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Dynamic immunisation does not imply cash flow matching: a hard application to Spain

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ABSTRACT

Immunisation is not a static strategy as the literature affirms: we argue that the conditions established for reaching immunisation are unbalanced in themselves as times go on. This paper presents a valid, comprehensive strategy with all the conditions and assumptions made. It is checked in the Spanish debt market with data from 2004 to 2013 using some immunised portfolios preset following these conditions so that there is no rebalancing. The authors find a strategy that eliminates the requirement of rebalancing because of time passing or due to the mere parallel shift of interest rates regarding the yields that should have been obtained under the hypothesis of the rational expectations theory.

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1. Introduction

The objective of most financial institutions with assets to invest is to fund some sort of liability. As a result, asset/liability management (ALM) should be the investment focus and the basis for selecting core portfolios. Insurance companies may have been the birthplace of ALM, and they have certainly been the reference model for ALM discipline, thanks to the stringent regulations imposed on them (International Association of Insurance Supervisors [IAIS], 2006).

In short, ALM can be defined as the process that deals with interest rate risk management. Banks and insurance companies have practised ALM since their inception. Their ALM approach centres on the interest rate risk management of assets versus liabilities such that their risk/reward behaviour is similar or matched (Brick, 2014). Insurance companies must meet their liabilities in the manner in which they have been underwritten, independent of their financial and economic trend. Having estimated the future outflows coming from the liabilities, the company must collect enough money and invest it to attain that target. It should never be forgotten that even an insurance company can use different strategies to achieve its goals. The first objective is to maximise the value of the company for shareholders

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and the second is to protect value for policy holders and guarantee the future payout of potential policy obligations (Plantinga & Huijgen, 2000): to pay when payment is due.

The factors that condition an ALM strategy are asset flows, liabilities and the interest rates used to assess them. There is a large body of literature on each of these factors though, in all cases, the fact that immunisation is a static strategy is taken into account. This means that when there are unexpected changes in interest rates an almost continuous rebalancing is required to attain the payment target. The present paper sets out to show that there is an immunisation strategy other than cash flow matching (CFM) that enables the payments due to be met and does not entail continual rebalancing in the face of variable interest rates and no defined law.

To that end the following section reviews the main literature on ALM, noting the main characteristics of CFM and immunisation. Most authors point out the static nature of the strategy, but there is a variant that enables the aim of payment to be fulfilled.

This general, dynamic strategy is presented in Section 3 (Methodology). Its main characteristics are outlined, along with the correct indicator for checking whether the strategy meets its goal. At this point immunisation risk is significant: to measure it we use the absolute immunisation risk (in Spanish *Riesgo de Inmunización Absoluto* (RIA)), which is an improvement on the usual measurements of M^2 and M^A.

To test its applicability we run a hard practical test of this strategy, taking Spanish interest rate data from 2004 to 2013 and checking the trends over nine years for seven different portfolios without a single rebalancing.

Before this test, the trend in interest rates over this very long, turbulent period is reviewed. As might be expected, in this troubled period the interest rates observed are far from those expected. Therefore, as the assumptions are not fulfilled, the trend in the portfolios could be disastrous. At any event, it is found that this trend is really threatening, proving that the strategy works even in the hardest conditions.

Finally, we offer proposals as to what an insurance company could do to avoid almost all interest risk while offering its customers interest rates near those of the debt market. In summary, the investment strategy makes it possible to raise payments to customers or, conversely, to promise higher interest rates (Kortleve, Mulder, & Pelsser, 2011).

2. Immunisation models: an overview

Immunisation models can be said to have originated in 1952, when two seminal proposals for dealing with interest rate risk were made: CFM (Haynes & Kirton, 1952), also called absolute or dedicated matching (Bader, 1983) and immunisation (Redington, 1952), also called duration matching.

Several authors have taken on the task of extending the original model to take into account more realistic real-world assumptions. In this sense, CFM is not always possible because sometimes the market might not have the bonds needed to implement this strategy, but when possible it has advantages, such as the complete elimination of interest rate risk. Future-value matching of liabilities (dedication) is most beneficial for accounting purposes when there is a guaranteed match of assets to liabilities (Ryan, 2013). To execute a certain match of liabilities requires zero-coupon bonds matched to the liability payment dates and amounts. But when zero-coupon bonds are not available it is mathematically difficult for dedication models using coupon bonds to be certain of a guaranteed match of liabilities because of reinvestment risk, call risk, low interest rates (Beer & Gnan, 2015; Focarelli, 2015) and other factors that would alter the cash flows.

Immunisation focuses on matching the interest rate sensitivity of liabilities in present values. As a result, it focuses on duration (or modified duration) in harmony with horizon analysis. The duration matching approach and its many modified approaches (Fong & Vasicek, 1984; Shiu, 1987a, Shiu, 1988; Reitano, 1996; Hürlimann, 2002) are generally used for interest rate risk management. However, they are not feasible in our case due to the lack of long-term bonds to discount the liabilities and compute the duration.

Several authors argue that CFM (Kocherlakota, Rosenbloom, & Shiu, 1988, 1990) is a class of bond immunisation technique where a stream of liabilities is matched perfectly by cash flows generated from a bond portfolio (Fisher & Weil, 1971; Hiller & Schaack, 1990; Zipkin, 1992). The resulting portfolio is thus truly immune from interest rate changes. Various extensions of the CFM technique have been studied. Hiller and Eckstein (1993) consider a stochastic programming approach in the spirit of mean variance portfolio optimisation (Cheng, 1962; Markowitz, 1952). To also take into account interest-sensitive cash flows, unknown random times of payments and other stochastic modelling assumptions, further extensions are needed. Several authors have already worked along these lines (Boyle, 1978; Albrecht, 1985; Castellani, DeFelice, & Moriconi, 1992; Shiu, 1993, Munk, 2011). On the other hand, Chun Ma and Ki Cheung (2013) apply a dynamic approach to future interest rates under lattice models.

Because immunisation strategies focus on matching present values, the question of what discount rates to use to calculate the present value of liabilities became a major consideration. The Financial Accounting Standards Board (Financial Accounting Standards Board [FASB], 1985) clarifies this point by asserting that the discount rate methodology used for liabilities should be based on a high-quality bond yield curve that settles the liabilities.

Fisher and Weil (1971) work with a non-flat term structure of interest rates and show how to immunise a single liability for parallel shifts in the term structure. This approach was subsequently generalised to multiple liabilities by Shiu (1988); who also finds interesting connections with linear programming (Shiu, 1987a, 1987b, 1988). The extending of this approach and a remarkably rigorous statement of Redington's theorem are discussed in Montrucchio and Peccati (1991); and Hülmimann (2002) extends the conditions set by Fong and Vasicek (1983a) and Shiu (1988) to a necessary and sufficient condition for immunisation under arbitrary convex shift factors of the term structure of interest rates.

From the very outset (Redington, 1952), the main limitation that has been attributed to immunisation is that it supposedly works only at the initial time, i.e., it is a static approach. Even when the conditions are set (Iyengar & Ma, 2009), the mere passage of time dismantles the initial balance. Almost all the previous literature about immunisation supports the opinion

> that it is possible to immunise a portfolio of default-free assets against unexpected interest rate changes so that at the end of the planning period the investor will realise at least the returns expected at purchase. However, this immunisation strategy is applicable for the case in which the change in unexpected interest rate occurs only once at the instant after the purchase of the asset. Obviously, the case depicted above is not likely to resemble the real world situation in at least two respects. First, the interest rate change is likely to occur at any time and, second, the interest rate change is likely to occur many times during the investor's planning period.

Khang (1983) analyses this case and establishes that the only way to enable dynamic immunisation is to rebalance constantly. He states that if nothing is done only CFM is able to secure the portfolio dynamically. The same conclusion is reached by Hürlimann (2012); who presents the immunisation against all possible shift directions and concludes that a portfolio of fixed assets and liabilities can only be immunised against arbitrary shifts of the interest rate curve through an exact matching strategy. However, as Navarro and Nave (1994) remark, there is just another way to secure the portfolio, following the 'Dynamic Global Immunisation Theorem enunciated by Khang in 1983', consisting of following 'a continuous portfolio rebalancing in order to keep portfolio duration equal to the length of the remaining planning period'. But, 'Khang's theorem was based on a set of hypothesis including… the absence of transaction costs', because continuous rebalancing would obviously entail incredibly high transaction costs, and this is why Navarro and Nave (1994, p. 397) establish 'that the optimal dynamic strategy against interest rate risk depends on the level of transaction costs'.

By contrast, Iturricastillo Plazaola and De La Peña Esteban (2003), find that there is a strategy that enables a portfolio to be immunised for a long time rather than merely at the time of implementation of the immunisation conditions: horizon matching. This strategy consists of a short initial period of CFM and a duration matching for the remainder of the portfolio's lifetime. Under several constraints, immunisation would remain perfect by itself in every portfolio that satisfies those constraints (Iturricastillo Plazaola, 2007; Iturricastillo Plazaola & De La Peña Esteban, 2008). So there is no need for exact matching or continuous rebalancing to immunise a portfolio for a long period.

3. Methodology

3.1. The model

The horizon matching strategy consists of an initial short period of CFM and then duration matching for the remainder of the portfolio's lifetime (Iturricastillo Plazaola & De La Peña Esteban, 2003).

The sole assumption is that the interest rate curve follows the hypothesis of rational expectations theory (HRE) except when there is a parallel shift. Thus, the future interest rate curves will be the curve of the implied forwards rates expected from the beginning or a curve parallel to them. It is proved that under these assumptions it does not matter at what stage the parallel shift happens, because it is transferred to the implied rate curves in a parallel manner (Iturricastillo Plazaola, 2007). The immunisation conditions therefore remain perfect by themselves following this strategy, even if there is a shift of the interest rate curve at any time, so long as the assumption is fulfilled.

This model can be called 'general' because the strategy presented for either of the last two dynamic immunisations, which are two special immunisation cases (Bierwag & Kaufman, 1985), is a generalisation of the classical model. Moreover, the classical model is also clearly a generalisation of the model presented by Redington (1952), because Redington's model performed the analysis from a unique interest rate, and can thus be considered to use a flat interest rate curve, so parallel shifts may be expected. Obviously, after such a shift the flat interest rates curve remains flat.

This is a special case of our model, since the implied forward rate curves of a flat spot rates curve is the same curve and, therefore, remains flat. And if the shifts are parallel all curves also remain flat.

3.2. Requirements for dynamic classic immunisation

In this case a portfolio is immunised without a net capital, where the liabilities of the portfolio have a present value equal to the money available for the company to invest in order to meet those liabilities. The goal is to avoid the assets having a lower value than the liabilities at any time.

Definitions:

$_t i_0$: Spot rate for the (0;t) period;
$A_0({}_t i_0)$: Current value of assets;
$L_0({}_t i_0)$: Current value of liabilities;
F_t: Inflow at t;
L_t: Outflow at t;
$MD_A({}_t i_0)$: Modified duration of asset at present time (with the current spots);
$MD_L({}_t i_0)$: Modified duration of liability at present time (with the current spots);
$MCX_A({}_t i_0)$: Modified convexity of asset at present time (with the current spots);
$MCX_L({}_t i_0)$: Modified convexity of liability at present time (with the current spots).

The requirements for this immunisation are:

1. Present value of the assets equal to the present value of the liabilities

$$A_0\left({}_t i_0\right) = \sum_{t=1}^{t=T} F_t \left(1 + {}_t i_0\right)^{-t} = \sum_{t=1}^{t=T} L_t \left(1 + {}_t i_0\right)^{-t} = L_0\left({}_t i_0\right) \tag{1}$$

2. Initial value of the modified duration of assets equal to the initial value of the modified duration of liabilities

$$MD_A\left({}_t i_0\right) = \frac{\sum_{t=1}^{t=T} t \cdot F_t \cdot \left(1 + {}_t i_0\right)^{-t} \cdot \left(1 + {}_t i_0\right)^{-1}}{\sum_{t=1}^{t=T} F_t \cdot \left(1 + {}_t i_0\right)^{-t}} \tag{2}$$

$$= \frac{\sum_{t=1}^{t=T} t \cdot L_t \cdot \left(1 + {}_t i_0\right)^{-t} \cdot \left(1 + {}_t i_0\right)^{-1}}{\sum_{t=1}^{t=T} L_t \cdot \left(1 + {}_t i_0\right)^{-t}} = MD_L\left({}_t i_0\right)$$

3. Initial value of the modified convexity of assets equal to or higher than the initial value of the modified convexity of liabilities

$$\mathrm{MCX_A}\left({}_t\mathrm{i}_0\right) = \frac{\sum_{t=1}^{t=T} t\,(t+1)\,\mathrm{F}_t \left(1+{}_t\mathrm{i}_0\right)^{-t}\left(1+{}_t\mathrm{i}_0\right)^{-2}}{\sum_{t=1}^{t=T} \mathrm{F}_t\left(1+{}_t\mathrm{i}_0\right)^{-t}} \geq \frac{\sum_{t=1}^{t=T} t\,(t+1)\,\mathrm{L}_t \left(1+{}_t\mathrm{i}_0\right)^{-t}\left(1+{}_t\mathrm{i}_0\right)^{-2}}{\sum_{t=1}^{t=T} \mathrm{L}_t\left(1+{}_t\mathrm{i}_0\right)^{-t}} = \mathrm{MCX_L}\left({}_t\mathrm{i}_0\right) \tag{3}$$

4. CFM period or absolute matching period longer than or equal to the period in which the company would like to avoid any rebalancing (no rebalancing period).

These conditions immunise the portfolio perfectly and dynamically, but a prudent portfolio manager would also like to control the immunisation risk. The best known tool for measuring this risk is M^2 (Fong & Vasicek, 1983b). This measure has come in for considerable criticism and there have been many proposals looking to surpass its performance. For example, although it is well known that financial risks are linked with time, money, etc., but not with their squares, some advocates of M^2 (Li & Panjer, 1994) present the fact that M^2 looks like a variance as one of its strengths. Furthermore, and linked with the above, although M^2 has been proved not to perform badly, it is clearly some distance from financial sense. There have been many proposals, knowing that any dispersion measure is an immunisation risk measure (Balbás & Ibáñez, 1995), and among them an interesting step forward could be M^A (Nawalkha & Chambers, 1996): this is rather similar to M^2, but it measures risk in financial units and not in its squares and is thus more closely tied in with financial reality. However, even this measure is bound to statistics and has no real financial sense, which is why it also has problems measuring immunisation risk.

Iturricastillo Plazaola (2007) proposes a new measure for quantifying and controlling immunisation risk: RIA. This measure classifies the risk of portfolios better and also has a clear financial sense, which means that the measure itself is quantitatively meaningful and easy to understand (Iturricastillo Plazaola & De La Peña Esteban, 2010).

The RIA measures the average time between liabilities and the inflows that serve to offset them, so it is consistent with its objective, i.e., immunisation risk, as it shows how far a portfolio is from (or how close it is to) the strategy without interest risk – absolute matching or CFM – in which outflows and inflows are perfectly matched. So, the manager can establish a maximum RIA to control immunisation risk. In making this choice it is always advisable to adopt a minimum margin of prudence.

For the sake of simplification Iturricastillo Plazaola (2007) sets out the following equation to calculate the RIA

$$\mathrm{RIA} = \frac{\sum_{h=0}^{n}\left|\sum_{j=1}^{h}\left(\mathrm{F}_j - \mathrm{L}_j\right)\cdot\left(1+{}_j\mathrm{i}_0\right)^{-j}\right|}{\sum_{h=1}^{n}\mathrm{F}_h\cdot\left(1+{}_t\mathrm{i}_0\right)^{-h}}\cdot\frac{1}{\mathrm{k}} \tag{4}$$

k Number of periods considered within each year.

This paper sets out a dynamic, complete, general immunisation model, which means that the immunisation risk measure must also be dynamic, but M^2 and M^A are not. If immunisation is established for a long time, the manager should at least know as accurately as possible how immunisation risk is likely to develop. During the absolute matching period the deployment of the RIA is understood (Iturricastillo, De la Peña, Moreno, & Trigo, 2011), so this growth can be programmed beforehand, making it easier to monitor this risk in following this strategy.

The RIA at x within the no rebalancing period is the following

$$\mathrm{RIA_x} = \frac{\mathrm{RIA_0}}{\mathrm{PFF_x}} \tag{5}$$

RIA_0: RIA at the initial time.

PFF_x: Proportional weight of the flows after x in the whole initial value

where PFF_0 is 1 and decreases as x grows, RIA_x is an increasing value.

Given that the value for a given time x of PFF_x is known when the portfolio is established and considering x to be the end of the absolute matching period, a fifth condition could also be established to control immunisation risk, even though it is not really needed to secure theoretical immunisation:

5. The initial *RIA* should not exceed the proportion PFF_x of the maximum RIA previously established by the manager.

This last condition can be stated as

$$RIA_0 = PFF_x \cdot RIA_x \leq PFF_x \cdot Maximum\ RIA \tag{6}$$

If the assumptions hold, then immunisation will remain perfect by itself on every portfolio that satisfies the above conditions (Iturricastillo Plazaola, 2007; Iturricastillo Plazaola & De La Peña Esteban, 2008).

4. Application to an economic crisis period: Spain

4.1. Trend in the interest rate curve

The objective of this paper is to test the performance of immunisation in one of the worst conditions ever. Spanish State debt has recently undergone severe hardships and has suffered a serious financial storm. The interest rate trend period chosen is therefore unique in terms of showing how immunisation implemented long before a deep crisis performs: this immunisation was expected to be free from rebalancing for a long period.

In this paper the interest rates curves considered are those for Spanish State debt at the beginning of 2004, 2009, 2011 and 2013. These curves have not been smoothed: they are the actual curves, with the problem of strange changes of interest rates from one term to another and the advantage that real investment may be found in those conditions.

There are intervals of 5 years, 2 years and 2 years, respectively, between these curves, and we want to show what would have happened if there had been an immunisation in 2004 following the criteria explained without a single rebalancing. As the period free from rebalancing is so long, the initial absolute matching period has to be even longer.

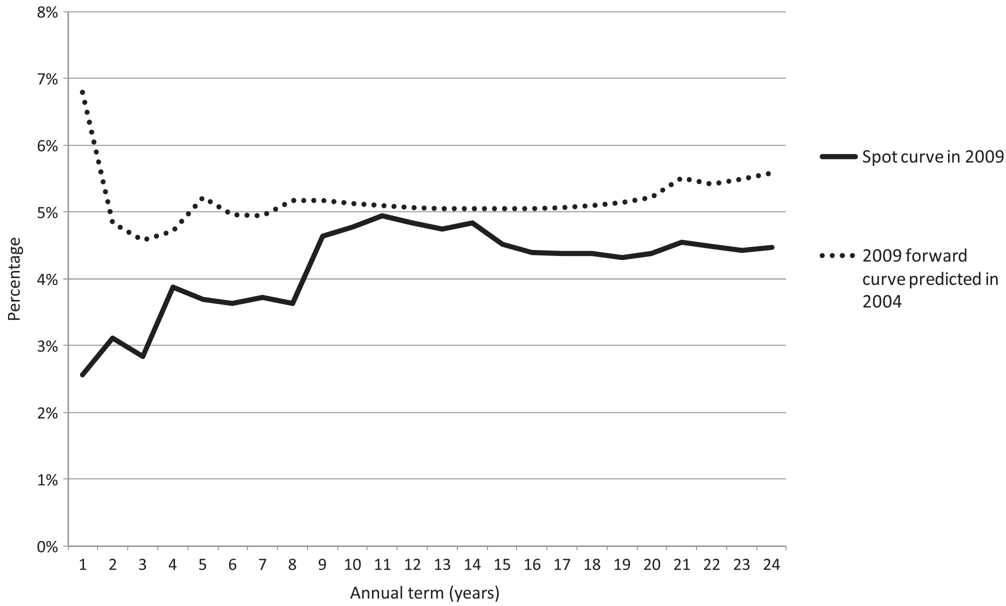

Figure 1. 2009 spot interest (realised) curve versus forward (predicted) interest curve. Source: Authors' calculation with data from Bank of Spain.

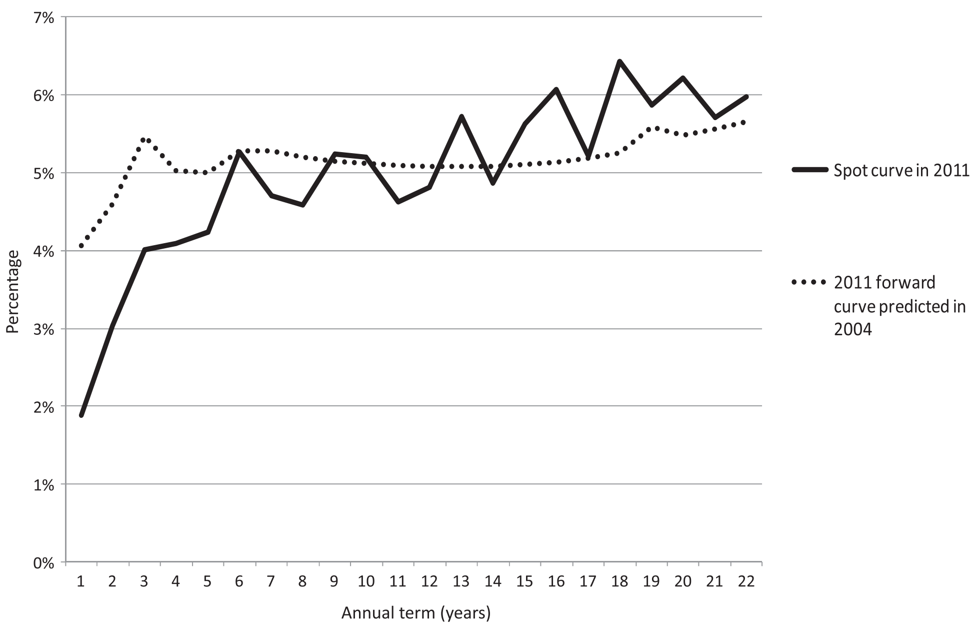

Figure 2. 2011 spot interest (realised) curve versus forward (predicted) interest curve. Source: Authors' calculations with data from Bank of Spain.

Appendix 1 shows the interest rate curves at these times. The starting point is the beginning of 2004, so that is the starting point for the comparison. Figures 1, 2 and 3 show the observed interest rate curves (spots) at each time to be tested and the interest rate curves predicted (forward) from the interest rate curve for 2004.

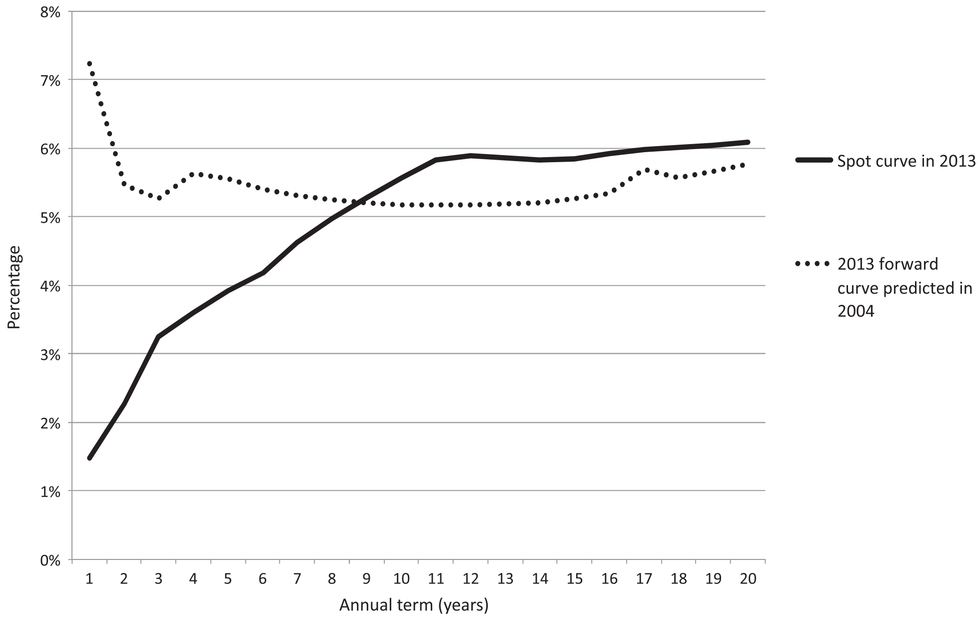

Figure 3. 2013 spot interest (realised) curve versus forward (predicted) interest curve. Source: Authors' calculations with data from Bank of Spain.

A glance at Figures 1, 2 and 3 is enough to confirm how different from a parallel movement (from the predicted curve) the interest rate curve is. Theoretically, the expected performance of the portfolios immunised with the conditions stated would be very poor.

4.2. *Trend in different portfolios*

In this section a test of the performance of seven different portfolios is developed. They are immunised against interest risk, and the initial outflows of the liabilities are chosen by a random procedure with the constraint that they must all be future outflows in multiples of €200,000, up to a maximum of €4 million.

The no-rebalancing period required, unforeseen circumstances notwithstanding, is 9 years, from 2004 to 2013. So, the portfolios must perfectly match the liabilities/outflows for this whole period and two further years because

> …in practice, the portfolio manager must leave time between the moment when he wants to rebalance and the term when the absolute matching ends. This will help to manage when there is a financial earthquake, because it will give time to rebalance, when more probably the short-term interest rates will not move in a parallel manner. While there are not non-zero net cash flows in the short term, the short-term interest rates will not affect the portfolio. (Iturricastillo Plazaola & De La Peña Esteban, 2012, p.12)

Table 1 shows the outflows originated by the liabilities and the inflows from the seven portfolios to be immunised. First, the portfolios have to meet the first four conditions of immunisation (the real conditions). Second, the basic difference between the portfolios is the limit chosen for the initial RIA. In theory, the final RIA should be limited and then the initial RIA at the beginning should be calculated following Equation (6), but for the sake of simplification the initial limit is set directly. So, the first five portfolios have a limit on

Table 1. Outflows of commitment and inflows to portfolios to be immunised.

		Portfolio						
Term	Liabilities	1	2	3	4	5	6	7
1	1,800,000	1,800,000	1,800,000	1,800,000	1,800,000	1,800,000	1,800,000	1,800,000
2	1,400,000	1,400,000	1,400,000	1,400,000	1,400,000	1,400,000	1,400,000	1,400,000
3	2,400,000	2,400,000	2,400,000	2,400,000	2,400,000	2,400,000	2,400,000	2,400,000
4	1,800,000	1,800,000	1,800,000	1,800,000	1,800,000	1,800,000	1,800,000	1,800,000
5	3,400,000	3,400,000	3,400,000	3,400,000	3,400,000	3,400,000	3,400,000	3,400,000
6	2,600,000	2,600,000	2,600,000	2,600,000	2,600,000	2,600,000	2,600,000	2,600,000
7	3,200,000	3,200,000	3,200,000	3,200,000	3,200,000	3,200,000	3,200,000	3,200,000
8	1,200,000	1,200,000	1,200,000	1,200,000	1,200,000	1,200,000	1,200,000	1,200,000
9	1,800,000	1,800,000	1,800,000	1,800,000	1,800,000	1,800,000	1,800,000	1,800,000
10	0	0	0	0	0	0	0	0
11	2,800,000	2,800,000	2,800,000	2,800,000	2,800,000	2,800,000	2,800,000	2,800,000
12	3,200,000	5,690,185	8,597,440	9,063,216	7,716,804	12,962,421	14,530,443	1,642,178
13	1,800,000	234,743	0	175,513	1,316,209	75,260	0	2,612,125
14	800,000	350,875	200,256	384,857	1,132,481	107,720	0	2,320,312
15	2,400,000	891,250	294,072	674,030	1,204,383	190,093	0	2,310,913
16	2,600,000	1,520,350	383,489	945,108	1,275,493	213,587	0	2,093,476
17	1,000,000	372,223	1,199,491	1,097,085	1,442,691	265,203	0	2,155,449
18	1,600,000	4,351,969	2,198,285	1,332,624	1,153,323	475,013	0	1,999,815
19	2,200,000	2,661,317	1,853,311	1,550,617	3,164,150	484,815	0	1,868,445
20	2,800,000	1,516,699	1,432,377	1,649,858	165,669	511,781	0	1,049,711
21	2,400,000	2,595,379	1,076,776	1,582,065	166,351	446,493	0	1,908,608
22	3,200,000	2,535,504	1,601,725	1,565,067	134,357	402,151	0	2,072,557
23	600,000	2,479,480	2,073,281	1,538,327	1,057,655	324,367	0	1,999,208
24	1,400,000	2,575,809	2,489,705	849,156	808,887	208,585	0	2,176,995
25	2,600,000	2,432,735	2,917,720	773,097	44,280	226,492	0	3,610,141
26	3,400,000	2,573,691	2,979,017	718,359	24,619	270,576	0	2,014,532
27	2,000,000	536,889	3,311,529	658,914	0	290,944	0	2,197,786
28	4,000,000	2,694,174	3,768,646	720,893	0	300,176	0	2,355,158
29	800,000	2,712,842	3,176,289	15,366,134	20,494,428	24,628,993	28,545,894	2,439,713

Source: Authors' calculations.

the initial RIA of 0.25, 0.5, 0.75, 1 and 1.5, and the sixth portfolio maximises the positive difference between modified convexities, with an RIA of 1.8. The maximisation of the RIA is linked to the maximisation of the net modified convexity (Iturricastillo Plazaola & De La Peña Esteban, 2010); this is good for making a profit if the assumptions are fulfilled, but it increases the immunisation risk.).

By contrast, the last portfolio is chosen with an additional constraint that could be chosen by managers, limiting the risk by restricting the inflow at any time, as the outflows are also spread over time. The seventh portfolio thus restricts inflow to four million. The last portfolio meets the conditions for immunisation and has an RIA close to 0.18, so the level of unexpected movements in the net value should be near but below the level of those in the first portfolio, though the direction of these movements might be different.

4.3. Theoretical trend and parallel shifts

This section begins by showing in Table 2 the basic data on the performance of the portfolios assuming that the predicted forwards become spot rates.

Table 3 then shows the performance that there would be if there was a parallel shift with respect to the forward figures. Thus, in both cases the assumptions are perfectly matched. For this last example a parallel increase of 3% is chosen for all the predicted interest rates.

Table 2. Theoretical trend in the portfolios.

Year	Measure	Portfolio 1	2	3	4	5	6	7
2004	Net value (S/A)	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	MDA-MDL	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	MCXA-MCXL	0.44	4.47	7.86	9.55	14.98	17.83	0.00
	RIA	0.25	0.50	0.75	1.00	1.50	1.80	0.18
2009	Net value (S/A)	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	MDA-MDL	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	MCXA-MCXL	0.61	6.31	11.09	13.46	21.13	25.14	0.00
	RIA	0.35	0.70	1.06	1.41	2.11	2.54	0.26
2011	Net value (S/A)	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	MDA-MDL	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	MCXA-MCXL	0.76	7.79	13.70	16.63	26.09	31.05	0.00
	RIA	0.43	0.87	1.31	1.74	2.61	3.14	0.32
2013	Net value (S/A)	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	MDA-MDL	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	MCXA-MCXL	0.85	8.77	15.42	18.72	29.38	34.97	0.00
	RIA	0.49	0.98	1.47	1.96	2.94	3.53	0.36

Notes: S: Surplus; A: Asset; MDA: Modified duration of asset; MDL: Modified duration of liability; MCXA: Modified convexity of asset; MCXL: Modified convexity of liability; RIA: Absolute immunisation risk.
Source: Authors' calculations.

Table 3. Theoretical trend in the portfolios with a parallel shift.

Year	Measure	Portfolio 1	2	3	4	5	6	7
2004	Net value (S/A)	0.02%	0.16%	0.26%	0.32%	0.50%	0.60%	0.00%
	MDA-MDL	−0.01	−0.09	−0.15	−0.18	−0.28	−0.34	0.00
	MCXA-MCXL	0.07	0.02	−0.28	−0.50	−0.54	−0.64	−0.06
	RIA	0.20	0.39	0.59	0.77	1.17	1.40	0.15
2009	Net value (S/A)	0.03%	0.25%	0.41%	0.49%	0.78%	0.93%	0.00%
	MDA-MDL	−0.02	−0.15	−0.25	−0.29	−0.47	−0.56	0.00
	MCXA-MCXL	0.26	1.27	1.60	1.63	3.02	3.59	−0.12
	RIA	0.32	0.63	0.94	1.23	1.86	2.22	0.23
2011	Net value (S/A)	0.04%	0.32%	0.54%	0.65%	1.03%	1.22%	0.00%
	MDA-MDL	−0.02	−0.20	−0.34	−0.40	−0.64	−0.76	0.00
	MCXA-MCXL	0.41	2.31	3.16	3.40	5.99	7.11	−0.17
	RIA	0.42	0.83	1.24	1.63	2.46	2.94	0.30
2013	Net value (S/A)	0.04%	0.37%	0.63%	0.75%	1.19%	1.42%	0.00%
	MDA-MDL	−0.03	−0.24	−0.40	−0.47	−0.76	−0.90	0.00
	MCXA-MCXL	0.58	3.53	5.06	5.58	9.57	11.35	−0.21
	RIA	0.49	0.98	1.45	1.91	2.88	3.44	0.35

Source: Authors' calculations.

For the sake of clarity, the net values are shown in terms of the ratio between the surplus and asset values.

If the interest rate curve follows HRE theory then immunisation ensures that the net value and the duration differences will remain constant (zero) and that the positive convexity difference and the RIA will increase as the zero net cash flows of the initial absolute matching period disappear (Table 2). This last trend depends on how many matched inflows/outflows have already been paid, since they reduce the value of the portfolio while the nominal risk faced is the same. Hence, the proportional risk gets higher.

Table 3 shows that there is no possibility of losses so long as the assumptions are fulfilled. In this case, the benefit of parallel shifts is higher when the risk is higher. However, the

Table 4. Trend in the basic immunised portfolios with no rebalancing.

		Portfolio						
Year	Measure	1	2	3	4	5	6	7
2004	Net value (S/A)	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	MDA-MDL	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	MCXA-MCXL	0.44	4.47	7.86	9.55	14.98	17.83	0.00
	RIA	0.25	0.50	0.75	1.00	1.50	1.80	0.18
2009	Net value (S/A)	0.01%	0.99%	1.61%	1.80%	3.19%	3.80%	−0.03%
	MDA-MDL	0.00	0.09	0.21	0.28	0.40	0.47	0.01
	MCXA-MCXL	0.54	8.69	17.78	22.50	33.20	39.25	0.25
	RIA	0.35	0.71	1.15	1.54	2.24	2.67	0.26
2011	Net value (S/A)	0.55%	−0.10%	0.49%	1.04%	0.84%	0.95%	−0.24%
	MDA-MDL	0.01	−0.08	−0.05	−0.03	−0.10	−0.12	−0.03
	MCXA-MCXL	0.85	5.83	11.90	14.89	22.63	26.89	−0.84
	RIA	0.47	0.87	1.31	1.74	2.60	3.12	0.33
2013	Net value (S/A)	−0.58%	−0.23%	0.50%	1.44%	1.41%	1.76%	0.45%
	MDA-MDL	0.00	−0.05	−0.05	−0.05	−0.11	−0.13	−0.01
	MCXA-MCXL	1.06	7.76	13.89	16.70	25.91	30.64	−0.32
	RIA	0.56	1.02	1.47	1.99	2.93	3.52	0.41

Source: Authors' calculations.

riskiest immunised portfolio can bring the highest losses (or profits as the case may be) if the assumptions are not fulfilled.

4.4. Real trend and consequences

Table 4 shows the trend in the seven immunised portfolios (linked to liabilities) at the very beginning of 2004 and after 5, 7 and 9 years without a single rebalancing. It is shown that there are just two cases where the final net value is slightly negative, and even then there has been a chance to make a profit first by rebalancing, so the real possibilities of this strategy are very clear.

The net values change over time in a way very different from what was expected, because the interest rates also move in an unexpected way. Nevertheless, the worst observed performance at any time tested is a loss of only 0.58%, which is a very good result, considering that this is a decrease on the final result after 9 years of capitalisation and having not spent a single euro in transaction costs, and even more so when there was a profit of 0.55% in a previous period, which could have provoked an in-between rebalancing to ensure this surplus.

In brief, the proposed immunisation strategy will meet its target, as shown in one of the toughest periods that could be found.

4.5. Results and proposed strategy

A customer looking to ensure a pension plan or an annuity has no possibility of investing in anything as profitable as, and less risky than, the immunising portfolio, and would in any event encounter larger transaction costs.

Article 2 of Spanish Royal Decree 239/2007 of 16 February (amending the Regulations on Planning and Supervision of Private Insurance approved by Royal Decree 2486/1998 of 20 November) includes a redrafting of Article 33, which regulates the rate of interest allowed to determine the provision of life insurance. There is now a maximum rate set by the supervisor as a weighted mean of the interest rates for five or more years, calculated

Table 5. Surplus offering the maximum interest rate.

Parameter	Value
Mean interest rate of portfolio 1	4.46113%
Mean market interest for 5 years or more	4.48824%
Maximum interest (60%)	2.69294%
Cost of insurance to customer	41,818,703.26 €
Net value for insurance company	8,250,791.06 €
Net value (percentage of real cost)	24.57940%

Source: Authors' calculations.

Table 6. Mean interest rates of each portfolio.

Portfolio	Mean interest rate
CFM	4.47360%
Porfolio 1	4.46113%
Porfolio 2	4.48803%
Porfolio 3	4.52805%
Porfolio 4	4.55782%
Porfolio 5	4.58578%
Porfolio 6	4.60798%
Porfolio 7	4.47644%

Source: Authors' calculations.

each month. Table 5 shows the interest that would have been in force in 2004 and the implications of offering this rate.

The data above show that a company offering this maximum interest rate would charge almost 25% over the fair price, even though the rate obtained is not so small for a customer unable to cover these risks on their own.

If there were no other options, companies would have to compete by other means to get such profitable business, but in fact the new regulations also enable them to promise higher interest rates, linking any investment with the appropriate duration to this liability.

Hence, the regulations allow even the highest interest rate obtained by the linked portfolio to be pledged, but a company should not act in this way due, among other things, to the following:

1. The company has to face burdens and costs to manage the portfolio, so a minimum extra charge is needed;
2. A cushion is also needed to overlay risks such as credit risk and immunisation risk, for example;
3. If the company promises the interest rate of the investment for its liabilities and if this is higher than the rate offered by the absolute matching portfolio, the company would collect less than the amount needed.

Table 6 shows the mean interest rate obtained by the seven immunised portfolios shown above and by the perfectly matched portfolio, with inflows equal to liabilities.

In view of Table 6 there must be a great need before a strategy other than absolute matching will be chosen, e.g., the impossibility of finding bonds for one or more of the terms, or the intention to make a profit by rebalancing every time there is a surplus. The latter might be interesting in the long term, when interest rates move enough to make a profit high enough to justify a rebalance.

Table 7. Surplus offering a higher interest rate by immunising.

Parameter	Value
Mean interest rate of portfolio 1	4.46113%
Reduction in promised interest	0.25000%
Promised interest	4.21113%
Cost of insurance to customer	34,572,101.75 €
Net value for insurance company	1,004,189.55 €
Net value (percentage of real cost)	2.99152%

Source: Authors' calculations.

Table 7 shows the situation if the company promises 0.25% less than the interest rate earned by portfolio 1, which has the lowest interest rate. Even in this case, the promised interest rate would be higher than 4.2%, which is clearly a high return in the long term, thus showing the high performance from immunising. In this case the net value would be almost 3%, which might be a high-enough cushion if the investment is made in a secure-enough bond type.

Finally, once the promised interest is set, the premium is set, and the strategy can then be implemented.

5. Conclusions

Generally, dynamic immunisation is designed to establish a strategy and exercise very light control, with no need to make changes in the portfolio for long periods, though it could also be implemented when a minimally significant profit can be assured by switching from one immunised portfolio to another.

The regulators and supervisor should follow the immunisation model proposed in this paper to establish the conditions for permitting an interest rate near that of the immunised portfolio, limiting the RIA to a prudent level and asking for a quite long absolute matching period both at the beginning and later.

The general, dynamic immunisation strategy developed proves to be a very useful tool, and performs well in reaching goals regardless of the hardships faced. Following the immunisation strategy, companies can offer higher interest rates, so they will be much more competitive. On the other hand, insurance holders see that their commitments are assured by guaranteed returns not influenced by market fluctuations.

The immunisation model proposed is the one that is most closely linked with the usual business of companies, so it should be the one chosen by the regulator and by companies themselves. To ensure solvency the regulator must set a required minimum difference between the interest of the immunising portfolio and the interest rate promised. It is thus possible to determine the solvency capital, which should be minimal due to the interest rate, as has been shown. This strategy enables companies to immunise themselves to unexpected, non-parallel changes in interest rates, and thus makes them stronger.

Long-term risks are common in the insurance industry: interest risk, mortality risk, longevity risk, etc. The current approach taken by life insurers in Spain has led to a high level of specialisation with regard to interest risk by means of duration-type immunisation systems. It also represents a degree of experience that is sure to help in the development of the new solvency systems envisaged in Solvency II.

After all, as shown in the paper, interest rates can describe the future earnings of a fixed-income portfolio rather well. As has been proved here, there is a strategy that really works in the long term, even in the Spanish debt market crisis: the dynamic, complete, general immunisation model.

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Appendix 1

Table 8. Spot interest rates.

Term (years)	2004 Spot	2009 Spot	2011 Spot	2013 Spot
1	2.49900000000%	2.55653047115%	1.88383205469%	1.47200000000%
2	2.59169722203%	3.10440152530%	3.04077795586%	2.27490519841%
3	2.95203500566%	2.83665465213%	4.01233942416%	3.25202780305%
4	3.08512471946%	3.87070112191%	4.09218870527%	3.60167169079%
5	3.36724647932%	3.69285008956%	4.24022162773%	3.91326484542%
6	3.93084063315%	3.63065565116%	5.26606724378%	4.18362034491%
7	3.78295918119%	3.72268771581%	4.70203830216%	4.61643558780%
8	3.81786764298%	3.63136706112%	4.57763106340%	4.97104229873%
9	3.96191846658%	4.64496869548%	5.23405181471%	5.28145621820%
10	4.28451456719%	4.77215354984%	5.19303819462%	5.56389829546%
11	4.23187367973%	4.94050705279%	4.62685744400%	5.82922365870%
12	4.28667075418%	4.83050300993%	4.81034029178%	5.88738482994%
13	4.47160909590%	4.74778442278%	5.71481383921%	5.86090805539%
14	4.52750583792%	4.83316552279%	4.86671088794%	5.82140592670%
15	4.53666800358%	4.51999901293%	5.63055677784%	5.84388645721%
16	4.54828365447%	4.39188300395%	6.06167024818%	5.91865483817%
17	4.56237198571%	4.37531643838%	5.20701096037%	5.97278214277%
18	4.57906164214%	4.37614581563%	6.42985189963%	6.01395469817%
19	4.59860934064%	4.32452227416%	5.85946706150%	6.04402250210%
20	4.62144307306%	4.38078817341%	6.21272072903%	6.07998142829%
21	4.64824982660%	4.55180825849%	5.70136275638%	6.10409969383%
22	4.68015875234%	4.48199300280%	5.96532069201%	6.09757035134%
23	4.71916720926%	4.42237126130%	6.09784976265%	6.06336447829%
24	4.76934315770%	4.47199631021%	5.59637187300%	6.00407516995%
25	4.84166223936%	4.62388199671%	5.47215439925%	5.93876433971%
26	5.09194529562%	4.77576768320%	5.36637302872%	5.89469972175%
27	5.03205732903%	4.92765336970%	6.39719257836%	5.86989419491%
28	5.11399967451%	5.07953905620%	6.37953745042%	5.97259555159%
29	5.19995162787%	5.10389417146%	6.34914176305%	

Source: Bank of Spain.