model of Fanti and Gori (2008a, 2008b), which used Becker's broader definition of child's quality, has the disadvantage that parents never invest in the quality of children if they do not get a child allowance. That means that in a world without child allowances, parents never invest in human capital building. If the total costs of rearing and educating a child matter as an argument in the utility function, then the pure child rearing costs have a utility-enhancing effect and they are a perfect substitute to investments in education. In our model, these costs do not enter the utility function directly and an increase of them decreases the utility of the representative individual indirectly. Our decision to use human capital as an argument in the utility function coincides with the assumptions of the Unified Growth Theory (Galor, 2005).

In our model, in the first period of life, an individual is relatively young, not yet prepared to work, and/or participate in economic decision-making, and hence undergoes education funded by her parents. In the second period of life, the individual supplies labour which is considered to be inelastic, gives birth to  $N_t$  children, rears and educates her off-spring, consumes  $c_t^1$  and saves  $s_t$  amounts respectively. In the third period of life, the individual is unable to work and lives from her savings and interest income, that is,  $c_{t+1}^2 = R s_t$ . Subsequently, the utility function of the individual over the three periods is defined as the following log-linear function:

$$U(c_t^1, c_{t+1}^2, N_t, h_{t+1}) = \ln c_t^1 + q \ln c_{t+1}^2 + \mu \ln N_t + \beta \ln(h_{t+1}). \quad (6)$$

The parameter  $q$  reflects the subjective discount factor. This form of the utility function is a variation of those used by de la Croix and Doepke (2003). In contrast to the approach of Fanti and Gori (2008a, 2008b) and Strulik (2003, 2004), we explicitly consider human capital of the children as an argument in the utility function.<sup>7</sup>

Moreover, we make the natural assumption that parents have a stronger preference for the quantity than the quality of children:  $\mu > \beta$ . This assumption guarantees that the law of demand holds for the quality and quantity of children. In addition, we assume that the parent treats all her children equally with respect to their education. A representative parent is constrained by the following budget:

The wage rate per human capital unit  $\tilde{w}$  and the interest factor  $R_t$  are determined on the world capital market since we consider a reasonably small country. The capital stock used in this economy adjusts according to the factor prices on the world market. Assuming that the physical capital is totally depreciated within one period, we get the following equations:

$$c_t^1 \leq w_t (1 - (1 - \tau)u_t N_t - b N_t - T_t) - s_t \quad (7)$$

$$c_{t+1}^2 \leq R_{t+1}s_t \tag{8}$$

In the second period of life, each individual has to allocate her available time between working time, time to educate the children  $(1 - \tau)u_tN_t$ , and time to rear the children  $bN_t$ . The variable  $\tau$  represents the subsidy rate. Assuming that it makes no difference if parents educate their children or if they are educated in public or private schools, then an alternative interpretation would be that parents pay for the schooling the amount  $(1 - \tau)u_t w_t$ .

Additionally, the individual has to pay a payroll tax where the tax rate is  $T_t$ . If we normalise the available time to one and define  $l_t$  as labour time, then  $l_t = (1 - (1 - \tau)u_tN_t - bN_t - T_t)$  and the net income becomes  $l_t w_t = l_t \tilde{w} h_t$ . A second interpretation is that the individual earns the gross labour income  $w_t$  and spends  $w_t(1 - \tau)u_tN_t$  units of her income for the education,  $w_t bN_t$  units for child-rearing, where the parameter  $0 < b < 1$  represents the constant time share or constant income share which is needed to rear a child and pays  $w_t T_t$  units as taxes. Equation (8) states that the individual lives in the third period of life from her savings and interest. Combining (7) and (8) gives us a single budget constraint:

$$w_t(1 - (1 - \tau)u_tN_t - bN_t - T_t) - c_t^1 - \frac{c_{t+1}^2}{R_{t+1}} = 0. \tag{9}$$

We assume that the government finances the education subsidy by issuing government bonds  $B_t$  with a term of one period and collects the income share of  $T_t$  as a payroll tax to finance its debt and interest payments. This means that the public debt per worker at the beginning of the current period  $D_t$  equals the subsidy per child of the previous period times the interest factor,  $R$ . This implies that a worker pays back the subsidy her parents received based on the market conditions. That is:

$$D_t = RB_{t-1} = R\tau u_{t-1} w_{t-1}. \tag{10}$$

It should be noted that parents are unable to get an analogous loan contract on the capital market because of legal restrictions and moral hazard problems. To guarantee a balanced government budget, the tax revenue per worker in period  $t$  has to be:

$$T_t = \frac{RB_{t-1}}{w_t} = R \frac{\tau u_{t-1} h_{t-1}}{h_t} = \frac{R\tau u_{t-1}}{1 + \varphi u_{t-1}}. \tag{11}$$

The government is financing the education subsidies by a public debt which will be covered by the tax revenue in the future. Using (2), we maximise (6) subject to (9) by constructing the following Lagrange function:

$$L(c_t^1, c_{t+1}^2, \lambda) = lnc_t^1 + qlnc_{t+1}^2 + \mu \ln N_t + \beta \ln(h_{t-1}(1 + \varphi u_t)) - \lambda \left( w_t(1 - (1 - \tau)u_tN_t - bN_t - T_t) - c_t^1 - \frac{c_{t+1}^2}{R_{t+1}} \right). \tag{12}$$

Accordingly, the first order conditions are:

$$\frac{1}{c_t^1} + \lambda = 0, \tag{13}$$

$$\frac{q}{c_{t+1}^2} + \frac{\lambda}{R} = 0, \tag{14}$$

$$\frac{\mu}{N_t} + \lambda w_t [(1 - \tau)u_t + b] = 0, \tag{15}$$

$$\frac{\varphi\beta}{1 + \varphi u_t} + \lambda w_t (1 - \tau)N_t = 0, \tag{16}$$

$$w_t (1 - (1 - \tau)u_t N_t - bN_t - T_t) - c_t^1 - \frac{c_{t+1}^2}{R} = 0. \tag{17}$$

Substituting (15) into (16) and solving for the investment in education gives:

$$u^* = \frac{\varphi\beta b - (1 - \tau)\mu}{(1 - \tau)(\mu - \beta)\varphi}. \tag{18}$$

From equation (18) we can derive the necessary condition for the existence of an interior solution for  $u^* \in [0, 1]$ :

$$\frac{\mu}{\varphi\beta} \leq \frac{b}{(1 - \tau)} < \left(\frac{\varphi + 1}{\varphi}\right) \left(\frac{\mu}{\beta}\right) - 1. \tag{19}$$

Notably, the inequality (19) will not hold for all  $\tau \in [0, 1]$  because the expression in the middle will strive to infinity if the subsidy rate converges to one. Therefore, we must restrict our analysis with respect to the subsidy rate. Especially, we assume throughout the paper that:

$$\frac{\mu}{\varphi\beta} \leq b < \left(\frac{\varphi + 1}{\varphi}\right) \left(\frac{\mu}{\beta}\right) - 1. \tag{20}$$

Given this condition, the existence of an interior equilibrium is guaranteed in this economy without any government intervention. Consequently, we analyse the effects of a subsidy rate  $\tau \in [0, \bar{\tau}]$  because all  $\tau > \bar{\tau}$  lead into an undesirable corner solution in the equilibrium. Due to the fact that  $b$  has to lie between 0 and 1, we can derive from (20) that  $\varphi\beta > \mu$  must hold in the case of no subsidy ( $\tau = 0$ ). In combination with the assumption that  $\mu > \beta$ , we conclude that  $\varphi b > \frac{\mu}{\beta} > 1$ . Subsequently, the growth rate of human capital is given by:

$$g^{h*} = \frac{\varphi\beta b - (1 - \tau)\mu}{(1 - \tau)(\mu - \beta)}. \tag{21}$$

Solving for the remaining independent variables, we get the following equilibrium quantities:



$$c_t^{1*} = \frac{h_{t-1} \tilde{w} [\varphi \beta (b(\varphi - \tau R) - (1 - \tau)) + (1 - \tau) \mu \tau R]}{\varphi (1 - \tau) (1 + q + \mu) (\mu - \beta)}, \tag{22}$$

$$c_{t+1}^{2*} = Rq \left( \frac{h_{t-1} \tilde{w} [\varphi \beta (b(\varphi - \tau R) - (1 - \tau)) + (1 - \tau) \mu \tau R]}{\varphi (1 - \tau) (1 + q + \mu) (\mu - \beta)} \right), \tag{23}$$

$$N_t^* = \frac{(\mu - \beta) [\varphi \beta (b(\varphi - \tau R) - (1 - \tau)) + (1 - \tau) \mu \tau R]}{(1 + q + \mu) \beta (1 - \varphi b - \tau)^2}. \tag{24}$$

Here we restrict our analysis to interior equilibria which are characterised by a non-negative equilibrium fertility rate ( $N_t^* \geq 0$ ), a non-negative and feasible investment in human capital ( $0 \leq u^* \leq 1$ ) and non-negative consumption levels in both periods of life. We relate the equilibrium values to  $h_{t-1}$  instead of  $h_t$  because we will need this later to calculate the long-run effects caused by a change of  $\tau$ . To ensure that the tax rate  $T_t$  is smaller than one, the maximum subsidy rate  $\bar{\tau}$  is given by the value of:

$$\bar{\tau} = \frac{\beta \varphi + R(\mu - b\beta \varphi) + \sqrt{\varphi^2 \beta^2 (bR - 1)^2 - 2R\mu \varphi \beta (1 + bR - 2b\varphi) + \mu^2 R^2}}{2\mu R}. \tag{25}$$

In addition, it can be shown that  $N_t^* \geq 0$  as long as  $\tau \in [0, \bar{\tau}]$ .

**Proposition 1:** The existence of a unique interior equilibrium without government intervention is guaranteed if condition (20) is fulfilled.

Proof:

In the absence of a government intervention (18), and (21)-(24) becomes:

$$u^* = \frac{\varphi \beta b - \mu}{(\mu - \beta) \varphi} \tag{18}$$

$$g^{h*} = \frac{\varphi \beta b - \mu}{(\mu - \beta)}. \tag{21}$$

$$c_t^{1*} = \frac{h_{t-1} \tilde{w} \beta (\varphi b - 1)}{(1 + q + \mu) (\mu - \beta)}, \tag{22}$$

$$c_{t+1}^{2*} = Rq \left( \frac{h_{t-1} \tilde{w} \beta (\varphi b - 1)}{(1 + q + \mu) (\mu - \beta)} \right), \tag{23}$$

$$N_t^* = \frac{(\mu - \beta) \varphi}{(1 + q + \mu) (\varphi b - 1)}. \tag{24}$$

As noted, condition (20) guarantees that  $0 < u^* < 1$ . Further, the condition ensures that  $\varphi b > 1$ . This, combined with the assumption  $\mu > \beta$  ensures that all other independent variables have positive values.

**Proposition 2:** A unique interior equilibrium described by equations (18) and (21)-(24) exists, if  $\tau \in [0, \bar{\tau}]$  and condition (20) is fulfilled.

Because of our relatively small country assumption, the capital intensity per human capital unit is determined on the international capital market. The private savings per capita is given by:

$$s_t^* = qh_{t-1}\tilde{w} \left( \frac{[\varphi\beta(b(\varphi - \tau R) - (1 - \tau)) + (1 - \tau)\mu\tau R]}{\varphi(1 - \tau)(1 + q + \mu)(\mu - \beta)} \right). \tag{26}$$

To calculate the amount of resources available to invest in physical capital, we subtract the amount of government bonds per head that are issued in the corresponding period. The government bonds per capita issued in the current period accumulate to:

$$B_t = \tau h_{t-1}\tilde{w}u_t^*N_t^* = \tau h_{t-1}\tilde{w} \frac{[\varphi\beta(b(\varphi - \tau R) - (1 - \tau)) + (1 - \tau)\mu\tau R][(1 - \tau)\mu - \varphi\beta b]}{(1 - \tau)(1 + q + \mu)\beta\varphi(\beta\varphi - 1 + \tau)^2}. \tag{27}$$

The net wealth per capita in the beginning of period  $t+1$ ,  $a_{t+1}$ , is the savings per capita of period  $t$  minus the government bonds per capita issued in period  $t$  and the result is divided by the number of children born in period  $t$ :

$$a_{t+1}^* = h_{t-1}\tilde{w} \frac{[\tau\beta^2\varphi b + \beta(\tau^2(q + \mu) + \tau(\varphi b - 1)(2q - \mu) + q(\varphi b - 1)^2) + (1 - \tau)\tau\mu^2]}{(1 - \tau)\varphi(\mu - \beta)^2}. \tag{28}$$

However, if the wealth per capita is smaller than the capital per capita, the country is a net borrower on the international capital market. Now we analyse the short- and long-run effects of an increasing subsidy rate  $\tau$  on the equilibrium values of this economy.

### 3. Short-run effects

If the government introduces an education subsidy in period 0, then the parents are offered a subsidy for education but they do not have to pay a tax like all following parent generations and their human capital stock is given. As a consequence, the budget constraint is somewhat different from (9) because  $T_0 = 0$ . Repeating the maximisation procedure with the adjusted budget restriction yields the following results:

$$u_0 = \frac{\varphi\beta b - (1 - \tau)\mu}{(1 - \tau)(\mu - \beta)\varphi}. \tag{29}$$

$$g_0^h = \frac{\varphi\beta b - (1 - \tau)\mu}{(1 - \tau)(\mu - \beta)}. \tag{30}$$

$$c_0^1 = \frac{h_0\tilde{w}}{(1 + q + \mu)}, \tag{31}$$

$$c_1^2 = Rq \frac{h_0 \tilde{w}}{(1 + q + \mu)}, \tag{32}$$

$$N_0 = \frac{(\mu - \beta)\varphi}{(1 + q + \mu)(\varphi b - 1 + \tau)}. \tag{33}$$

Obviously, the consumption behaviour and therefore the savings behaviour, is not influenced by the introduction of the subsidy. However, the subsidy influences the level of education, the number of children and the per capita growth rate. Differentiating (29), (30) and (31) gives us:

$$\frac{\partial u_0}{\partial \tau} = \frac{\beta b}{(1 - \tau)^2(\mu - \beta)} > 0, \tag{34}$$

$$\frac{\partial g_0^h}{\partial \tau} = \frac{\varphi \beta b}{(1 - \tau)^2(\mu - \beta)} > 0, \tag{35}$$

$$\frac{\partial c_0^1}{\partial \tau} = \frac{\partial c_1^2}{\partial \tau} = 0. \tag{36}$$

$$N_0 = \frac{(\mu - \beta)\varphi}{(1 + q + \mu)(\varphi b - 1 + \tau)}. \tag{37}$$

As expected, the subsidy enhances the level of education and therefore the growth rate of human capital accumulation with a simultaneous decline in the number of children. The result that the aggregate consumption expenditures and the share of income spent for consumption remain unchanged is due to the additive separability of the utility function. This characteristic of the utility function leads to the result that only a substitution effect between the quantity and quality of children occurs. The subsidy decreases the relative price of the child quality and raises the relative price of the quantity of children. The outcome is a lower number of children who receive a higher level of education. These effects lead to a change of the parents' welfare:

$$\frac{\partial U(c_0^1, c_1^2, N_0, h_1)}{\partial \tau} = \frac{\varphi \beta b - (1 - \tau)\mu}{(1 - \tau)(\varphi b - 1 + \tau)} > 0. \tag{38}$$

The less surprising result is an increase of parents' utility.

**Proposition 3:** In the short run, the introduction of an education subsidy leads to an increase in the growth rate of human capital accumulation, the utility of the parents, and a decline of the fertility rate.

Before proceeding to the analysis of the long-run effects, note that the aggregated non-financial wealth of the economy tends to decline because a part of the savings will be invested in the government bonds to finance the subsidies. However, the change of the non-financial wealth per capita does not necessarily decrease because the number of children is also lower than in the previous periods. The non-financial wealth in period 1 is defined by:

$$a_1 = \frac{s_0 - \tau h_0 \tilde{w} u_0 N_0}{N_0} = \frac{h_0 \tilde{w} (q(\varphi b - 1) - \tau^2(q + \mu) - \tau(b\beta\varphi - \mu - q(2 - \varphi b)))}{(1 - \tau)(\mu - \beta)\varphi}. \quad (39)$$

If an education subsidy will be introduced, the wealth per capita can be determined by differentiating (39) with respect to the subsidy rate and evaluating the derivative at  $\tau = 0$ :

$$\left. \frac{\partial a_1}{\partial \tau} \right|_{\tau=0} = \frac{h_0 \tilde{w} (q + \mu - b\beta\varphi)}{\varphi(\mu - \beta)} \leq 0. \quad (40)$$

**Proposition 4:** The introduction of an education subsidy results in an increase of the non-financial wealth per capita in the following period, only if the subjective time preference for the future is sufficiently large ( $q > b\beta\varphi - \mu$ ).

This also means that the net borrowing position of the country will improve provided the condition of proposition 4 holds.

#### 4. Long-run effects

This part of the analysis is relevant for all generations born in period zero and thereafter. In contrast to the short-run analysis, we take into account that each individual has to pay a tax, which is used to cover the public debt and the corresponding interest. Differentiating (18) and (21) leads to the following results:

$$\frac{\partial u^*}{\partial \tau} = \frac{\beta b}{(1 - \tau)^2(\mu - \beta)} > 0. \quad (41)$$

$$\frac{\partial g_h^*}{\partial \tau} = \frac{\varphi\beta b}{(1 - \tau)^2(\mu - \beta)} > 0. \quad (42)$$

The long-run effects regarding the educational level and the growth rate of human capital are identical to the corresponding effects in the short run.

Because the subsidy rate has to be sufficiently small as noted in Proposition 2, we evaluate the following derivatives at  $\tau = 0$ . As opposed to the educational time and growth rate, the number of children will decline more than in the short run. This is induced by the payroll tax which reduces the net income, and the assumption that the costs of rearing children grow proportionally to the gross income.

$$\left. \frac{\partial N^*}{\partial \tau} \right|_{\tau=0} = -\frac{(\mu - \beta)(\varphi\beta + R(\varphi b\beta - \mu))}{(\varphi b - 1)^2(1 + q + \mu)\beta} < 0. \quad (43)$$

As distinct from the short-run effects, the consumption and hence the individual savings in the first and second period of life will change as follows:

$$\left. \frac{\partial c_t^{1*}}{\partial \tau} \right|_{\tau=0} = \frac{\tilde{w}(\varphi^2 b \beta - R(\varphi b \beta - \mu))}{(1 + q + \mu)(\mu - \beta)\varphi} \leq 0, \tag{44}$$

$$\left. \frac{\partial c_t^{1*}}{\partial \tau} \right|_{\tau=0} = \frac{\tilde{w}(\varphi^2 b \beta - R(\varphi b \beta - \mu))}{(1 + q + \mu)(\mu - \beta)\varphi} \leq 0, \tag{44}$$

$$\left. \frac{\partial c_{t+1}^{2*}}{\partial \tau} \right|_{\tau=0} = \frac{\tilde{w}Rq(\varphi^2 b \beta - R(\varphi b \beta - \mu))}{(1 + q + \mu)(\mu - \beta)\varphi} \leq 0. \tag{45}$$

The sign of the derivatives (43) and (44) depends on, among other parameter values, the ratio between the education coefficient  $\varphi$ , and the interest factor  $R$ .

**Proposition 5:** The introduction of an education subsidy financed by a public debt increases the consumption in both periods of life and the private savings only if,  $\frac{\varphi}{R} > 1 - \frac{\mu}{\varphi b \beta}$ .

Since the R.H.S. of the condition in Proposition 5 is assumed to be less than 1, the consumption can increase even though the interest factor exceeds the education coefficient. This is caused by the reduction of the number of children.

Further, we examine how the education subsidy affects the non-financial wealth per capita. Differentiating (28) subject to the subsidy rate and evaluating it at  $\tau = 0$ , gives:

$$\left. \frac{\partial a_{t+1}^*}{\partial \tau} \right|_{\tau=0} = \frac{\tilde{w}(\beta(\varphi \beta b + q(\varphi^2 b^2 - 1) - \mu(1 + \varphi b)) + \mu^2)}{\varphi(\mu - \beta)^2} \leq 0. \tag{46}$$

Whether the non-financial wealth is increasing or decreasing depends strongly on the time preference for future consumption. Solving for the critical value of  $q$  leads to:

$$\bar{q} = \frac{(\mu - \beta)(\varphi b \beta - \mu)}{\beta(\varphi b - 1)^2}. \tag{47}$$

**Proposition 6:** The introduction of an education subsidy raises the non-financial wealth *per capita* only if the subjective time preference factor  $q$  exceeds the critical value  $\bar{q}$ .

If the condition of Proposition 6 holds, the net borrowing position of the country will improve, and if the country was a debtor country, it will become a lending country after the introduction of the education subsidy, albeit in the long run.

Next, we analyse the long-run welfare effects. Accordingly, we consider whether all the generations in the (very) long run are better off because of the increased growth rate. Therefore, the relevant generation to consider is the one which receives a minimal gain from the growth rate increase but has to bear a higher tax burden caused by the increased subsidy.

Hence, by inserting the corresponding equilibrium values (18) and (21)–(24) in the utility function (6), differentiating it with respect to the subsidy rate, and evaluating the expression at  $\tau = 0$  yields:

$$\left. \frac{\partial U(c_t^{1*}, c_{t+1}^{2*}, N_t^*, h_{t+1})}{\partial \tau} \right|_{\tau=0} = \frac{(1 + \mu + q) \left[ \varphi \beta b (\varphi - R) + \mu \left( R - \frac{\varphi \beta}{1 + \mu + q} \right) \right]}{\varphi \beta (\varphi b - 1)} \stackrel{!}{\leq} 0. \quad (48)$$

Setting the R.H.S. of the derivative to 0 and solving for  $R$  gives us:

$$\bar{R} = \frac{\varphi \beta (\varphi b (1 + \mu + q) - \mu)}{(1 + \mu + q)(\varphi b \beta - \mu)}. \quad (48)$$

**Proposition 7:** The introduction of an education subsidy financed by a public debt generates an A-Pareto-improvement<sup>8</sup> only if the market interest factor is sufficiently low ( $R < \bar{R}$ ).

If the interest rate is too high, the tax burden outweighs the positive effect of increased growth and the additional utility generated by the extra human capital. Of course, as time elapses since the introduction of the education subsidy, the more important is the growth effect. However, when the interest factor is not sufficiently large, an A-Pareto improvement can be realised.

## 5 Conclusions

In this article, we show that building human capital financed through a public debt can provide a beneficial outcome both on the current and future generations. An important feature of the modelling and theoretical analysis presented is that, unlike previous studies, we reach results without directly including the positive externality generated through human capital accumulation.<sup>9</sup> Thus, our results are interpreted to show that public education should be financed by a government debt to internalise the intertemporal positive externality caused through human capital building. Specifically, we have shown that the proposed finance mechanism reduces the fertility rate and enhances the human capital accumulation and hence the per capita growth rate of income.

It should be noted that in the results it is not necessary that the rate of return on human capital  $\varphi$  exceeds the rate of return of physical capital  $R$ . However, Barro and Lee (2010) note that in the period 1950–2010, the rate of return of each year of additional schooling exceeded in all parts of the world by 6%, and by 12% in developed countries. These results coincide with earlier ones of 6–10% rate of return (Acemoglu and Angrist 2000; Card 1999). However, these results do not take into account the positive externality which is omitted in our analysis but exists. Further, we have shown that the net borrowing position of the economy improves and the welfare increases under certain conditions. Therefore, it is strongly recommended that the SGP of the EU should consider subtracting the public expenditures for education from the government deficit before the 3% deficit to GDP criterion is calculated.<sup>10</sup> Otherwise, any austerity measures will create more harm in the long run than they create benefits. For example, Ježić (2012) recommended that Croatia needs to increase its human capital investment. However, we note that the government reduced it in 2014 by 1.95% as a consequence of the European austerity policy, although the educational expenditures was only 4.2% of the Croatian GDP (2011) which was far less than the EU and Organisation for Economic Cooperation and Development (OECD) average of 5.28%.

Additionally, from a development aid perspective, it would make sense to give concessional loans to small and developing countries to finance education and human capital development in areas of resource deficit.

## Notes

1. See Eurostat: [http://ec.europa.eu/eurostat/statistics-explained/index.php/File:Public\\_expenditure\\_on\\_education,\\_2011\\_%28%C2%B9%29\\_%28%25\\_of\\_GDP%29\\_YB15.png](http://ec.europa.eu/eurostat/statistics-explained/index.php/File:Public_expenditure_on_education,_2011_%28%C2%B9%29_%28%25_of_GDP%29_YB15.png) (accessed 23 November 2015).
2. See Eurostat: <http://ec.europa.eu/eurostat/tgm/graph.do?tab=graph&plugin=1&language=en&pcode=tps00068&toolbox=type> (accessed 23 November 2015).
3. For example, the following member states (regions) cut their educational expenditure in real terms according to Eurostat in 2014: Czech Republic (-3.33%), Ireland (-1.53%), Greece (-2.11), Croatia (-1.95%), Finland (-2.39%), Austria (-2.72%), UK (Wales) (-1.88%), Belgium (French region -0.07%; German region -18.22%).
4. Expressed in per human capital unit the *production function becomes to*  $f(\tilde{k}_t = F\left(\frac{\tilde{k}_t}{H_t}, 1\right)$ . We assume that the corresponding Inada conditions hold:  $f(0) = 0$ ;  $f(\infty) = \infty$ ;  $f'(\infty) = 0$ ; and  $f'(0) = \infty$ .
5. Then  $u_t$  must be interpreted as the share of income which is spent for education.
6. Li, Zhang, and Zhu (2008), Rosenzweig and Wolpin (1980) and Rosenzweig and Zhang (2009) confirm the existence of the quality-quantity trade-off of children econometrically.
7. Unfortunately, in all four papers the suspected interior maxima are no maxima, because they suffer to fulfil the second order conditions. The corresponding proofs for this statement can be requested from the authors.
8. The A-Pareto criterion was introduced by Golosov, Jones, and Tertilt (2007) and refers to the assumption that only the utility of the actually born individuals are to be taken into account.
9. See the references cited below.
10. Or more precisely, the criterion should be reformulated as: (total government expenditures-public educational expenditures-total tax revenues)/GDP<3%. Accordingly, the debt to GDP ratio has also to be adjusted.

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No potential conflict of interest was reported by the authors.

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