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THE STRATEGY OF FINANCIAL INVESTMENTS IN SECURITIES

The paper presents the financial analysis which has to be performed by every successful financial investor while creation of the optimal portfolio of securities, with the aim to minimize or diverse financial risks. According to the limitless number of possible portfolio of securities available to investors, this paper concerns only portfolio of securities which are a part of effective series of securities.

Key words: financial investment, investor, diversification of risk, optimal portfolio of securities.

INTRODUCTION

To make strategic investment decision from the field of financial investments in securities means to choose optimal financial possibility between available alternatives. The estimate of relative value of every available alternative needs valuation of actual conditions of business activity, about which there's usually not enough knowledge, and the estimate of future events, which include several degrees of risk and uncertainties, especially in the field of financial consequences.

Risk presents the variation of average expected value, which comes by chance. The bigger the variation, the bigger the risk.

From the fundamental financial motive, which is profit for the firm, respectively net income per share, of course with as much as possible less risk, less uncertainty and in shorter period as much as possible, comes out the two main financial principles. The first principle demand the maximum use of capital, which must be addressed and controlled to give maximum income per unit of capital invested. The second principle is close related to the first, and demand that the value of net income per every unit of income earned needs also to be maximised.

The question now, to the financial managers is how much, where and how to invest their capital? Thus, we must make investment decision about which shares to buy, which portfolio to choose. To give answers to these questions investors must analyse the movement in the financial markets and alternative possibilities. Of course that, except for the objective circumstances that the markets gives together with the existing alternatives of current prices, the investors subjective preferences can also have big impact.

In this paper, which is based upon the fundamental analysis of financial investment in securities, we will closely demonstrate the main questions and their alternative answers, so that the adequate financial investment decision can be made. After the decision, investing investor's funds follows for the purpose of acquisition bigger income per unit of invested assets, that is, bigger income with the minimum expenses.

1. THE EFFICIENT SET THEOREM

As mentioned earlier, an infinite number of portfolios can be formed from a set of N securities. Consider the situation with company A, B and C where N is equal to 3. The investor could purchase just shares of company A, or just shares of company B. Alternatively, the investor could purchase a combination of shares of company A and B. For example, the investor could put 50% of his or her money in each company, or 25% in one company and 75% in the other, or 33% in one and 67% in the other, or any percent (between 0% and 100%) in one company with the rest going into the other company. Without even considering investing in company C, there are already an infinite number of possible portfolios that could be purchased.[1 2]

Does the investor need to evaluate all these portfolios? Fortunately, the answer to this question is "no." The key to why the investor needs to look at only a subset of the available portfolios lies in the efficient set theorem, which states that: An investor will choose his or her optimal portfolio from the set of portfolios that offer maximum expected return for varying levels of risk and offer minimum risk for varying levels of expected return.

The set of portfolios meeting these two conditions is known as the efficient set or efficient frontier.[3]

1.1 The Feasible Set

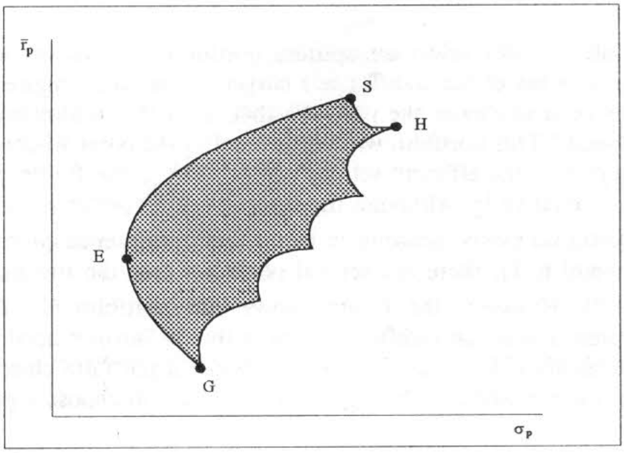
Figure 1 provides an illustration of the location of the feasible set, also known as the opportunity set, from which the efficient set can be identified.

The feasible set simply represents all portfolios that could be formed from a group of N securities. That is, all possible portfolios that could be formed from the Nsecurities lie either on or within the boundary of the feasible set (the points denoted G, E, S, and Hin the figure are examples of such portfolios). In general, this set will have an umbrella-type shape similar to the one shown in the figure. Depending on the

[1] See, Ivanka Avelini Holjevac, *Kontroling – Upravljanje poslovnim rezultatom*, Hotelijerski fakultet Opatija, 1998.
[2] T.J. Callagher and Andrew, J.D. Jr., *Financial Management – Principles and Practice*, Prentice Hall, Inc., New Jersey, 1997, p. 137.
[3] See, Hanke, J.E. and Reitsch, A.G., *Understanding Business Statistics*, Irwin, Homewood, Il., Boston, 1991.

particular securities involved, it may be more to the right or left, or higher or lower, or fatter or skinnier than indicated here. The point is that its shape will, except in perverse circumstances, look similar to what appears here.

Figure 1.: Feasible and Efficient Sets



1.2 The Efficient Set Theorem Applied to the Feasible Set

The efficient set can now be located by applying the efficient set theorem to this feasible set. First, the set of portfolios that meet the first condition of the efficient set theorem must be identified. Looking at Figure 1, there is no portfolio offering less risk than that of portfolio E. This is because if a vertical line were drawn through E, there would be no point in the feasible set that was to the left of the line. Also, there is no portfolio offering more risk than that of portfolio H. This is because if a vertical line were drawn through H, there would be no point in the feasible set to the right of the line. Thus the set of portfolios offering maximum expected return for varying levels of risk is the set of portfolios lying on the "northern" boundary of the feasible set between points E and H.

Considering the second condition next, there is no portfolio offering an expected return greater than portfolio S, because no point in the feasible set lies above a horizontal line going through S. Similarly, there is no portfolio offering a lower expected return than portfolio G, because no point in the feasible set lies below a horizontal line going through G. Thus the set of portfolios offering minimum risk for varying levels of expected return is the set of portfolios lying on the "western" boundary of the feasible set between points G and S.

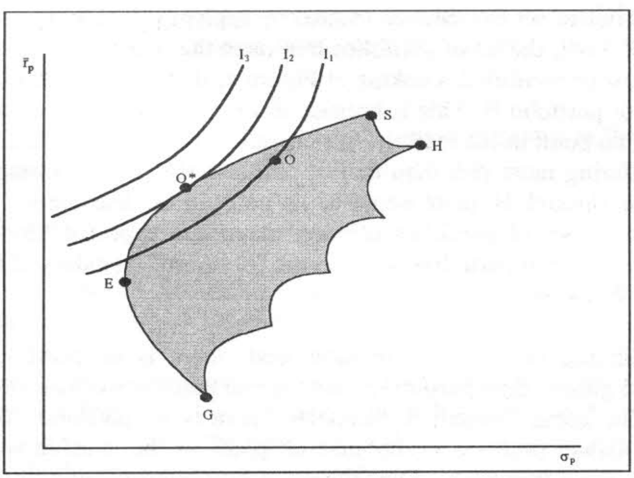
Remembering that both conditions have to be met in order to identify the efficient set, it can be seen that only those portfolios lying on the "northwest" boundary between points E and S do so.

Accordingly, these portfolios form the efficient set, and it is from this set of efficient portfolios that the investor will find his or her optimal one. All the other feasible portfolios are inefficient portfolios and can be safely ignored.

1.3. Selection of the Optimal Portfolio

How the investor select an optimal portfolio? As shown in Figure 2, the investor should plot his or her indifference curves on the same figure as the efficient set and then proceed to choose the portfolio that is on the indifference curve that is "furthest northwest." This portfolio will correspond to the point where an indifference curve isjust tangent to the efficient set. As can be seen in the figure, this is portfolio O* on indifference curve I_2. Although the investor would prefer a portfolio on I_3, no such feasible portfolio exists; wanting to be on this indifference curve is just wishful thinking. In regard to I_1, there are several portfolios that the investor could choose (for example, 0). However, the figure shows that portfolio O* dominates such portfolios, because it is on an indifference curve that is "further northwest." Figure 3 shows how the highly riskaverse investor will choose a portfolio close to E. Figure 4 shows that the investor who is only slightly risk-averse will choose a portfolio close to S.[4]

Figure 2.: Selecting an Optimal Portfolio



[4] The risk-neutral investor will choose portfolio S, while the risk-seeking investor will choose either S or H.

Figure 3.: Portfolio Selection for a Highly Risk-Averse Investor

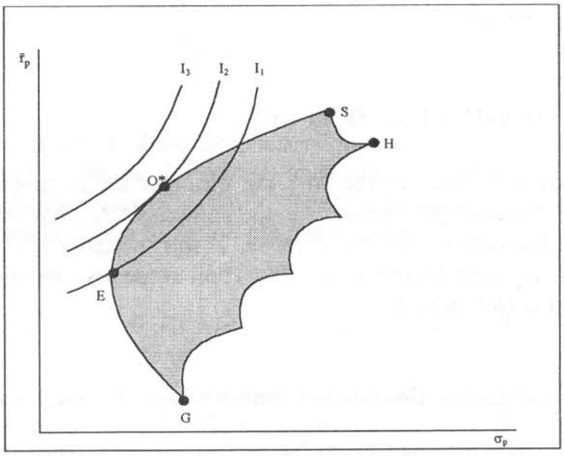
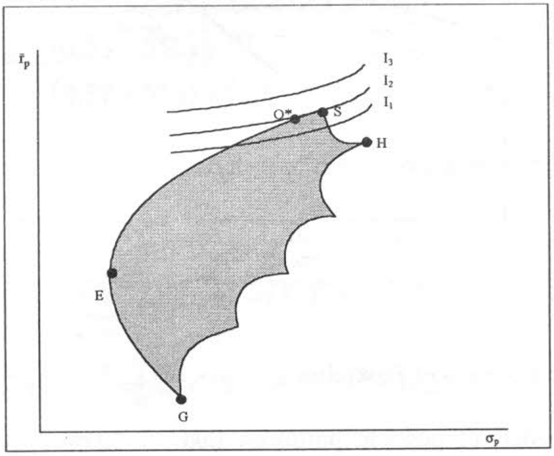


Figure 4.: Portfolio Selection for a Slightly Risk-Averse Investor



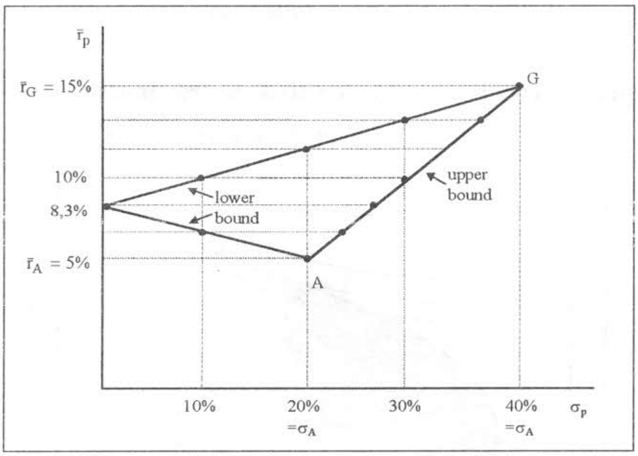
Upon reflection, the efficient set theorem is quite rational. Investor should select the portfolio that put him or her on the indifference curve "furthest northwest." The efficient set theorem, stating that the investor need not be concerned with portfolios that do not lie on the "northwest" boundary of the feasible set, is a logical consequence.

Efficient set is generally positively sloped and concave, meaning that if a straight line is drawn between any two points on the efficient set, the straight line will lie below the efficient set. This feature of the efficient set is important because it means that there will be only one tangency point between the investor's indifference curves and the efficient set.

2. CONCAVITY OF THE EFFICIENT SET

In scientifically order to see why the efficient set is concave, consider the following two-security example. Security 1, the company "A", has an expected return of 5% and standard deviation of 20%. Security 2, the company "G", has an expected return of 15% and standard deviation of 40%. Their respective locations are indicated by the letters A and G in Figure 5.

Figure 5.: Upper and Lower Bounds to Combinations of Securities A and G



2.1 Bounds on the Location of Portfolios

Now consider all possible portfolios that an investor could purchase by combining these two securities. Let X1 denote the proportion of the investor's funds invested in company "A" and X2 (= 1 – X1) the proportion invested in company "G". Thus, if the investor purchased just company "A", then X1 = 1 and X2 = 0. Alternatively, if the investor purchased just company "G", then X1 = 0 and X2 = 1. A combination of .1 7 in company "A" and .83 in company "G" is also possible, as are the respective combinations of .33 and .67, and .50 and .50. While there are many other possibilities, only the following seven portfolios will be considered:

	Portfolio A	Portfolio B	Portfolio C	Portfolio D	Portfolio E	Portfolio F	Portfolio G
X_1	1,00	0,83	0,67	0,50	0,33	0,17	0,00
X_2	0,00	0,17	0,33	0,50	0,67	0,83	1,00

In order to consider these seven portfolios for possible investment, their expected returns and standard deviations must be calculated. All the necessary information to calculate the expected returns for these portfolios is at hand because all that is needed in order to utilize Equation (1) has been provided:

$$\bar{r}_p = \sum_{i=1}^{N} X_i \bar{r}_i = \sum_{i=1}^{2} X_i \bar{r}_i = X_i \bar{r}_1 + X_2 \bar{r}_2 = (X_1 \times 5\%) + (X_2 \times 15\%). \quad (1)$$

for portfolios A and G, this calculation is trivial, because the investor is purchasing shares ofjust one company. Thus their expected returns are 5% and 15%, respectively. For portfolios B, C, D, E, and F, the expected returns are, respectively:

$$\bar{r}_B = (0{,}83 \times 5\%) + (0{,}17 \times 15\%) = 6{,}79\%$$
$$\bar{r}_C = (0{,}67 \times 5\%) + (0{,}33 \times 15\%) = 10{,}00\%$$
$$\bar{r}_D = (0{,}50 \times 5\%) + (0{,}50 \times 15\%) = 10{,}00\%$$
$$\bar{r}_E = (0{,}33 \times 5\%) + (0{,}67 \times 15\%) = 11{,}70\%$$
$$\bar{r}_F = (0{,}17 \times 5\%) + (0{,}83 \times 15\%) = 13{,}30\%.$$

In calculating the standard deviation of these seven portfolios, Equation (2) must be utilized:

$$\sigma_p = \left[\sum_{i=1}^{N}\sum_{j=1}^{N} X_i X_j \sigma_{ij}\right]^{1/2} = \left[\sum_{i=1}^{2}\sum_{j=1}^{2} X_i X_j \sigma_{ij}\right]^{1/2} = \left[X_1 X_1 \sigma_{11} + X_1 X_2 \sigma_{12} + X_2 X_1 \sigma_{21} + X_2 X_2 \sigma_{22}\right]^{1/2}$$
$$= \left[X_1^2 \sigma_2^2 + X_2^2 \sigma_2^2 + 2X_1 X_2 \sigma_{12}\right]^{1/2} = \left[(X_1^2 \times 20\%^2) + (X_2^2 \times 40\%^2) + 2X_1 X_2 \sigma_{12}\right]^{1/2}. \quad (2)$$

Again, for portfolios A and G, this calculation is trivial, because the investor is purchasing shares of just one company. Thus their standard deviations are 20% and 40%, respectively.

For portfolios B, C, D, E, and F, application of Equation (2) indicates that the standard deviations depend on the magnitude of the covariance between the two securities. As shown in Equation (3), this covariance term is equal to the correlation between the two securities multiplied by the product of their standard deviations:

$$\sigma_{ij} = \rho_{ij} \times \sigma_i \times \sigma_j \tag{3}$$

so letting i = 1 and i = 2,

$$\sigma_{12} = \rho_{12} \times \sigma_1 \times \sigma_2 = \rho_{12} \times 20\% \times 40\% = 0.800\rho_{12}.$$

This means that the standard deviation of any portfolio consisting of company "A" and company "G" can be expressed as:

$$\begin{aligned}\sigma_{12} &= \left[\left(X_1^2 \times 20\%^2\right) + \left(X_2^2 \times 40\%^2\right) + \left(2X_1X_2 \times 800\rho_{12}\right)\right]^{1/2} \\ &= \left[400X_1^2 + 1{,}600X_2^2 + 1{,}600X_1X_2\rho_{12}\right]^{1/2}.\end{aligned} \tag{4}$$

Consider portfolio D first. The standard deviation of this portfolio will be somewhere between 10% and 30%, the exact value depending upon the size of the correlation coefficient. How were these bounds of 10% and 30% determined? First, note that for portfolio D, Equation (4) reduces to:

$$\sigma_D = \left[(400 \times 0{,}25) + (1{,}600 \times 0{,}25) + (1{,}600 \times 0{,}5 \times 0{,}5\rho_{12})\right]^{1/2} = \left[500 \times 400\rho_{12}\right]^{1/2}. \tag{5}$$

Inspection of Equation (5) indicates that σ_D will be at a minimum when the correlation coefficient, ρ_{12}, is at a minimum. Now remembering that the minimum value for any correlation coefficient is - 1, it can be seen that the lower bound on σ_D is:

$$\sigma_D = \left[500 + (400 \times -1)\right]^{1/2} = 10\%.$$

Similarly, inspection of Equation (5) indicates that σ_D will be at a maximum when the correlation coefficient is at a maximum, which is + 1. Thus the upper bound on σ_D is:

$$\sigma_D = \left[500 + (400 \times 1)\right]^{1/2} = 30\%.$$

In general, it can be seen from Equation (4) that for any given set of weights X, and X2, the lower and upper bounds will occur when the correlation between the two securities is - 1 and + 1, respectively. Proceeding to apply the same analysis to the other portfolios reveals that their lower and upper bounds are:

Table 1.: Standard Deviation of Portfolio

Portfolio	Lower Bound %	Upper Bound %
A	20,00	20,00
B	10,00	23,33
C	0,00	26,67
D	10,00	30,00
E	20,00	33,33
F	30,00	36,67
G	40,00	40,00

These values are shown in Figure 5.

Interestingly, the upper bounds all lie on a straight line connecting points A and G. This means that any portfolio consisting of these two securities cannot have a standard deviation that plots to the right of a straight line connecting the two securities. Instead, the standard deviation must lie on or to the left of the straight line. This observation suggests a motivation for diversifying a portfolio.[5] Namely, diversification generally leads to risk reduction, because the standard deviation of a portfolio will generally be less than a weighted average of the standard deviations of the securities in the portfolio.

Also interesting is the observation that the lower bounds all lie on one of two line segments that go from point A to a point on the vertical axis corresponding to 8.30% and then to point G. This means that any portfolio consisting of these two securities cannot have a standard deviation that plots to the left of either of these two line segments. For example, portfolio B must lie on the horizontal line going through the vertical axis at 6.70%, but bounded at the values of 10% and 23.33%.

In sum, any portfolio consisting of these two securities will lie within or on the boundary of the triangle shown in Figure 5, with its actual location depending on the magnitude of the correlation coefficient between the two securities.

2.2 Actual Locations of the Portfolios

What if the correlation were zero? In this case, Equation (4) reduces to:

$$\sigma\rho = \left[\left(400X_1^2\right) + \left(1.600X_2^2\right) + \left(1.600X_1X_2 \times 0\right)\right]^{1/2} = \left[400X_1^2 + 1.600X_2^2\right]^{1/2}.$$

[5] J.C. Francis, *Investments – Analysis and Management*, Fifth Edition, McGraw-Hill, New York, 1991, pp. 228-256.

Applying the appropriate weights for X, and X2, the standard deviation for portfolios B, C, D, E, and F can therefore be calculated as follows:

$$\sigma_B = \left[\left(400 \times 0{,}83^2\right) + \left(1.600 \times 0{,}17^2\right)\right]^{1/2} = 17{,}94\%$$
$$\sigma_C = \left[\left(400 \times 0{,}67^2\right) + \left(1.600 \times 0{,}33^2\right)\right]^{1/2} = 18{,}81\%$$
$$\sigma_D = \left[\left(400 \times 0{,}50^2\right) + \left(1.600 \times 0{,}50^2\right)\right]^{1/2} = 22{,}36\%$$
$$\sigma_E = \left[\left(400 \times 0{,}33^2\right) + \left(1.600 \times 0{,}67^2\right)\right]^{1/2} = 27{,}60\%$$
$$\sigma_F = \left[\left(400 \times 0{,}17^2\right) + \left(1.600 \times 0{,}83^2\right)\right]^{1/2} = 33{,}37\%.$$

Figure 6.: Portfolios Formed by Combining Securities A and G

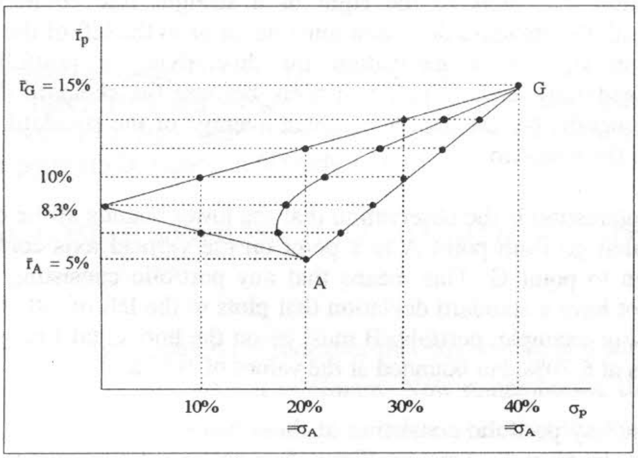


Figure 6 indicates the location of these portfolios, along with the upper and lower bounds that were shown in Figure 5. As can be seen, these portfolios, as well as all other possible portfolios consisting of company "A" and company "G", lie on a line that is curved or bowed to the left. Mile not shown here, if the correlation were less than zero, the line would curve more to the left. If the correlation were greater than zero, it would not curve quite as much to the left. The important point about this figure is that as long as the correlation is less than + 1 and greater than - 1, the line representing the set of portfolios consisting of various combinations of the two securities will have some degree of curvature to the left. Furthermore, the "northwest" portion will be concave.

Similar analysis can be applied to a situation where there are more than two securities under consideration. When this is done, as long as the correlations are less than + 1 and greater than - 1, the "northwest" portion must be concave, just as it is in the two-security example.[6] Thus in general the efficient set will be concave.

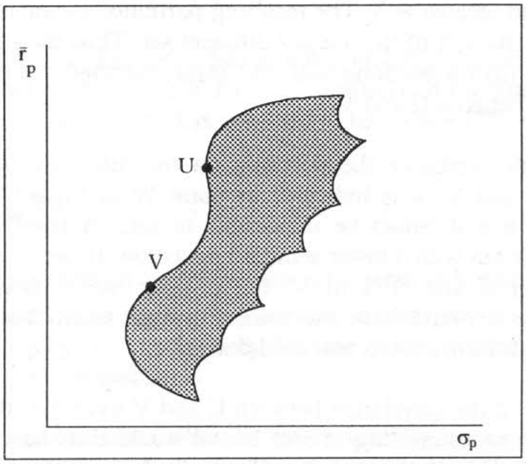
2.3 Impossibility of Having -Dents" in the Efficient Set

The previous example indicated what happens when two securities (such as company A and company G) are combined to form a portfolio. It is important to recognize that the same principles hold if two portfolios are combined to form a third portfolio. That is, point A in Figure 6 could represent a portfolio of securities with an expected return of 5% and a standard deviation of 20%, and point G could represent another portfolio of securities with an expected return of 15% and a standard deviation of 40%.

Combining these two portfolios will result in a third portfolio that has an expected return and standard deviation dependent on the proportions invested in A and G. Assuming that the correlation between A and G is zero, the location of the third portfolio will lie on the curved line connecting A and G.

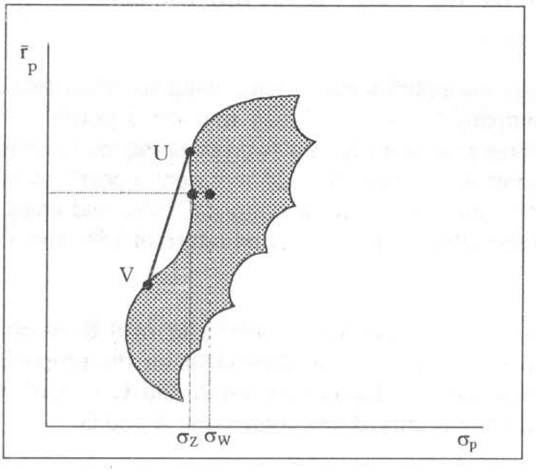
Recognizing this, it can now be shown why the efficient set is concave. One way to do this is to show that it cannot have any other shape.

Figure 7.: Concavity of the Efficient Set



[6] This "curvature property" can also be used to explain why the right-hand side of the feasible set has the umbrella shape noted in Figure 2.

Figure 8.: Removing a "Dent" from The Efficient Set



Consider the efficient set shown in Figure 7. Note that there is a "dent" in it between points U and V That is, between U and V there is a region on the efficient set where it is not concave. Can this truly be an efficient set? No, because an investor could put part of his or her funds in the portfolio located at U and the rest of his or her funds in the portfolio located at V The resulting portfolio, a combination of U and V, would have to lie to the left of the alleged efficient set. Thus the new portfolio would be "more efficient" than a portfolio with the same expected return that was on the alleged efficient set between U and V.

For example, consider the portfolio on the alleged efficient set that lies halfway between U and V; it is indicated as point W in Figure 8. If it truly is an efficient portfolio, then it would be impossible to form a portfolio with the same expected return as W but with a lower standard deviation. However, by putting 50% of his or her funds in U and 50% in V, the investor would have a portfolio that dominates W, since it would have the same expected return but a lower standard deviation. Why will it have a lower standard deviation?

Remember, if the correlation between U and V were + 1, this portfolio would lie on the straight line connecting U and V and would thus have a lower standard deviation than W. Z denotes this point in Figure 8. Because the actual correlation is less than or equal to + 1, it would have a standard deviation as low as or lower than Z's standard deviation. This means that the alleged efficient set was constructed in error, because it is easy to find "more efficient" portfolios in the region where it is not concave.

3. THE MARKET MODEL

Suppose that the return on a common stock over a given time period (say, a month) is related to the return over the same period that is earned on a market index. That is, if the market has gone up then it is likely that the stock has gone up, and if the market has gone down then it is likely that the stock has gone down. One way to capture this relationship is with the market model:

$$r_i = \alpha_{iI} + \beta_{iI} r_I + \varepsilon_{iI} \tag{6}$$

r_i = return on security i for some given period,
r_I = return on market index I for the same period,
α_{iI} = intercept term,
β_{iI} = slope term,
ε_{iI} = random error term.

Assuming that the slope term β_{iI}, is positive, Equation (6) indicates that the higher the return on the market index, the higher the return on the security is likely to be (note that the expected value of the random error term is zero).

Consider stock A, for example, which has α_{iI} = 2 % and β_{iI} 8i, = 1. 2. This means that the market model for stock A is:

$$r_A = 2\% + 1{,}2 r_I + \varepsilon_{AI} \tag{7}$$

So that if the market index has a return of 10%, the return on the security is expected to be 14% [= 2% + (1.2 X 10%) 1. Similarly, if the market index's return is - 5%, then the return on security A is expected to be - 4% = [2% + (1.2 X -5%)].

3.1 Random Error Terms

The term ε_{iI} , known as the random error term in Equation (7), simply shows that the market model does not explain security returns perfectly. That is, when the market index goes up by 10% or down by 5%, the return on security A is not going to be exactly 14% or - 4%, respectively.

The difference between what the return actually is and what it is expected to be, given the return on the market index, is attributed to the effect of the random error term. Hence, if the security's return were 9% instead of 14%, the 5% difference would be attributed to the random error term (that is, ε_{iI} = - 5%; this will be illustrated shortly in Figure 11). Similarly, if the security return were - 2% instead of - 4%, the 2% difference would be attributed to the random error term (that is, ε_{iI} = + 2%).

The random error term can be viewed as a random variable that has a probability distribution with a mean of zero and a standard deviation denoted $\sigma_{\varepsilon i}$.[7] That is, it can be viewed as the outcome that results from the spin of a special kind of roulette wheel.

For example, security A might be thought of as having a random error term corresponding to a roulette wheel with integer values on it that range from - 10% to + 10%, with the values evenly spaced.[8] This means that there are 21 possible outcomes, each of which has an equal probability of occurring. Given the range of numbers, it also means that the expected outcome of the random error term is zero:

$$[-10\times 1/21] + [-9\times 1/21] + \ldots + [9\times 1/21] + [10\times 1/21] = 0.$$

As can be seen, this calculation involves multiplying each outcome by its probability of occurring and then summing up the resulting products. The standard deviation of this random error term can now be shown to equal 6.06%:

$$\{[(-10-0)^2\times 1/21]+[(-9-0)^2\times]+\ldots+[(9-0)^2\times 1/21]+[(10-0)^2\times 1/21]\}^{1/2} = 6{,}06\%.$$

This calculation involves subtracting the expected outcome from each possible outcome, then squaring each one of these differences, multiplying each square by the probability of the corresponding outcome occurring, adding the products, and finally taking the square root of the resulting sum.

Figure 9 illustrates the roulette wheel corresponding to this random error term. In general, securities will have random error terms whose corresponding roulette wheels have different ranges and different forms of uneven spacing. While all of them will have an expected value of zero, they will typically have different standard deviations. For example, security B may have a random error term whose expected value and standard deviation are equal to zero and 4.76%.[9]

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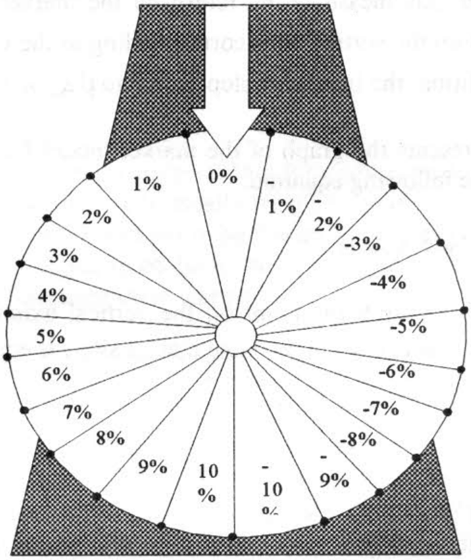
[7] To be technically correct, the standard deviation of the random error term should be denoted $\sigma_{\varepsilon i,}$ because it is measured relative to market index I. The subscript I is not shown here for ease of exposition.

[8] Because the range refers to the possible outcomes and the spacing refers to the probabilities of the various outcomes occurring, it can be seen that the roulette wheel is just a convenient way of referring to the random error term's probability distribution. Typically, it is assumed that a random error term has a normal distribution.

[9] This would be the case if security B had a random error term whose roulette wheel had integers form –9% to +9% on it, but the spacing for each integer between –5% and +5% was twice as large as the spacing for each integer from –9% to –6% and +6% to +9%. This means that the probability of any specific integer between –5% and +5% occurring is equal to 2/30, while the probability of any specific integer from –95 to –6% and +6% to +9% occurring is equal to 1/30.

Figure 9.: Security As Random Error Term

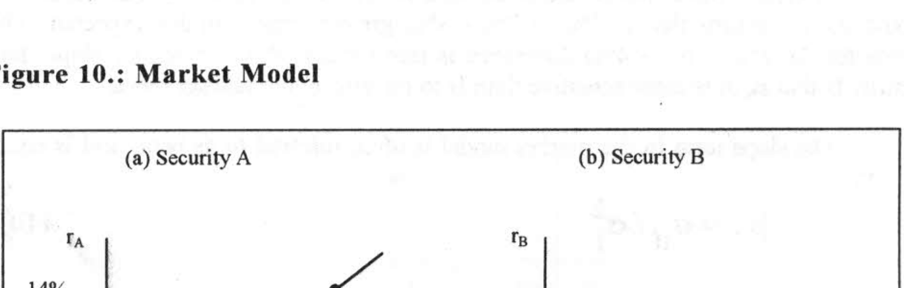


3.2 Graphical Representation of the Market Model

The solid line in panel (a) of Figure 10 provides a graph of the market model for security A. This line corresponds to Equation (7), but without the random error term. Accordingly, the line that is graphed for security A is:

$$r_A = 2\% + 1{,}2\ r_I. \tag{8}$$

Figure 10.: Market Model



Here the vertical axis measures the return on the particular security (r_A) whereas the horizontal axis measures the return on the market index (r_I). The line goes through the point on the vertical axis corresponding to the value of α_{AI} which in this case is 2%. In addition, the line has a slope equal to β_{AI} or 1.2.

Figure 10b presents the graph of the market model for security B. The line can be expressed as the following equation:

$$r_B = -1\% + 0{,}8\ r_I. \tag{9}$$

This line goes through the point on the vertical axis corresponding to the value of α_{BI}, which in this case is - 1 %. Note that its slope is equal to β_{BI} , or .8.

3.3 Beta

At this point it can be seen that the slope in a security's market model measures the sensitivity of the security's returns to the market index's returns. Both lines in Figure 10 have positive slopes, indicating that the higher the returns of the market index, the higher the returns of the two securities. However, the two securities have different slopes, indicating that they have different sensitivities to the returns of the market index. Specifically, A has a higher slope than B, indicating that the returns of A are more sensitive than the returns of B to the returns of the market index.

For example, assume that the market index's expected return is 5%. If the market index subsequently has an actual return of 10%, it will have returned 5% more than expected. Figure 10a shows that security A should have a return that is 6% (= 14% - 8%) greater than initially expected. Similarly, Figure 10b shows that security B should have a return that is 4% (= 7% - 3%) greater than initially expected. The reason for the 2% (= 6% - 4%) difference is that security A has a higher slope than security B-that is, A is more sensitive than B to returns on the market index.

The slope term in the market model is often referred to as beta, and is equal to:

$$\beta_{iI} = \sigma_{iI} / \sigma_I^2 \tag{10}$$

where σ_{iI} , denotes the covariance of the returns on stock i and the market index, and σ_I^2 , denotes the variance of returns on the market index. A stock that has a return that mirrors the return on the market index will have a beta equal to one (and an intercept of zero, resulting in a market model that is $r_i = r_i + \varepsilon_{iI}$). Hence stocks with betas greater than one (such as A) are more volatile than the market index and are known as aggressive stocks. In contrast, stocks with betas less than one (such as B) are less volatile than the market index and are known as defensive stocks.[10]

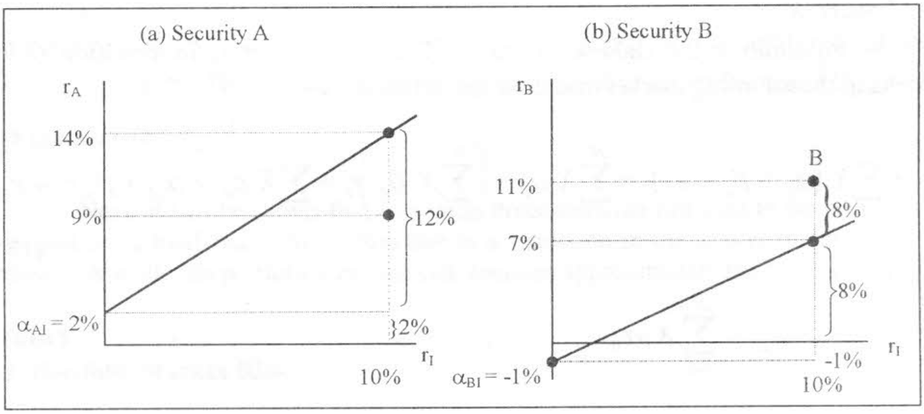
3.4 Actual Returns

The random error term suggests that for a given return on the market index, the actual return on a security will usually lie off its market model line. If the actual returns on securities A and B turn out to be 9% and 11%, respectively, and the market index's actual return turns out to be 10%, then the actual return on A and B could be viewed as having the following three components:

	Security A	Security B
Intercept	2%	1%
Actual return on the market indey x beta	12%=10%×1,2	8%=10%×0,8
Random error outcome	-5%=9%-(2%+12%)	4%=11%-(-1%+8%)
Actual return	9%	11%

In this case, the roulette wheels for A and B can be thought of as having been "spun" resulting in values (that is, random error outcomes) of - 5 % for A and + 4% for B. These values can be viewed as being equal to the vertical distance by which each security's actual return ended up being off its market model line, as shown in Figure 11.

Figure 11.: Market Model and Actual Returns



[10] Zoran Ivanović, Financijski menedžment, drugo izdanje, Hotelijerski fakultet, Opatija, 1997.

4. DIVERSIFICATION

According to the market model, the total risk of any security i, measured by its variance and denoted σ_i^2 consists of two parts: (1) market (or systematic) risk; and (2) unique (or unsystematic) risk. That is, σ_i^2 equals the following:

$$\sigma_i^2 = \beta_{iI}^2 \sigma_I^2 + \sigma_{\varepsilon i}^2 \tag{11}$$

where σ_i^2 denotes the variance of returns on the market index. Thus $\beta_{iI}^2 \sigma_I^2$ denotes the market risk of security i, and $\sigma_{\varepsilon i}^2$ denotes the unique risk of security i as measured by the variance of the random error term, appearing in Equation (6).

4.1 Portfolio Total Risk

When the return on every risky security in a portfolio is related to the return on the market index as specified by the market model, what can be said about the total risk of the portolio? If the proportion of funds invested in security i for a given portfolio p is denoted X_i, then the return on this portfolio will be:

$$r_p = \sum_{i=1}^{N} X_i r_i \tag{12}$$

Substituting the right-hand side of Equation (6) for r_i in Equation (8.9) results in the following market model for the portfolio:

$$r_p = \sum_{i=1}^{N} X_i \left(\alpha_{iI} + \beta_{iI} + \varepsilon_{iI}\right) = \sum_{i=1}^{N} X_i \alpha_{iI} + \left(\sum_{i=1}^{N} X_i \beta_{iI}\right) r_I + \sum_{i=1}^{N} X_i \varepsilon_{iI} = \alpha_{pI} + \beta_{pI} r_I + \varepsilon_{pI} \tag{13}$$

where:

$$\alpha_{pI} = \sum_{i=1}^{N} X_i \alpha_{iI} \tag{13b}$$

$$\beta_{pI} = \sum_{i=1}^{N} X_i \beta_{iI} \tag{13c}$$

$$\varepsilon_{pI} = \sum_{i=1}^{N} X_i \varepsilon_{iI} \tag{13d}$$

In Equations (13b) and (13c), the portfolio's vertical intercept (α_{pI}) and beta (β_{pI}) are shown to be weighted averages of the intercepts and betas of the securities, respectively, using their relative proportions in the portfolio as weights. Similarly, in Equation (13d), the portfolio's random error term (ε_{pI}) is a weighted average of the random error terms of the securities, again using the relative proportions in the portfolio as weights. Thus the portfolio's market model is a straightforward extension of the market model for individual securities given in Equation (6).

From Equation (13a), it follows that the total risk of a portfolio, measured by the variance of the portfolio's returns and denoted σ^2_p will be:

$$\sigma_p^2 = \beta_I^2 \sigma_I^2 + \sigma_{\varepsilon p}^2 \tag{14a}$$

where:

$$\beta_{pI}^2 = \left[\sum_{i=1}^{N} X_i \beta_{iI} \right] \tag{14b}$$

and, assuming the random error components of security returns are uncorrelated:

$$\sigma_{\varepsilon p}^2 = \sum_{i=1}^{N} X_i^2 \sigma_{\varepsilon i}^2 \tag{14c}$$

Equation (14a) shows that the total risk of any portfolio can be viewed as having two components, similar to the two components of the total risk of an individual security. These components are again referred to as market risk $\left(\beta_p^2, \sigma_I^2\right)$ and unique risk $\left(\sigma_{\varepsilon p}^2\right)$.

Next, it will be shown that increased diversification can lead to the reduction of a portfolio's total risk. This occurs due to a reduction in the size of the portfolio's unique risk while the portfolio's market risk remains approximately the same size.

4.2 Portfolio Market Risk

Generally, the more diversified a portfolio (that is, the larger the number of securities in the portfolio), the smaller will be each proportion X_i. This will not cause β_{pI}, to either decrease or increase significantly unless a deliberate attempt is made to do so by adding either relatively low or high beta securities, respectively, to the portfolio. That is, because a portfolio's beta is an average of the betas of its securities, there is no reason to suspect that increasing the amount of diversification will cause the portfolio beta, and thus the market risk of the portfolio, to change in a particular direction. Accordingly, Diversification leads to averaging of marketrisk.

This makes sense because when prospects for the economy turn sour (or rosy), most securities will fall (or rise) in price. Regardless of the amount of diversification, portfolio returns will always be susceptible to such marketwide influences.

4.3 Portfolio Unique Risk

The situation is entirely different for unique risk. In a portfolio, some securities will go up as a result of unexpected good news specific to the company that issued the securities (such as an unexpected approval of a patent). Other securities will go down as a result of unexpected company-specific bad news (such as an industrial accident). Looking forward, approximately as many companies can be expected to have good news as bad news, leading to little anticipated net impact on the return of a "well-diversified" portfolio. This means that as a portfolio becomes more diversified, the smaller will be its unique risk and, in turn, its total risk.

This can be quantified precisely if the random error components of security returns are assumed to be uncorrelated, as was done when Equation (14c) was written. Consider the following situation. If the amount invested in each security is equal, then the proportion X_i will equal 1/N, and the level of unique risk, as shown in Equation (14c), will be equal to:

$$\sigma_{\varepsilon p}^2 = \sum_{i=1}^{N} \left[\frac{1}{N}\right]^2 \sigma_{\varepsilon i}^2 \tag{15a}$$

$$= \frac{1}{N}\left[\frac{\sigma_{\varepsilon 1}^2 + \sigma_{\varepsilon 2}^2 + \ldots + \sigma_{\varepsilon \mathrm{N}}^2}{\mathrm{N}}\right]. \tag{15b}$$

The value inside the square brackets in Equation (15a) is simply the average unique risk of the component securities. But the portfolio's unique risk is only one-Nth as large as this, since the term 1/N appears outside the square brackets. Now as the portfolio becomes more diversified, the number of securities in it (that is, N) becomes larger. In turn, this means that 1/N becomes smaller, resulting in the portfolio having less unique risk. That is diversification can substantially reduce unique risk.

Roughly speaking, a portfolio that has 30 or more randomly selected securities in it will have a relatively small amount of unique risk. This means that its total risk will be only slightly greater than the amount of market risk that is present. Thus such portfolios are "well diversified."

Figure 12.: Risk and Diversification

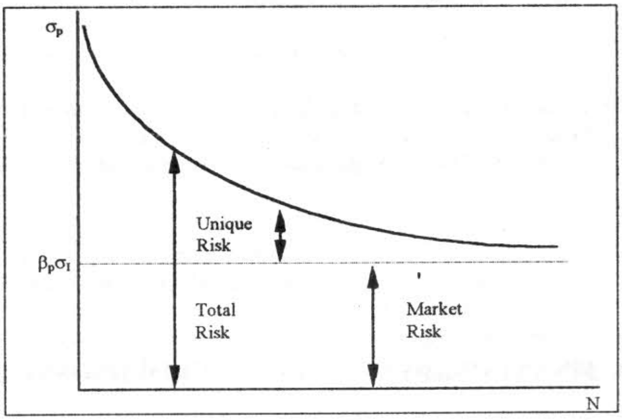


Figure 12 illustrates how diversification results in the reduction of unique risk but the averaging of market risk.

CONCLUSION

Based on analysis made investors make decisions about financial investments of his or foreign capital in the portfolio of securities. He can also distribute his capital funds and invest them in different amounts in several portfolios of securities. Analysis of financial investments is made combining different alternative possibilities, meanwhile, though with graphic presentations of possible events in the capital markets.

Thanks to the every day work and resolving pragmatically financial situations, financial investors gain experience relatively fast and learn how to deal with current financial problems in practice. Meanwhile, only the best and theoretically educated are capable to answer in an efficient and fast way to the unusual and unexpected financial problems and challenges that brings modern financial business. To do these financial investors needs bigger and wider knowledge than knowing standard and stereotype rules, to understand deeper meaning of financial laws and the logic of financial investment, so that they could derive from their theoretical knowledge initiative for efficient investing. Good theoretical base will, beside this, enable them to reasonably judge the financial world that's surrounding them, purposefully diving into joints, recombining and synthesising actual financial and economic problems, getting accustomed to the new business situations, making logical questions and giving logical financial answers, elimination and avoidance of insignificant factors and ephemera phenomena, and getting out the right and good financial decisions.

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Sažetak

STRATEGIJA FINANCIJSKIH ULAGANJA U VRIJEDNOSNE PAPIRE

U ovom članku prikazana je analiza iz domene financijske analize koja treba biti izvedena od strane svakog uspješnog financijskog investitora prilikom stvaranja optimalnog portfelja vrijednosnih papira, a sve u cilju minimiziranja odnosno diverzifikacije financijskog rizika. Na osnovu neograničenog broja mogućih portfelja vrijednosnica dostupnih investitoru, on se usredotočuje samo na onaj portfelj vrijednosnica koji se nalazi u efikasnoj seriji vrijednosnica.

Ključne riječi: financijsko ulaganje, investitor, diverzifikacija rizika, optimalni portfelj vrijednosnih papira.