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Kaiye HU
Rui WANG
Shan MA
Wenyang DUAN
Wenhao XU
Rui DENG

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NUMERICAL MODELLING AND STUDY OF PARAMETRIC ROLLING FOR C11 CONTAINERSHIP IN REGULAR HEAD SEAS USING CONSISTENT STRIP THEORY

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Summary

In this paper, a numerical model was proposed to simulate the parametric rolling of ships in head seas. The method was developed in time-domain based on strip theory, in which a consistent way of estimating the radiation forces was applied using impulse response function method. To take the coupling effect into account, the heave and pitch motions were solved together with the rolling motion. Also, the Froude-Krylov forces and hydrostatic forces were evaluated on the instantaneously wetted surface of the ship, in order to model the time varied restoring rolling moment in waves. Based on the developed numerical model, the parametrically roll motions of C11 containership was simulated. The influence of roll damping was investigated using two different methods, and the numerical results were compared with model tests. The comparative study shows that results obtained by the proposed method generally agree well with experimental data. Discussions and possible improvements of the current numerical model were also presented in this paper, with regard to the numerical deviation between the numerical and experimental results when the wave steepness was larger than 0.04.

Key words: parametric rolling modelling; consistent strip theory in time domain; non-linear ship motions; impulse response function; roll damping modelling; comparative study with model test;

1. Introduction

In a seaway, the parametrically excited roll motion of a ship generated unexpectedly in either following or head sea conditions is quite a dangerous phenomenon due to its occurrence with large rolling amplitudes after long periods of quiescence. Therefore, the quantitative prediction of the parametric roll phenomenon is absolutely essential to ensure the safety of life and property on ships.

Parametric rolling in head seas, resulting from time-varying of restoring rolling moment, is a nonlinear phenomenon. The hull forms with pronounced flare at fore and aft extremities, or a flat transom together with wall sided ship near the amidships are most vulnerable to parametric rolling [1]; the typical hull form is the containership which is prone to parametric rolling both in longitudinal and oblique waves.

The prediction of the parametric roll phenomenon and its impact on ship's intact stability and safety has attracted great scientific interest. Munif et al. [2] had proposed a 6 degree-of-freedom (DOF) nonlinear mathematical model for predicting extreme roll behaviors and capsizing for a ship in astern waves with a moderate forward velocity. Belenky et al. [3] used Large Amplitude Motion Program (LAMP) to simulate the motions of C11 container ship. LAMP is a time domain simulation system based on a 3-D potential flow panel method. In the said program the hydrostatics and Froude-Krylov (F-K) forces are computed over the instantaneous wetted hull surface while the perturbation potential, which includes radiation and diffraction effects, is computed over the mean wetted surface. Ribeiro e Silva et al. [4] provided a simple method to identify whether a ship is susceptible to extreme rolling. Obreja et al. [5] use typical differential coupled equations in time domain for heave, roll and pitch motions to simulate the parametric rolling. They present the simulation results for the ship motions in regular following waves, at zero speed, their results have a good agreement with experimental results. Bulian et.al [6] develop a 1.5 DOF nonlinear mathematical model of the parametric rolling in longitudinal waves with particular regard to the head sea condition. In his model, the roll motion is modelled dynamically using a single degree of freedom, whereas the additional half DOF indicates that the coupling with heave and pitch is taken into account by means of hydrostatic calculations without considering dynamic effects. Neves et.al [7] developed a non-linear mathematical model of higher order for strong parametric resonance of the roll motion of ships in waves. The model is employed to describe the coupled restoring actions up to third order. The third order model can give good results and a better comparison with the experiments than a second order model.

In head seas, prediction of parametric rolling is not so easy because parametric roll is more likely influenced by and coupled with heave and pitch motions [8]. A better numerical model to solve the heave, pitch motions coupled together with parametric roll is quite necessary. To consider the memory effect of hydrodynamic loads, ship motions can be established in the time domain base on the impulse response function method (IRFM). In IRFM method, the simulated hydrodynamic loads on ships are not dependent directly on encounter frequency of waves, and this method is especially applicable when the ship motions are estimated in irregular waves. By the use of the method of IRFM, the heave/pitch/roll motions can be solved together in time domain. The time-varying restoring rolling moment due to the geometrical change of hull forms under wave from heave/pitch motions is considered explicitly as well. By this way, the coupling effect of heave/pitch motions on parametric rolling is taken into account properly.

In the numerical simulations of parametric rolling in waves, the IRFM methods have been widely used. Park et al. [9] simulate the parametric roll in regular waves by two different time-domain methods: the impulse-response-function method and the 3D Rankine panel method. The results show that these two methods are in good agreement with the theoretical approach using the Mathieu equation. Liu et al. [10] adopt a 3D nonlinear time domain numerical simulation method, which is based on the impulse response function concept, is applied to the investigation of parametric rolling of the ITTC-A1 ship. Ahmed et al. [11] employed 4 DOF (sway, heave, roll, pitch) numerical model to investigate parametric rolling in regular waves. The non-linear incident wave and hydrostatic restoring forces/moments are incorporated considering the instantaneous wetted surface while the hydrodynamic forces and moments, including diffraction, are expressed in terms of convolution integrals based on the mean wetted hull surface.

In this paper, the IRMF method was used to solve the parametric rolling in head regular waves. The parametric resonance on the C11 containership sailing in regular head seas is investigated using a partly non-linear numerical model with three degree of freedom (heave, roll and pitch). In the presented numerical model, the incident wave and hydrostatic restoring forces/moments are evaluated under the instantaneous submerged surface below the incident wave surface, while hydrodynamic forces and moments from radiation and diffraction are assumed linear. The Radiation and diffraction forces/moments are expressed in terms of convolution integrals using impulse response function method. The requisite impulse response functions were obtained from Fourier transformation performed on hydrodynamic coefficients and diffraction force components evaluated from STF strip theory in the frequency domain. Here the STF strip theory was developed by Salvesen et al. [12].

The unique feature of this paper is that the consistent formulation of time domain radiation forces developed by using IRFM from transformation of the frequency-domain hydrodynamic coefficients of strip theory has been employed to simulate parametric rolling. The hydrodynamic coefficients from strip theory were used to estimate the IRF (impulse response function), the IRF can also be obtained by the Inverse Fourier Transformation of ship hydrodynamic coefficients. During the transformation of ship, hydrodynamic coefficients from frequency domain to time domain, the basic requirement was that the time-domain hydrodynamic load using the IRF should be equal to those obtained in frequency domain. The hydrodynamic coefficients in the time domain and frequency domain should satisfy the K–K relationships in the forward-speed case, instead of the two-dimensional added-mass and damping coefficients obtained from strip theory satisfying the K–K relationships at zero forward speed only. In this paper, the Cummins's method [13] combined with STF strip theory was used to calculate the ship motion in time domain. In order to keep the radiation forces in time domain have the consistent results with those from frequency domain hydrodynamics, some necessary correction to radiation restoring force coefficients are proposed. In this way, the linear radiation force in the time domain using the IRMF can be made to be consistent with (or equal to) those obtained from the frequency domain within the framework of the strip theory, so this method is called "consistent strip theory" [14,15].

Although some part of the content in this paper, like the motion equations, the use of consistent way of IRFM had been shortly discussed in the paper by Ma et al. [15]. There is some distinct difference between that paper and the current work. The difference includes the following aspects. 1. The numerical prediction accuracy on the parametric rolling in this paper had been significantly improved compared with the paper by Ma et al.[15] due to the correction of numerical rolling damping calculation. 2. The diffraction force was also modelled here using impulse response function methods which make it more feasible for current numerical model to predict the parametric rolling in irregular wave. 3. More discussions and numerical comparison with the model test was presented in this paper and the influence of roll damping modelling on the parametric rolling was also discussed. 4. The consistent treatment of radiation force using IRFM derived in a more rigorous way in the paper Ma et al.[14] had been referred which is different with the method in the paper[15].

2. Mathematical Modelling of Parametric Rolling

2.1 Ship Motion Equations

A right-handed inertial coordinate system *oxyz* fixed with respect to the mean position of the ship was established with *oz* in the vertical upward direction and passing through the center of gravity of the ship and *ox* directed to the bow. The origin 'O' is in the plane of the undisturbed free surface. This coordinate moves with the ship but remains unaffected by its oscillatory motions. In order to express the ship motions and represent the geometry property of the ship, two other coordinate systems were established with the origin c at the center of mass of the ship. One coordinate system was the inertial coordinate system *cxyz* which was parallel with *oxyz* and used to express the ship motions. Another coordinate system was the body-fixed coordinate system $cx_by_bz_b$, which was used to represent the hull geometry and the inertial moment of the ship. The oscillatory displacements in the *cz* direction is the heave η_3. The rotational displacements about the *cx* and *cy* axes are the roll η_4 and the pitch η_5 respectively.

The unrestrained 3-DOF rigid body motions of a vessel with forward speed were considered. The ship motions in time domain were formed as followed:

$$\begin{aligned}
&(M+\mu_{33})\ddot{\eta}_3+b_{33}\dot{\eta}_3+\int_0^t K_{33}(t-\tau)\dot{\eta}_3(\tau)d\tau+c_{33}\eta_3+\mu_{35}\ddot{\eta}_5+b_{35}\dot{\eta}_5+\\
&\qquad\int_0^t K_{35}(t-\tau)\dot{\eta}_5(\tau)d\tau+c_{35}\eta_5=F_3^I+F_3^S+F_3^D-Mg\\
&(I_{44}+\mu_{44})\ddot{\eta}_4+b_{44}\dot{\eta}_4+\int_0^t K_{44}(t-\tau)\dot{\eta}_4(\tau)d\tau+c_{44}\eta_4=F_4^I+F_4^S+F_4^D\\
&\mu_{53}\ddot{\eta}_3+b_{53}\dot{\eta}_3+\int_0^t K_{53}(t-\tau)\dot{\eta}_3(\tau)d\tau+c_{53}\eta_3+(I_{55}+\mu_{55})\ddot{\eta}_5+b_{55}\dot{\eta}_5+\\
&\qquad\int_0^t K_{55}(t-\tau)\dot{\eta}_5(\tau)d\tau+c_{55}\eta_5=F_5^I+F_5^S+F_5^D
\end{aligned} \tag{1}$$

where M is the mass of the ship; I_{44}, I_{55} are the inertial moments of the ship about the cx_b and cy_b axes. Radiation forces/moments were expressed by convolution integrals, accounting for the memory effect. Diffraction forces/moments F_3^D , F_4^D , F_5^D were also expressed by convolution integrals, which will be shown in section 2.3. Both radiation and diffraction forces/moments were represented on *cxyz* coordinate system. The incident wave excitations F_3^I , F_4^I , F_5^I and restoring forces/moments F_3^S, F_4^S, F_5^S were initially obtained in the coordinate system *cxyz* and then expressed in the coordinate system $cx_by_bz_b$ via appropriate transformations between the two axis systems [16].

2.2 Consistent model of Radiation forces and moments using IRFM

According to Cummins's theory [13], the linear radiation forces in time domain can be written as follows:

$$F_{jk}(t)=-\mu_{jk}\ddot{\eta}_k(t)-b_{jk}\dot{\eta}_k(t)-c_{jk}\eta_k(t)-\int_0^t K_{jk}(t-\tau)\dot{\eta}_k(\tau)d\tau \tag{2}$$

where, $\eta_k(t)$ represents the oscillation motion in k -mode and the overdot represents the derivative with respect to time. K_{jk} is the impulse response function (IRF) representing the memory effect of fluid, μ_{jk} and b_{jk} are the asymptotic values of the added mass and damping coefficients in frequency domain at high frequency, and c_{jk} is the radiation restoring force coefficient. The IRF K_{jk} can be directly related to the frequency-domain hydrodynamic coefficient:

$$K_{jk}(\tau) = \frac{2}{\pi}\int_0^\infty (B_{jk}(\omega) - b_{jk})\cos\omega\tau d\omega \tag{3a}$$

$$K_{jk}(\tau) = \frac{2}{\pi}\int_0^\infty \left(\omega\mu_{jk} - \omega A_{jk}(\omega) - \frac{1}{\omega}c_{jk}\right)\sin\omega\tau d\omega \tag{3b}$$

where ω is the encounter frequency of ship in waves.

Eq. (3) means that the hydrodynamic impulse response function K_{jk} can be expressed using frequency domain hydrodynamic coefficients without solving the problem directly in time domain. In the present paper, strip theory was used to calculate the hydrodynamic coefficients and estimate the IRF K_{jk}. It is commonly known that this theory is not fully rigorous to solve the hydrodynamics of ship with forward speed in the frequency domain. If the hydrodynamic coefficients obtained by STF were directly used to calculate the IRF based on Eq. (3a) or Eq. (3b), some of the numerical approximation to K_{jk} had already been introduced. If the approximated K_{jk} was used to calculate the hydrodynamic forces due to ship motions, the hydrodynamics given by Eq. (2) could fail to produce the equal (consistent) numerical results with those from frequency-domain strip theory. In order to keep the radiation forces in time domain have the consistent results with those from frequency domain hydrodynamics, some necessary theoretical analysis and numerical method was needed to calculate the hydrodynamic coefficients μ_{jk}, b_{jk} and c_{jk}. In fact, the related theoretical study by us on getting the consistent time domain radiation forces using the IRFM from frequency domain hydrodynamic coefficients had been performed for several years. In the paper by Ma et. al. [15], the proper representation of the radiation restoring coefficients had been briefly introduced. After that, the thoroughly theoretical analysis work are presented in Ma et. al. [14], which was the further development of that work. This paper will not enter into the details on how to keep the consistency between time domain results with those from frequency domain.

2.3 Diffraction forces and moments

Similarly, the diffraction forces/moments contribution can also be represented using convolution integrals [17]. The wave diffraction impulse response function can be expressed as:

$$h_j(\tau) = \frac{1}{\pi}\int_0^\infty \left\{\Xi_j^R(\omega)\cos(\omega\tau) - \Xi_j^I(\omega)\sin(\omega\tau)\right\} d\omega \tag{4}$$

and the diffraction force/moment is:

$$F_j^D(t) = \int_{-\infty}^t h_j(\tau)\,\zeta(\mathrm{t}-\tau)d\tau \qquad j = 3,4,5 \tag{5}$$

where, ω was the encounter frequency of ship in waves, $\Xi_j(\mathrm{i}\omega)$ was the frequency domain complex wave diffraction fore/moment for unit wave amplitude and $\zeta(\mathrm{t})$ was the wave elevation. In the present method, the strip theory in the frequency domain was employed to calculate the diffraction force transfer function $\Xi_j(\mathrm{i}\omega)$.

2.4 Froude–Krylov and restoring forces/moments

In order to capture the ship rolling restoring moment variation in wave, the main cause for parametric rolling, the restoring force/moment should be accurately modelled accounting for the exact ship geometry and the position on waves at each time step.

Traditionally in strip theory, the fluid loads from F-K and hydrostatic restoring forces can be obtained by integration of two dimensional sectional fluid loads along the longitudinal directions. The two-dimensional hydrostatic forces and moments calculations were made using the pressure integration technique along each segment (Cx) of each transverse section of the ship hull. Ribeiro e Silva et al.[1] used this method to predict the contribution of Froude–Krylov plus hydrostatic force/moments contribution in the numerical simulation of parametric rolling.

In the present method, in order to adapt to the complex geometry variation over the instantaneous wetted hull surface, the non-linear Froude–Krylov fluid loads and hydrostatic restoring forces $F_j^I + F_j^S$ are determined by integration of the incident wave pressure and hydrostatic pressure over the actual submerged part of the hulls (t) as shown in the following Eq. (6).

$$F_j^I + F_j^S = \iint_{s(\mathrm{t})} (\mathrm{p}_I + \mathrm{p}_s) n_j ds = \iint_{s(\mathrm{t})} \rho \mathrm{g}(\zeta_a \cos(\omega_e \mathrm{t} + \mathrm{kx}) \mathrm{e}^{k(z-\zeta(\mathrm{t}))} - \mathrm{z}) n_j ds \qquad j = 3,4,5 \qquad (6)$$

In Eq. (6), the linear incident wave pressure in infinite water depth had been estimated using the Wheeler stretching method over the instantaneous wetted hull surface under the incident wave free surface. The $n_j, j = 1,2...6$ represents the generalized normal vector for the ship hull surface. Among them n_1, n_2, n_3 were the component of the normal vector $\vec{n}$ of the instantaneous hull surface in the coordinate system *cxyz*, n_4, n_5, n_6 were the components of the vector $\vec{r}_c \times \vec{n}$ and $\vec{r}_c$ was the vector of the hull surface in the coordinate system *cxyz*.

During numerical calculation, the entire surface of the ship hull (up to deck line) was discretized with quadrilateral or triangular panels. At each time step, panels which were above the instantaneous incident wave free surface were directly ignored while panels below the instantaneous free surface were taken into account. In particular, panels which were crossing the free surface were subdivided and the smaller panels were newly formed, as shown in Fig. 2. The pressure acting on each panel was assumed uniform and equal to that acting at the centroid of the panel. At each time step, the total ship rolling moment was given by summing up contributions of all the accounted panels.

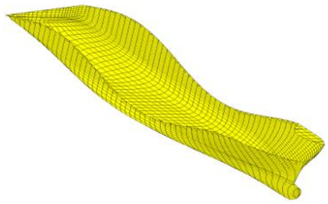

Fig. 1 The instantaneous wetted ship surface under incident wave profile.

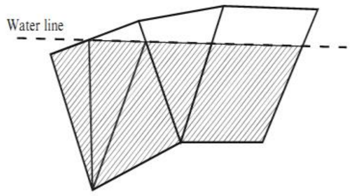

Fig. 2 Panels subdivision across the incident wave free surface.

2.5 Ship roll damping modelling

In general, the viscous effect contributes quite a large amount to the roll damping, and it cannot be calculated using the traditional potential flow theory. Besides, another factor responsible for the difficulty in predicting the roll damping was its strong dependence on forward speed effect of the vessel. In this paper, in order to evaluate the effect of the roll damping modelling on the ship parametric rolling, two different methods to predict roll damping were employed, i.e. model tests, simplified Ikeda's formula [18]. The Ikeda's simplified formula [18] was obtained from the regression analyses of original Ikeda's method [19].

The advantage of the Ikeda's simplified formula was that only the ship principal particulars and the bilge keel dimensions were needed for roll damping estimation. Recommended by IMO, during the vulnerable assessment of ships to the parametric rolling stability failure mode, the Ikeda's simplified formula was adopted to predict the roll damping in absence of experimental data [20].

2.5.1 The roll damping estimation from free decay rolling model test

In this study, a series of free-decay experiment with different forward speed were conducted on a scaled model of C11 containership. The extinction curve method was employed to estimate the roll damping coefficients from the measured free decay rolling. This method was based on the principle of conservation of energy that for a half cycle the change of the total energy in roll motion equals to the energy dissipated by the roll damping due to radiated waves and viscous effects.

For large amplitude roll motion, the extinction curve was fitted using a third-degree polynomial [21]:

$$\Delta\eta = a\eta_m + c\eta_m^3 \tag{7}$$

where a and c are extinction coefficients; $\Delta\eta$ and η_m denote the decrement and mean value of the adjacent roll amplitude in a half cycle, respectively. The roll angles used in this process were in degree.

Corresponding to Eq. (7), the nonlinear rolling damping B_η was expressed as a series expansion of $\dot{\eta}_4$ in the form:

$$B_\eta = -(B_{\eta_1}\dot{\eta}_4 + B_{\eta_3}\dot{\eta}_4^3) \tag{8}$$

The linear roll damping coefficient B_{η_1} and cubic roll damping coefficient B_{η_3} in the above equation were considered constant during a periodic oscillation. According to the energy balance principle, the relationship between the damping coefficients and the extinction coefficients was shown as followed [22]:

$$a = \frac{\pi}{2}\frac{\omega_\eta}{D\overline{GM}}B_{\eta_1}$$
$$c\left(\frac{180}{\pi}\right)^2 = \frac{3\pi}{8}\frac{\omega_\eta^3}{D\overline{GM}}B_{\eta_3} \tag{9}$$

where D is the ship displacement in Newton and $\overline{GM}$ is the transverse metacenter height of the ship.

2.5.2 The roll damping estimation from simplified Ikeda's formula

Following the Ikeda's simplified formula [18], it is known that the estimated equivalent roll damping coefficient varies with the roll amplitude.

Corresponding to Eq. (8), the equivalent damping coefficient can be defined as follows [22]:

$$B_{\eta_E} = B_{\eta_1} + \frac{3}{4}\omega_\eta^2\eta_a^2 B_{\eta_3} \tag{10}$$

Therefore, the equivalent linear roll damping coefficient B_{η_E} with different roll amplitude η_a was estimated firstly, then the least-square fitting method based on Eq. (10) had been adopted to obtain the roll damping coefficients B_{η_1} and B_{η_3} in this paper, the fitting procedure can be referred in [23]. During the fitting procedure, a series of roll amplitudes up to 30 degrees and relevant equivalent roll damping coefficients B_{η_E} had been applied.

During the modelling of nonlinear roll damping, the following roll equation were considered, where the predicted damping term expressed as in Eq. (8) was directly added to the left side of the roll equation:

$$(I_{44} + \mu_{44})\ddot{\eta}_4 + b_{44}\dot{\eta}_4 + B_{\eta_1}\dot{\eta}_4 + B_{\eta_3}\dot{\eta}_4^3 + \int_0^t K_{44}(t-\tau)\dot{\eta}_4(\tau)d\tau + c_{44}\eta_4 = F_4^I + F_4^S + F_4^D \tag{11}$$

where, B_{η_1} is the linear damping coefficient, and B_{η_3} is the cubic damping coefficient. They were determined by methods discussed above.

3. The parametric rolling model test for C11 containership

The free running experiment with a 1/65.5 scaled model of the post Panamax C11 containership in regular waves was conducted at the seakeeping basin of China Ship Science Research Center (CSSRC) in Wuxi China, in which the ship model was propelled by one propeller whose revolution was controlled to keep the same mean forward speed with the carriage [24]. The principal particulars and the line plan of the C11 class containership were shown in Table 1 and Fig. 3 respectively.

Table 1 Main particulars of the C11 class containership

Principal particular	Value
Length between perpendiculars (L_{pp})	262.0 m
Breadth (B)	40.0 m
Mean draught (T)	11.8 m
Block coefficient (C_b)	0.566
Pitch radius of gyration (I_{yy})	$0.24L_{pp}$
Longitudinal position of center of gravity from amidship (x_{CG})	5.768 m aft
Transverse metacentric height in calm water ($\overline{GM}$)	1.9 m
Natural roll period (T)	24.5 s

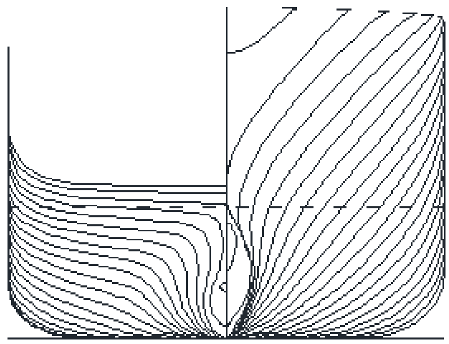

Fig. 3 Lines plan for the C11 containership

The test carried out covering a range of Froude numbers of 0.0, 0.05, 0.1, 0.15 and a range of wave steepness varying from 0.01 to 0.07. The wavelength was equal to ship length between perpendiculars.

4. Numerical results and discussions

4.1 The numerical results of radiation and diffraction forces

In this paper, the radiation forces of ship motion were calculated using Eq. (2). A consistent method to predict the radiation fluid loads had been applied. In Ma et al. [14-15], the comparisons of hydrodynamic loads and ship motions had been made between time domain and frequency domain, and it was shown that the consistent method applied here was theoretical consistent with that from strip theory in frequency domain. It can be concluded that the consistent method should be applied if the impulse response function method was used to predict hydrodynamic loads in the numerical simulation of parametric rolling.

To validate the approach of using impulse response functions to determine diffraction forces/moments, comparisons of vertical diffraction force and pitch diffraction moment due to harmonic incident wave of unit amplitude are shown in Fig 4 and Fig 5. The results were from time domain analysis using Eq. (5) and frequency domain analysis using Eq. (12) in the strip theory of Salvesen et al. [12] respectively.

$$F_j^D(\mathrm{t}) = \mathrm{Re}\left\{\Xi_j(\mathrm{i}\omega)e^{i\omega t}\right\} \qquad j = 3,5 \tag{12}$$

Results in Fig 4 and Fig 5 show excellent agreement, it was thus shown that the way of estimating the diffraction force using impulse response function had been dealt with properly.

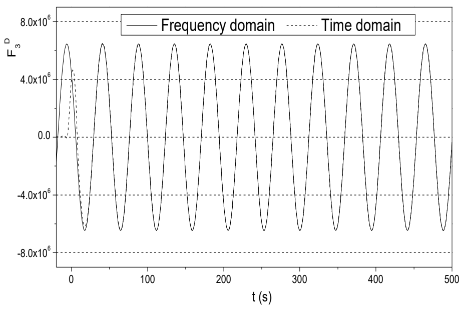

Fig.4 Comparison of heave diffraction force between frequency domain and time domain analysis, Fr=0.1, $\omega_e = 0.14$

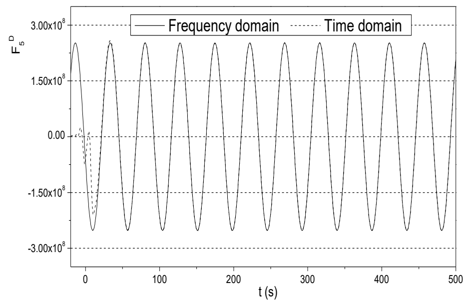

Fig.5 Comparison of pitch diffraction moment between frequency domain and time domain analysis, Fr=0.1, $\omega_e = 0.14$

4.2 The numerical comparative study of roll damping modelling

It was widely acknowledged that the roll damping had evident influence on the final amplitude of parametric rolling. Before conducting the simulation of parametric rolling, it was of great importance to calibrate the numerical modelling of roll damping. The roll damping calibration was performed by comparing the time history of the numerical free-decay rolling with that from the experiment.

In this paper, using the available roll damping from model test and Ikeda's simplified formula, the influence of estimation methods had been investigated. The methods used to predict the roll damping introduced shortly here. "Method 1" in the following results means that the roll damping were obtained from free decay rolling in model test based on extinction curve method; " Method 2" denotes the roll damping obtained by the Ikeda's simplified formula[18] where the lift roll damping component was neglected presently.

Table 2 Comparison between roll damping coefficients

Fn.	**Damping coefficients**	**Method 1 (Model test)**	**Method 2 (Ikeda's Simplified formula)**
0.0	B_{η_1}	1.623E+8	1.021E+8
	B_{η_3}	1.066E+11	1.110E+11
	B_{η_E}	1.164E+9	1.144E+9
0.05	B_{η_1}	2.211E+8	1.021E+8
	B_{η_3}	1.012E+11	1.110E+11
	B_{η_E}	1.17E+9	1.144E+9
0.1	B_{η_1}	5.440E+8	1.021E+8
	B_{η_3}	6.504E+10	1.110E+11
	B_{η_E}	1.155E+9	1.144E+9
0.15	B_{η_1}	8.573E+8	1.021E+8
	B_{η_3}	7.241E+10	1.110E+11
	B_{η_E}	1.537E+9	1.144E+9

The comparison of the free decay rolling of the C11 class containership between the experimental data and simulation results were illustrated in Fig. 6 and Fig. 7. Particularly, the simulation results were obtained by using different roll damping coefficients estimated by the methods mentioned before. As it can be observed, at zero-forward-speed cases, the Ikeda’s simplified formula slightly underestimate the roll damping and results in relatively larger roll motion. At forward speed case with Fn=0.05, as the rolling amplitude decays to smaller after three cycles, the predicted roll amplitude by the Ikeda’s simplified formula was evidently higher than the experimental data. If we take a look at the numerical difference of linear and cubic damping coefficients between those from method 1 and method 2, the reason can be understood. The linear roll damping coefficient played a leading role when the roll amplitude was smaller while the cubic roll damping coefficient will take the dominant part when the roll amplitude was relatively larger. Because the linear damping coefficient obtained by method 2 was smaller than method 1, the corresponding amplitude decreases relatively slower after a few cycles. From the comparisons of free roll decay between numerical and experimental data, it was also seen that the damping coefficients estimated from free roll decay model test had been well calibrated and can be used for the following parametric rolling prediction in waves. The influence of roll damping estimation will be further shown in the predicted steady parametric roll amplitudes.

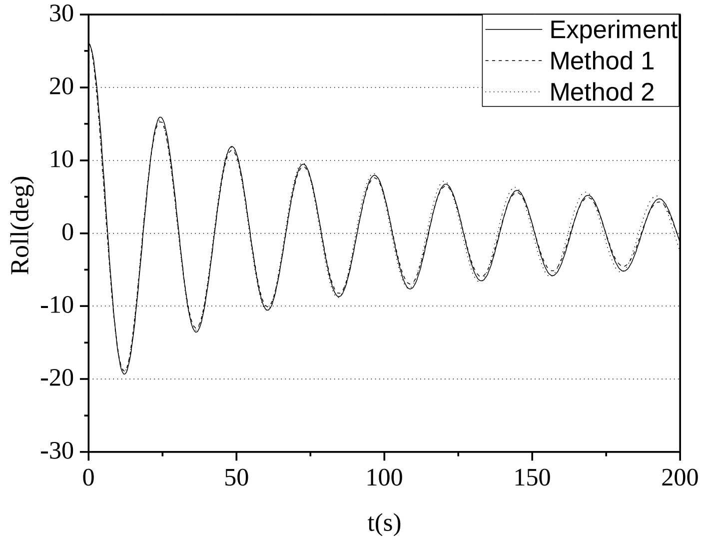

Fig. 6 Comparisons of roll decay curves between the experimental data and simulation results using different roll damping prediction method, Fn=0.0.

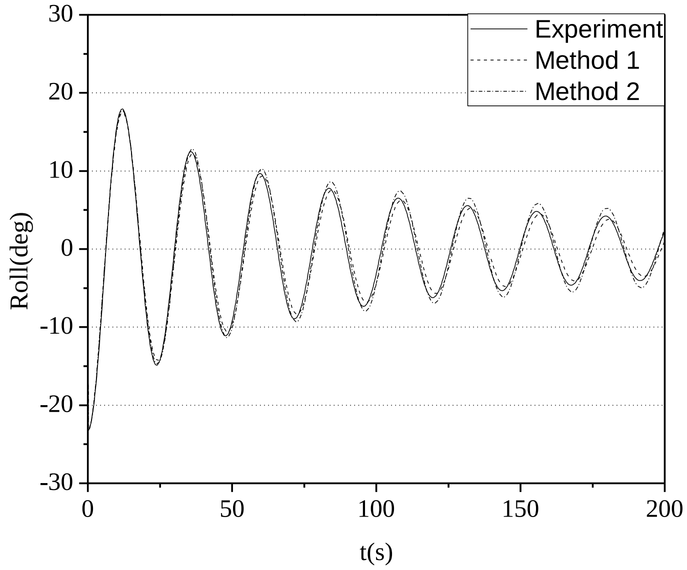

Fig. 7 Comparisons of roll decay curves between the experimental data and simulation results using different roll damping prediction method, Fn=0.05.

4.3 The simulations of parametric rolling and comparison with model test

Based on the numerical model for parametric rolling prediction, the time-domain simulation of vessel's motions was carried out based on Eq. (1). The three DOF motions, i.e., heave, pitch and roll are solved using the fourth order Runge-Kutta method.

In order to validate the present numerical model, both measured and simulated parametric rolling results of the ship in regular head-waves are demonstrated in Figs. 8-11 respectively. The associated pitch motions were also illustrated. In the model test, four different forward speed with Froude number Fn=0.0, 0.05, 0.1, 0.15 were performed. In Figs.

8-11, the incident wave height H_w 7.86 m (wave steepness 0.03) and wave period T_w 12.95 s were used. For the simulated results, roll damping used here were obtained from experimental data.

It can be observed from the results that the presented numerical model successfully predicts the occurrences of parametric roll. In Figs 8-11, the relatively numerical difference for the average roll amplitude was about 3.7%, 3.5%, 4.1%, 7.8% respectively, which presents quite well agreement with the model test. With regard to the pitch motions, the relatively numerical difference for the average pitch amplitude was about 27%, 17%, 17.6%, 13.7% respectively, whose numerical differences were larger than those of steady parametric rolling amplitude.

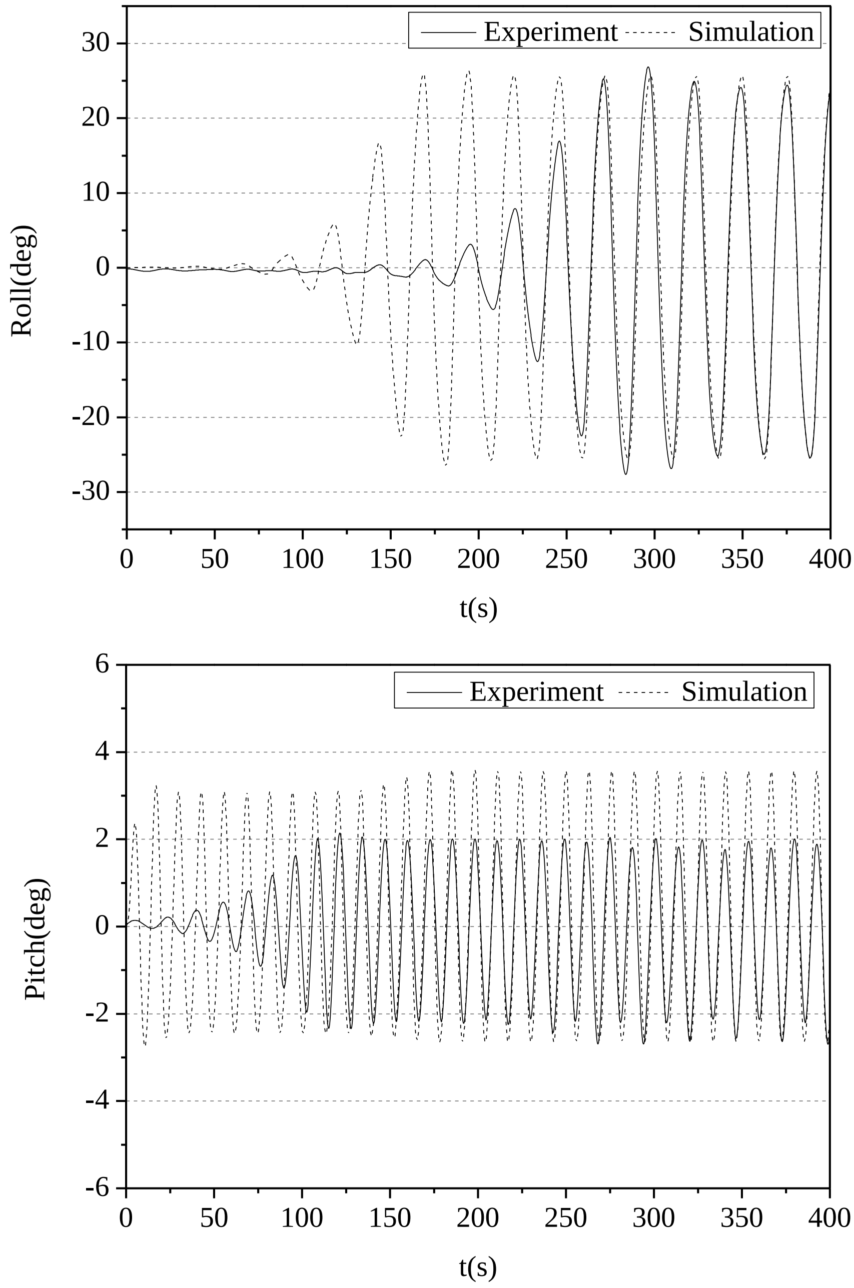

Fig.8 Comparison of time history between experimental measurements and numerical simulations, Fn=0.0, H_w=7.86m

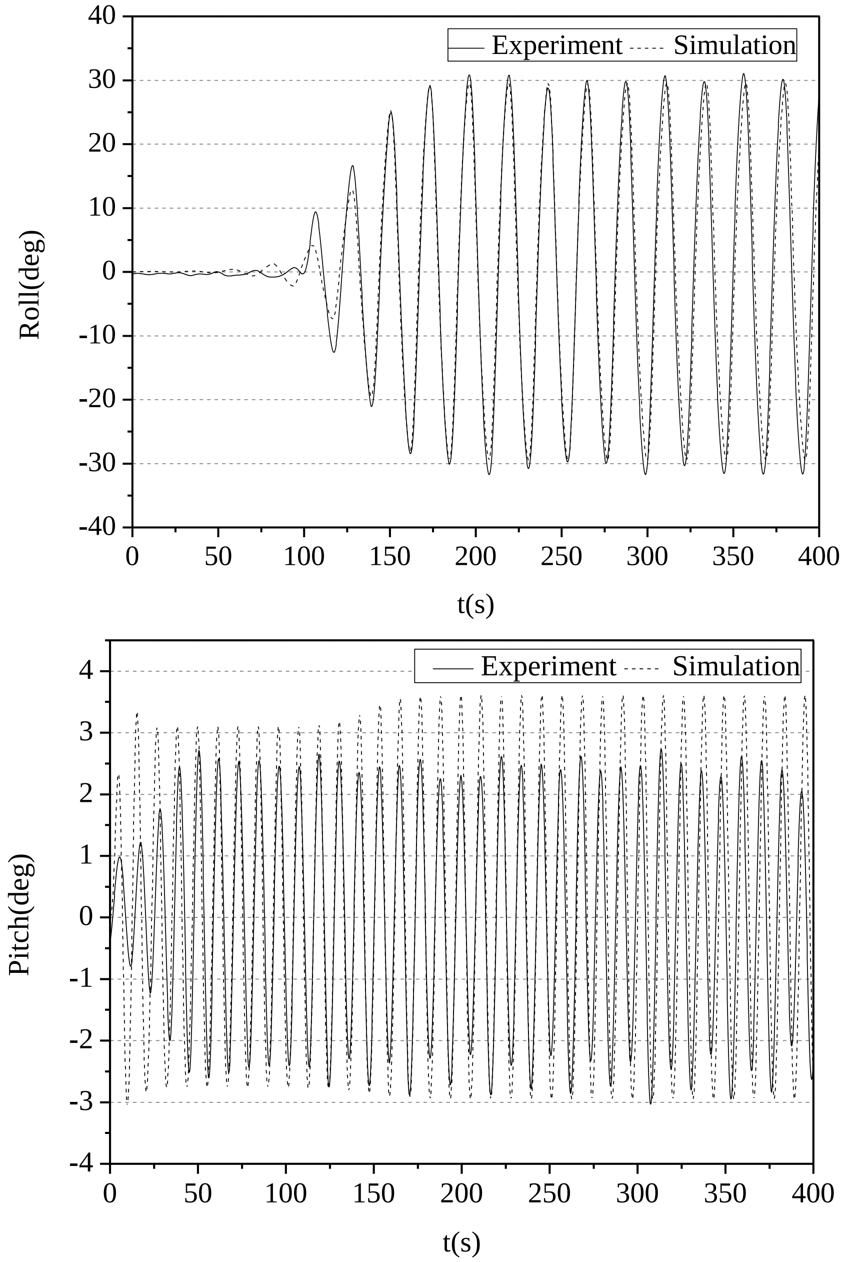

Fig.9 Comparison of time history between experimental measurements and numerical simulations, Fn=0.05, H_w=7.86m

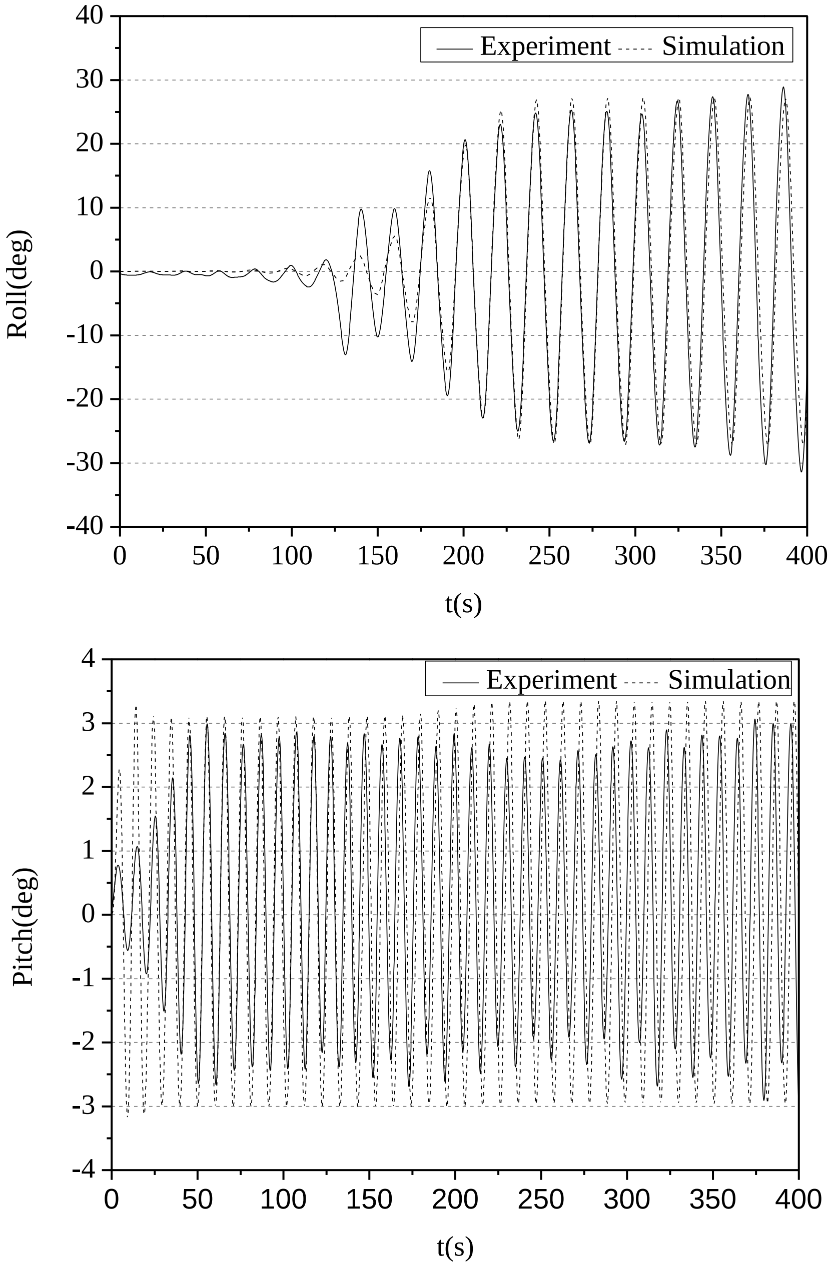

Fig.10 Comparison of time history between experimental measurements and numerical simulations, Fn=0.1, H_w=7.86m

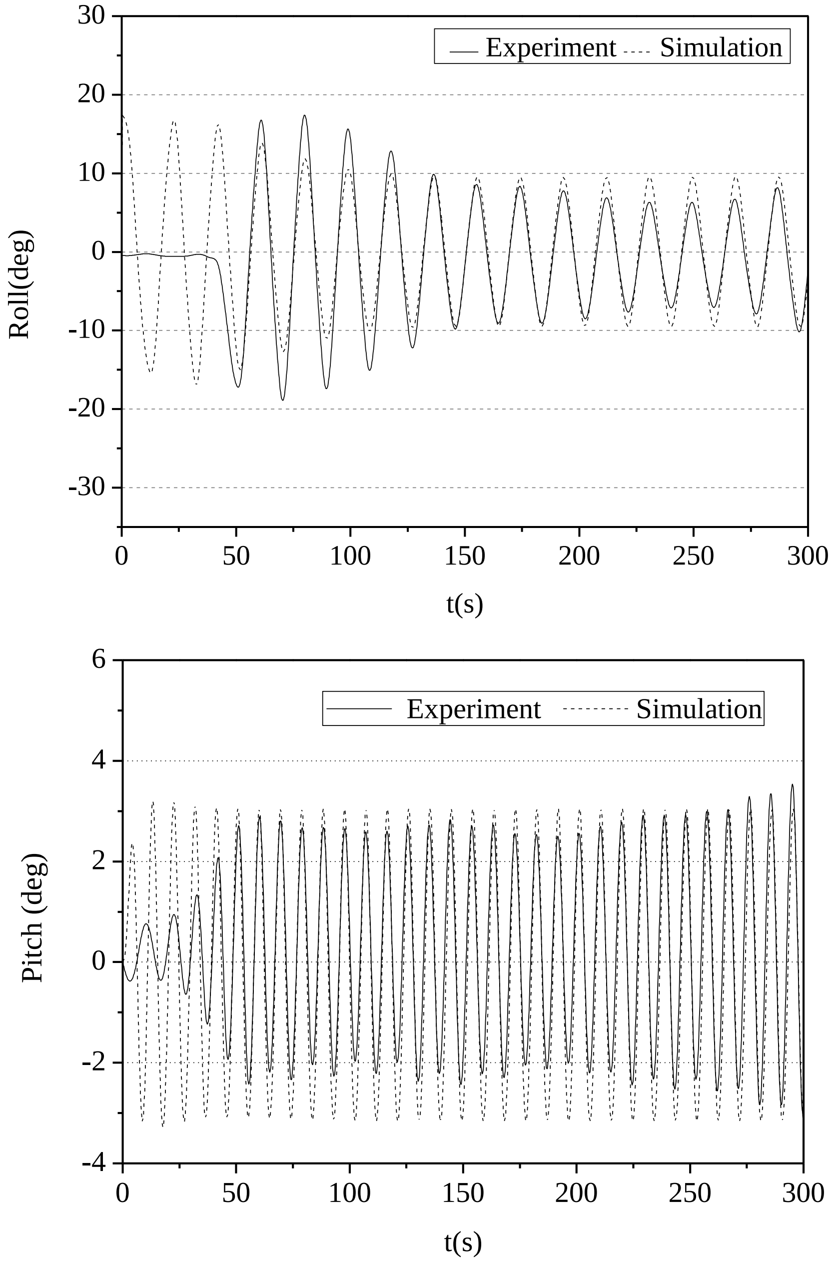

Fig.11 Comparison of time history between experimental measurements and numerical simulations, Fn=0.15, H_w=7.86m

A direct comparison of steady roll amplitudes between the numerical and experiment results of the C11 containership at four forward speeds in regular head wave are provided in Figs. 12-13. In the comparisons, the incident wave length was equal to ship length and regular wave steepness were from 0.01 to 0.04. As observed, the numerical code generally presents quite good predictions on the steady amplitude of parametric rolling. There was relatively smaller difference between the numerical results using the roll damping from Methods 1 and 2.

For the present case from model test, with Froude numbers of 0.0, 0.05, 0.1, 0.15 and wave length L_w = L_{pp}, the encounter frequency was 0.485 rad/s, 0.546 rad/s, 0.607rad/s and

0.667 rad/s, respectively, which was equal or close to twice the natural roll frequency of 0.26 rad/s. At each wave steepness, the steady roll amplitude was generally larger with the Froude number 0.05 whose corresponding encounter frequency was closer to twice of the natural roll frequency. In addition, only when the incident wave heights exceed certain value the parametric rolling will occur, like the scenarios for Froude numbers 0.1 and 0.2. In general, the presented numerical model was capable of presenting prediction which was qualitatively and quantitatively consistent with those from model test.

In Fig. 14, more comprehensive influence of the wave steepness on the parametric roll amplitude is presented for Fn=0.05. The wave steepness varied from 0.01 to 0.07. Both the experimental data and numerical results shows that the reduction of the amplitude of parametric roll beyond certain wave steepness. This phenomenon was possible due to the "nonlinear stiffness" or "hardening" effect which had been discussed by Neves and Rodriguez [25, 26]. The nonlinear stiffness was a non-oscillatory term in the restoring moment, which was proportional to wave amplitude squared, and this term can be treated as an additional (nonlinear) stiffness of the dynamic system. The nonlinear stiffness comes from the coupling of roll, pitch, heave and also the wave excitation force. In some cases, when the incident wave amplitude increases, the limits of unstable region bends to right which was not as one expected that the unstable region will increase when the incident wave amplitude increase. That implies that at a given frequency, above a certain level of wave amplitude, the increase in restoring moment rigidity causes the decrease in the amplitude of parametric excitation.

In the current numerical model, the nonlinear variation of parametric excitation on the ship was taken into account properly due to the incident wave and coupled effect from heave and pitch motions . Therefore the numerical results show the decrease of the parametric roll amplitude beyond some certain amplitude.

From the comparison in Fig. 14, we can see that when the wave steepness was larger than 0.04, the model test shows the steady roll amplitude reached more than 30 degrees. The numerical results underestimate the experimental data. It was possibly related to the numerical uncertainty from both roll damping and restoring force variation, contribution from the nonlinear vertical radiation and diffraction forces/moments.

In Fig. 14, the roll damping coefficients was estimated from experimental free roll decay data. In the numerical model, the roll damping coefficients were obtained from free decay data corresponding to maximum 25 roll degrees. These damping coefficients were used to predict the parametric rolling amplitude at higher roll amplitude. The roll damping coefficients were possibly overestimated for large roll amplitude which cause the amplitude of PR was smaller than experimental data.

Another reason was possibly from the dynamic coupling effect between heave/pitch and roll motions. At such large rolling angle, there should be some coupling effects for the radiation forces between heave/pitch and roll motions, because the instantaneous ship cross sections were in the form of asymmetric relative to the *oxz* plane induced by the large rolling motions. The nonlinear dynamic radiation and diffraction forces due to heave and pitch motion at such large rolling angle should produce the rolling restoring force variation and influence on the final predicted magnitude of parametric rolling. Actually Gu et al. [24], Bu. et al [27] have pointed the influence of nonlinear radiation and diffraction forces from heave/pitch motion on restoring force and parametric roll motions. Their study was performed by experimental data and numerical model using the extended nonlinear strip theory. The present numerical model estimates the hydrodynamic forces in a linear way which cannot reflect such influences.

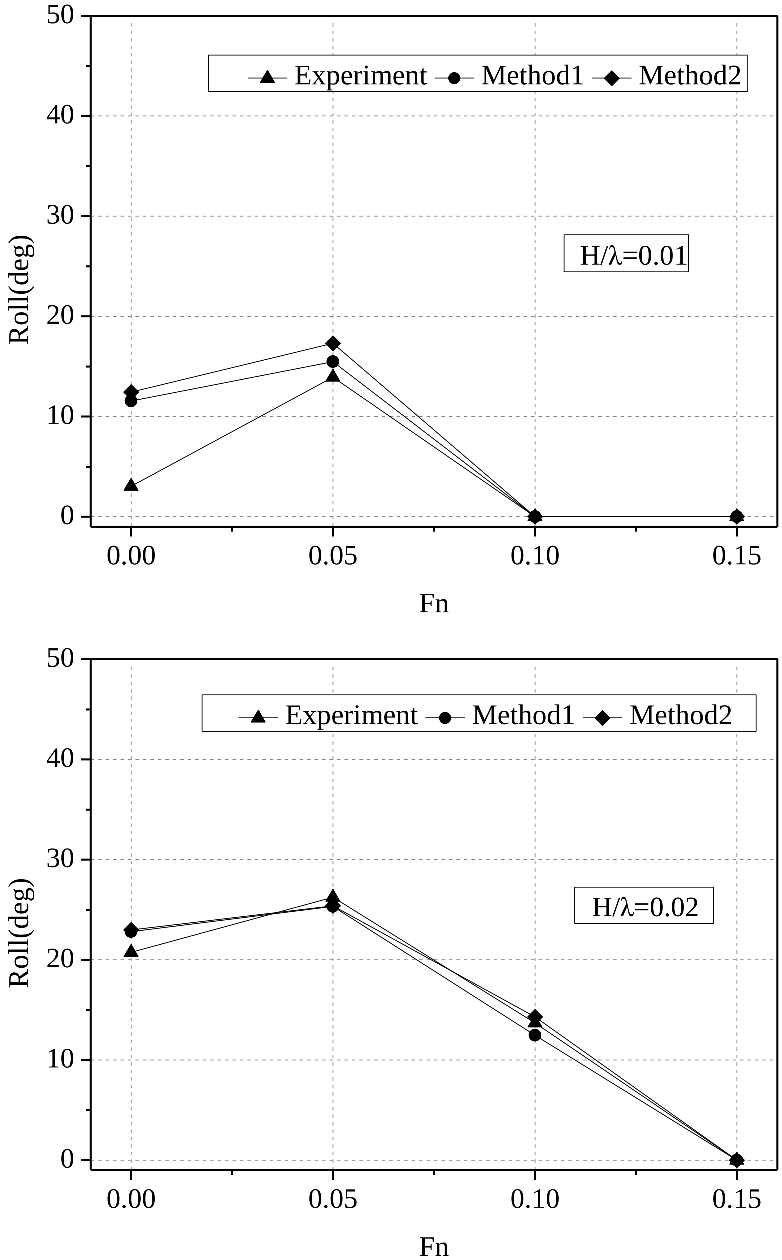

Fig. 12 Comparison of steady roll amplitudes between experimental measurements and numerical simulations using different roll damping prediction methods for the incident wave steepness H/ λ =0.01, 0.02.

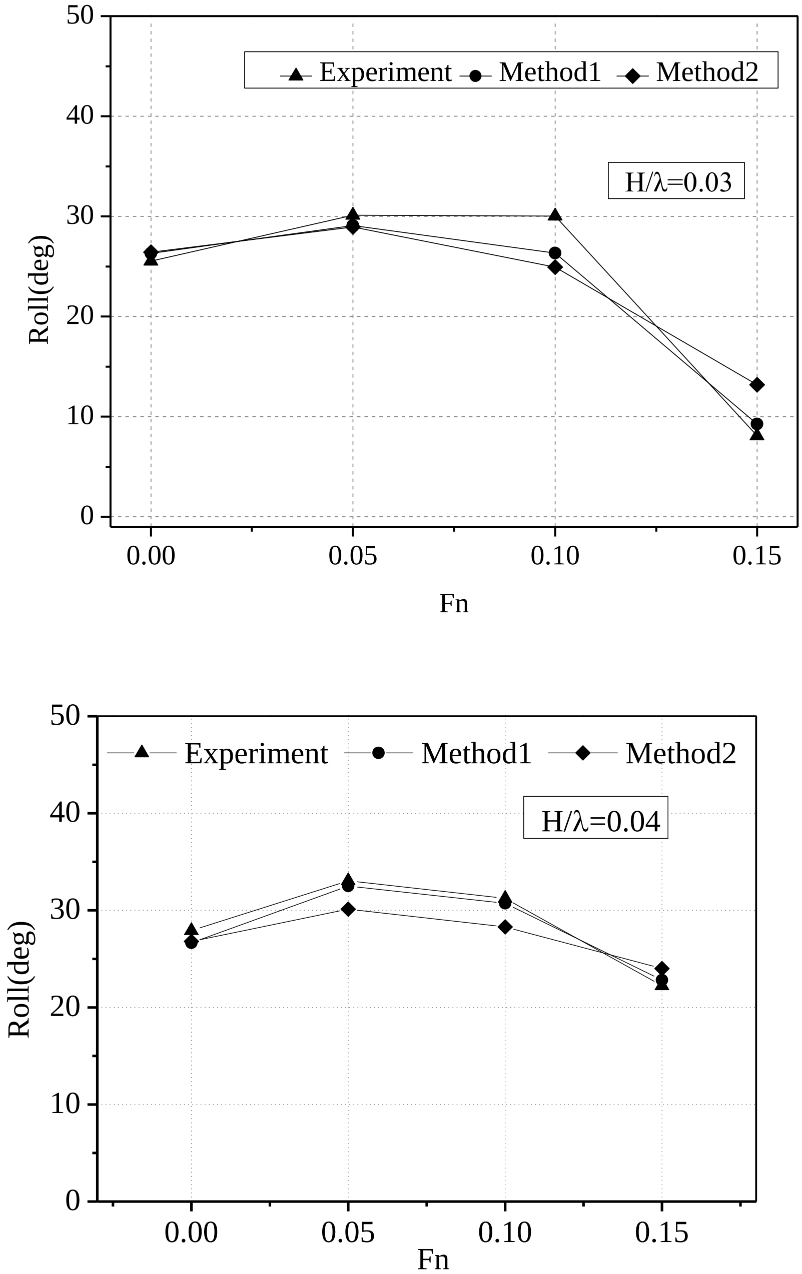

Fig.13 Comparison of steady roll amplitudes between experimental measurements and numerical simulations using different roll damping prediction methods for the incident wave steepness H/ λ =0.03,0.04.

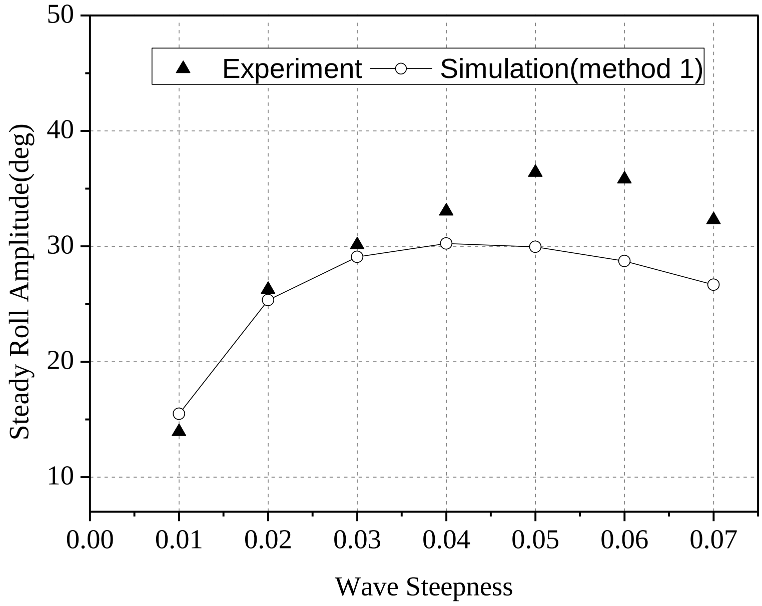

Fig.14 Illustration of the nonlinear stiffness/hardening effect for the C11 Containership in head waves, Fn=0.05

5. Conclusion

In this paper, a partially non-linear time-domain numerical model was presented and utilized to simulate parametric excited rolling resonance in regular head waves. In the present numerical model, the impulse response function method is used to model the time-domain radiation and diffraction forces which was obtained from the strip theory. Via theoretical analysis, a consistent transformation for the radiation force from frequency-domain to time-domain has been applied.

Results obtained for C11 class containership demonstrate that the proposed model succeeds in obtaining the steady roll angles of parametric rolling and agrees well with experimental data, both qualitatively and quantitatively. So far as the present simulated results, the numerical model can produce quite good predictions of parametric rolling of C11 Containership when the wave steepness is less than 0.03 and the roll amplitude is less than about 30 degrees. Otherwise the numerical model deviates from the model test and needs further developments.

To predict the total roll damping accurately, it is crucial for any methods to simulate parametric rolling resonance. The Ikeda’s simplified formula has proved to be a good method for predicting relatively reasonable roll damping coefficient, which can be used as a good estimation in the absence of experimental data.

Further developments are needed to improve the capability of the presented model which includes: considering the nonlinearities of vertical radiation and diffraction forces/moments and advanced numerical modelling or model test study of ship roll damping at large amplitude roll motions.

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Kaiye HU
Rui WANG
Shan MA, *mashan01@hrbeu.edu.cn*
Wenyang DUAN
Wenhao XU
Rui DENG
College of Shipbuilding Engineering , Harbin Engineering University
No.145 Nantong Street, Nangang District, Harbin, Heilongjiang 150001, P.R. China