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NEW STRESS-STRAIN MODEL FOR CONCRETE AT HIGH TEMPERATURES

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Original scientific paper
Concrete is one of the most widely used materials in civil engineering structures. Extremely high temperatures can seriously damage the concrete structure, leading to degradation of its mechanical properties. Considering high temperatures in civil engineering applications is strongly connected with the fire scenarios in which temperatures in concrete can reach 1000 °C. For using computer software for analysis of the reinforced concrete structures, it is essential to formulate constitutive stress-strain models of steel and concrete, which can be done using fundamental approach or by fitting curves to experimental data. In this paper, new stress-strain model was proposed using the two-parameter equation for fitting curves to experimental data. Verification of the model is done using experimental results available in the literature for temperatures up to 800 °C. Comparison with the models available in the literature was provided. The new model showed better agreement with experimental results, especially for the temperatures higher than 500 °C.

Keywords: *concrete; high temperature; stress-strain curve*

Novi model za opisivanje krivulje naprezanje-deformacija betona pri povišenim temperaturama

Izvorni znanstveni članak
Beton je jedan od najrasprostranjenijih materijala koji se koriste za izradu građevinskih objekata. Djelovanje visokih temperatura može ozbiljno oštetiti strukturu betona, što dovodi do degradacije njegovih mehaničkih karakteristika. Razmatranje glavnih karakteristika ponašanja građevinskih konstrukcija pri visokim temperaturama je povezano s djelovanjem požara, pri kojima se javljaju temperature u betonu i do 1000 °C. Za uporabu računarskih softvera za analizu armirano betonskih elemenata izloženih požarima neophodno je formulirati konstitutivne σ-ε modele kako čelika tako i betona, što se može postići fundamentalnim pristupom, ali i prilagodba krivulja prema eksperimentalno dobivenim podacima. U ovom radu predložen je novi dvoparametarski model za opisivanje krivulje naprezanje-deformacija. Novi model je potvrđen na eksperimentalnim podacima dostupnim u literaturi za temperature do 800 °C. Dana je njegova usporedba s drugim modelima dostupnim u literaturi. Rezultati pokazuju bolje poklapanje s eksperimentalnim podacima, posebice pri temperaturama većim od 500 °C.

Ključne riječi: *beton; krivulja naprezanje-deformacija; visoka temperatura*

1 Introduction

During their lifetime, civil engineering structures are subjected to different effects. To ensure their safe performance, the designers must take into consideration all possible actions that can occur during the construction and exploitation period. Some of them can cause the change of the properties of construction materials. Concrete is one of the most common materials used for civil engineering structures. It is well known that fires and high temperatures in general can seriously damage concrete structure, which leads to degradation of its mechanical properties [1]. Behaviour of concrete under high temperatures has been investigated by many researchers in past decades. The most relevant parameters that have been investigated are compressive strength, modulus of elasticity, strength in indirect tension, spalling and stress-strain curves. Extensive literature review of researches on these parameters is given in [2].

For using advanced computer software tools to analyse reinforced concrete elements subjected to fires, it is necessary to obtain constitutive stress-strain models for both steel and concrete. This can be done by using fundamental principles or by fitting curves to experimentally obtained data. Although it is preferable for the first approach to be used, many authors have proposed stress-strain models by using fitting procedures [7÷9]. The reason for this is either the paucity of data needed to establish a fundamental model or practical aspects in that the model is not required to be portable, but applicable to a single identified material [3]. In this paper, several known stress-strain models for concrete subjected to high temperatures are presented and, based on them, a new two-parameter model is proposed. For validating the model, experimental results published in [4] are used. A comparison between the new model and models from the literature is given. The models are compared using linear regression method, by R^2 value. The results are plotted as a function of the temperature.

2 Stress-strain models

2.1 Furamura

The first one who proposed the complete stress-strain curve for concrete was Furamura [5]. He concluded that besides the compressive strength reduction, there was a reduction of the slope of descending part of the curve. Later, it was shown in [6] that if his model is normalized with the peak stress and the strain corresponding to the peak stress, all the curves can be represented by the following expression:

$$\frac{\sigma_0}{\sigma_{0,c}} = \frac{\varepsilon_c}{\varepsilon_{0,c}} \cdot \mathrm{e}^{\left(1-\frac{\varepsilon_c}{\varepsilon_{0,c}}\right)}, \tag{1}$$

where σ_c is stress, ε_c is strain, $\sigma_{0,c}$ is the peak stress and $\varepsilon_{0,c}$ is strain corresponding to the peak stress.

2.2 Popovics

In [7] Popovics proposed the model that can be fitted to any concrete stress-strain curve. He defined stress-strain curve with the following expression:

$$\frac{\sigma_0}{\sigma_{0,c}} = \frac{\varepsilon_c}{\varepsilon_{0,c}} \cdot \frac{n}{n-1+\left(\frac{\varepsilon_c}{\varepsilon_{0,c}}\right)^n}, \quad (2)$$

where n is determined by fitting curve to experimental data. He also suggested that n was only dependant on the concrete compression strength and proposed that it is equal to $0{,}4\times10^{-3}\sigma_{0,c}$. This was later shown not to be entirely correct, which was also deduced by the authors of this paper, and that it is dependent on the size of the aggregate and the aggregate-cement ratio [4].

2.3 Anderberg and Thelandersson

In [8] Anderberg and Thelandersson proposed the constitutive law valid at transient conditions, that can be formulated in terms of four strain components, thermal strain, instantaneous stress-related strain, creep (as measured at constant temperature and constant stress) and transient strain. They assumed instantaneous stress-related strain as a parabolic function, considering ascending and descending part of the stress-strain curve separately. For ascending part they proposed the expression:

$$\frac{\sigma_0}{\sigma_{0,c}} = \frac{\varepsilon_c}{\varepsilon_{0,c}} \cdot \left(2-\frac{\varepsilon_c}{\varepsilon_{0,c}}\right), \quad (3)$$

and for descending part the straight line was assumed. Additionally, they proposed temperature independent slope of the descending part, which is not quite accurate when compared to the experimental results available in the literature.

2.4 Blagojević

Blagojević realized that stress-strain curve of the concrete at high temperatures can be considered as an impulse function. In [9] he proposed the model which actually represents modified expression (1) with additional parameter ν:

$$\frac{\sigma_0}{\sigma_{0,c}} = \left(\frac{\varepsilon_c}{\varepsilon_{0,c}} \cdot \mathrm{e}^{\left(1-\frac{\varepsilon_c}{\varepsilon_{0,c}}\right)}\right)^{\nu}, \quad (4)$$

where parameter ν can be obtained by fitting curves to experimentally obtained data. When this parameter is equal to 1, the model becomes the same as Furamuras normalized model. In this paper authors proposed modification of Blagojević's model in order to achieve better agreement with experimental results available in the literature.

2.4 Eurocode 2

In Eurocode 2 [10] the complete stress-strain curve is defined by the compressive strength, and the strain corresponding to the peak stress, for a defined temperature. For ascending part of the curve the following expression is given

$$\sigma(\theta) = 3 \cdot \frac{\varepsilon_c}{\varepsilon_{c1,\theta}} \cdot \frac{f_{c,\theta}}{2+\left(\frac{\varepsilon_c}{\varepsilon_{c1,\theta}}\right)^3}, \quad (5)$$

where $\sigma(\theta)$ is stress of the concrete at the temperature θ, ε_c is strain, $f_{c,\theta}$ is the peak stress and $\varepsilon_{c1,\theta}$ is the strain corresponding to the peak stress.

It is suggested that for numerical purposes a descending branch should be adopted, whether linear or nonlinear (Fig. 1). For linear descending branch the values of different parameters of $\varepsilon_{cu1,\theta}$ at different temperatures are provided in Eurocode 2 [10].

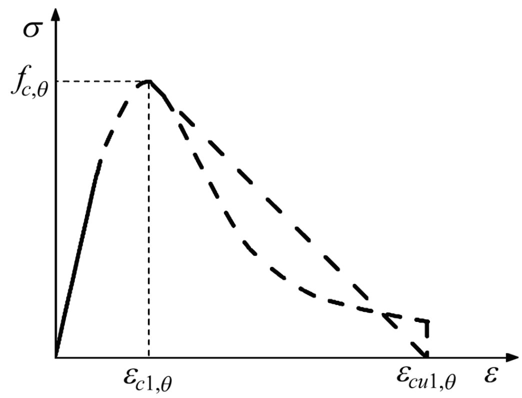

Figure 1 Stress-strain curve according to Eurocode 2 [10]

2 New stress-strain model

Although Eq. (4) fits generally well to the shape of stress-strain curve of concrete at both normal and high temperatures, some flaws can be noticed when compared to the experimental data. They are related to the nature of the parameter ν. In Fig. 2, it is shown how this parameter affects the shape of the stress-strain curve.

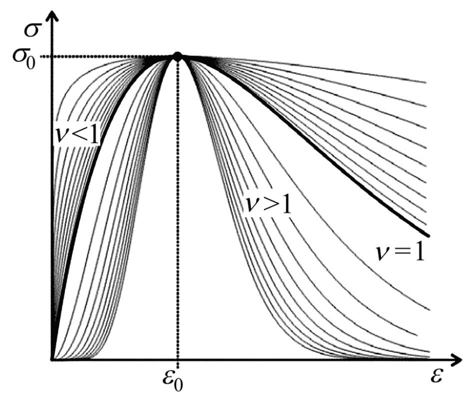

Figure 2 Influence of the parameter ν on the shape of the stress-strain curve [9]

In order to explain above mentioned, the model was compared to experimental data published in [4] (Fig. 3). The results were obtained by testing standard concrete cylinders with the dimensions ∅15×30 cm, made with Portland cement of Type I and siliceous aggregate. The compressive strength of the concrete was 40 MPa at 20

°C. For fitting curves, i.e. obtaining parameter ν, the least square method was used [11]. For different temperatures, parameters could be obtained by fitting the model to entire curve or just for ascending branch. If the latter is the case, different parameter or different function could be used for descending branch.

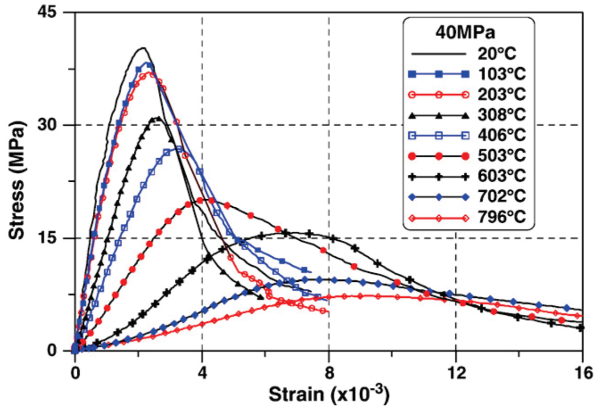

Figure 3 Experimental stress strain curves of the concrete at different temperatures [4]

In order to show the full potential of Blagojevics model, the parameters ν were obtained by fitting Eq. (4) only to ascending branch of the experimental stress-strain curve published in [4]. The obtained values of parameter ν are 2,12; 2,05; 3,09 and 2,66 for 308 °C, 406 °C, 702 °C and 796 °C respectively. These temperatures were chosen because in those curves the potential for improving the model was clearly visible.

Fig. 4 shows comparison between experimental stress-strain curves and curves fitted by using Blagojević's model for ascending branch of the curve. It can be seen that in the first part of the ascending branch of the curve parameter ν seems to be higher and in the second part lower than the one that corresponds to experimental data. This fact brings the authors to conclusion that better results would be obtained if its values varied through the ascending branch of the curve. The variable parameter ν is proposed, with the value α at the beginning and value β at the end. The value varies with the value $\varepsilon_c/\varepsilon_{0,c}$. This can be represented by the following expression:

$$\frac{\sigma_0}{\sigma_{0,c}}=\left(\frac{\varepsilon_c}{\varepsilon_{0,c}}\cdot \mathrm{e}^{\left(1-\frac{\varepsilon_c}{\varepsilon_{0,c}}\right)}\right)^{\left(\alpha+\frac{\varepsilon_c}{\varepsilon_{0,c}}\cdot(\alpha-\beta)\right)}, \tag{6}$$

where σ_c is stress, ε_c is strain, $\sigma_{0,c}$ is the peak stress, $\varepsilon_{0,c}$ is strain corresponding to the peak stress, and the parameters α and β can be obtained by fitting the curve to experimental data. In Eq. (6) it is clear that for the value ε_c=0 parameter ν would be equal to α and for $\varepsilon_c=\varepsilon_{0,c}$ the value of ν would be β.

In order to use the proposed model, the values of the peak stress and corresponding strain must be obtained from the experimental stress-strain curve. Parameters α and β can be obtained by using least square method or any other convenient fitting procedure. Least square method has shown itself to be very efficient for this purpose. No problems with convergence were experienced.

The mathematical nature of the proposed model is such that it can have a concave-up part at the beginning of loading which can be seen in stress-strain curves of the concrete at high temperatures. In [4] this phenomenon was explained by the closure of pre-existing cracks caused by heating and cooling.

4 Validation of the new model and comparison

In order to validate the proposed two-parameter model, the stress-strain curves were obtained by fitting the model to experimental data published in [4]. The parameters were obtained by fitting the model to ascending branch and the whole curve separately. It is worth noticing that the parameters α and β obtained by fitting the curve to the ascending branch give much better agreement with the experimental data than those

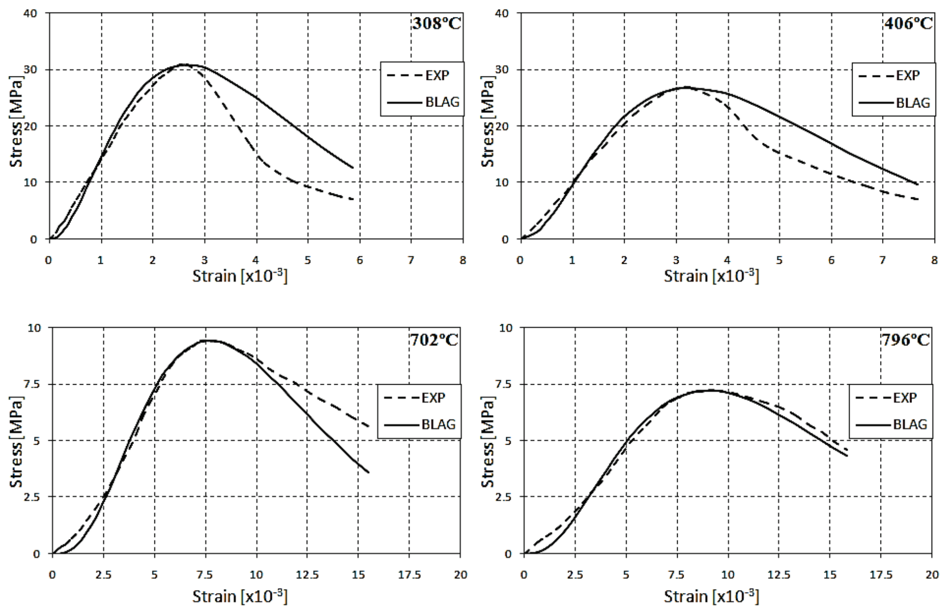

Figure 4 Stress strain curves of concrete at different temperatures, Blagojevićs model vs. experimental data from [4]

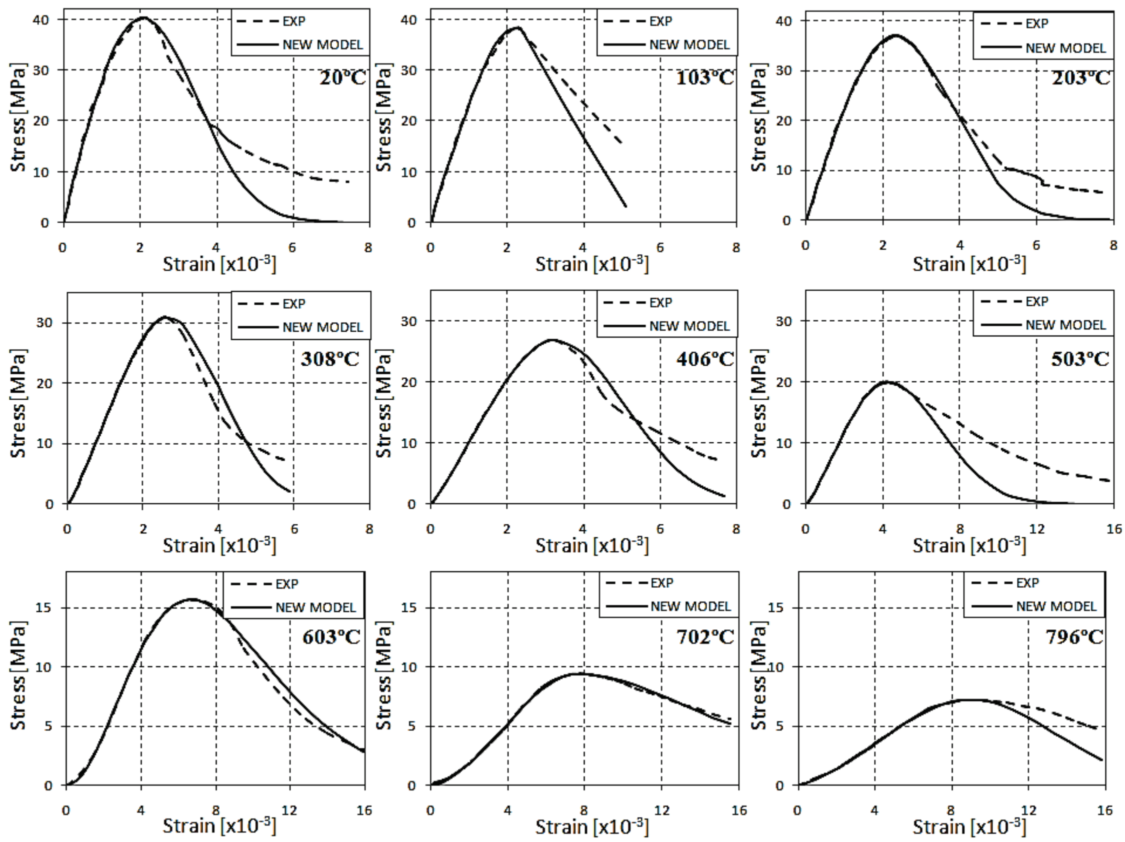

Figure 5 Stress-strain curves at different temperatures, new model vs. experimental results from [4]

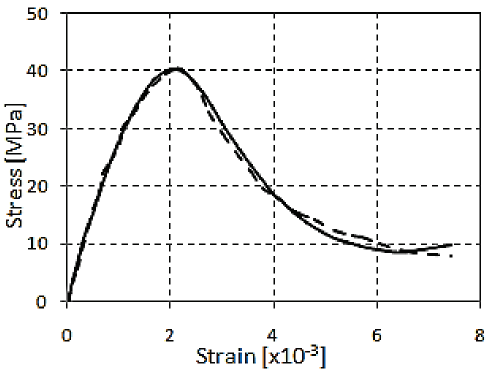

Figure 6 Stress-strain curve at 20 °C
(two-parameter used model for both branches)

obtained by fitting to the entire curve. Due to this fact, the first option is considered in this paper and the values of the parameters are given in Tab. 1.

Fig. 5 shows the comparison between experimental stress-strain curves and curves fitted by using Eq. (6) for ascending branch of the curve. As it can be seen, the new model fits perfectly to the ascending branch of the experimental stress-strain curve. However, in descending branch the agreement is not that good. Thus, the authors recommend that the curve should be divided into two parts.

The parameters should be obtained for both parts separately. However, using two-parameter model for descending branch has shown some disadvantages. As shown in Fig. 6, the curve obtained by using the two-parameter model for both parts of the stress-strain curve at 20 °C has the tendency of going upward at the end part of the descending branch. Due to this fact, the authors recommend Eq. (6) to be used for ascending branch and either Eq. (4) should be used for descending branch or it should be assumed linear.

By fitting ascending branch by using Eq. (6) and descending by Eq. (4) parameters α, β and ν were obtained for each temperature. For each parameter the linear function of dependency on the temperature was approximated. With the values of parameters calculated by these functions stress-strain curves were plotted and shown in Fig. 7 compared to experimental curves.

Approximated functions of dependency on the temperature are the following:

$$\alpha = 1{,}072 + 0{,}001 \cdot \theta, \qquad (7a)$$

$$\beta = 2{,}442 + 0{,}002 \cdot \theta, \qquad (7b)$$

$$\nu = 2{,}988 + 0{,}0006 \cdot \theta, \qquad (7c)$$

where θ is the temperature of the concrete.

It can be seen in Fig. 7 that the proposed model fits perfectly to the experimental curves.

Table 1 The values of parameters obtained by fitting the new model to ascending branch of the stress-strain curve

T	20 °C	103 °C	203 °C	308 °C	406 °C	503 °C	603 °C	702 °C	796 °C
α	1,157	1,175	1,204	1,357	1,383	1,538	2,212	1,923	1,627
β	2,320	2,982	2,617	3,647	3,364	2,906	3,250	4,678	4,233

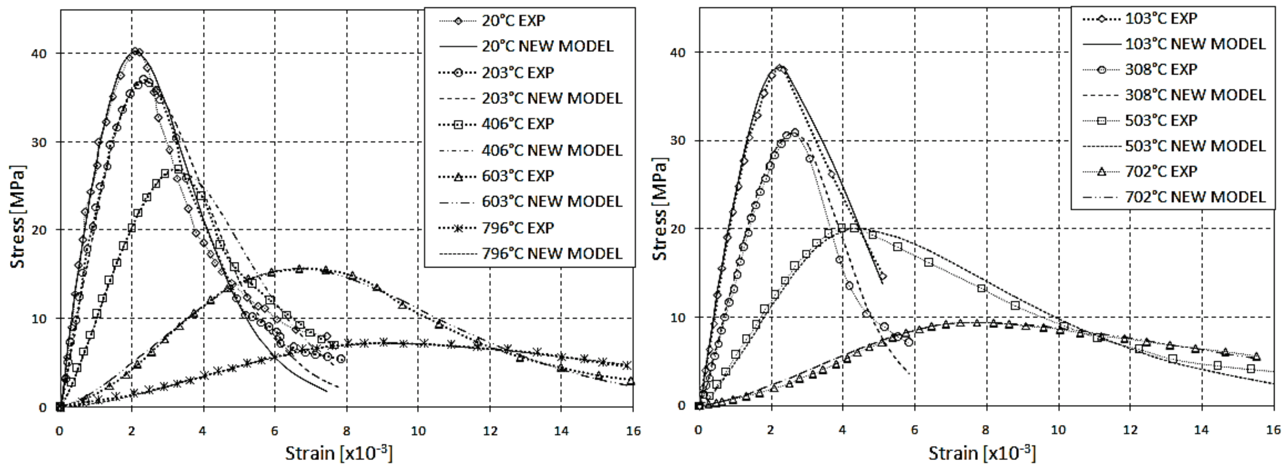

Figure 7 Stress-strain curves at different temperatures: new model vs. experimental results from [4]

For additional validation of the proposed model a comparison with the models presented in chapter 2 has been provided. Since the models of Furamura and Blagojević represent special cases of the proposed model, the comparison was done only with other models. Additionally, stress-strain curves were compared only for the ascending branch, since two of the three presented models do not cover the descending one.

The stress-strain curves were fitted to experimental data shown in Fig. 8 for temperatures of 503 °C, 603 °C, 702 °C and 796 °C. These four temperatures were selected because in other cases the difference in the plotted curves could not be seen clearly without magnification. Since the suggestion that Popovics gave for the parameter n was shown to be non-satisfactory, it was obtained by fitting the curves for each temperature level separately. Additionally, linear regression analysis was performed and R^2 values were plotted in Fig. 9 as a function of the temperature.

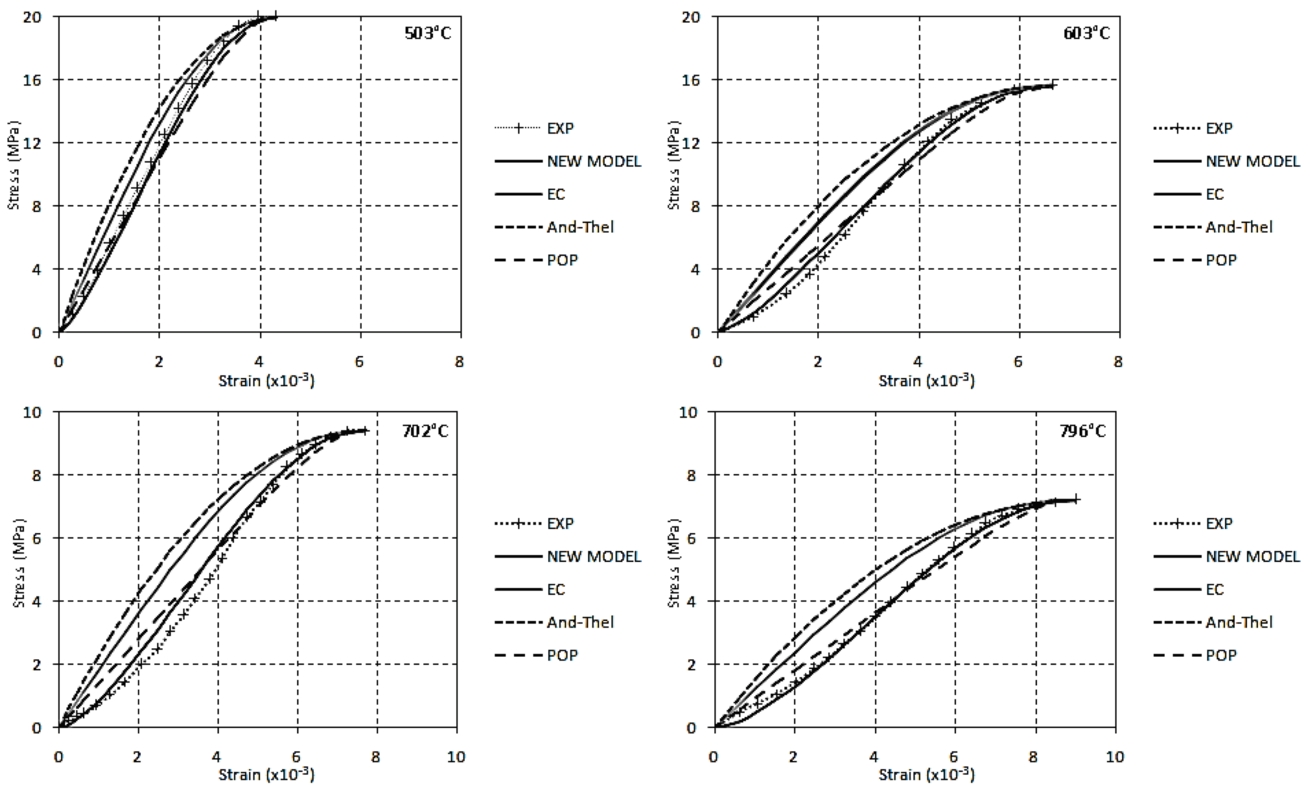

Figure 8 Comparison of ascending branches of stress-strain curves at different temperatures

It can be seen that the proposed model gave the best agreement with the experimental data. Both the new and the model proposed by Popovics gave good results for the whole range of temperatures. However, the advantage of the proposed model is that it can follow the concave up shape of the curves and slightly better agreement with experimental data at temperatures higher than 500 °C. In the case of the model of Anderberg and Thelandersson and the model given in Eurocode 2, the advantage of the proposed model is even bigger through the whole range of temperature.

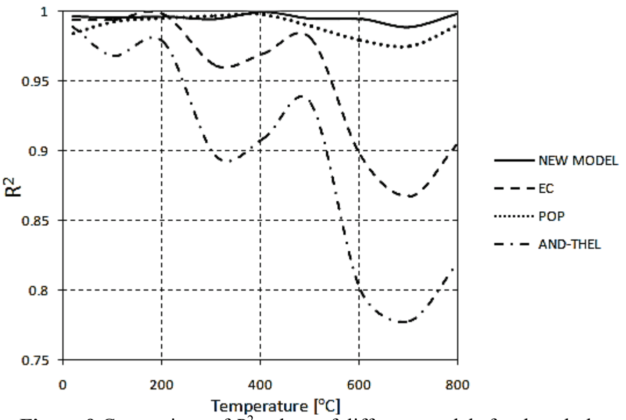

Figure 9 Comparison of R^2 values of different models for the whole range of the temperature

5 Conclusion

The use of modern software for calculating stresses and deformations in concrete elements requires knowledge of its stress-strain relation. For the mechanical properties of the concrete when subjected to high temperatures it is very important to know how it changes the shape of the stress-strain curve, and how this change can be mathematically represented. It was shown that the approach to obtaining stress strain curve using the proposed model for ascending branch gives very good agreement with the experimental results. Its mathematical nature is convenient for representing stress-strain curves of the concrete at both normal and high temperatures. Temperature dependence of the model parameters was presented and the values obtained by those functions gave very good results when compared to the experimental data. The fact that with the increase of temperature ascending branch becomes more linear, as well as the phenomenon that at temperatures higher than 500 °C the beginning part can be concave up, can be also represented by using the proposed model. Additionally, it was shown by regression analysis that it gives better results than other models presented in this paper.

In the future work, the dependency of α, β and ν on different parameters other than the temperature could be investigated.

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