DEVELOPING A DYNAMIC MAXIMAL COVERING LOCATION PROBLEM BY CONSIDERING CAPACITATED FACILITIES AND SOLVING IT USING HILL CLIMBING AND A GENETIC ALGORITHM

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Abstract:

The maximal covering location problem maximizes the total number of demands served within a maximal service distance given a fixed number of facilities or budget constraints. Most research papers have considered this maximal covering location problem in only one period of time. In a dynamic version of maximal covering location problems, finding an optimal location of P facilities in T periods is the main concern. In this paper, by considering the constraints on the minimum or maximum number of facilities in each period and imposing the capacity constraint, a dynamic maximal covering location problem is developed and two related models (A, B) are proposed. Thirty sample problems are generated randomly for testing each model. In addition, Lingo 8.0 is used to find exact solutions, and heuristic and meta-heuristic approaches, such as hill climbing and genetic algorithms, are employed to solve the proposed models. Lingo is able to determine the solution in a reasonable time only for small-size problems. In both models, hill climbing has a good ability to find the objective bound. In model A, the genetic algorithm is superior to hill climbing in terms of computational time. In model B, compared to the genetic algorithm, hill climbing achieves better results in a shorter time.

1 Introduction

Location problems with covering objectives are one of the main types of facility location problems [1]. Network covering problems have a rich history [2]. Revelle et al. [3] have provided a comprehensive bibliography of recent papers in median, center and covering models as three important types of facility location problem. In covering problems, if the distance between the demand point and the facility is less than a threshold, the demand can be served by that facility [4]. This threshold is called the covering radius. The three main assumptions of covering problems are all-or-nothing coverage, individual

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coverage and fixed coverage radius. With relaxation of these assumptions, the gradual covering model [5], cooperative covering model [6] and variable radius model [7] are proposed respectively [8]. The maximal covering location problem (MCLP) and set covering location problem (SCLP) are two distinct categories of covering location problems [9]. While a SCLP calls for covering all demand points with the minimum number of facilities, MCLP seeks the maximum possible covering with a fixed number of facilities. MCLP was introduced by Church and Revelle [10]. Allocated resources (e.g. budgets) in many practical applications are not sufficient to cover all demand points [11], so MCLP is used as a powerful tool for the optimal distribution of limited resources to reach maximum covering [12]. Examples of this problem appear in determining the optimal location for intersection safety cameras on an urban traffic network [13], determining optimal police patrol areas [14], determining the optimal location of fire stations [15], [16] and the optimal location of emergency facilities [17], [18].

The MCLP has been a highly attractive area of study, but most researchers have considered MCLP in only one period. Dynamic MCLP considers one time horizon that includes T periods and finds the optimal location of P facilities. To the best of our knowledge, the most recent publications similar to our paper are those by Fazel Zarandi et al. [9] and Dell'Olmo et al. [13]. Fazel Zarandi et al. [9] considered a large-scale dynamic MCLP and applied a simulated annealing algorithm to solve large size problems, whereas Dell'Olmo et al. [13] proposed a multi-period MCLP for finding the optimal location of intersection safety cameras. However, it is worth noting that the current paper differs from both of these papers in terms of its problem definition and its solution method.

This paper develops the dynamic MCLP of Fazel Zarandi et al. [9] by considering the maximum capacity constraint on the facilities and the minimum and maximum number of facilities in each period of the time horizon. Up to now, capacitated MCLP have assumed only one fixed capacity level for the facility at each potential site. In this paper, sample problems are solved by an exact method with Lingo 8.0. The exact solutions are compared with the solutions from genetic and hill climbing algorithms.

Differences between this research paper and the one written by Fazel Zarandi et al. (2013) is presented in Table 1.

The rest of the paper is organized as follows: First, a concise literature review of covering problems and related issues is presented in Section 2. Section 3 defines the problem, and the solution algorithms are introduced in Section 4. Parameter settings and numerical examples appear in Section 5, together with an analysis and discussion of the results. Finally, conclusions and outlooks for potential future research are offered in Section 6.

2 Literature review

While static problems consider only one period, dynamic problems refer to problems with multiple planning periods for which some information is initially unknown and becomes available over time. The concept of a dynamic covering location problem is not new in the literature [9]. Schilling [19] proposed a dynamic multi-objective model for emergency facilities such as ambulances. In fact, it combined T MCLPs. In this model, a constraint was imposed on the number of facilities in each period, and it was supposed that, if a facility was located in each period, it would serve until the end of the planning horizon.

Table 1. Difference between this paper and the one by Fazel Zarandi et al. (2013)

difference		Fazel Zarandi et al. (2013)	This paper	
			Model A	Model B
problem definition	capacity constraint	-	✓	✓
	number of facility constraints	In whole time horizon	In whole time horizon	In each period
	constraints on the minimum or maximum number of facilities in each period	-	✓	-
solution method		Simulated annealing (SA)	Lingo Genetic algorithm hill Climbing.	Lingo Genetic algorithm hill Climbing

Afterwards, Gunawardane [20] proposed several multi-period public facility planning decision problems. In addition to proposing a dynamic model for the SCLPs, he proposed two dynamic models for the MCLPs. In the first model, the objective function minimized uncovered demands. The constraint on the number of facilities was imposed in each period. It was assumed that, once opened, a new facility would have to remain open and, once closed, an existing facility would have to remain closed. This assumption was not considered in the second model. But costs for uncovered demands and the opening or closing of facilities were considered. The purpose of the second model was to minimize those costs. Chrissis et al. [21] addressed the dynamic version of the set covering formulation for facility location problems. The problems were characterized by binary cover coefficients that possibly changed in value from one time period to the next. Repede and Bernardo [22] developed a maximal expected covering location model by considering time variations.

Antunes and Peeters [23] proposed a dynamic (multi-period) optimization model, allowing for facility closing or size reduction as well as facility opening and size expansion according to a predefined size. Encompassing specifications of a dynamic optimization model for public facilities planning, this model has been applied in Portugal for school networks.

Gendreau et al. [24] considered a dynamic model for ambulance relocation, thus maximizing backup coverage and minimizing relocation costs. Rajagopalan et al. [25] proposed a multi-period SCLP for dynamic redeployment of ambulances, likewise minimizing the number of ambulances needed to provide a given level of coverage. The location of these ambulances are determinated in different time periods. Başar et al. [17] applied a multi-period double coverage approach for emergency medical service (EMS) stations in Istanbul, wherein the maximum number of EMS stations in each period is predefined. Fazel Zarandi et al. [9] proposed a simulated annealing algorithm to solve large-scale dynamic MCLP. In their model, a constraint on the number of facilities is imposed on the whole time horizon. Dell'Olmo et al. [13] proposed a multi-period MCLP for the optimal location of intersection safety cameras on an urban traffic network. According to this model, wherein the positions of available cameras are changed periodically in a given time horizon, the constraint on the number of facilities is imposed in each period and no cost for relocation is considered. This model has been studied in road accidents occurring on a portion of the urban traffic network of the city of Rome.

Due to the dynamic nature of multi-period models, the word "dynamic" is used to describe multi-period in most research studies. Multi-period location problems consider a time horizon that includes a couple of time periods. These models propose better plans to respond to predictable demand fluctuations by time and space [9], [26]. Although dynamic covering models are not new and different types of MCLPs have been studied by researchers, as Fazel Zarandi et al. [9] stated, a literature review confirms that not enough attention has been paid to dynamic cases. As a result, dynamic MCLP seem to be a worthwhile research topic.

Capacity is an important property of facilities. Facility capacity determines how much demand it can meet. The capacity of facilities may be limited or unlimited [27]. Although researchers such as Yin and Mu [28] have considered capacitated facilities in MCLPs, all these researchers considered only one period. In this paper, capacitated facilities are considered in multi-period MCLPs.

In dynamic MCLPs, constraints on the number of facilities have been imposed in different ways. Dynamic MCLPs are shown in terms of the number of facility constraints in Table 2.

Table 2. Classification of dynamic MCLPs in terms of the number of facility constraints

Number of facility constraints in dynamic MCLPs		
In whole time horizon	In each period of the time horizon	
	$P_1 = P_2 = \cdots = P_T$	$P_1 \neq P_2 \neq \cdots \neq P_T$
Fazel Zarandi et al. [9]	Dell'Olmo et al. [13]	Başar et al. [17] Schilling [19] Gunawardane [20]

In previous studies, when a constraint on the number of facilities is imposed on the whole time horizon, the dynamic MCLP does not consider a preference on the number of facilities to be located in each period. In fact, in addition to the number of facility constraints in the time horizon, constraints on the minimum and maximum number of facilities in each period may be imposed. In this paper, this issue is considered as it appears/seems to be a gap in our understanding of this issue.

3 Problem definition

MCLP arises from the fact that the total number of facilities to be located is restricted due to budget limitations. When the planning horizon includes multiple periods, budget limitations vary from period to period [29]. In addition, in real world situations, some data may change over time because of the dynamic nature of the business. Multi-period location problems have been proposed to approach situations in which parameters change over time according to predictable patterns [29]. In multi-period MCLP, the number of facilities is an important constraint. It may simply be assumed that the number of facilities in each period is known. Thus, each period can be considered an independent MCLP. When the number of facilities is limited by the available budget in the whole time horizon, determining how many of these facilities should be located in each period is a challenge that concerns policy makers. Previous studies have considered the number of facility constraints in each period or in the whole time horizon with no priority about the number of facilities in each period. The number of facility constraints in multi-period MCLP could be extended to allow other constraints, such as limitations on the maximum or minimum number of capacitated facilities in some period.

On the basis of the time horizon considered, it is possible to identify periods in which events are most likely to happen. Although some periods are not eventful, due to emergency nature it may be necessary to locate a certain number of facilities in these periods; for example, it might be necessary to locate at least one facility in some periods. It is assumed that the minimum number of facilities are to to be added in this period. It is obvious that the minimum number of facilities in each period cannot be greater than the total number of facilities that can be added over a planning horizon. It may be that there is no information to determine the minimum number of facilities in some period since in that case the minimum number of facilities which can be added in this period is zero. In addition, it may be for some reason such as budget limitation in a period, it is not possible to add more than a certain number of facilities. Thus, the maximum number of facilities in such periods is known. If the maximum number of facilities in a period is not predefined, it should be noted that the maximum number of facilities cannot be more than the available facilities. In the first period, P is the number of available facilities. In the next period, the number of available facilities is P minus the number of facilities located in previous periods. In this paper, we consider comprehensive constraints on the number of facilities as follows:

1. The constraint on the number of facilities is imposed over the whole time horizon. P is the total number of facilities which is located over a time horizon (Model A). This way: If minimum number of needed facilities in period t is predetermined (m_t) then $m_t \leq$ the number of facilities in period t; otherwise $0 \leq$ the number of facilities in period t

If the maximum number of facilities that can be located in period t is predetermined (n_t), then the number of facilities in period

$t \leq P - \sum_{t=1}^{t-1}$ located facilities; otherwise the number of facilities in period $t \leq P$.

2. The constraint on the number of facilities is imposed for each period (Model B). P_t is the number of facilities for period t.

On the other hand, each facility has capacity constraints which limit the number of demands it can serve. Consideration of incapacitated facilities limits the application of covering models [30].

Capacitated facility location problems ensure that the total demand assigned to a facility doesn't exceed the capacity of that facility. Until now, multi-period MCLPs haven't considered capacitated facilities. In the models that have been proposed, facilities have only one fixed capacity level and demand coverage is binary, i.e., a demand point is either completely covered or not covered at all. As Fig. 1 illustrates, different demand types can be defined according to capacity constraints and coverage radius:

1. A demand which is located beyond the covering radius of the facilities, so it is not allocated to any facility (e.g., d_d in Fig. 1);

2. A demand which is located within the covering radius of at least one facility. Here there are two cases:

A) The demand at this point is more than the total capacity of the facilities which can cover this demand point. In such a situation, although this demand point

can be assigned to these facilities, it would not be covered (e.g., d_a and d_e in Fig. 1);
B) The demand at this point is less than or equal to the total capacity of the facilities which can cover this demand point. Consequently, the demand at this point would be covered by at least one facility (e.g., d_b and d_c in Fig. 1).

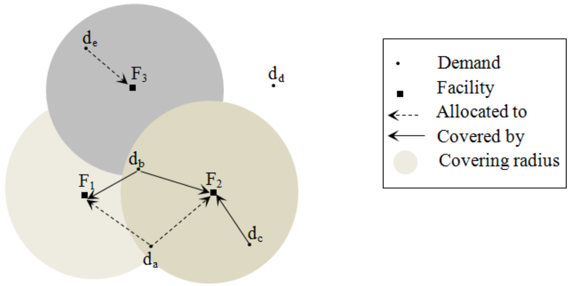

Figure 1. Demand.

It is assumed that demand points and eligible facility sites are considered identical in all periods; each demand point cannot host a facility. Only one facility may be located in each potential location.
It is to be noted that location models are classified, based on their objectives, as private and public sectors. While a function of cost is to be optimized in private sector models, minimizing costs is not a concern in the public sector models [9]. Proposed models are attributed to the public sector. So, opening/closing facility costs are not considered. Maximizing covered demand over a time horizon by a given number of capacitated facilities is a strategic goal of proposed models.
The proposed modification is motivated by the following observations. In problems such as locating facilities in fire stations, police stations or emergency rescue centers, demands are not the same at all sites during all periods. According to demand pattern, a planning horizon can be divided into multiple periods. Due to some reasons such as seasonal pattern of tourism demand, holidays, weather conditions, and local traditions, emergency events risk such as car accidents occurred increasingly dramatically in some period. So policy makers can predict some areas that have high event risk during particular periods. In order to achieve an effective emergency response system, managers may decide to add new facilities and to locate them over a planning horizon by maximizing covered demand. Due to budget limitations, MCLP has attracted and policy makers should decide how many faculties are located in each period and where these facilities are located. Effectively planning, the addition of these facilities can significantly reduce uncovered demand. It is clear that policy makers prefer to allocate more facilities during periods when events are most likely to happen. Due to budget limitations for each period, policy makers may not locate enough facilities. In fact, policy makers consider the fact that in a certain period only a certain number of facilities can be added. On the other hand, due to the importance of some periods, policy makers may decide to establish at least a certain number of facilities. This situation describes the difficulty that policy makers face when they try to maximize covered demand by considering the minimum and maximum number of facilities in each period. To consider this situation, two models (A, B) are proposed as follows:

3.1 Model A

For simplicity it is assumed that each facility serves in only one period of the time horizon and facilities relocation is not considered in model A. Considering relocation of facilities in model B is simple.
The main assumption of this model is that the facilities serve in only one period of the time horizon. In other words, if a facility is located in one period, it will be closed at the end of that period and relocation of the facilities over that time horizon will not be considered. All facilities are closed at the end of each time horizon, therefore no facilities are located in eligible locations at the beginning of each time horizon ($x_{j0} = 0$). In this model, the constraint on the number of facilities is imposed over the whole time horizon. A constraint on the minimum and maximum number of facilities in each period may be imposed as well. Furthermore, in this particular model if the decision makers do not impose a constraint on the minimum number of facilities for that period t, it will not be necessary to locate a facility in that period. Moreover, the minimum number of facilities in that period is zero ($m_t = 0$). If the decision makers do not impose a constraint on the maximum number of facilities in period t (in this model, each facility is closed at the end of each period and is not relocated), it would be clear that the maximum number of facilities in period t cannot be more than the total number of facilities ($n_t = P$). In this model, it is assumed that the minimum or maximum number of facilities in each period is certain.
Constraints on the minimum or maximum number of facilities could be imposed simultaneously in a period. For example, it might be necessary to locate

at least one facility in all periods or for some reasons, such as budget limitations, facilities might be located gradually. Therefore, all P facilities might not be available in period t. In such a situation, the maximum number of facilities that can be located in period t is dependent on the total number of available facilities in period t. It is assumed that the decision maker determines the minimum number of facilities in each period in such a way that the sum of these minima would not be more than the total number of facilities in the time horizon. On the other hand, the maximum number of facilities in each period has to be more than the minimum number of facilities in that period. In this paper, the number of facilities constraint is formulated in such a way that it encompasses all possible situations. Herein, a proposed dynamic MCLP is presented. First, the problem parameters and variables are defined.

Sets and parameters
i, I: The index and set of demand points.
j, J: The index and set of eligible facility sites.
t, T: The index and set of time periods.
a_{it}: The population/demand at point i in period t.
d: The Euclidean distance from demand point i to the facility at j.
S: The distance (or time) standard within which coverage is desired.
$N= \{j|\ d \leq S\}$: The set of points that are within a distance that is less than S from point i.
P: The number of facilities to be located within the whole time horizon.
m_t: The minimum number of facilities in period t ($\sum_{t=1}^{T} m_t \leq P$).
n_t: The maximum number of facilities in period t ($n_t \geq m_t\ \forall t$).
c: The capacity of each facility.

Variables
x_{jt}: A binary variable that equals one when a facility is sited at location j in period t and zero otherwise.
y_{it}: A binary variable which equals one if demand point i in period t is covered by one or more facilities stationed within S and zero otherwise.
Then, the proposed model will be as follows:

$$\max Z = \sum_{t=1}^{T}\sum_{i=1}^{I} a_{\mathrm{it}} y_{\mathrm{it}} \tag{1}$$

$$a_{\mathrm{it}} y_{\mathrm{it}} \leq \sum_{j \in N_i} C x_{\mathrm{jt}} \quad \forall \mathrm{t}, \forall \mathrm{i} \tag{2}$$

$$\sum_{j=1}^{J} C x_{\mathrm{jt}} \geq \sum_{i=1}^{I} a_{\mathrm{it}} y_{\mathrm{it}} \quad \forall \mathrm{t} \tag{3}$$

$$\sum_{t=1}^{T}\sum_{j=1}^{J} C x_{\mathrm{jt}} = P \tag{4}$$

$$m_t \leq \sum_{j=1}^{J} x_{\mathrm{jt}} \leq \min\left\{ n_t, \left(P - \sum_{j=1}^{J}\sum_{t=1}^{t} x_{\mathrm{jt\text{-}1}} \right) \right\} \forall t \tag{5}$$

$$y_{\mathrm{it}}, x_{\mathrm{jt}} \in \{0,1\} \tag{6}$$

The objective function (1) maximizes the overall covered demand. Constraints (2) illustrate that the demand point i in period t will be covered if its demand is less than or equal to the total capacity of the facilities which are located within the service distance from demand point i. Constraints (3) show that in each period, the total covered demand cannot be more than the total capacity of located facilities in that period (capacity constraint). Constraint (4) confines the total number of facilities in the whole time horizon up to P facilities. According to Constraints (5), if the minimum and maximum number of facilities were defined in period t, the number of located facilities in period t would be in the related interval. Otherwise, it would be between zero and the total number of available facilities (which are not located yet) in period t. Constraints (6) show that decision variables are binary.

3.1.1 Linearization

Constraints (5) cause non-linearization of the model. If we have a non-linear constraint in the form of $y \leq min(x_1, x_2)$, it could be linearized by Eq. (7-10) where δ is a binary variable and G is a sufficiently large positive value ($G \geq P$).

$$x_1 \leq x_2 + G\delta \tag{7}$$

$$x_2 \leq x_1 + G(1-\delta) \tag{8}$$

$$y \leq x_2 + G(1-\delta) \tag{9}$$

$$y \leq x_1 + G\delta \tag{10}$$

3.2 Model B

In this model, facilities are closed at the end of each period but may be relocated in the subsequent periods if available. As such, a facility may serve in more than one period. In model B, the number of facilities at the beginning of the first period, and the change in this number at the beginning of each new period, are predefined. The number of facilities might increase if new facilities are required or decrease due to failures or access limitations. It is assumed that the number of facilities used at the beginning of each period is certain and predefined. In other words, a constraint is imposed on the number of facilities used in each period of the time horizon.

Unlike model A, where a MCLP is proposed for the time horizon as a whole, we propose a MCLP for each separate period in model B. In other words, there is T MCLP applicable to model B, which provides maximum coverage of the whole time horizon by providing maximum coverage of each period. The main objective of this model is to provide the maximum coverage of the time horizon. In this way, a MCLP can be defined for each period.

In each period of the time horizon, P_t facilities (in specific cases, p facilities) are located ($P_{t+1} = P_t + d_t$). In this model, we are faced with the location of added facilities ($d_t > 0$) and the relocation of facilities from the previous period (if the facility is available). From another viewpoint, model B could also be applied to a situation in which a certain number of facilities are located in each period and, at the end of each period are closed thus becoming unavailable.

Parameters and decision variables for this model are similar to those in model A. The only difference is that in this model, P_t is defined instead of P. P_t is the number of facilities in period t. Therefore, this model does not have m_t and n_t.

$$\max Z = \sum_{t=1}^{T} \sum_{i=1}^{I} a_{\text{it}} Y_{\text{it}} \tag{11}$$

$$Y_{\text{it}} a_{\text{it}} \leq \sum_{j \in N_i} CX_{\text{jt}} \quad \forall \text{t}, \forall \text{i} \tag{12}$$

$$\sum_{j=1}^{J} CX_{\text{jt}} \geq \sum_{i=1}^{I} a_{\text{it}} Y_{\text{it}} \quad \forall \text{t} \tag{13}$$

$$\sum_{j=1}^{J} CX_{\text{jt}} = P_t \tag{14}$$

$$Y_{\text{it}}, X_{\text{jt}} \in \{0,1\} \tag{15}$$

The objective function and constraints in model B are exactly the same as those in model A. The only difference is the substitution of constraint (14) with constraints (4) and (5). Constraint (14) specifies the number of facilities in each period of the time horizon.

4 Solution methods

The MCLP is NP-hard, as shown by Church and Revelle [31] and Garey and Johnson [32]. Therefore, exact methods such as branch and bound can reach a solution within reasonable time only for small-size problems. In this paper, genetic algorithm and hill climbing heuristic are employed to solve the numerical problems.

4.1 Genetic algorithm (GA)

GA is one of the best methods for solving facility location problems. It was first proposed by John Holland in 1975. The main purpose of GA is to improve generations gradually using operators such as crossover and mutation. Each generation involves a set of individual solutions. Each iteration involves the selection of a set of chromosomes based on their fitness value and application of reproduction schemes to generate a set of new chromosomes. A selection strategy updates the population, and the process continues until the termination criterion is met. The rest of this section elaborates on the proposed GA [33].

4.1.1 Encoding scheme

Many different approaches are capable of representing a solution for MCLP. Considering that Matlab has a powerful matrix-processing capability [34], a chromosome is represented with a binary matrix. This paper employs a multi-chromosome technique, and two binary matrices are defined. One of the matrices has I rows and T columns. Each element in this matrix represents the status (covered/uncovered) of demand point i in period t. The other matrix has J rows and T columns. In this matrix, each element represents the facility status for location j in period t. A value of 1 in the jth position means that there is a facility in location j in period t. The initial population for each chromosome is created randomly.

4.1.2 Selection

This paper uses roulette wheel selection to select parents for crossovers. For mutations, chromosomes are selected randomly. Random selection is the simplest way to select chromosomes without considering fitness values.

4.1.3 Crossover

Crossover is a genetic operator that combines two chromosomes (parents) to produce a new chromosome (offspring). P_c is the crossover probability. If there is no crossover, the offspring is an exact copy of its parents. In this paper, one of two methods, one-point crossover and two-point crossover are randomly chosen. One-point crossover randomly selects a crossover point within a chromosome and then interchanges the two parent chromosomes at this point to produce two new offspring. The crossover point can be an element or a column of a matrix. Two-point crossover randomly selects two crossover points within a chromosome and then interchanges the two parent chromosomes at these points to produce two new offspring. Fig. 2 illustrates the different types of crossover.

4.1.4 Mutation

Mutation serves to ensure that a population does not converge to a local minimum by changing the sequences of one or more genes within a chromosome at random. Although the probability of a mutation arising is usually at a very low frequency per thousand base pairs, several authors have alluded to a higher mutation rate when the GA has converged [35]. In this paper, three different methods for mutation are addressed, namely binary, swap and reversion mutation. In a binary mutation, a number of elements are selected at random and their values are said to change from one to zero or from zero to one [36]. In a swap mutation two elements are selected at random and their position is exchanged. In a reversion mutation the positions of two elements are reversed at random. The three different forms of mutation are depicted in Fig. 3.

4.1.5 Termination criteria

The algorithm will iterate until the maximum number of iterations is attained.

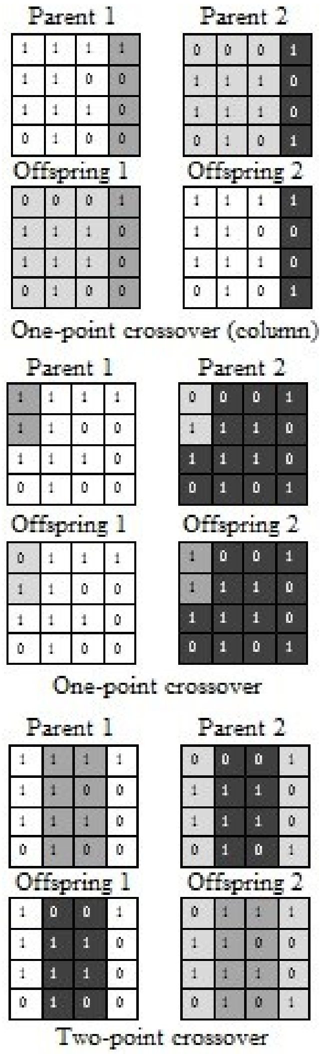

Figure 2. Crossover.

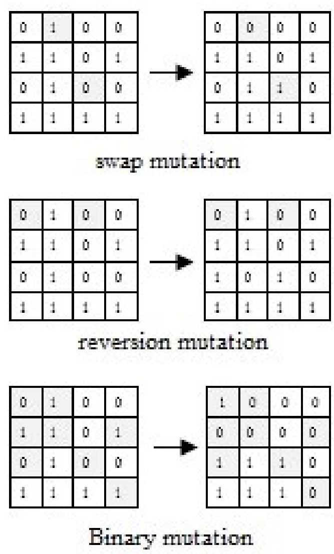

Figure 3. Mutation.

4.2 Hill climbing heuristic (HC)

The hill climbing heuristic is a path-based local search method and is strongly dependent upon the starting positions for the search [37]. For the purposes of this paper, the hill climbing heuristic was applied due to its inherent simplicity and effectiveness. Moreover, it is frequently preferred in comparison with more complex search algorithms such as GA [38-41].

Hill Climbing uses a kind of gradient to guide the direction of search. Each iteration consists in choosing randomly a solution in the neighborhood of the current solution and retains this new solution only if it improves the fitness function. Stochastic Hill Climbing converges towards the optimal solution if the fitness function of the problem is continuous and has only one peak (unimodal function). On functions with many peaks (multimodal functions), the algorithm is likely to stop on the first peak it finds even if it is not the highest one. Once a peak is reached, hill climbing cannot progress anymore, and that is problematic when this point is a local optimum. Stochastic hill climbing usually starts from a random select point. A simple idea to avoid getting stuck on the first local optimal consists in repeating several hill climbs each time starting from a different randomly chosen point. This method is sometimes known as iterated hill climbing. By discovering different local optimal points, it gives more chance to reach the global optimum. It works well if there is not too many local optima in the search space. But if the fitness function is very "noisy" with many small peaks, stochastic hill climbing is definitely not a good method to use. Nevertheless, such methods have the great advantage to be really easy to implement and to give fairly good solutions very quickly [33].

In this paper an initial population was first generated at random and its representation in hill climbing solutions was similar to those obtained from GA. Some neighbours are generated for each solution within each iteration. The hill climbing heuristic includes local searches, so finding a neighbour is the primary concern. Neighbors are generated via mutation methods in GA, whereas in the hill climbing heuristic initial solutions and generated neighbours are sorted based upon the fitness function and the initial solution is replaced by a 'best solution'. This process continues until the algorithm is unable to find better neighbours to satisfy current solutions. For the purposes of this paper 10 initial solutions were generated randomly, and in each iteration for each solution 5 neighbours were generated (a total of 50 neighbors for each solution). In this algorithm, a termination criterion is used to reach the maximum number of iterations. The maximum number of iterations for model A was 20 and for model B 10.

5 Numerical examples

5.1 Test problems

To generate test problems, a similar approach to Revelle et al. [42] is used. According to this approach, the locations of demand points and eligible facility sites are randomly generated using a uniform distribution between 0 and 30 for both x and y coordinates. Populations on the demand points in each time period are randomly generated using a uniform distribution between 0 and 100. Then, the distances between the points are defined as their Euclidean distance. Revelle et al. [42] used this method to generate one-period problems. Fazel Zarandi et al. [9] used this method to generate sample problems. Since they considered a dynamic version of the problem, it was necessary to consider time scale. They considered all sample problems for five periods. It should be noticed that in both papers aiming to solve large problems, they indicate the number of demand points. In this paper, the number of demand points is being determined according to the complexities of the proposed models. As the proposed models are dynamic, it is necessary to specify the time scale. Therefore, the problems are being considered in different time periods. The minimum number of facilities in each period is randomly generated using a uniform distribution between 0 and p (in such a way that $\sum_t^T m_t \leq P$). The maximum number of facilities in each period is generated by random numbers equal to and more than the minimum number of facilities in that period. As a result, for each model, 30 sample problems are generated in defined intervals.

Lingo 8.0 is used to solve these problems, and results are compared against those obtained using GA and hill climbing algorithm.

5.2 Parameter setting

Although appropriate selection of parameters and operators in each algorithm depends on the type of problems, most researchers neglect this point and set algorithm parameters based on the reference values of the previous similar studies [43]. There are several static methods for designing experiments to tune the algorithm. Among these methods, the full factorial method is used most frequently. The Taguchi method is used to reduce the number of required experiments. In the Taguchi method, orthogonal arrays are used to survey numerous decision variables with a small number of experiments. Taguchi transformed the repetitive data to another value called the measure of variation. This transformation is defined as the signal to noise (*S/N*) ratio. The purpose is the maximization of the *S/N* ratio [44]. In this study, objective functions are "the larger the better". The formula used for calculating the *S/N* ratio (the large the better) is given by Eq. (16).

$$S/N \text{ ratio} = -10\log\left(\frac{1}{n}\sum_{i=1}^{n}\frac{1}{x_i^2}\right)_t \tag{16}$$

Where x_i = observed response value and n = number of replication.

In this study, four parameters at three levels are considered for each GA. The factors and their levels are presented in Tables 3 and 4. According to the standard table of orthogonal arrays, L9 is selected as the fittest orthogonal array design that satisfies all the minimum requirements. For each algorithm setting, six sample problems are considered and each problem is iterated five times. Size of the sample problems is different; therefore, a substantial difference exists between their objective functions. Therefore, the *S/N* ratio is calculated after converting the raw data to a relative deviation index (*RDI*). RPD_{ijk} is calculated using Eq. (17).

$$RDI_{ijk} = \begin{cases} \left|\dfrac{OF_{ijk} - u_i}{u_i - l_i}\right| & u_i \neq l_i \\ 0 & u_i = l_i \end{cases} \quad \forall i,j,k \tag{17}$$

In Eq. (17), OF_{ijk} is the objective function value attributed to iteration j in sample problem i in scenario k. The values l_i and u_i are the minimum and maximum values of the objective function for the ith sample problem.

In the Taguchi method, *S/N* is considered the first criteria. A meaningful difference might not exist between different levels of *S/N*. Therefore, another criteria named $\overline{RDI_k}$ is defined for scenario k, which is calculated by Eq. (18). $\overline{RDI_k}$ is considered a "smaller-the-better" criteria.

$$\overline{RDI_k} = \frac{1}{n}\sum_{i=1}^{I}\sum_{j=1}^{J} RPI_{ijk} \quad k = 1\ldots9 \tag{18}$$

Finally, the best combination of parameters is selected for GA according to *S/N* (Fig. 4 and Fig. 6) and *RPD* (Fig. 5 and Fig. 7) charts. Selected levels are colored in Tables 3 and 4.

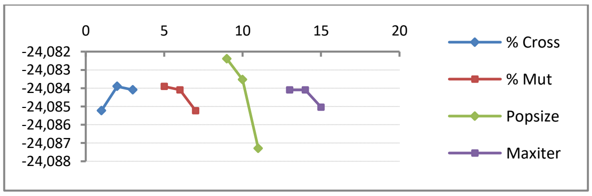

Figure 4. S/N ratio (model A)

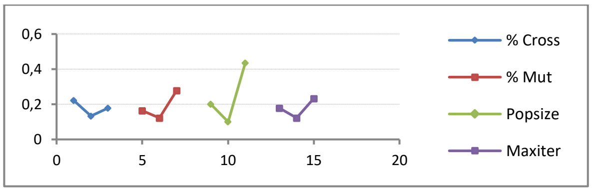

Figure 5. RDI (model A)

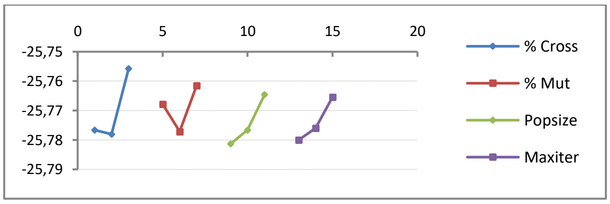

Figure 6. S/N ratio (model B)

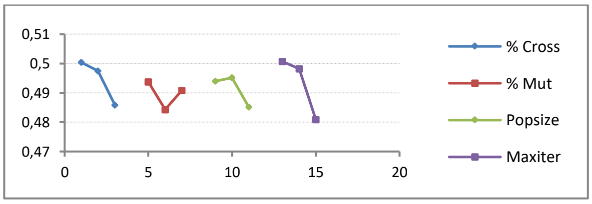

Figure 7. RDI (model B)

5.3 Results and discussions

Test problems are solved using three approaches. First, each problem is solved using Lingo 8.0. Then, solutions are compared against the results of the proposed GA and hill climbing heuristic. Computational results of the problems are summarized in Tables 5, 6 and 7. In each case, average computational time and objective function in five iterations are reported. Lingo uses the branch and bound technique to solve problems. Objective bound illustrates the theoretical bound of the objective function. This bound is limit which shows how much the solver can improve the objective function. In some cases, the best values of the objective function and the objective bound might be very close to each other. The best value of the objective function cannot exceed the objective bound [45]. Whereas the problems are NP-hard, Lingo 8.0 can reach the solution in a reasonable time only for small-size problems. Coloured lines indicate problems in which Lingo 8.0 is unable to find the optimal solution in one hour. In such cases, instead of the optimal value, the objective bound and best feasible solutions found in one hour are reported. Heuristic and meta-heuristic algorithms might achieve a better objective value than Lingo 8.0 in one hour. In such a situation, the gap will be negative ($\mathrm{Gap} < 0$) [46], [47] and gap can be calculated using Lingo objective bound [48].

Table 3. Parameter and their levels in GA (model A)

Parameter	Level 1	Level 2	Level 3
P_C	0/5	0/55	0/6
P_m	0/01	0/05	0/1
population	10	15	20
Max iteration	20	40	60

Table 4. Parameter and their levels in GA (model B)

Parameter	Level 1	Level 2	Level 3
P_C	0/75	0/85	0/95
P_m	0/01	0/03	0/05
population	20	30	40
Max iteration	20	25	30

5.3.1 Computational results of model A

Computational results of the problems are summarized in Table 5. According to the results of 30 sample problems of model A, in two thirds of sample problems Lingo is unable to find an optimal solution in one hour. In four problems, GA can find better solutions than the best solution of Lingo in only 12 seconds ($\mathrm{Gap_{GA}} < 0$). In other problems, except for one problem, GA finds the solution with a gap less than 1.9% in 12 seconds. The hill climbing heuristic can always find a solution better than or equal to the best solution of Lingo in less than 60 seconds ($\mathrm{Gap_{HC}} \leq 0$). This can be stated as follows:

$$\mathrm{Gap_{HC}} \leq \mathrm{Gap_{GA}}\,, \mathrm{Gap_{HC}} \leq 0$$

$$\mathrm{Time_{GA}} < \mathrm{Time_{HC}}$$

Therefore, it can be concluded that although hill climbing has good ability in finding Lingo objective bound, GA is superior to hill climbing in terms of computational time.

5.3.2 Computational results of model B

As shown in Tables 6 and 7, sample problems in model B are considered for six time horizons. For each time horizon, five sample problems in different sizes are solved. In six sample problems, Lingo can find the optimal solution in a short time**.** As the number of periods increases, exact computational time increases too. In more than two-thirds of sample problems, Lingo is unable to find the optimal solution in one hour. For more than half of the sample problems, GA can find a better solution than (or equal to) the best solution of Lingo in one hour ($\mathrm{Gap_{GA}} < 0$). In other problems except two, GA achieves a gap less than 1.1%. Hill climbing algorithm in more than 70% of the problems can find a solution better than or equal to the best solution of Lingo in one hour ($\mathrm{Gap_{HC}} < 0$). In other problems except two, it can achieve a gap less than 0.8%. The computational time for each algorithm is presented in Table 8. Overall, it can be stated as follows:

$$\mathrm{Time_{HC}} < \mathrm{Time_{GA}}$$

$$\mathrm{Gap_{HC}} \leq \mathrm{Gap_{GA}}$$

Therefore, compared with GA, hill climbing can achieve better results in a shorter time.

The structure of model and the size of sample problems affect Performance of algorithms. The number of facilities is an important constraint in MCLPs. In model A, we defined the minimum or maximum number of facilities in each period in addition to number of facility in whole of time horizon. But in model B, number of facility in each period is simply predefined. So in this model, time periods are independent and we have a capacitated MCLP in each period. Although model B is simply summation of some MCLP, model A is single MCLP that during all time periods depends on each other. In short, Complexity of model A and B is different.

According to the results of sample problems, algorithm hill climbing can find better solutions (in term of gap and computational time) in the model B by local search but the model A is complex and genetic algorithm reaches better results**.**

6 Conclusion and future research areas

In this paper, the dynamic MCLP has been extended to the capacitated dynamic MCLP. Capacity facility has been considered and a new constraint defined for the number of facilities. The developed models were solved by GA and hill climbing, and the results were compared with exact solutions of Lingo 8.0. We have shown that while GA and hill climbing heuristics are superior to the exact method in terms of runtime, there are negligible errors compared to the optimal solutions. Although GA and hill climbing heuristics show great performance in solving capacitated dynamic MCLP, one may assess the performance of other methods in finding solutions to the same problem.

Table 5. Computational results of model A

Test problems			Lingo			GA			Hill climbing heuristic		
S	*C*	*P*	Z^*	Objective bound	Time (s)	Z^*	Time (s)	Gap (%)	Z^*	Time (s)	Gap (%)
10	**10**	10	99	100	3600	97	11/79	2/02	100	37/77	**-1/01**
		15	143	150	3600	142/6	11/81	0/27	8/149	38/07	**-4/75**
		20	197	200	3600	193/2	11/84	1/92	198/6	38/32	**-0/812**
		25	237	250	3600	236/6	11/97	0/16	294/4	38/56	**-5/23**
		30	279	300	3600	284/4	12/12	**-1/93**	295/8	38/81	**-6/02**
	12	10	120	-	629	118/4	12/02	1/33	120	37/94	0
		15	179	180	3600	175/8	11/93	1/78	179/8	29/84	**-0/449**
		20	229	240	3600	227/8	11/99	0/52	239	38/83	**-0/446**
		25	276	300	3600	288	12/00	**-4/34**	299	40/18	**-8/333**
		30	328	360	3600	438/4	12/39	**-6/21**	357/8	40/85	**-9/085**
15	10	10	100	-	239	99	12/26	1	100	39/78	0
		15	149	150	3600	147/6	12/40	0/93	150	40/00	**-0/671**
		20	200	-	1835	198/4	12/49	0/8	200	40/25	0
		25	249	250	3600	247	12/75	0/8	250	40/72	**-0/40**
		30	299	300	3600	295/6	12/24	1/13	299/8	43/70	**-0/26**
	12	10	120	-	756	119/2	11/77	0/66	120	38/77	0
		15	180	-	1519	179/4	11/49	0/33	180	39/15	0
		20	239	240	3600	237/6	11/58	0/58	192	39/22	**-0/41**
		25	297	300	3600	296/8	11/60	0/06	300	39/62	**-1/01**
		30	359	360	3600	358	11/87	0/27	360	39/81	**-0/27**
20	10	10	100	-	180	99/2	11/72	0/8	100	38/75	0
		15	149	150	3600	148/8	11/81	0/13	150	39/18	**-0/67**
		20	199	200	3600	199	11/81	0	200	39/32	**-0/50**
		25	249	250	3600	247/2	11/92	0/72	250	39/58	**-0/40**
		30	299	300	3600	297/4	12/01	0/53	300	39/80	**-0/33**
	12	10	120	-	166	119/8	11/78	0/16	120	38/65	0
		15	180	-	442	178/4	11/83	0/88	180	39/07	0
		20	240	-	2094	238/2	11/92	0/75	240	39/57	0
		25	300	-	3600	298/6	12/03	0/46	300	39/65	0
		30	355	360	3600	355/6	12/15	**-0/16**	360	40/22	**-1/40**

Table 6. Computational results of model B (I=J=300)

Test problems			Lingo			GA			Hill climbing		
S	*C*	*T*	*Z**	Objective bound	Time (s)	*Z**	Time (s)	Gap (%)	*Z**	Time (s)	Gap (%)
5	5	3	175	-	1	175	50/58	0	175	21/99	0
		5	298	300	3600	8/297	84/81	067/0	8/298	37/03	**26/0-**
		7	399	400	3600	8/395	121/80	80/0	398	57/08	25/0
		9	573	575	3600	6/569	182/59	59/0	2/569	80/79	66/0
		10	515	525	3600	8/518	175/54	**73/0-**	6/530	72/57	**029/3-**
		12	721	725	3600	713	31/235	10/1	718	99/06	416/0
	7	3	245	-	2	245	33/52	0	245	45/21	0
		5	420	-	2	420	89/87	0	420	34/36	0
		7	555	560	3600	6/558	14/125	**64/0-**	8/559	92/52	**86/0-**
		9	803	805	3600	8/801	68/180	14/0	6/803	63/82	**07/0-**
		10	734	735	3600	734	60/156	0	8/734	50/72	**10/0-**
		12	1014	1015	3600	8/1011	05/233	21/0	8/1013	78/100	01/0

Table 7. Computational results of model B (I=400, J=300)

Test problems			Lingo			GA			Hill climbing heuristic		
S	*C*	*T*	*Z**	Objective bound	Time (s)	*Z**	Time (s)	Gap (%)	*Z**	Time (s)	Gap (%)
5	3	3	120	-	138	120	78/92	0	120	33/63	0
		5	180	-	471	176/4	132/82	2	178/4	56/45	0/88
		7	246	255	3600	247/6	132/29	**-0/65**	249/8	82/03	**-1/54**
		9	334	345	3600	330	272/67	1/19	334	124/34	0
		10	314	315	3600	306	236/68	2/54	305/4	120/93	2/73
		12	459	465	3600	446/2	332/94	2/78	452/8	151/44	1/35
	5	3	200	-	2	200	75/00	0	200	34/08	0
		5	298	300	3600	299/6	125/45	**-0/53**	300	57/29	**-0/67**
		7	424	425	3600	424/4	178/71	**-0/09**	425	82/01	**-0/23**
		9	567	575	3600	573/2	248/95	**-1/09**	574/2	129/38	**-1/26**
		10	524	525	3600	522/8	235/78	0/22	524/2	110/34	**-0/03**
		12	773	775	3600	771/4	322/13	0/20	773/8	150/68	**-0/10**
	7	3	280	-	2	280	76/175	0	280	32/66	0
		5	420	-	3182	420	129/27	0	420	54/79	0
		7	593	595	3600	594/8	180/27	**-0/30**	595	83/52	**-00/33**
		9	804	805	3600	804	261/75	0	805	124/04	**-0/12**
		10	734	735	3600	735	241/00	**-0/13**	735	112/38	**-0/13**
		12	1079	1085	3600	1081/2	329/73	**-0/20**	1084/8	161/89	**-0/53**

A possible future study could be to compare using various heuristics/meta-heuristics on this problem. Another avenue for future research could be to assess the performance of the hill climbing heuristic for other variants of MCLP, or considering some parameters of the problem as fuzzy variables. Fuzzy theory can be utilized in this model where input parameters such as minimum and maximum numbers of facilities in each period cannot be estimated with certainty. This model can also be investigated in conditions where each facility has a failure probability. Considering probabilistic demand is a real contribution to the model.

Table 8. Computational time in model B

Time periods	3	5	7	9	10	12
Hill climbing	< 30	< 60	$\cong 60$	$\cong 90$	<120	$\cong 120$
GA	$\cong 60$	< 120	$\cong 150$	< 210	< 240	< 300

References

[1] Berman, O., Drezner, Z., Krass, D.: *Cooperative cover location problems: the planar case*, IIE Transactions, 42 (2009), 3, 232-246.

[2] Batanović, V., Petrović, D., Petrović, R.: *Fuzzy logic based algorithms for maximum covering location problems*, Information Sciences, *179* (2009), 1-2, 120–129.

[3] Revelle, C. S., Eiselt, H. A., Daskin, M. S.: *A bibliography for some fundamental problem categories in discrete location science*, European Journal of Operational Research, 184 (2008), 3, 817–848.

[4] Hosseininezhad, S. J., Jabalameli, M. S., Jalali Naini, S. G.: *A continuous covering location model with risk consideration*, Applied Mathematical Modelling, 37 (2013), 23, 9665-9676.

[5] Berman, O., Kalcsics, J., Krass, D., & Nickel, S.: *The ordered gradual covering location problem on a network*, Discrete Applied Mathematics, 157 (2009), 18, 3689-3707.

[6] Berman, O., Drezner, Z., Krass, D.: *Generalized coverage: New developments in covering location models*. Computers & Operations Research, 37 (2010), 10, 1675-1687.

[7] Berman, O., Drezner, Z., Krass, D., Wesolowsky, G. O.: *The variable radius covering problem*, European Journal of Operational Research, 196 (2009), 2, 516-525.

[8] Berman, O., Drezner, Z., Krass, D.: *Generalized coverage: New developments in covering location models*, Computers & Operations Research, 37 (2010), 10, 1675-1687.

[9] Fazal Zarandi, M. H., Davari, S., Haddad Sisakht, S. A.: *The large-scale dynamic maximal covering location problem,* Mathematical and Computer Modelling, 57 (2013), 3, 710-719.

[10] Church, R., Revelle, C. R.: *The maximal covering location problem*. Papers in regional science, 32 (1974), 1, 101-118.

[11] Zanjirani Farahani, R., Asgari, N., Heidari, N., Hosseininia, M. Goh, M.: *Covering problems in facility location: A review*, Computers & Industrial Engineering, 62 (2012), 1, 368-407.

[12] Balcik, B., Beamon, B. M.: *Facility location in humanitarian relief.* International Journal of Logistics, 11 (2008), 2, 101-121.

[13] Dell'Olmo, P., Ricciardi, N., Sgalambro, A.: *A multiperiod maximal covering location model for the optimal location of intersection safety cameras on an urban traffic network,* Procedia-Social and Behavioral Sciences, 108 (2014), January, 106-117.

[14] Curtin, K.M., Hayslett-McCall, K. Qiu, F.: *Determining optimal police patrol areas with maximal covering and backup covering location models,* Networks and Spatial Economics, 10 (2010), 1, 125-145.

[15] Murray, A.T.: *Optimising the spatial location of urban fire stations*, Fire Safety Journal, 62 (2013), November, 64-71.

[16] Indriasari, V., Mahmud, A. R., Ahmad, N., Shariff, A. R. M.: *Maximal service area problem for optimal siting of emergency facilities,* International Journal of Geographical Information Science, 24 (2010), 2, 213-230.

[17] Başar, A., Çatay, B., Ünlüyurt, T: *A multi-period double coverage approach for locating the emergency medical service stations in Istanbul,* Journal of the Operational Research Society, 62 (2011), 4, 627-637.

[18] Ratick, S. J., Osleeb, J. P., Hozumi, D: *Application and extension of the Moore and Revelle hierarchical maximal covering model,* Socio-Economic Planning Sciences, 43 (2009), 2, 92-101.

[19] Schilling, D. A.: *Dynamic location modeling for public-sector facilities: A multicriteria approach*,* Decision Sciences, 11 (1980), 4, 714–724.

[20] Gunawardane, G.: *Dynamic versions of set covering type public facility location problems,* European Journal of Operational Research, 10 (1982), 2, 190-195.

[21] Chrissis, J. W., Davis, R. P., Miller, D. M.: *The dynamic set covering problem*, Applied Mathematical Modelling, 6 (1982), 1, 2-6.

[22] Repede, J. F., Bernardo, J. J.: *Developing and validating a decision support system for locating emergency medical vehicles in Louisville, Kentucky.* European Journal of Operational Research, 75 (1994), 3, 567-581.

[23] Antunes, A., Peeters, D.: *A dynamic optimization model for school network planning,* Socio-Economic Planning Sciences, 34 (2000), 2, 101–120.

[24] Gendreau, M., Laporte, G., Semet, F.: *A dynamic model and parallel tabu search heuristic for real-time ambulance relocation*, Parallel computing, 27 (2001), 12, 1641-1653.

[25] Rajagopalan, H. K., Saydam, C., Xiao, J.*: A multiperiod set covering location model for dynamic redeployment of ambulances,* Computers & Operations Research, 35 (2008), 3, 814-826.

[26] Gelareh, S., Neamatian Monemi, R. N., Nickel, S.*: Multi-period hub location problems in transportation,* Transportation Research Part E: Logistics and Transportation Review, 75 (2015), March, 67-94.

[27] Arifin, S. Location allocation problem using genetic algorithm and simulating annealing: A case study based on school in *Enschede*. Master's thesis, *University* of *Twente (2010).*

[28] Yin, P., Mu, L.: *Modular capacitated maximal covering location problem for the optimal siting of emergency vehicles,* Applied Geography, 34 (2012), May, 247-254.

[29] Melo, M. T., Nickel, S., Saldanha-Da-Gama, F.: *Facility location and supply chain management–A review*, European journal of operational research, 196 (2009), 2, 401-412.

[30] Current, J. R., Storbeck, J. E.: *Capacitated covering models,* Environment and planning B: planning and Design, *15* (1988), 2, 153-163.

[31] Church, R. L., Revelle, C. S.: *Theoretical and Computational Links between the p-Median, Location Set-covering, and the Maximal Covering Location Problem,* Geographical Analysis, 8 (1976), 4, 406-415.

[32] Garey, M. R., Johnson, D. S.: *Computers and intractability: a guide to the theory of NP-completeness,* W. H. Freeman and Company, *San Francisco*, 1979.

[33] Sivanandam, S. N., Deepa, S. N.: *Introduction to genetic algorithms*, Springer, 2007.

[34] Liu, W., Zhu, H., Wang, Y., Zhou, S., Bai, Y., Zhao, C.: *Topology optimization of support structure of telescope skin based on bit-matrix representation NSGA-II*, Chinese Journal of Aeronautics, 26 (2013), 6, 1422-1429.

[35] Jaramillo, J. H., Bhadury, J., Batta, R.: *On the use of genetic algorithms to solve location problems*. Computers & Operations Research, 29 (2002), 6, 761-779.

[36] Kumar, R., Pillai, E., Annamalai, K.: *Investigation into optimal fixturing cost of an assembly using genetic algorithm*, Engineering Review, *34* (2014), 2, 85-91.

[37] Barros, M. D. O.: *An experimental evaluation of the importance of randomness in hill climbing searches applied to software engineering problems.* Empirical Software Engineering, 19 (2014), 5, 1423-1465.

[38] Doval, D., Mancoridis, S., & Mitchell, B. S. (1999). *Automatic clustering of software systems using a genetic algorithm*. In Software Technology and Engineering Practice, *1999. STEP'99. IEEE Proceedings* (pp. 73-81).

[39] Antoniol, G., Di Penta, M., Harman, M..: *Search-based techniques applied to optimization of project planning for a massive maintenance project*, Proceedings of the 21st IEEE International Conference on Software Maintenance, (2005), 240–249.

[40] Li, Z., Harman, M., Hierons, R. M.: *Search algorithms for regression test case prioritization*, Software Engineering, 33(2007), 4, 225-237.

[41] Praditwong, K., Harman, M., Yao, X.: Software module clustering as a multi-objective search problem. *IEEE Transactions on Software Engineering, 37* (2011), 2, 264-282.

[42] Revelle, C., Scholssberg, M., Williams, J.: *Solving the maximal covering location problem with heuristic concentration,* Computers & Operations Research, 35 (2008), 2, 427–435.

[43] Zoraghi, N., Najafi, A. A., Akhavan Niaki, S. T. A.: *An integrated model of project scheduling and material ordering: A hybrid simulated annealing and genetic algorithm,* Journal of Optimization in Industrial Engineering, 5 (2012), 10, 19–27.

[44] Naderi, B., Zandieh, M., Khaleghi Ghoshe Balagh, A., Roshanaei, V.: *An improved*

simulated annealing for hybrid flow shops with sequence-dependent setup and transportation times to minimize total completion time and total tardiness. Expert systems with Applications, 36 (2009), 6, 9625-9633.

[45] Nobil, A., Kazemi, A., Alinejad, A.: *A two objective model for location-allocation in a supply chain,* The Journal of Mathematics and Computer Science, 4 (2012), 3, 392-401.

[46] Mehdizadeh, E., & Afrabandpei, F.: *Design of a mathematical model for logistic network in a multi-stage multi-product supply chain network and developing a meta-heuristic algorithm*, Journal of Optimization in Industrial Engineering, 5 (2012), 10, 35-43.

[47] Pereira, M. A., Coelho, L. C., Lorena, L. A., & De Souza, L. C.: *A hybrid method for the probabilistic maximal covering location–allocation problem,* Computers & Operations Research, 57 (2015), May, 51-59.

[48] Bayat, A.: *Integrating capacitated lot-sizing and lot streaming in flow shop schedules with time varying demand.* Master's thesis, Concordia university (2008).