Novel Queueing Model for Multimedia Over Downlink in 3.5G Wireless Network

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Abstract **- In this paper, a model for multimedia transmission over downlink shared channel in 3.5G wireless network is presented. The multimedia stream consists of multiple substreams that are aggregated into one real-time and one nonreal-time flows. Correlation with each flow and between flows is assumed. Additionally, we propose a combined time-space priority buffer management scheme to optimise quality of service requirements for each flow. The problem is formulated in terms of a queue with two priority classes, one of which has time priority while the another has space priority. The input is described by the Batch Marked Markovian Arrival Process (*BMMAP*). Service time distributions are of *PH* (phase) type dependent on the class of a customer. The buffer is finite, but the customers of a class having higher priority for taking into the service from a buffer (time priority) can occupy only a part of this buffer. Queueing system's behavior is described in terms of multi-dimensional continuous time skip-free to the left Markov chain. It allows to exploit an effective algorithm for calculation of the stationary distribution of the queueing system. Loss probability for customers of both classes is calculated. Waiting time distribution for priority customers is calculated.**

Index terms**: 3.5G wireless networks, multimedia traffic, stochastic models, performance modelling**

I. INTRODUCTION

While Third Generation (3G) wireless systems are being intensively deployed worldwide, new proposals for enhanced data rates and quality of service provision are being standardised. One of the most promising enhancements to the widely deployed Universal Mobile Telecommunication System (UMTS) which is based on Wideband Code Division Multiple Access (W-CDMA) is called High Speed Downlink Packet Access (HSDPA) [1] which is also referred to as 3.5G. HSDPA aims at providing data rates around 2-10Mbps on the downlink to mobile users mainly for multimedia services such as real-time and streaming video in packet-switched common channel.

On one hand, modelling of multimedia traffic over shared channel is rather complicated. Therefore, most of the studies typically investigate the problem using packet-level simulation [4] or as data flows which can be real-time (voice or video) or non-real-time (www browsing, e-mail, ftp, or data access). Other works considered performance study of the HSDPA system taking into account system details rather than the multimedia traffic characteristics [15]. Attempts to capture some characteristics of the multimedia traffic analytically were made by several authors such as using self-similar traffic [6], [10], Generalised Exponential (*GE*) Distribution [3], or Batch Markov Arrival Process [5], [7], [13]. A common weakness in all previous models is that even if they capture correlation which an important feature of the multimedia traffic (*GE* does not even capture correlation), they consider the two flows independent of each other. However, even the single multimedia stream is composed of two types of traffic; real-time (voice and/or video) and non-real-time (data) traffic flows. These flows cannot be considered as independent flows. This implies the necessity of considering two types of correlation; intra-flow and inter-flow correlation within the same multimedia stream.

On the other hand, providing Quality of Service (QoS) for multimedia traffic requires differentiated consideration of the different flows within the single multimedia stream. The real-time traffic flow requires low delay and low jitter but can tolerate some packet loss while the data component (non-real-time) is very sensitive to packet loss but can tolerate some delay or jitter. In [2], the authors presented a multi-service class buffer model that can provide time and space prioritization based on thresholds Both priorities, however, can only be given to the same service or flow. In addition, the multimedia traffic was modelled by the *GE* distribution.

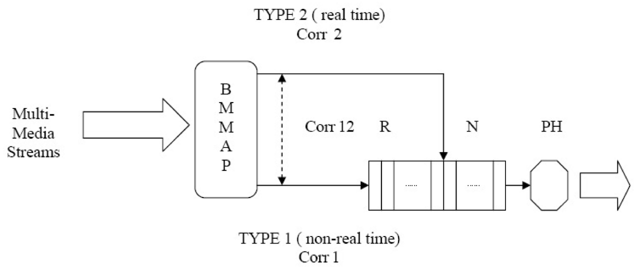

Fig. 1. A simplified block diagram of the proposed buffer architecture

The paper reports on initial results of a new line of research that will bring new dimension to the analytical modelling of multimedia traffic over wireless channels. We consider a multimedia stream that is composed of two types of traffic; real-time (voice and/or video) and non-real-time (data) traffic streams (See Figure 1). Unlike most recent studies, our models present two novel features that aim at providing necessary prioritization for QoS provisioning of multimedia services.

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First, we consider a novel queuing model for the buffer in which the two classes of traffic that compose the one multimedia stream enjoy different priority types that suits their QoS requirement; namely time priority for real-time traffic which a threshold to control the delay and jitter while we provide space priority for the data in order to minimize packet loss. In our project, we consider a number of distributions to model both the arrival process and the service process of the multimedia traffic. In this paper, we consider a novel application of the Batch Marked Markovian Arrival Process (*BMMAP*) to represent the multimedia stream that consists of two classes of traffic. The main advantage of using *BMMAP* is that it is capable of capturing both the intra-flow and inter-flow correlation.

This paper represents the report on the results of this project, and contains the formulation of the model, its analysis and the results of numerical experiments. It provides the solution and formulae for calculating packet loss probabilities for both classes and derivation of waiting time distribution for type-2 flow (real-time) will provide the tool to calculate the delay and jitter. This will then be the base for our optimization goal in which we use the threshold value to obtain desirable QoS for the multimedia traffic.

II. Mathematical Model

We have a single-server queue with a finite buffer of a capacity $R, R > 0$. The input flow is described by the *BMMAP* [9]. Customers arrival in the *BMMAP* is directed by the irreducible continuous time Markov chain $\nu_t, t \ge 0$, with a finite state space $\{0,1,\ldots,W\}$. Sojourn time of the chain $\nu_t, t \ge 0$, in the state ν has exponential distribution with a parameter λ_ν. After this time expires, with probability $p_0(\nu,\nu')$ the chain jumps into the state ν' without generation of customers and with probability $p_k^{(l)}(\nu,\nu')$ the chain jumps into the state ν' and a batch consisting of k customers of type l (type-l customers) is generated, $k \ge 1$. Here we will assume that only two types of customers are served in the system, so $l = 1, 2$. The introduced probabilities satisfy conditions:

$$p_0(\nu,\nu) = 0,$$

$$\sum_{l=1}^{2}\sum_{k=1}^{\infty}\sum_{\nu'=0}^{W} p_k^{(l)}(\nu,\nu') + \sum_{\nu'=0}^{W} p_0(\nu,\nu') = 1, \nu = \overline{0,W}.$$

The parameters defining the *BMMAP* can be kept in the square matrices $D_0, D_k^{(1)}, D_k^{(2)}$, $k \ge 1$, of size $\overline{W} = W + 1$ defined by their entries:

$$(D_0)_{\nu,\nu} = -\lambda_\nu,$$

$$(D_0)_{\nu,\nu'} = \lambda_\nu p_0(\nu,\nu'),$$

$$\left(D_k^{(l)}\right)_{\nu,\nu'} = \lambda_\nu p_k^{(l)}(\nu,\nu'),$$

$$\nu,\nu' = \overline{0,W}, k \ge 1, l = 1, 2.$$

Denote

$$D(1) = D_0 + \sum_{l=1}^{2}\sum_{k=1}^{\infty} D_k^{(l)},$$

$$\hat{D}_k^{(l)} = \sum_{i=k}^{\infty} D_i^{(l)}, k \ge 1, l = 1, 2,$$

$\boldsymbol{\theta}$ is the stationary probability vector of the states of the Markov chain $\nu_t, t \ge 0$.

Vector $\boldsymbol{\theta}$ is the unique solution to the system

$$\boldsymbol{\theta} D(1) = \mathbf{0}, \boldsymbol{\theta}\mathbf{e} = 1.$$

Here and below $\mathbf{0}$ is the row vector of dimension which should be clear from context, $\mathbf{e}$ is the column vector of appropriate dimension consisting of all 1's.

Intensity λ_l of type-l customers arrival is calculated by

$$\lambda_l = \boldsymbol{\theta}\sum_{k=1}^{\infty} k D_k^{(l)}\mathbf{e}, l = 1, 2.$$

Intensity $\lambda_l^{(b)}$ of batches of type-l customers arrival is calculated by

$$\lambda_l^{(b)} = \boldsymbol{\theta}\hat{D}_1^{(l)}\mathbf{e}, l = 1, 2.$$

Variation v_l of inter-arrival times of batches of type-l customers arrival is calculated by

$$v_l = \frac{2\boldsymbol{\theta}\left(-D_0 - \hat{D}_1^{(\bar{l})}\right)^{-1}\mathbf{e}}{\lambda_l^{(b)}} - \left(\frac{1}{\lambda_l^{(b)}}\right)^2, \bar{l} \ne l, \bar{l}, l = 1, 2.$$

Squared coefficient of correlation $C_{cor}^{(l)}$ of two successive intervals between type-l customers arrival is computed by

$$C_{cor}^{(l)} = \left[\frac{\boldsymbol{\theta}\left(-D_0 - \hat{D}_1^{(\bar{l})}\right)^{-1}}{\lambda_l^{(b)}}\hat{D}_1^{(l)}\left(-D_0 - \hat{D}_1^{(\bar{l})}\right)^{-1}\mathbf{e} - \left(\frac{1}{\lambda_l^{(b)}}\right)^2\right] v_l^{-1},$$

$$\bar{l} \ne l, \bar{l}, l = 1, 2.$$

Coefficient of correlation $C_{cor}^{(l,\bar{l})}$ of two successive intervals between type-l and type-$\bar{l}$ customers arrival (coefficient of cross correlation) is computed by

$$C_{cor}^{(l,\bar{l})} = \left[\frac{\boldsymbol{\theta}\left(-D_0 - \hat{D}_1^{(\bar{l})}\right)^{-1}}{\lambda_l^{(b)}}\hat{D}_1^{(l)}\left(-D_0 - \hat{D}_1^{(l)}\right)^{-1}\mathbf{e} - \frac{1}{\lambda_l^{(b)}\lambda_{\bar{l}}^{(b)}}\right] \times$$

$$\times\left(\sqrt{v_l}\sqrt{v_{\bar{l}}}\right)^{-1}, \bar{l} \ne l, \bar{l}, l = 1, 2.$$

Type-1 customers are accepted into the system if the buffer is not full. If the size of arriving batch exceeds the available space in a buffer we assume that the corresponding part of the batch is accepted into the buffer while the rest is lost. Such a discipline is called partial admission. Disciplines of complete admission or complete rejection where the whole batch is accepted or rejected correspondingly can be handled in a similar way.

Type-2 customers have a priority with respect to type-1 customers. If at least one type-2 customers presents in the system at the service completion epoch, then type-2 customer will get the service. Type-1 customers have a chance to get service only if no one type-2 customer presents in the system. Interruption of the service is not allowed.

However, type-2 customers have more restricted access into the system. No more then $N, 1 \le N < R$, type-2 customers can be accepted into the buffer. Discipline of partial admission is applied as well.

The service time of customers has *PH* (phase) type distribution. It means the following. Service of type-l customer is defined as a time until the continuous-time Markov chain $\eta_t^{(l)}, t \ge 0$, having the states $\{1, \ldots, M_l\}$ as the transient and state 0 as absorbing one reaches the absorbing state. The initial state of the chain is selected in a random way, according to the probability distribution defined by the row-vector $(\boldsymbol{\beta}_l, 0)$, where $\boldsymbol{\beta}_l$ is the stochastic row vector of dimension M_l. Transitions of the Markov chain $\eta_t^{(l)}, t \ge 0$, are described by the generator $\begin{pmatrix} S^{(l)} & S_0^{(l)} \\ 0 & 0 \end{pmatrix}$, where the matrix $S^{(l)}$ is sub-generator and the column vector $S_0^{(l)}$ is defined by $S_0^{(l)} = -S^{(l)}\mathbf{e}$. The average service time is given by $\boldsymbol{\beta}_l \left(-S^{(l)}\right)^{-1} \mathbf{e}$. For more details about the *PH* type distribution see [14].

Assumption that the arriving flow is the *BMMAP* allows to catch correlation in the input process that is very important in modelling multimedia traffic. Assumption about *PH* type service time distribution is some kind of trade-off between desire to consider the most general service process and possibility to have still analytically tractable Markov process as the model of the system behavior.

The proper selection of the buffer size R and the threshold N, which restricts access of the priority customers, can allow effectively control the main performance measures of the system (delay and jitter for type-2 customers and loss probability for type-1 customers). As a first step in this direction, the problem of calculation of the stationary state distribution of the Markov chain, which will be described in the next section, should be solved. Mention that this stationary distribution exists always as the state space of the Markov chain is finite, the process $\nu_t, t \ge 0$, is assumed to be irreducible and the so called $\left(\boldsymbol{\beta}_l, S^{(l)}\right)$ representations of *PH* are suggested to be irreducible.

III. Markovian Process of the System States

Let

- $i_t^{(l)}$ be the number of type-l customers presenting in a buffer at the epoch $t, l = 1, 2$;
- ξ_t be the type of the customer is service at epoch t;
- ν_t be the state of the directing process of the *BMMAP* at epoch t;
- $\eta_t^{(\xi_t)}$ be the state of the process which defines the service at the epoch t.

It is clear that the process

$$\zeta_t = \left\{i_t^{(2)}, i_t^{(1)}, \xi_t, \nu_t, \eta_t^{(\xi_t)}\right\}, t \ge 0,$$

$$i_t^{(2)} = \overline{0, N}, i_t^{(1)} = \overline{0, R - i_t^{(2)}}, \nu_t = \overline{0, W}, \eta_t^{(l)} = \overline{1, M_l}, l = 1, 2,$$

$$\xi_t = \begin{cases} l, & \text{if type} - l \text{ customer is in the service}, l = 1, 2, \\ 0, & \text{if the server is idle}, \end{cases}$$

is the Markov chain. Denote the stationary state probabilities of this Markov chain by:

$$p(0, 0, 0, \nu) = \lim_{t \to \infty} P\left\{i_t^{(2)} = 0, i_t^{(1)} = 0, \xi_t = 0, \nu_t = \nu\right\},$$

$$p\left(i_2, i_1, r, \nu, \eta^{(r)}\right) = \lim_{t \to \infty} P\left\{i_t^{(2)} = i_2, i_t^{(1)} = i_1, \xi_t = r, \nu_t = \nu, \eta_t^{(r)} = \eta^{(r)}\right\},$$

$$i_2 = \overline{0, N}, i_1 = \overline{0, R - i_2}, \nu = \overline{0, W}, \eta^{(r)} = \overline{1, M_r}, r = 1, 2.$$

To simplify operation with the probabilities, we enumerate the states of the processes in the lexicographic order and introduce the vectors of stationary probabilities:

$$\mathbf{p}(0, 0, 0) = \left(p(0, 0, 0, 0), p(0, 0, 0, 1), \ldots, p(0, 0, 0, W)\right),$$

$$\mathbf{p}(i_2, i_1, r) = \left(p(i_2, i_1, r, 0, 1), \ldots, p(i_2, i_1, r, 0, M_r), p(i_2, i_1, r, 1, 1), \ldots, p(i_2, i_1, r, 1, M_r), \ldots, p(i_2, i_1, r, W, M_r)\right), \quad r = 1, 2.$$

So, the row vector $\mathbf{p}(0, 0, 0)$ has dimension $\overline{W}$, the row vector $\mathbf{p}(i_2, i_1, r)$ has dimension $\overline{W} M_r, r = 1, 2$.

In what follows, we use the following denotations: I is the identity matrix of dimension defined by the suffix, $\otimes$ and $\oplus$ are the symbols of Kronecker product and sum of the matrices correspondingly, $\delta_{i, j}$ is Kronecker delta:

$$\delta_{i, j} = \begin{cases} 1, & \text{if } i = j, \\ 0, & \text{if } i \ne j. \end{cases}$$

Theorem 1. *The vectors* $\mathbf{p}(0, 0, 0)$, $\mathbf{p}(i_2, i_1, r)$ *satisfy the following system of equations*:

$$\mathbf{0} = \mathbf{p}(0, 0, 0) D_0 + \mathbf{p}(0, 0, 1)\left(I_{\overline{W}} \otimes S_0^{(1)}\right) + \mathbf{p}(0, 0, 2)\left(I_{\overline{W}} \otimes S_0^{(2)}\right), \quad (1)$$

$$\mathbf{0} = \mathbf{p}(0, i_1, 1)\left(D_0 \oplus S^{(1)}\right) + \mathbf{p}(0, 0, 0)\left(D_{i_1+1}^{(1)} \otimes \boldsymbol{\beta}_1\right) + \sum_{m=0}^{i_1-1} \mathbf{p}(0, m, 1)\left(D_{i_1-m}^{(1)} \otimes I_{M_1}\right) + \mathbf{p}(0, i_1+1, 1)\left(I_{\overline{W}} \otimes S_0^{(1)} \boldsymbol{\beta}_1\right) +$$

$$\mathbf{p}(0, i_1+1, 2)\left(I_{\overline{W}} \otimes S_0^{(2)}\boldsymbol{\beta}_1\right), i_1 = \overline{0, R-1}, \quad (2)$$

$$\mathbf{0} = \mathbf{p}(i_2, i_1, 1)\left(D_0 \oplus S^{(1)}\right) + \sum_{m=0}^{i_1-1}\mathbf{p}(i_2, m, 1)\left(D_{i_1-m}^{(1)} \otimes I_{M_1}\right) + \sum_{m=0}^{i_2-1}\mathbf{p}(m, i_1, 1)\left(D_{i_2-m}^{(2)} \otimes I_{M_1}\right),$$
$$i_1 = \overline{0, R-i_2-1}, i_2 = \overline{1, N-1}, \quad (3)$$

$$\mathbf{0} = \mathbf{p}(N, i_1, 1)\left(\left(D_0 + \hat{D}_1^{(2)}\right) \oplus S^{(1)}\right) + \sum_{m=1}^{i_1-1}\mathbf{p}(N, m, 1)\left(D_{i_1-m}^{(1)} \otimes I_{M_1}\right) + \sum_{i_2=0}^{N-1}\mathbf{p}(i_2, i_1, 1)\left(\hat{D}_{N-i_2}^{(2)} \otimes I_{M_1}\right), i_1 = \overline{0, R-N-1}, \quad (4)$$

$$\mathbf{0} = \mathbf{p}(i_2, R-i_2, 1)\left(D(1) \oplus S^{(1)}\right) + \sum_{m=0}^{R-i_2-1}\mathbf{p}(i_2, m, 1)\left(\hat{D}_{R-i_2-m}^{(1)} \otimes I_{M_1}\right) + \sum_{l=0}^{i_2-1}\mathbf{p}(l, R-i_2, 1)\left(\hat{D}_{i_2-l}^{(2)} \otimes I_{M_1}\right), i_2 = \overline{1, N}, \quad (5)$$

$$\mathbf{0} = \mathbf{p}(0, R, 1)\left(D(1) \oplus S^{(1)}\right) + \mathbf{p}(0, 0, 0)\left(\hat{D}_{R+1}^{(1)} \otimes \boldsymbol{\beta}_1\right) + \sum_{m=0}^{R-1}\mathbf{p}(0, m, 1)\left(\hat{D}_{R-m}^{(1)} \otimes I_{M_1}\right), \quad (6)$$

$$\mathbf{0} = \mathbf{p}(i_2, i_1, 2)\left(D_0 \oplus S^{(2)}\right) + \sum_{m=0}^{i_1-1}\mathbf{p}(i_2, m, 2)\left(D_{i_1-m}^{(1)} \otimes I_{M_2}\right) + \mathbf{p}(0, 0, 0)\left(D_{i_2+1}^{(2)} \otimes \boldsymbol{\beta}_2\right)\delta_{i_1, 0} + \sum_{m=0}^{i_2-1}\mathbf{p}(m, i_1, 2)\left(D_{i_2-m}^{(2)} \otimes I_{M_2}\right) + \mathbf{p}(i_2+1, i_1, 1)\left(I_{\overline{W}} \otimes S_0^{(1)}\boldsymbol{\beta}_2\right) + \mathbf{p}(i_2+1, i_1, 2)\left(I_{\overline{W}} \otimes S_0^{(2)}\boldsymbol{\beta}_2\right),$$
$$i_2 = \overline{0, N-1}, i_1 = \overline{0, R-i_2-1}, \quad (7)$$

$$\mathbf{0} = \mathbf{p}(N, i_1, 2)\left(\left(D_0 + \hat{D}_1^{(2)}\right) \oplus S^{(2)}\right) + \sum_{m=1}^{i_1-1}\mathbf{p}(N, m, 2)\left(D_{i_1-m}^{(1)} \otimes I_{M_2}\right) + \sum_{i_2=0}^{N-1}\mathbf{p}(i_2, i_1, 2)\left(\hat{D}_{N-i_2}^{(2)} \otimes I_{M_2}\right) + \mathbf{p}(0, 0, 0)\left(\hat{D}_{N+1}^{(2)} \otimes \boldsymbol{\beta}_2\right)\delta_{i_1, 0}, i_1 = \overline{0, R-N-1}, \quad (8)$$

$$\mathbf{0} = \mathbf{p}(i_2, R-i_2, 2)\left(D(1) \oplus S^{(2)}\right) + \sum_{m=0}^{R-i_2-1}\mathbf{p}(i_2, m, 2)\left(\hat{D}_{R-i_2-m}^{(1)} \otimes I_{M_2}\right) + \sum_{l=0}^{i_2-1}\mathbf{p}(l, R-i_2, 2)\left(\hat{D}_{i_2-l}^{(2)} \otimes I_{M_2}\right), i_2 = \overline{0, N}. \quad (9)$$

Proof. Proof of the theorem is implemented by means of analysis of intensity of transition of the process $\zeta_t, t \geq 0$.

Combine now the states corresponding to a fixed value of components i_2 and r and introduce macro-vectors of stationary probabilities:

$$\tilde{\mathbf{p}}(0) = \mathbf{p}(0, 0, 0),$$

$$\mathbf{p}(i_2, r) = \left(\mathbf{p}(i_2, 0, r), \mathbf{p}(i_2, 1, r), \ldots, \mathbf{p}(i_2, R-i_2, r)\right),$$

$$i_2 = \overline{0, N}, r = 1, 2.$$

Introduce the following notations of matrices:

- $\mathfrak{B}_i^{(l,2)}$ is the matrix of dimension $(R+1-i)\overline{W}M_l \times (R+2-i)\overline{W}M_2$, $l = 1, 2$. For $i = \overline{1, N}$, this matrix represents the block diagonal matrix with diagonal blocks $I_{\overline{W}} \otimes S_0^{(l)}\boldsymbol{\beta}_2$ supplemented from the right side by the zero block column of dimension $(R+1-i)\overline{W}M_l \times \overline{W}M_2$;
- $\mathfrak{B}_i^{(l,1)}$ is the matrix of dimension $(R+1)\overline{W}M_l \times (R+2)\overline{W}M_1$, $l = 1, 2$. The matrix represents the block diagonal matrix with R diagonal blocks $I_{\overline{W}} \otimes S_0^{(l)}\boldsymbol{\beta}_1$ supplemented from the right side by the zero block column of dimension $(R+1)\overline{W}M_l \times \overline{W}M_1$ and from above by the zero block row of dimension $\overline{W}M_l \times (R+1)\overline{W}M_1$;
- $\mathfrak{Z}^{(1)}$ has dimension $\overline{W} \times (R+1)\overline{W}M_1$ and is defined by
$$\mathfrak{Z}^{(1)} = \left(D_1^{(1)} \otimes \boldsymbol{\beta}_1, \ldots, D_R^{(1)} \otimes \boldsymbol{\beta}_1, \hat{D}_{R+1}^{(1)} \otimes \boldsymbol{\beta}_1\right);$$
- $\mathfrak{T}^{(l)}$ has dimension $(R+1)\overline{W}M_l \times \overline{W}$, $l = 1, 2$, and is defined as the block column having zero blocks except the first block which is equal to $I_{\overline{W}} \otimes S_0^{(l)}$;
- $\mathfrak{D}_{i,j}^{(l)}$ is the matrix of dimension $(R-i+1)\overline{W}M_l \times (R-i-j+1)\overline{W}M_l$, $j = \overline{1, N-i-1}$, $i = \overline{0, N-1}$, $l = 1, 2$. It consists of the block diagonal matrix with the diagonal blocks $\left\{D_j^{(2)} \otimes I_{M_l}, \ldots, D_j^{(2)} \otimes I_{M_l}, \hat{D}_j^{(2)} \otimes I_{M_l}\right\}$ supplemented from below by the zero matrix;
- $\mathfrak{D}_{i,N-i}^{(l)}$ has the analogous structure, but the blocks of the diagonal matrix are equal to $\hat{D}_{N-i}^{(2)} \otimes I_{M_l}$, $l = 1, 2$;
- $\mathfrak{Z}_i^{(2)}$ has dimension $\overline{W} \times (R-i+2)\overline{W}M_2$ and is calculated by
$$\mathfrak{Z}_i^{(2)} = \left(D_i^{(2)} \otimes \boldsymbol{\beta}_2, O, \ldots, O\right), i = \overline{1, N},$$
$$\mathfrak{Z}_{N+1}^{(2)} = \left(\hat{D}_{N+1}^{(2)} \otimes \boldsymbol{\beta}_2, O, \ldots, O\right);$$
- $\mathfrak{A}_i^{(l)}$ is the square matrix of dimension $(R-i+1)\overline{W}M_l$, $i = \overline{0, N-1}, l = 1, 2$, defined as follows:

$$\mathcal{A}_i^{(l)} = \begin{pmatrix} D_0 \oplus S^{(l)} & D_1^{(1)} \otimes I_{M_l} & D_2^{(1)} \otimes I_{M_l} & \cdots & D_{R-i-1}^{(1)} \otimes I_{M_l} & \hat{D}_{R-i}^{(1)} \otimes I_{M_l} \\ O & D_0 \oplus S^{(l)} & D_1^{(1)} \otimes I_{M_l} & \cdots & D_{R-i-2}^{(1)} \otimes I_{M_l} & \hat{D}_{R-i-1}^{(1)} \otimes I_{M_l} \\ O & O & D_0 \oplus S^{(l)} & \cdots & D_{R-i-3}^{(1)} \otimes I_{M_l} & \hat{D}_{R-i-2}^{(1)} \otimes I_{M_l} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ O & O & O & \cdots & D_0 \oplus S^{(l)} & \hat{D}_1^{(1)} \otimes I_{M_l} \\ O & O & O & \cdots & O & D(1) \oplus S^{(l)} \end{pmatrix}; \tag{10}$$

- $\mathcal{A}_N^{(l)}$ is the square matrix of dimension $(R-N+1)\overline{W}M_l$, $l=1,2$, having the same structure, as the matrix (10), but the matrix D_0 on diagonal blocks is replaced with $D_0 + \hat{D}_1^{(2)}$.

Corollary 1. *The macro-vectors* $\tilde{\mathbf{p}}(0)$, $\mathbf{p}(i,r)$, $r=1,2$, *satisfy the following system of equations*:

$$\mathbf{0} = \tilde{\mathbf{p}}(0) D_0 + \mathbf{p}(0,1)\mathcal{T}^{(1)} + \mathbf{p}(0,2)\mathcal{T}^{(2)}, \tag{11}$$

$$\mathbf{0} = \tilde{\mathbf{p}}(0) Z^{(1)} + \mathbf{p}(0,1)\left(\mathcal{A}_0^{(1)} + \mathcal{B}_0^{(1,1)}\right) + \mathbf{p}(0,2)\mathcal{B}_0^{(2,1)}, \tag{12}$$

$$\mathbf{0} = \mathbf{p}(i,1)\mathcal{A}_i^{(1)} + \sum_{l=0}^{i-1} \mathbf{p}(l,1)\mathcal{D}_{l,\,i-l}^{(1)}, \ i=\overline{1,N}, \tag{13}$$

$$\mathbf{0} = \mathbf{p}(i,2)\mathcal{A}_i^{(2)} + \tilde{\mathbf{p}}(0)\mathcal{Z}_{i+1}^{(2)} + \mathbf{p}(i+1,1)\mathcal{B}_{i+1}^{(1,2)} + \mathbf{p}(i+1,2)\mathcal{B}_{i+1}^{(2,2)} + \sum_{m=0}^{i-1} \mathbf{p}(m,2)\mathcal{D}_{m,\,i-m}^{(2)}, \ i=\overline{1,N-1}, \tag{14}$$

$$\mathbf{0} = \mathbf{p}(N,2)\mathcal{A}_N^{(2)} + \tilde{\mathbf{p}}(0)\mathcal{Z}_{N+1}^{(2)} + \sum_{m=0}^{N-1} \mathbf{p}(m,2)\mathcal{D}_{m,\,N-m}^{(2)}. \tag{15}$$

Denote

$$\mathbf{p}(i) = (\mathbf{p}(i,1), \mathbf{p}(i,2)), \ i=\overline{0,N},$$

$$\mathbf{p} = (\tilde{\mathbf{p}}(0), \mathbf{p}(0), \mathbf{p}(1), \ldots, \mathbf{p}(N)).$$

Corollary 2. *The macro-vector* $\mathbf{p}$ *satisfies equation:*

$$\mathbf{p}Q = \mathbf{0}, \tag{16}$$

where Q *is generator of the Markov chain* $\zeta_t, t \ge 0$, *having the following structure*:

$$Q = \begin{pmatrix} D_0 & \mathcal{Z}_0 & \mathcal{Z}_1 & \mathcal{Z}_2 & \mathcal{Z}_3 & \cdots & \mathcal{Z}_N \\ \mathcal{T} & \mathcal{A}_0 & \mathcal{D}_{0,1} & \mathcal{D}_{0,2} & \mathcal{D}_{0,3} & \cdots & \mathcal{D}_{0,N} \\ O & \mathcal{B}_1 & \mathcal{A}_1 & \mathcal{D}_{1,1} & \mathcal{D}_{1,2} & \cdots & \mathcal{D}_{1,N-1} \\ O & O & \mathcal{B}_2 & \mathcal{A}_2 & \mathcal{D}_{2,1} & \cdots & \mathcal{D}_{2,N-2} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ O & O & O & O & O & \cdots & \mathcal{A}_N \end{pmatrix}, \tag{17}$$

where

$$\mathcal{T} = \begin{pmatrix} \mathcal{T}^{(1)} \\ \mathcal{T}^{(2)} \end{pmatrix},$$

$$\mathcal{A}_0 = \begin{pmatrix} \mathcal{A}_0^{(1)} + \mathcal{B}_0^{(1,1)} & O \\ \mathcal{B}_0^{(2,1)} & \mathcal{A}_0^{(2)} \end{pmatrix},$$

$$\mathcal{A}_i = diag\{\mathcal{A}_i^{(1)}, \mathcal{A}_i^{(2)}\}, \ i=\overline{1,N},$$

$$\mathcal{B}_i = \begin{pmatrix} O & \mathcal{B}_i^{(1,2)} \\ O & \mathcal{B}_i^{(2,2)} \end{pmatrix}, \ i=\overline{1,N},$$

$$\mathcal{D}_{i,j} = diag\{\mathcal{D}_{i,j}^{(1)}, \mathcal{D}_{i,j}^{(2)}\}, \ j=\overline{1,N-i}, \ i=\overline{0,N-1},$$

$$\mathcal{Z}_0 = \begin{pmatrix} \mathcal{Z}^{(1)} & \mathcal{Z}_1^{(2)} \end{pmatrix},$$

$$\mathcal{Z}_i = \begin{pmatrix} O & \mathcal{Z}_{i+1}^{(2)} \end{pmatrix}, \ i=\overline{1,N},$$

$diag\{C_1, \ldots, C_N\}$ denotes the block diagonal matrix with the diagonal entries $\{C_1, \ldots, C_N\}$.

Direct solution of the system (16) is possible as it is the finite system of linear algebraic equations with normalization condition

$$\tilde{\mathbf{p}}(0)\mathbf{e} + \sum_{i=0}^{N} \mathbf{p}(i)\mathbf{e} = 1. \tag{18}$$

However, the blocking matrix (17) (generator of the process $\zeta_t, t \ge 0$) has only non-zero blocks below the sub-diagonal. So, the effective and stable algorithm for solving system (16), (18), which is presented in [11], [12], can be applied for solving this system.

Having the stationary distribution been computed we can easy calculate, e.g., the mean queue lengths and their variations.

Corollary 3. *Average number* $L_1^{(l)}$ *of type-*l *customers in the buffer is calculated by*

$$L_1^{(l)} = \sum_{r=1}^{2} \sum_{i_2=0}^{N} \sum_{i_1=0}^{R-i_2} i_l \mathbf{p}(i_2, i_1, r), \ l=1,2.$$

Variation $V^{(l)}$ of the number of type-l customers in the buffer is calculated by

$$V^{(l)} = \sum_{r=1}^{2} \sum_{i_2=0}^{N} \sum_{i_1=0}^{R-i_2} i_l^2 \mathbf{p}(i_2, i_1, r) - \left(L_1^{(l)}\right)^2, \ l=1,2.$$

IV. Loss Probability

We will assume below that customers of the same priority class are served according to *FIFO* (first in - first out) discipline and the customers in an arriving batch of size k are numbered in such a way as an arbitrary customer gets the number $m, m=\overline{1,k}, k\ge 1$, with equal probability $\frac{1}{k}$.

Having the stationary distribution of the Markov chain $\zeta_t, t\ge 0$, been computed, different performance measures of the system can be calculated and optimization problems can be solved. In particular, probabilities $P_{loss}^{(l)}$ that the arbitrary type-l customer, $l=1,2$, is lost are calculated as given in the following statement.

Theorem 2. *The loss probabilities* $P_{loss}^{(l)}$, $l=1,2$, *are calculated by*

$$P_{loss}^{(1)}=\lambda_1^{-1}\Bigg(\mathbf{p}(0,0,0)\sum_{k=R+2}^{\infty}(k-R-1)D_k^{(1)}\mathbf{e}+\sum_{l=1}^{2}\sum_{i_2=0}^{N}\sum_{i_1=0}^{R-i_2}\mathbf{p}(i_2,i_1,l)\sum_{k=R-i_2-i_1+1}^{\infty}(k-R+i_2+i_1)D_k^{(1)}\otimes I_{M_l}\mathbf{e}\Bigg), \tag{19}$$

$$P_{loss}^{(2)}=\lambda_2^{-1}\Bigg(\mathbf{p}(0,0,0)\sum_{k=N+2}^{\infty}(k-N-1)D_k^{(2)}\mathbf{e}+\sum_{l=1}^{2}\sum_{i_2=0}^{N}\sum_{i_1=0}^{R-i_2}\mathbf{p}(i_2,i_1,l)\sum_{k=\min\{N,R-i_1\}-i_2+1}^{\infty}\big(k-\min\{N,R-i_1\}-i_2\big)D_k^{(2)}\otimes I_{M_l}\mathbf{e}\Bigg). \tag{20}$$

Proof. The statement of the theorem is proved using the formula of total probability and the following evident assertions.

- Probability that arbitrary type-l customer arrives during small time interval $(u,u+\Delta t)$ in a batch consisting of k customers and a state of the directing process $\nu_t, t\ge 0$, of the *BMMAP* changes to the value ν', conditional it had a value ν at epoch u, is the (ν,ν')th entry of the matrix

$$\lambda_l^{-1}kD_k^{(l)}\Delta t, k\ge 1, l=1,2; \tag{21}$$

- Due to assumption that an arbitrary customer arriving in a batch of size k gets the number m, $m=\overline{1,k}$, with equal probability $\frac{1}{k}$, it is clear that: (i) arbitrary type-2 customer arriving in a batch consisting of k customers is lost with probability $\frac{k-(N+1)}{k}$, $k>N+1$, if the batch meets the empty system and is lost with probability $\frac{k-\min\{N,R-i_1\}-i_2}{k}$ if the batch meets the system in the state (i_2,i_1,l); (ii) arbitrary type-1 customer arriving in a batch consisting of k customers is lost with probability $\frac{k-(R+1)}{k}$, $k>R+1$, if the batch meets the empty system and is lost with probability $\frac{k-(R-i_2-i_1)}{k}$ if the batch meets the system in the state (i_2,i_1,l).

V. Waiting Time Distribution

Denote $W_2(x)$ waiting time distribution function for type-2 (real-time) flow. Let $\omega_2(v)$ be its Laplace-Stieltjes transform $\omega_2(v)=\int_0^{\infty}e^{-vx}dW_2(x)$, $\mathrm{Re}\,v>0$, $W_1^{(2)}$ be the mean waiting time, $W_2^{(2)}$ be the second initial moment of distribution function and J_2 be a jitter for type-2 customers.

Theorem 3. *Laplace-Stieltjes transform* $\omega_2(v)$ *of waiting time distribution function for type-2 customers is calculated as follows:*

$$\omega_2(v)=\lambda_2^{-1}\Bigg(P_{loss}^{(2)}+\mathbf{p}(0,0,0)\sum_{k=1}^{\infty}D_k^{(2)}\mathbf{e}\times\sum_{i=0}^{\min\{k-1,N\}}\left(\boldsymbol{\beta}_2\left(vI-S^{(2)}\right)^{-1}S_0^{(2)}\right)^i+\sum_{l=1}^{2}\sum_{i_2=0}^{N-1}\sum_{i_1=0}^{R-i_2-1}\mathbf{p}(i_2,i_1,l)\sum_{k=1}^{\infty}D_k^{(2)}\mathbf{e}\otimes\boldsymbol{\gamma}^{(l)}(v)\times\sum_{m=0}^{\min\{k,\min\{N,R-i_1\}-i_2\}-1}\left(\boldsymbol{\beta}_2\times\left(vI-S^{(2)}\right)^{-1}S_0^{(2)}\right)^{i_2+m}\Bigg), \tag{22}$$

where

$$\boldsymbol{\gamma}^{(l)}(v)=\left(vI-S^{(l)}\right)^{-1}S_0^{(l)}, l=1,2.$$

Proof. To prove the theorem, we use the method of collective marks (method of catastrophes). We temporarily assume that the parameter v is real and interpret it as an intensity of some imaginary stationary Poisson flow of catastrophes. Thus, $\omega_2(v)$ has sense of probability that no one catastrophe arrives during the waiting time of an arbitrary type-2 customer. Then, we use formula of total probability. We take into account formula (21) and information that the entries of the vector $\boldsymbol{\gamma}^{(l)}(v)$ define probability of no catastrophe arrival during the residual service of a type-l customer being in service at an arbitrary customer arrival epoch and the function $\boldsymbol{\beta}_2\left(vI-S^{(2)}\right)^{-1}S_0^{(2)}$ define probability of no catastrophe arrival during the total service time of type-2 customer. Sure, no one catastrophe arrives during the waiting time of an arbitrary type-2 customer if it is lost. As the result, we have formula (22) been proven.

Corollary 4. *The mean waiting time* $W_1^{(2)}$, *the second initial moment* $W_2^{(2)}$ *of distribution function and a jitter* J_2 *for type-2 customers are calculated by*

$$W_1^{(2)} = -\omega_2'(0) = \lambda_2^{-1}\Bigg(\mathbf{p}(0,0,0)\sum_{k=1}^{\infty} D_k^{(2)}\mathbf{e}\boldsymbol{\beta}_1^{(2)} \times$$
$$\times \frac{\min\{k, N+1\}\min\{k-1, N\}}{2} +$$
$$\sum_{l=1}^{2}\sum_{i_2=0}^{N-1}\sum_{i_1=0}^{R-i_2-1} \mathbf{p}(i_2, i_1, l)\sum_{k=1}^{\infty} D_k^{(2)} \otimes I_{M_l} \times$$
$$\times\Big(\mathbf{e}\otimes\bar{\boldsymbol{\gamma}}^{(l)} + \mathbf{e}\boldsymbol{\beta}_1^{(2)} \min\{k, \min\{N, R-i_1\} - i_2\}\times$$
$$\times\Big[i_2 + \frac{1}{2}\min\{k, \min\{N, R-i_1\} - i_2\} - 1\Big]\Big)\Bigg), \qquad (23)$$

$$W_2^{(2)} = -\omega_2''(0) = \lambda_2^{-1}\Bigg(\mathbf{p}(0,0,0)\sum_{k=1}^{\infty} D_k^{(2)}\mathbf{e}\times$$
$$\times \sum_{i=0}^{\min\{k-1, N\}}\left(i\boldsymbol{\beta}_2^{(2)} + \frac{i(i-1)}{2}\left(\boldsymbol{\beta}_1^{(2)}\right)^2\right) +$$
$$\sum_{l=1}^{2}\sum_{i_2=0}^{N-1}\sum_{i_1=0}^{R-i_2-1} \mathbf{p}(i_2, i_1, l)\sum_{k=1}^{\infty} D_k^{(2)} \otimes I_{M_l}\Big(\mathbf{e}\otimes\bar{\bar{\boldsymbol{\gamma}}}^{(l)} +$$
$$\sum_{m=i_2}^{\min\{k+i_2,\, \min\{N,\, R-i_1\}\}-1}\Big(2\mathbf{e}\otimes\bar{\boldsymbol{\gamma}}^{(l)}\boldsymbol{\beta}_1^{(2)} m + m\boldsymbol{\beta}_2^{(2)}\mathbf{e} +$$
$$m(m-1)\left(\boldsymbol{\beta}_1^{(2)}\right)^2\mathbf{e}\Big)\Big)\Bigg), \qquad (24)$$

$$J_2 = W_2^{(2)} - \left(W_1^{(2)}\right)^2,$$

where

$$\bar{\boldsymbol{\gamma}}^{(l)} = \left(-S^{(l)}\right)^{-1}\mathbf{e},\ \ \bar{\bar{\boldsymbol{\gamma}}}^{(l)} = 2\left(-S^{(l)}\right)^{-2}\mathbf{e},$$

$$\boldsymbol{\beta}_1^{(l)} = \boldsymbol{\beta}^{(l)}\left(-S^{(l)}\right)^{-1}\mathbf{e},\ \ \boldsymbol{\beta}_2^{(l)} = 2\boldsymbol{\beta}^{(l)}\left(-S^{(l)}\right)^{-2}\mathbf{e}.$$

Waiting time distribution $W_2(x)$ defines probability that waiting time of an arbitrary type-2 customer is less or equal to x. Denote by $\widetilde{W}_2(x)$ the waiting time distribution for an arbitrary type-2 customer which was not lost due to the buffer overfull. It is easy to see that the distribution functions $W_2(x)$ and $\widetilde{W}_2(x)$ are related as follows:

$$\widetilde{W}_2(x) = \frac{W_2(x) - P_{loss}^{(2)}}{1 - P_{loss}^{(2)}}.$$

So, the following corollary is obvious.

Corollary 5. *The mean waiting time* $\widetilde{W}_1^{(2)}$*, the second initial moment* $\widetilde{W}_2^{(2)}$ *of distribution function and a jitter* $\widetilde{J}_2$ *for type-2 customers are calculated by*

$$\widetilde{W}_1^{(2)} = \frac{W_1^{(2)}}{1 - P_{loss}^{(2)}},$$
$$\widetilde{W}_2^{(2)} = \frac{W_2^{(2)}}{1 - P_{loss}^{(2)}},$$
$$\widetilde{J}_2 = \widetilde{W}_2^{(2)} - \left(W_1^{(2)}\right)^2.$$

VI. Numerical Examples

Analytical results, which are presented above, have transparent algorithmic form. So, their realization on computer does not meet essential problems. Realization was developed in frameworks of software "SIRIUS++", see, e.g. [8].

Basic parameters of the buffer, flow and service processes are the following.

We assume that capacity R of a buffer is equal to 32 and the number N of places available in a buffer for priority customers is equal to 6.

Type-1 packets can arrive in batches consisting of 1, 2, 3 or 4 packets and type-2 customers can arrive in batches consisting of 1 or 2 packets. The matrices $D_0, D_k^{(l)}$, which define the *BMMAP*, are given by

$$D_0 = \begin{pmatrix} -0.1646 & 0 \\ 0 & -0.1651 \end{pmatrix},$$

$$D_1^{(1)} = \begin{pmatrix} 0.002 & 0.0022 \\ 0.0019 & 0.002 \end{pmatrix},\ D_2^{(1)} = \begin{pmatrix} 0.004 & 0.0031 \\ 0.0038 & 0.0042 \end{pmatrix},$$
$$D_3^{(1)} = \begin{pmatrix} 0.0055 & 0.0053 \\ 0.0051 & 0.0059 \end{pmatrix},\ D_4^{(1)} = \begin{pmatrix} 0.006 & 0.0061 \\ 0.0062 & 0.0059 \end{pmatrix},$$
$$D_1^{(2)} = \begin{pmatrix} 0.0301 & 0.0301 \\ 0.0302 & 0.0299 \end{pmatrix},\ D_2^{(2)} = \begin{pmatrix} 0.035 & 0.0352 \\ 0.0351 & 0.0349 \end{pmatrix}.$$

These matrices provide the values of intensity λ_l of type-l customers arrival as

$$\lambda_1 = 0.1,\ \lambda_2 = 0.2.$$

Service time distribution of type-1 packets is characterized by irreducible representation $\left(\boldsymbol{\beta}_1, S^{(1)}\right)$, where

$$\boldsymbol{\beta}_1 = \begin{pmatrix} 0.3 & 0.2 & 0.5 \end{pmatrix},\ S^{(1)} = \begin{pmatrix} -1.7 & 1.4 & 0.1 \\ 1.3 & -2.6 & 1.2 \\ 1.7 & 1.5 & -3.4 \end{pmatrix}.$$

Mean service time $\boldsymbol{\beta}_1^{(1)}$ is equal to 6.

Service time distribution of type-2 packets is characterized by irreducible representation $\left(\boldsymbol{\beta}_2, S^{(2)}\right)$, where

$$\boldsymbol{\beta}_2 = \begin{pmatrix} 0.4 & 0.6 \end{pmatrix},\ S^{(2)} = \begin{pmatrix} -1.4 & 1.1 \\ 1.2 & -1.9 \end{pmatrix}.$$

Mean service time $\boldsymbol{\beta}_1^{(2)}$ is equal to 2.

Five experiments were made to give insight into dependence of the main performance measures of the system on its parameters.

Experiment 1. The aim of this experiment is to analyze dependence of the main performance measures of the system on the average intensity λ_1 of type-1 packets arrival.

In the Table 1, values of loss probabilities $P_{loss}^{(1)}$, $P_{loss}^{(2)}$, mean waiting time of type-2 packets $W_1^{(2)}$, second moment $W_2^{(2)}$ of type-2 packets waiting time distribution and jitter J_2 as well as the analogous values $\widetilde{W}_1^{(2)}$, $\widetilde{W}_2^{(2)}$, $\widetilde{J}_2$ for waiting

TABLE I

DEPENDENCE OF THE MAIN PERFORMANCE MEASURES ON THE AVERAGE INTENSITY λ_1 OF NON-PRIORITY PACKETS ARRIVAL

λ_1	$P_{loss}^{(1)}$	$P_{loss}^{(2)}$	$W_1^{(2)}$	$W_2^{(2)}$	J_2	$\widetilde{W}_1^{(2)}$	$\widetilde{W}_2^{(2)}$	$\widetilde{J}_2$	$L_1^{(1)}$	$L_1^{(2)}$
0.020	3.1e-08	0.0176	2.9458	31.1340	22.4562	2.9985	31.6910	22.6999	0.4381	0.6965
0.069	0.0031	0.0401	4.7717	61.8983	39.1296	4.9708	64.4822	39.7729	4.6222	1.1780
0.118	0.1379	0.1167	5.2634	70.9553	43.2517	5.9589	80.3308	44.8224	19.7817	1.3237
0.167	0.3307	0.2335	4.3733	57.6298	38.5040	5.7055	75.1853	42.6323	26.2667	1.1101
0.216	0.4563	0.3169	3.7511	48.5265	34.4556	5.4914	71.0392	40.8841	28.0097	0.9555
0.265	0.5386	0.3863	3.2731	41.7131	31.0001	5.3334	67.9710	39.5254	28.9130	0.8343
0.314	0.5994	0.4384	2.9395	37.0724	28.4319	5.2338	66.0083	38.6155	29.4175	0.7483
0.363	0.6437	0.4912	2.6198	32.7167	25.8532	5.1487	64.2975	37.7883	29.8407	0.6647
0.412	0.6807	0.5242	2.4292	30.1673	24.2660	5.1061	63.4098	37.3372	30.0684	0.6143
0.461	0.7098	0.5553	2.2575	27.9002	22.8037	5.0765	62.7391	36.9681	30.2669	0.5655
0.510	0.7345	0.5820	2.1136	26.0223	21.5551	5.0562	62.2523	36.6871	30.4213	0.5271
0.559	0.7547	0.6042	1.9966	24.5114	20.5250	5.0441	61.9241	36.4815	30.5419	0.4958
0.608	0.7722	0.6245	1.8913	23.1630	19.5858	5.0371	61.6881	36.3161	30.6469	0.4675
0.657	0.7875	0.6410	1.8074	22.0952	18.8284	5.0344	61.5440	36.1988	30.7284	0.4450
0.706	0.8005	0.6593	1.7156	20.9353	17.9918	5.0353	61.4430	36.0892	30.8158	0.4203
0.755	0.8122	0.6735	1.6449	20.0464	17.3406	5.0381	61.3973	36.0152	30.8810	0.4013
0.804	0.8224	0.6884	1.5718	19.1314	16.6610	5.0443	61.3993	35.9540	30.9475	0.3816
0.853	0.8315	0.7021	1.5052	18.3036	16.0378	5.0525	61.4381	35.9100	31.0066	0.3637
0.902	0.8398	0.7142	1.4465	17.5754	15.4831	5.0618	61.5033	35.8810	31.0577	0.3480
0.951	0.8472	0.7252	1.3936	16.9214	14.9794	5.0716	61.5826	35.8612	31.1027	0.3339
1.000	0.8526	0.7635	1.2115	14.6866	13.2190	5.1217	62.0908	35.8592	31.2513	0.2868

time distribution and jitter of type-2 packets, which are not lost, the average number $L_1^{(1)}$ of type-1 customers in the buffer, the average number $L_1^{(2)}$ of type-2 customers in the buffer, are presented along with the corresponding value of the average intensity λ_1. Intensity λ_2 is equal to 0.2.

Figures 2 and 3 present the dependence of the characteristics $P_{loss}^{(1)}$, $P_{loss}^{(2)}$, $\widetilde{J}_2$ on the average intensity λ_1 in the graphical form.

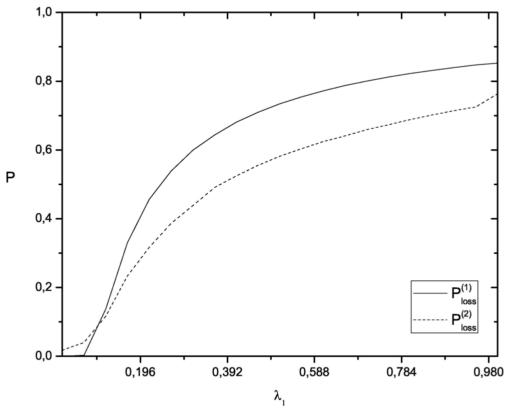

Fig. 2. Dependence of the loss probabilities $P_{loss}^{(1)}$, $P_{loss}^{(2)}$ on the average intensity λ_1 of non-priority packets

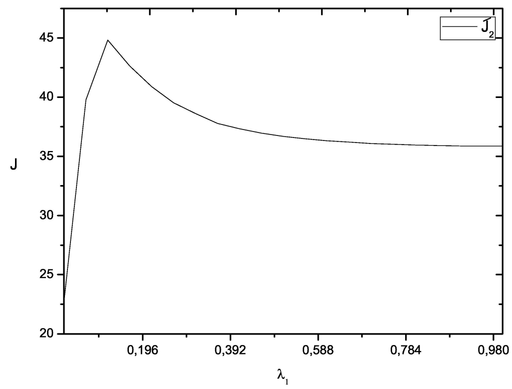

Fig. 3. Dependence of the jitter $\widetilde{J}_2$ of priority packets on the average intensity λ_1 of non-priority packets

Experiment 2. The aim of this experiment is to analyze dependence of the main performance measures of the system on the average intensity λ_2 of type-2 packets arrival.

In the Table 2, values of loss probabilities $P_{loss}^{(1)}$, $P_{loss}^{(2)}$, mean waiting time of type-2 packets $W_1^{(2)}$, second moment $W_2^{(2)}$ of type-2 packets waiting time distribution and jitter J_2 as well as the analogous values $\widetilde{W}_1^{(2)}$, $\widetilde{W}_2^{(2)}$, $\widetilde{J}_2$ for waiting

TABLE II

DEPENDENCE OF THE MAIN PERFORMANCE MEASURES ON THE AVERAGE INTENSITY λ_2 OF PRIORITY PACKETS ARRIVAL

λ_2	$P_{loss}^{(1)}$	$P_{loss}^{(2)}$	$W_1^{(2)}$	$W_2^{(2)}$	J_2	$\widetilde{W}_1^{(2)}$	$\widetilde{W}_2^{(2)}$	$\widetilde{J}_2$	$L_1^{(1)}$	$L_1^{(2)}$
0.020	0.0003	0.0002	3.1825	38.6870	28.5589	3.1831	38.6941	28.5623	2.7645	0.0090
0.069	0.0023	0.0039	4.1793	53.3927	35.9261	4.1959	53.6042	35.9990	4.5414	0.3688
0.118	0.0092	0.0207	4.6195	61.8992	40.5594	4.7173	63.2096	40.9567	7.0767	0.7413
0.167	0.0284	0.0533	4.9933	68.7129	43.7795	5.2743	72.5799	44.7612	10.8407	1.1355
0.216	0.0690	0.1030	5.1339	71.5784	45.2218	5.7233	79.7959	47.0402	15.4958	1.4979
0.265	0.1300	0.1593	5.0461	70.2211	44.7581	6.0020	83.5231	47.4995	19.7999	1.7780
0.314	0.1983	0.2151	4.8297	66.6055	43.2799	6.1534	84.8617	46.9969	22.7659	1.9857
0.363	0.2635	0.2658	4.5820	62.4736	41.4787	6.2409	85.0918	46.1427	24.5378	2.1473
0.412	0.3222	0.3107	4.3473	58.5576	39.6583	6.3070	84.9533	45.1756	25.5819	2.2812
0.461	0.3740	0.3503	4.1323	55.0180	37.9420	6.3602	84.6806	44.2283	26.2238	2.3973
0.510	0.4205	0.3858	3.9361	51.7987	36.3059	6.4081	84.3306	43.2664	26.6549	2.4966
0.559	0.4619	0.4176	3.7566	48.8866	34.7744	6.4499	83.9362	42.3344	26.9558	2.5829
0.608	0.4994	0.4462	3.5934	46.2341	33.3214	6.4885	83.4835	41.3823	27.1808	2.6578
0.657	0.5321	0.4727	3.4342	43.7702	31.9765	6.5129	83.0091	40.5916	27.3454	2.7236
0.706	0.5625	0.4965	3.2925	41.5198	30.6792	6.5396	82.4673	39.7004	27.4825	2.7797
0.755	0.5900	0.5183	3.1608	39.4412	29.4507	6.5622	81.8851	38.8228	27.5945	2.8280
0.804	0.6148	0.5386	3.0352	37.4922	28.2799	6.5781	81.2563	37.9851	27.6883	2.8689
0.853	0.6371	0.5572	2.9049	35.6996	27.1864	6.5899	80.6298	37.2031	27.7653	2.9177
0.902	0.6573	0.5746	2.8053	34.0057	26.1362	6.5951	79.9462	36.4510	27.8336	2.9347
0.951	0.6756	0.5909	2.6982	32.4139	25.1339	6.5951	79.2290	35.7342	27.8941	2.9592
1.000	0.6913	0.6064	2.5922	30.9211	24.2015	6.5858	78.5578	35.1852	27.9423	2.9812

time distribution and jitter of type-2 packets, which are not lost, the average number $L_1^{(1)}$ of type-1 customers in the buffer, the average number $L_1^{(2)}$ of type-2 customers in the buffer, are presented along with the corresponding value of the average intensity λ_2. Intensity λ_1 is equal to 0.1.

Figures 4 and 5 present the dependence of the characteristics $P_{loss}^{(1)}$, $P_{loss}^{(2)}$, $\widetilde{J}_2$ on the average intensity λ_2 in the graphical form.

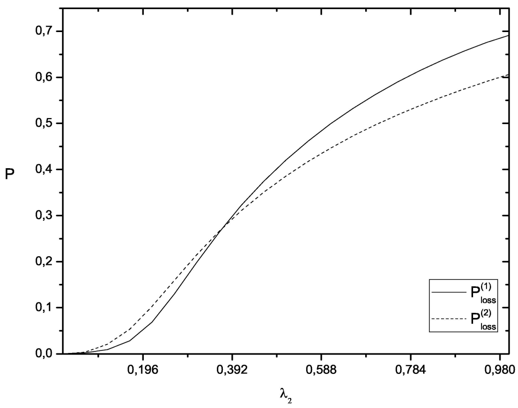

Fig. 4. Dependence of the loss probabilities $P_{loss}^{(1)}$, $P_{loss}^{(2)}$ on the intensity λ_2 of priority packets

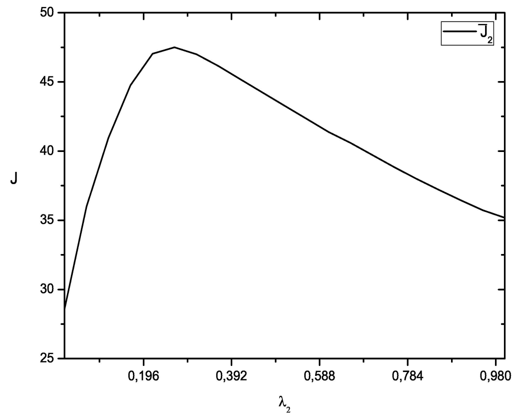

Fig. 5. Dependence of the jitter $\widetilde{J}_2$ of priority packets on the intensity λ_2 of priority packets

Experiment 3. The aim of this experiment is to analyze dependence of the main performance measures of the system on the capacity R of the buffer. Starting from the value $R = 6$, we increase the buffer capacity one-by-one.In the Table 3, values of loss probabilities $P_{loss}^{(1)}$, $P_{loss}^{(2)}$, mean waiting time of type-2 packets $W_1^{(2)}$, second moment $W_2^{(2)}$ of type-2 packets waiting time distribution and jitter J_2

TABLE III
DEPENDENCE OF THE MAIN PERFORMANCE MEASURES ON THE TOTAL CAPACITY R OF THE BUFFER

R	$P_{loss}^{(1)}$	$P_{loss}^{(2)}$	$W_1^{(2)}$	$W_2^{(2)}$	J_2	$\widetilde{W}_1^{(2)}$	$\widetilde{W}_2^{(2)}$	$\widetilde{J}_2$	$L_1^{(1)}$	$L_1^{(2)}$
6	0.2556	0.1701	3.4224	40.7038	28.9910	4.1241	49.0493	32.0416	1.6252	0.8553
7	0.2273	0.1539	3.7281	45.6005	31.7017	4.4065	53.8980	34.4810	2.0319	0.9329
8	0.2040	0.1413	3.9792	49.6531	33.8193	4.6338	57.8223	36.3498	2.4614	0.9961
9	0.1845	0.1314	4.1836	52.9718	35.4689	4.8167	60.9877	37.7868	2.9090	1.0473
10	0.1682	0.1237	4.3510	55.6921	36.7612	4.9649	63.5508	38.9002	3.3700	1.0892
11	0.1543	0.1174	4.4899	57.9563	37.7972	5.0869	65.6622	39.7861	3.8404	1.1239
12	0.1424	0.1121	4.6072	59.8689	38.6430	5.1890	67.4292	40.5039	4.3180	1.1532
13	0.1322	0.1077	4.7069	61.4955	39.3403	5.2752	68.9197	41.0921	4.8010	1.1782
14	0.1233	0.1040	4.7928	62.8958	39.9244	5.3489	70.1934	41.5822	5.2880	1.1996
15	0.1156	0.1007	4.8677	64.1163	40.4216	5.4128	71.2961	41.9976	5.7782	1.2183
16	0.1087	0.0978	4.9335	65.1884	40.8490	5.4686	72.2588	42.3533	6.2708	1.2347
17	0.1026	0.0953	4.9917	66.1374	41.2200	5.5177	73.1061	42.6611	6.7654	1.2492
18	0.0972	0.0931	5.0437	66.9837	41.5451	5.5613	73.8580	42.9301	7.2617	1.2622
19	0.0924	0.0911	5.0903	67.7434	41.8322	5.6002	74.5298	43.1671	7.7595	1.2738
20	0.0880	0.0892	5.1324	68.4290	42.0876	5.6353	75.1337	43.3776	8.2585	1.2844
21	0.0840	0.0876	5.1706	69.0509	42.3163	5.6669	75.6793	43.5656	8.7587	1.2939
22	0.0803	0.0861	5.2053	69.6177	42.5221	5.6956	76.1749	43.7347	9.2599	1.3026
23	0.0770	0.0847	5.2372	70.1363	42.7082	5.7218	76.6269	43.8874	9.7620	1.3105
24	0.0740	0.0834	5.2664	70.6127	42.8775	5.7459	77.0409	44.0261	10.2651	1.3178
25	0.0711	0.0823	5.2934	71.0518	43.0320	5.7679	77.4215	44.1526	10.7689	1.3245
26	0.0685	0.0812	5.3183	71.4578	43.1735	5.7883	77.7726	44.2684	11.2734	1.3308
27	0.0661	0.0802	5.3414	71.8344	43.3037	5.8071	78.0975	44.3748	11.7787	1.3365
28	0.0639	0.0793	5.3629	72.1846	43.4238	5.8246	78.3990	44.4729	12.2847	1.3419
29	0.0618	0.0784	5.3830	72.5111	43.5349	5.8409	78.6795	44.5637	12.7913	1.3469
30	0.0598	0.0776	5.4017	72.8162	43.6380	5.8560	78.9412	44.6479	13.2985	1.3516
31	0.0580	0.0768	5.4192	73.1020	43.7340	5.8702	79.1859	44.7262	13.8063	1.3559
32	0.0563	0.0761	5.4357	73.3702	43.8235	5.8835	79.4152	44.7992	14.3147	1.3601

as well as the analogous values $\widetilde{W}_1^{(2)}$, $\widetilde{W}_2^{(2)}$, $\widetilde{J}_2$ for waiting time distribution and jitter of type-2 packets, which are not lost, the average number $L_1^{(1)}$ of type-1 customers in the buffer, the average number $L_1^{(2)}$ of type-2 customers in the buffer, are presented along with the corresponding value of R.

Figures 6 and 7 present the dependence of the characteristics $P_{loss}^{(1)}$, $P_{loss}^{(2)}$, $\widetilde{J}_2$ on the buffer capacity R in the graphical form.

Experiment 4. The aim of this experiment is to analyze dependence of the main performance measures of the system on the number N of places in a buffer which are available for priority packets. The value of the total buffer capacity R is assumed to be equal to 32. Starting from the value $N = 6$, we increase the buffer capacity available for the priority packets one-by-one.

In the Table 4, values of loss probabilities $P_{loss}^{(1)}$, $P_{loss}^{(2)}$, mean waiting time of type-2 packets $W_1^{(2)}$, second moment $W_2^{(2)}$ of type-2 packets waiting time distribution and jitter J_2

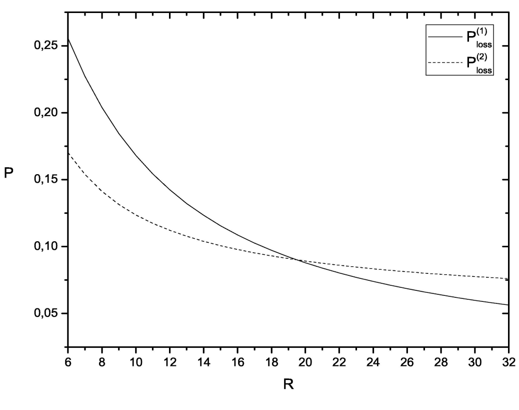

Fig. 6. Dependence of the loss probability $P_{loss}^{(1)}$, $P_{loss}^{(2)}$ on the total capacity R of the buffer

as well as the analogous values $\widetilde{W}_1^{(2)}$, $\widetilde{W}_2^{(2)}$, $\widetilde{J}_2$ for waiting time distribution and jitter of type-2 packets, which are not lost, the average number $L_1^{(1)}$ of type-1 customers in the

TABLE IV
DEPENDENCE OF THE MAIN PERFORMANCE MEASURES ON THE AVAILABLE CAPACITY N OF THE BUFFER FOR PRIORITY PACKETS

N	$P_{loss}^{(1)}$	$P_{loss}^{(2)}$	$W_1^{(2)}$	$W_2^{(2)}$	J_2	$\widetilde{W}_1^{(2)}$	$\widetilde{W}_2^{(2)}$	$\widetilde{J}_2$	$L_1^{(1)}$	$L_1^{(2)}$
2	0.0118	0.3145	2.5817	28.9124	22.2474	3.7659	42.1750	27.9928	7.8928	0.6785
3	0.0255	0.2065	3.6327	42.9386	29.7422	4.5783	54.1163	33.1551	10.5006	0.9284
4	0.0393	0.1387	4.4055	54.8828	35.4747	5.1146	63.7175	37.5581	12.4283	1.1258
5	0.0494	0.0995	5.0088	65.1451	40.0566	5.5621	72.3410	41.4039	13.6040	1.2627
6	0.0563	0.0761	5.4357	73.3702	43.8235	5.8835	79.4152	44.7992	14.3147	1.3601
7	0.0605	0.0626	5.7356	79.7129	46.8156	6.1186	85.0351	47.5982	14.7265	1.4257
8	0.0631	0.0546	5.9391	84.4653	49.1922	6.2824	89.3477	49.8789	14.9665	1.4700
9	0.0647	0.0500	6.0749	87.9102	51.0056	6.3949	92.5408	51.6460	15.1044	1.4991
10	0.0656	0.0474	6.1637	90.3538	52.3625	6.4701	94.8456	52.9831	15.1836	1.5180
11	0.0661	0.0458	6.2208	92.0432	53.3446	6.5195	96.4626	53.9586	15.2285	1.5301
12	0.0664	0.0449	6.2570	93.1891	54.0395	6.5513	97.5727	54.6533	15.2539	1.5377
13	0.0666	0.0444	6.2795	93.9504	54.5182	6.5714	98.3176	55.1343	15.2680	1.5424
14	0.0667	0.0441	6.2934	94.4475	54.8411	6.5839	98.8081	55.4600	15.2758	1.5453
15	0.0668	0.0440	6.3018	94.7662	55.0540	6.5916	99.1247	55.6756	15.2800	1.5470
16	0.0668	0.0439	6.3068	94.9672	55.1917	6.5962	99.3256	55.8155	15.2823	1.5481
17	0.0668	0.0438	6.3097	95.0919	55.2790	6.5990	99.4507	55.9043	15.2835	1.5487
18	0.0668	0.0438	6.3115	95.1678	55.3333	6.6006	99.5272	55.9596	15.2841	1.5490
19	0.0668	0.0438	6.3124	95.2132	55.3663	6.6015	99.5732	55.9934	15.2844	1.5492
20	0.0668	0.0438	6.3130	95.2399	55.3860	6.6020	99.6003	56.0136	15.2846	1.5493
21	0.0668	0.0438	6.3133	95.2552	55.3975	6.6023	99.6159	56.0254	15.2846	1.5494
22	0.0668	0.0438	6.3135	95.2638	55.4040	6.6025	99.6246	56.0321	15.2847	1.5494
23	0.0668	0.0438	6.3135	95.2686	55.4077	6.6025	99.6294	56.0358	15.2847	1.5495
24	0.0668	0.0438	6.3136	95.2711	55.4096	6.6026	99.6320	56.0378	15.2847	1.5495
25	0.0668	0.0438	6.3136	95.2723	55.4106	6.6026	99.6333	56.0388	15.2847	1.5495
26	0.0668	0.0438	6.3136	95.2730	55.4111	6.6026	99.6340	56.0393	15.2847	1.5495
27	0.0668	0.0438	6.3136	95.2733	55.4113	6.6026	99.6343	56.0396	15.2847	1.5495
28	0.0668	0.0438	6.3136	95.2734	55.4115	6.6026	99.6344	56.0397	15.2847	1.5495
29	0.0668	0.0438	6.3136	95.2734	55.4115	6.6026	99.6345	56.0397	15.2847	1.5495
30	0.0668	0.0438	6.3136	95.2735	55.4115	6.6026	99.6345	56.0397	15.2847	1.5495
31	0.0668	0.0438	6.3136	95.2735	55.4115	6.6026	99.6345	56.0398	15.2847	1.5495
32	0.0668	0.0438	6.3136	95.2734	55.4115	6.6026	99.6344	56.0397	15.2847	1.5495

buffer, the average number $L_1^{(2)}$ of type-2 customers in the buffer, are presented along with the corresponding value of N.

Figures 8 and 9 present the dependence of the characteristics $P_{loss}^{(1)}$, $P_{loss}^{(2)}$, $\widetilde{J}_2$ on the buffer capacity N available for the priority packets in the graphical form.

Experiment 5. The objective of the experiment is to find the buffer capacity N for type-2 customers which minimizes the economic criterion I_C called Weighted Grade of Service (WGoS). This criterion as assumed having the following form:

$$I_C = \frac{\lambda_1}{\lambda_1+\lambda_2} c_{loss}^{(1)} P_{loss}^{(1)} + \frac{\lambda_2}{\lambda_1+\lambda_2}\left(c_{loss}^{(2)} P_{loss}^{(2)} + c_{W_1^{(2)}} W_1^{(2)} + c_{J_2} J_2\right),$$

where $c_{W_1^{(2)}}$ is the penalty for the waiting of type-2 customer per unit of time, c_{J_2} is the penalty for waiting time jitter of

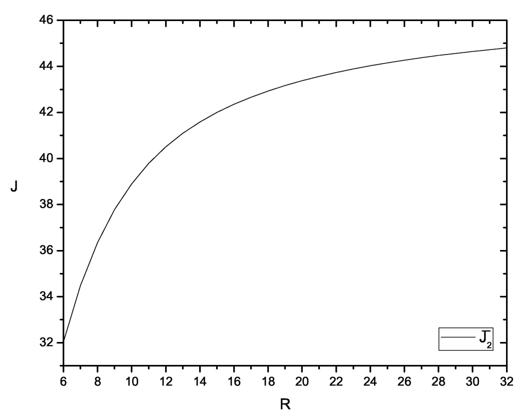

Fig. 7. Dependence of the jitter $\widetilde{J}_2$ of priority packets on the total capacity R of the buffer

type-2 customer per the squared time unit; $c_{loss}^{(l)}$, $l = 1, 2$, is the penalty for a type- l customer loss.

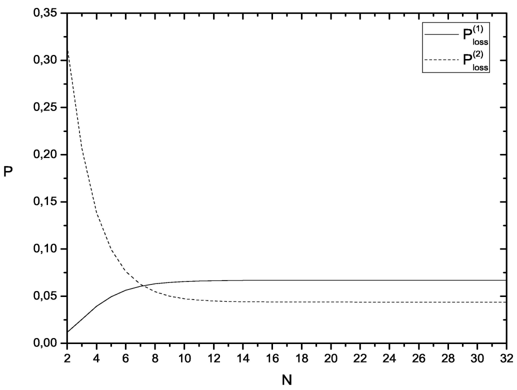

Fig. 8. Dependence of the loss probabilities $P_{loss}^{(1)}$, $P_{loss}^{(2)}$ on the available capacity N of the buffer for priority packets

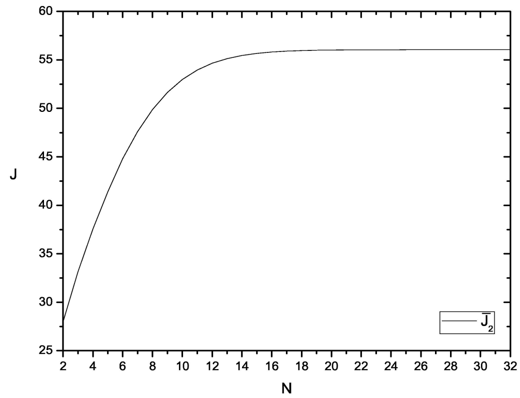

Fig. 9. Dependence of the jitter $\tilde{J}_2$ on the available capacity N of the buffer for priority packets

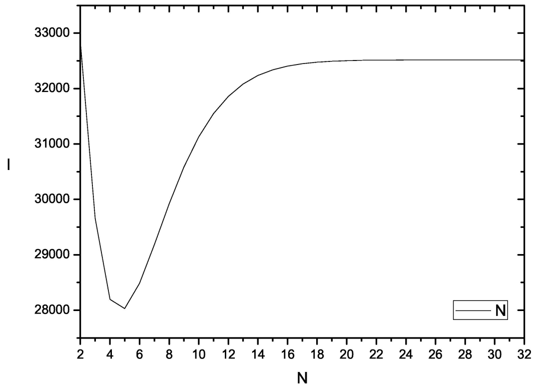

Fig. 10. Dependence of the economic criterion I_C on the available capacity N of the buffer for priority packets

In this experiment, the cost values are taken as follows:

$$c_{W_1^{(2)}} = 10,\ c_{J_2} = 800,\ c_{loss}^{(1)} = 100,\ c_{loss}^{(2)} = 10^5.$$

As it can be seen from Figure 10, the value of the WGoS criterion essentially depends on the buffer capacity N and it is minimal when $N = 5$.

VII. Conclusions

The paper presents novel buffer management that combines both space and time priorities for different streams of the same multimedia traffic. The paper also presents a queueing model that captures some features of multimedia traffic that were not captured of any previous model in the literature including intra- and inter-stream correlation. The QoS measures obtained by the model allow real-time optimization of the buffer to fulfill the changing traffic requirements.

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