ISSN 1846-6168 (Print), ISSN 1848-5588 (Online)
ID: TG-20170223061859

Original scientific article

DEFLECTION AND BENDING MOMENTS AMPLITUDE DISTRIBUTION AT THE FORCED OSCILLATIONS OF THE EULER-BERNOULLI BEAM

Yurii KRUTII, Nikolay SURYANINOV

Abstract: The paper considers forced oscillations of a simply supported Euler-Bernoulli beam with inner inelastic resistance. Maximum values of non-dimensional amplitudes of bending moments and deflections which are invariants with respect to dimensional parameters of beam are calculated. For beams with any dimensional parameters calculation of the maximum amplitudes corresponding to the set frequency of forced vibrations, reduces to multiplication by appropriate dimensional factor already calculated invariant dimensionless values. In a specific example, a comparative analysis of accurate amplitude values for dynamic bending moments and deflections is calculated according to the author's method, with the same calculation in the ANSYS program complex. Values of the amplitudes in the vicinity of the resonance frequencies are clarified.

Keywords: amplitude of the bending moments and deflections, Euler-Bernoulli beam, transverse vibrations, frequency of forced oscillations, resonance.

1 INTRODUCTION

One of the major problems in the theory of oscillations of elastic systems is the study of the influence of external and internal resistances to oscillation processes. The calculations on the free oscillations consist in definition of own frequencies and forms of oscillations, as well as calculations on forced vibrations away from the resonance zones. The resistance forces are often neglected. This is due to the fact that when such calculations accounting the forces of resistance do not significantly influence the final result. Another thing is calculation of oscillation near resonance. This calculation requires the consideration of the resistance forces, since their effect is appearing in the greatest measure.

Effect of external and internal resistance to oscillations is different and depends on many factors: the oscillatory characteristics of the system, the material of which the elements of the system are made, parameters of environment. However, in this case, internal resistance of inelastic material is of particular importance.

2 BACKGROUND AND ANALYSIS OF RESEARCH

Presence of internal friction in the material was firstly found by Coulomb in experiments with a torsion balance. The study of internal friction involved W. Thomson. It was established experimentally that the resilient material does not strictly follow Hooke’s law even if the elastic deformations within the limits of elasticity. This explains the internal energy loss fluctuations.

Many scientists talk about the importance of internal friction in the material studies conducted in the beginning of the 20th century. Experiments of Guye on the internal losses in the material during torsional vibration of metal wires showed a completely insignificant role of air friction as compared with losses in the metal. Rowett, exploring damping vibrations in machines, found that the share of domestic energy dissipation in the material accounts for at least two-thirds of all losses during vibration.

In the dynamic structural analysis, the hypothesis of Kelvin-Voigt, which is based on the idea of the viscosity of solids, is widespread when taking into account the internal resistance of inelastic material [1, 2]. The internal friction proportional to the velocity was used by the founders of the applied theory of vibrations A.N. Krylov [3] and S.P. Timoshenko [4].

However, as it is well known [2], the Kelvin-Voigt’s hypothesis in its pure form has a number of drawbacks. The main point is the fact that the hypothesis leads to a conclusion about the frequency-independent internal friction in the material, which contradicts the experimental data. This disadvantage is eliminated by taking the corrected Kelvin-Voigt hypothesis, according to which the damping is taken into account in proportion to the strain rate, divided by the oscillation frequency [2].

Among the fundamental studies on this issue, particularly noteworthy are the works of A.N. Krylov [3], S.P. Tymoshenko [4], J.G. Panovko [5] and E.S. Sorokin [6].

Current studies are characterized by the extensive use of computer methods of mechanics. Works related to considering variable mass for different kinds of friction are published by V.P. Olshansky and S.V. Olshansky [7, 8]. The work of N.N. Berendeev is devoted to the problem of the influence of the internal friction on the system of forced oscillations [9].

3 MAIN CHAPTER

3.1 The main symbols and formulas

Consider the forced harmonic oscillations of a hinged beam internal forces taking into account the inelastic resistance. The general scheme of the oscillation is shown in Fig. 1. Fig. 2 shows a diagram of forces acting on the oscillations of the beam element.

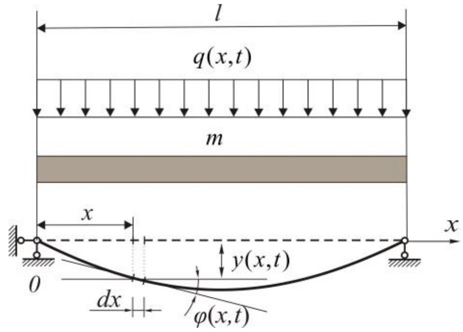

Figure 1 Forced transverse vibrations of a rod

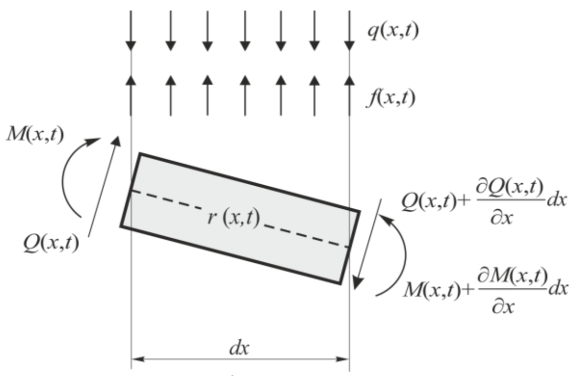

Figure 2 Driving forces acting on the rod element

The following designations are accepted: $q(x,t) = q\sin\theta t -$ external dynamic load, where $q -$ constant amplitude, $\theta -$ the frequency of the disturbing force; $m -$ the intensity of the distributed mass (mass per unit length) of the rod; $y(x,t) -$ lateral movement of the point with the coordinate axis of the rod at the time (dynamic bending); $\varphi(x,t) -$ dynamic rotation angle; $M(x,t) -$ dynamic bending moment; $Q(x,t) -$ dynamic shear force; $r(x,t) -$ the intensity of the internal forces of resistance; $f(x,t) = -m\dfrac{\partial^2 y}{\partial t^2} -$ the intensity of the inertial forces that arise in the course of the oscillation (the power of D'Alembert).

Assuming, according to the corrected hypothesis of Kelvin-Voigt [1, 2],

$$r(x,t) = \frac{\gamma}{\theta} EI \frac{\partial^5 y}{\partial t \partial x^4},$$

the equation of forced oscillations of the beam can be written as [1, 2]

$$EI\left(1 + \frac{\gamma}{\theta}\frac{\partial y}{\partial t}\right)\frac{\partial^4 y}{\partial x^4} + m\frac{\partial^2 y}{\partial t^2} = q\sin\theta t, \tag{1}$$

where $\gamma -$ inelastic resistance coefficient (dimensionless constant for a given material); $EI -$ the bending stiffness of the beam. To this we add the equation as defined boundary conditions:

$$y(0,t) = 0;\ M(0,t) = 0;\ y(l,t) = 0;\ M(l,t) = 0. \tag{2}$$

In the absence of the scientific literature of the exact solution of the Eq.(1), for the study of forced oscillations of the beam based on the internal resistance to the present, as a rule, we used approximate methods. An exception may be the publication of M. Abu-Hilal [10], where the definition of the dynamic deflection of the beam is based on the method of Green's functions.

In [11] an exact solution of the Eq. (1) and fully defined dynamic parameters of the beam is found. In particular, formulas for the deflection and bending moments

$$y(x,t) = y_1(x)\sin\theta t + y_2(x)\cos\theta t \tag{3}$$

$$\begin{aligned} M(x,t) &= M_1(x)(\sin\theta t + \gamma\cos\theta t) + \\ &+ M_2(x)(\cos\theta t - \gamma\sin\theta t), \end{aligned} \tag{4}$$

where $y_k(x),\ M_k(x)\ (k = 1,2) -$ the so-called constituent functions of their parameters. These functions with boundary conditions (2) at a point $x = 0$, defined by the formula [11]:

$$\begin{aligned} & y_k(x) = \\ & = (-1)^{k+1}\left(\varphi_1(0)\, l X_{2,k}(x) - \frac{Q_1(0)\, l^3}{EI} X_{4,k}(x)\right) + \\ & + \varphi_2(0) l X_{2,3-k}(x) - \frac{Q_2(0)\, l^3}{EI} X_{4,3-k}(x) + \\ & + (-1)^{k+1}\frac{ql^4}{(1+\gamma^2)EI}\left(X_{5,k}(x) + (-1)^k \gamma X_{5,3-k}(x)\right); \end{aligned} \tag{5}$$

$$\begin{aligned} & M_k(x) = \\ & = (-1)^k\left(\frac{\varphi_1(0)EI}{l} X^*_{4,k}(x) - Q_1(0)\, l X_{2,k}(x)\right) - \\ & - \frac{\varphi_2(0)EI}{l} X^*_{4,3-k}(x) + Q_2(0)\, l X_{2,3-k}(x) + \\ & + (-1)^k \frac{ql^2}{1+\gamma^2}\left(X_{3,k}(x) + (-1)^k \gamma X_{3,3-k}(x)\right), \end{aligned} \tag{6}$$

where

$$\begin{aligned} & X_{n,1}(x) = \\ & = \frac{1}{(n-1)!}\left(\frac{x}{l}\right)^{n-1} + \sum_{k=1}^{\infty}\frac{L^{2k}\cos k\delta}{(4k+n-1)!}\left(\frac{x}{l}\right)^{4k+n-1}, \end{aligned} \tag{7}$$

$$X_{n,2}(x) = \sum_{k=1}^{\infty}\frac{L^{2k}\sin k\delta}{(4k+n-1)!}\left(\frac{x}{l}\right)^{4k+n-1}\ (n = 2,3,4,5), \tag{8}$$

$$X_{4,1}^*(x) = \sum_{k=0}^{\infty} \frac{L^{2k+2}\cos(k+1)\delta}{(4k+3)!}\left(\frac{x}{l}\right)^{4k+3}, \tag{9}$$

$$X_{4,2}^*(x) = \sum_{k=0}^{\infty} \frac{L^{2k+2}\sin(k+1)\delta}{(4k+3)!}\left(\frac{x}{l}\right)^{4k+3}; \tag{10}$$

$$L = \frac{\theta}{\sqrt[4]{1+\gamma^2}} l^2 \sqrt{\frac{m}{EI}},\ \delta = \operatorname{atan}\gamma\ \left(0 \le \delta < \frac{\pi}{2}\right). \tag{11}$$

It is important to note that the parameters L, δ and all functions (7)–(10) are dimensionless.

Constituent functions (5), (6) represented in a complex form [11]:

$$y_1(x) + iy_2(x) = (\varphi_1(0) + i\varphi_2(0))\, lX_2(x) - $$
$$-(Q_1(0) + iQ_2(0))\frac{l^3}{EI}X_4(x) + \frac{q}{(1+i\gamma)}\frac{l^4}{EI}X_5(x); \tag{12}$$

$$M_1(x) + iM_2(x) =$$
$$= -(\varphi_1(0) + i\varphi_2(0))\frac{EI}{l}K^2X_4(x) + \tag{13}$$
$$+(Q_1(0) + iQ_2(0))\, lX_2(x) - \frac{q}{(1+i\gamma)}l^2X_3(x),$$

where

$$X_n(x) = X_{n,1}(x) - iX_{n,2}(x)\ (n = 2,3,4,5),$$
$$K^2X_4(x) = X_{4,1}^*(x) - iX_{4,2}^*(x).$$

3.2 The dimensionless amplitude of dynamic deflections and bending moments

Unidentified initial parameters $\varphi_1(0),\ \varphi_2(0)$, $Q_1(0),\ Q_2(0)$ are determined from the boundary conditions (2) at the point $x = l$. These boundary conditions on the basis of formulas (3), (4) are equivalent to

$$y_1(l) = 0,\ y_2(l) = 0,\ M_1(l) = 0,\ M_2(l) = 0,$$

and their implementation, by the formulas (12), (13), leads to a system of equations:

$$X_2(l)(\varphi_1(0) + i\varphi_2(0)) -$$
$$-\frac{l^2}{EI}X_4(l)(Q_1(0) + iQ_2(0)) = -\frac{ql^3}{(1+i\gamma)EI}X_5(l);$$
$$-\frac{EI}{l}K^2X_4(l)(\varphi_1(0) + i\varphi_2(0)) +$$
$$+lX_2(l)(Q_1(0) + iQ_2(0)) = \frac{ql^2}{1+i\gamma}X_3(l).$$

Hence we find the complex initial settings:

$$\varphi_1(0) + i\varphi_2(0) =$$
$$= \frac{ql^3}{(1+i\gamma)EI}\frac{X_3(l)X_4(l) - X_2(l)X_5(l)}{X_2^2(l) - K^2X_4^2(l)}; \tag{14}$$

$$Q_1(0) + iQ_2(0) =$$
$$= \frac{ql}{1+i\gamma}\frac{X_2(l)X_3(l) - K^2X_4(l)X_5(l)}{X_2^2(l) - K^2X_4^2(l)}. \tag{15}$$

To determine the required initial parameters $\varphi_1(0),\ \varphi_2(0),\ Q_1(0),\ Q_2(0)$ allocate the real and imaginary terms in the right-hand sides of the formulas (14), (15). The result

$$\varphi_k(0) = \frac{ql^3}{EI(1+\gamma^2)}H_k,\ \ Q_k(0) = \frac{ql}{1+\gamma^2}S_k\ \ (k = 1,2),$$

where H_k, S_k – dimensionless constants, calculated according to the formulas:

$$H_1 = \frac{a_1c_1 + a_2c_2 + \gamma(a_2c_1 - a_1c_2)}{c_1^2 + c_2^2};$$
$$H_2 = \frac{a_2c_1 - a_1c_2 - \gamma(a_1c_1 + a_2c_2)}{c_1^2 + c_2^2};$$
$$S_1 = \frac{b_1c_1 + b_2c_2 + \gamma(b_2c_1 - b_1c_2)}{c_1^2 + c_2^2};$$
$$S_2 = \frac{b_2c_1 - b_1c_2 - \gamma(b_1c_1 + b_2c_2)}{c_1^2 + c_2^2};$$
$$a_1 = X_{4,1}(l)X_{3,1}(l) - X_{4,2}(l)X_{3,2}(l) -$$
$$-X_{2,1}(l)X_{5,1}(l) + X_{2,2}(l)X_{5,2}(l);$$
$$a_2 = X_{2,1}(l)X_{5,2}(l) + X_{2,2}(l)X_{5,1}(l) -$$
$$-X_{4,1}(l)X_{3,2}(l) - X_{4,2}(l)X_{3,1}(l);$$
$$b_1 = X_{2,1}(l)X_{3,1}(l) - X_{2,2}(l)X_{3,2}(l) -$$
$$-X_{4,1}^*(l)X_{5,1}(l) + X_{4,2}^*(l)X_{5,2}(l);$$
$$b_2 = X_{4,1}^*(l)X_{5,2}(l) + X_{4,2}^*(l)X_{5,1}(l) -$$
$$-X_{2,1}(l)X_{3,2}(l) - X_{2,2}(l)X_{3,1}(l);$$
$$c_1 = X_{2,1}^2(l) - X_{2,2}^2(l) - X_{4,1}^*(l)X_{4,1}(l) + X_{4,2}^*(l)X_{4,2}(l);$$
$$c_2 = X_{4,1}^*(l)X_{4,2}(l) + X_{4,2}^*(l)X_{4,1}(l) - 2X_{2,1}(l)X_{2,2}(l).$$

Substituting the values of the initial parameters in the formula (5), (6) we have:

$$y_k(x) = \frac{ql^4}{(1+\gamma^2)EI}y_k^*(x); \tag{16}$$

$$M_k(x) = \frac{ql^2}{1+\gamma^2}M_k^*(x)\ \ (k = 1,2), \tag{17}$$

where $y_k^*(x)$, $M_k^*(x)$ – dimensionless functions, which have the form

$$y_k^*(x) =$$
$$= (-1)^{k+1}(H_1 X_{2,k}(x) - S_1 X_{4,k}(x)) + H_2 X_{2,3-k}(x) -$$
$$- S_2 X_{4,3-k}(x) + (-1)^{k+1}(X_{5,k}(x) + (-1)^k \gamma X_{5,3-k}(x)),$$
$$M_k^*(x) =$$
$$= (-1)^k (H_1 X_{4,k}^*(x) - S_1 X_{2,k}(x)) - H_2 X_{4,3-k}^*(x) +$$
$$+ S_2 X_{2,3-k}(x) + (-1)^k (X_{3,k}(x) + (-1)^k \gamma X_{3,3-k}(x)).$$

To study the vibrations, the formula, which clearly highlighted the amplitude function, will be more convenient. Rearranging equation (3), (4) to a desired form, taking into consideration the representations (16), (17) we finally obtain:

$$y(x,t) = \mathrm{Am}\, y(x,t) \sin(\theta t + \chi_y(x));$$
$$\mathrm{Am}\, y(x,t) = \frac{ql^4}{(1+\gamma^2)EI} y(x);$$
$$y(x) = \sqrt{(y_1^*(x))^2 + (y_2^*(x))^2},\ \chi_y(x) = \mathrm{atan}\frac{y_2^*(x)}{y_1^*(x)};$$
$$M(x,t) = \mathrm{Am}\, M(x,t) \sin(\theta t + \chi_M(x));$$
$$\mathrm{Am}\, M(x,t) = \frac{ql^2}{1+\gamma^2} M(x);$$
$$M(x) = \sqrt{(M_1^*(x) - \gamma M_2^*(x))^2 + (M_2^*(x) + \gamma M_1^*(x))^2};$$
$$\chi_M(x) = \mathrm{arctg}\frac{M_2^*(x) + \gamma M_1^*(x)}{M_1^*(x) - \gamma M_2^*(x)}.$$

As can be seen, the formula for the amplitude contains dimensionless factors $y(x)$ and $M(x)$, that is independent of the load. This factors up to size ratio $\frac{ql^4}{(1+\gamma^2)EI}$ and $\frac{ql^2}{1+\gamma^2}$ it defines the main forms for the dynamic deflection and dynamic bending moment. The functions $y(x)$ and $M(x)$ can also be interpreted as a dimensionless amplitude.

The maximum value of the amplitude will obviously be achieved in the middle of the beam, i.e.

$$\max \mathrm{Am}\, y(x,t) = \frac{ql^4}{(1+\gamma^2)EI} y\left(\frac{l}{2}\right), \tag{18}$$

$$\max \mathrm{Am}\, M(x,t) = \frac{ql^2}{1+\gamma^2} M\left(\frac{l}{2}\right). \tag{19}$$

Thus, to determine the maximum amplitude, it is necessary to calculate the corresponding value of the dimensionless amplitude in the middle of the beam. This will represent a special interest in the calculation of the resonance zones, when the frequency θ of forced oscillations will be located in the vicinity of the frequency of free vibrations of the beam.

As is known [1], for the frequency of free oscillations of the beam, excluding the resistances, there is the formula

$$\omega_j = \frac{K_j}{l^2}\sqrt{\frac{EI}{m}}\ (j = 1, 2, 3, \ldots), \tag{20}$$

where K_j – dimensionless coefficients of free oscillations. In the case of simply supported beam $K_j = (j\pi)^2$.

As the frequency of forced oscillations set the following values:

in the interval $0 < \theta \le \omega_1$ we believe

$$\theta = k\frac{\omega_1}{10}\ (k = 1, 2, 3, \ldots, 10);$$

in the interval $\omega_1 < \theta \le \omega_2$ we believe

$$\theta = \omega_1 + k\frac{\omega_2 - \omega_1}{10}\ (k = 1, 2, \ldots, 10);$$

in the interval $\omega_2 < \theta \le \omega_3$ we believe

$$\theta = \omega_2 + k\frac{\omega_3 - \omega_2}{10}\ (k = 1, 2, 3, \ldots, 10);$$

in the interval $\omega_3 < \theta < \omega_4$ we believe

$$\theta = \omega_3 + k\frac{\omega_4 - \omega_3}{10}\ (k = 1, 2, 3).$$

Substituting here instead of frequency ω_1, ω_2, ω_3, ω_4 their values (16), we obtain the representation

$$\theta = \frac{K^*}{l^2}\sqrt{\frac{EI}{m}}, \tag{21}$$

where K^* – dimensionless ratio of forced oscillations for which we have

$$K^* = \begin{cases} 0{,}1k\pi^2, \text{ when } \theta = k\dfrac{\omega_1}{10}; \\ (1+0{,}3k)\pi^2, \text{ when } \theta = \omega_1 + k\dfrac{\omega_2-\omega_1}{10}; \\ (4+0{,}5k)\pi^2, \text{ when } \theta = \omega_2 + k\dfrac{\omega_3-\omega_2}{10} \\ (k = 1, 2, 3, \ldots, 10); \\ (9+0{,}7k)\pi^2, \text{ when } \theta = \omega_3 + k\dfrac{\omega_4-\omega_3}{10} \\ (k = 1, 2, 3). \end{cases}$$

When this parameter L, determined by the first of the formulas (11), taking into account (21), we obtain $L = \frac{K^*}{\sqrt[4]{1+\gamma^2}}$.

Tab. 1 shows the results of calculations of the dimensionless deflection and bending moment amplitudes in the middle of the beam, corresponding to different values of coefficient of forced oscillation K^*. Especially note that these values are invariant relative to dimensional values l, EI, m, q and depend only on beams ends restraints. In fact, they describe the very essence of the phenomenon.

Table 1 Maximum values of dimensionless amplitudes

K^*	For deflection	For bending moment
	$y(l/2)$	$M(l/2)$
$0.1\pi^2$	0.013204	0.127292
$0.2\pi^2$	0.013614	0.131360
$0.3\pi^2$	0.014358	0.138729
$0.4\pi^2$	0.015546	0.150496
$0.5\pi^2$	0.017393	0.168793
$0.6\pi^2$	0.020338	0.197974
$0.7\pi^2$	0.025397	0.248102
$0.8\pi^2$	0.035476	0.347992
$0.9\pi^2$	0.062745	0.618348
π^2	0.148025	1.466401
$1.3\pi^2$	0.018987	0.191677
$1.6\pi^2$	0.008483	0.087690
$1.9\pi^2$	0.005097	0.054210
$2.2\pi^2$	0.003483	0.038293
$2.5\pi^2$	0.002564	0.029264
$2.8\pi^2$	0.001982	0.023597
$3.1\pi^2$	0.001587	0.019803
$3.4\pi^2$	0.001306	0.017153
$3.7\pi^2$	0.001099	0.015250
$4\pi^2$	0.000941	0.013865
$4.5\pi^2$	0.000752	0.012349
$5\pi^2$	0.000622	0.011532
$5.5\pi^2$	0.000532	0.011248
$6\pi^2$	0.000468	0.011453
$6.5\pi^2$	0.000426	0.012213
$7\pi^2$	0.000402	0.013748
$7.5\pi^2$	0.000401	0.016610
$8\pi^2$	0.000434	0.022275
$8.5\pi^2$	0.000539	0.035211
$9\pi^2$	0.000629	0.054233
$9.7\pi^2$	0.000185	0.025754
$10.4\pi^2$	0.000051	0.013671
$11.1\pi^2$	0.000017	0.009066

Fig. 3, 4 are graphs of the maximum amplitude of the dimensionless deflection and bending moment on the coefficient of beam forced vibrations. As can be seen, the highest values of the amplitudes are achieved when the value of the coefficient of oscillations is π^2, which corresponds to the frequency of free oscillations ω_1.

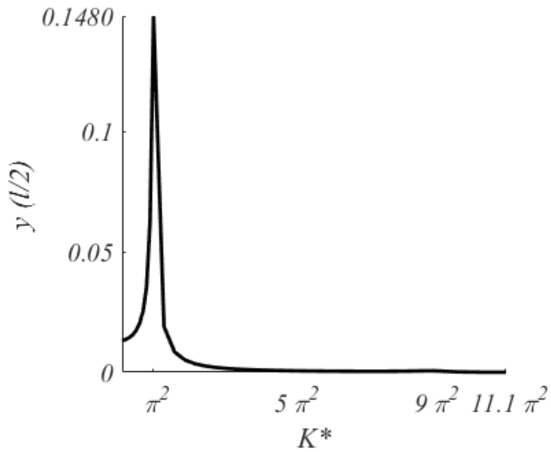

Figure 3 Dependence of the dimensionless deflection amplitude of the oscillation rate

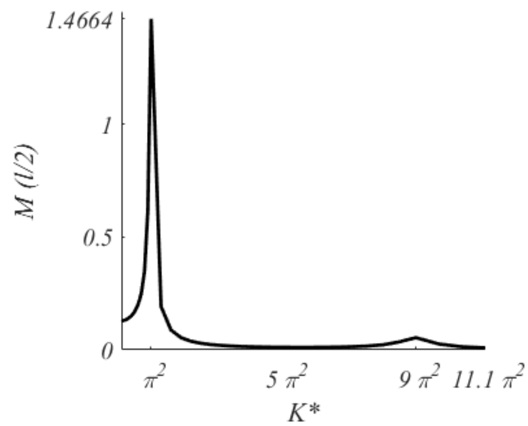

Figure 4 Dependence of the dimensionless bending moment amplitude of the oscillation rate

Thus, according to the formulas (21), (18), (19) the calculation of the maximum amplitudes of the dynamic deflection and bending moments of the beam caused by external dynamic load $q(x,t) = q\sin\theta t$ reduces to multiplication dimensionless value K^*, $y\left(\frac{l}{2}\right)$, $M\left(\frac{l}{2}\right)$, contained in Table 1, on the corresponding dimensional multipliers $\frac{1}{l^2}\sqrt{\frac{EI}{m}}$, $\frac{ql^4}{(1+\gamma^2)EI}$, $\frac{ql^2}{1+\gamma^2}$.

3.3 Example

Let us find the distribution of the amplitudes of the dynamic deflection and bending moments at the hinge beam for given values of higher frequency of forced oscillations. Inelastic material resistance factor is $\gamma = 0.089$. The force of inertia that occurs during the equipment operation is assumed to be q = 20 kN/m, mass per unit length of the beam m = 2.5 kNs^2/m^2, bending stiffness EI = 79615.11 kNm^2.

With such design data, the first four frequencies of oscillation beams excluding resistance according to (20) are equal:

$\omega_1 = 48.9243\ \text{s}^{-1}$; $\omega_2 = 195.6974\ \text{s}^{-1}$; $\omega_3 = 440.3191\ \text{s}^{-1}$; $\omega_4 = 782.7895\ \text{s}^{-1}$.

To calculate the required amplitude of dynamic deflections and bending moments corresponding to a given frequency θ, according to the proposed method, we multiply the values that appear in the first, second and third columns of Tab. 1, respectively, on the dimensional ratios

$$\frac{1}{l^2}\sqrt{\frac{EI}{m}} = 4.9571\ \text{s}^{-1},\ \ \frac{ql^4}{(1+\gamma^2)EI} = 0.3230\ \text{m},$$
$$\frac{ql^2}{1+\gamma^2} = 714.3417\ \text{kNm}.$$

Results are shown in Tabs. 2 and 3. For comparison, the results of calculations for a given beam finite element method are obtained using ANSYS [12] software package.

Table 2 Maximum values of dimensioned amplitudes

Frequencies θ, s^{-1}	For deflection, m	
	The author's method	ANSYS
4.8924	0.004265	0.004312
9.7849	0.004398	0.004402
14.6773	0.004638	0.004683
19.5697	0.005022	0.005074
24.4622	0.005618	0.005691
29.3546	0.006569	0.006611
34.2470	0.008204	0.008316
39.1395	0.011459	0.011700
44.0319	0.020267	0.020211
ω_1 = 48.9243	0.047813	0.041122
63.6016	0.006133	0.005918
78.2789	0.002740	0.002212
92.9562	0.001646	0.001533
107.6336	0.001125	0.001094
122.3109	0.000828	0.000800
136.9882	0.000640	0.000637
151.6655	0.000513	0.000510
166.3428	0.000422	0.000412
181.0201	0.000355	0.000349
ω_2 = 195.6974	0.000304	0.000293
220.1595	0.000243	0.000213
244.6217	0.000201	0.000180
269.0839	0.000172	0.000143
293.5461	0.000151	0.000139
318.0082	0.000137	0.000111
342.4704	0.000130	0.000100
366.9326	0.000129	0.000099
391.3947	0.000140	0.000100
415.8569	0.000174	0.000114
ω_3 = 440.3191	0.000203	0.000112
474.5661	0.000060	0.000058
508.8132	0.000017	0.000013
543.0602	0.000005	0.000005

Table 3 Maximum values of dimensioned bending moments amplitudes

Frequencies θ, s^{-1}	For bending moment, kNm	
	The author's method	ANSYS
4.8924	90.9303	90.9393
9.7849	93.8362	93.8732
14.6773	99.0996	99.1469
19.5697	107.5053	107.9006
24.4622	120.5757	120. 9934
29.3546	141.4211	142.1433
34.2470	177.2297	180.0014
39.1395	248.5850	256.4465
44.0319	441.7116	480.9984
ω_1 = 48.9243	1047.5114	1210.1123
63.6016	136.9229	134.4555
78.2789	62.6410	60.9702
92.9562	38.7246	38.3356
107.6336	27.3544	26.7447
122.3109	20.9048	20.8733
136.9882	16.8566	16.8341
151.6655	14.1464	14.0050
166.3428	12.2530	12.0405
181.0201	10.8938	10.9666
ω_2 = 195.6974	9.9044	9.8888
220.1595	8.8213	9.0403
244.6217	8.2375	8.7641
269.0839	8.0346	8.7445
293.5461	8.1815	11.2345
318.0082	8.7243	13.1477
342.4704	9.8206	19.2345
366.9326	11.8649	28.4332
391.3947	15.9119	45.3221
415.8569	25.1526	91.3576
ω_3 = 440.3191	38.7406	111.3787
474.5661	18.3970	76.3744
508.8132	9.7656	41.3222
543.0602	6.4760	26.8564

4 CONCLUSIONS

The maximum values of the amplitudes of the dimensionless dynamic deflections and bending moments are calculated. Analysis of the results shows that the sharp increase in the amplitude values of deflections and dynamic bending moments occur in the resonance regions corresponding to the first natural frequency of oscillation. At higher frequencies this effect is practically absent. At frequencies of the external load, close to the third natural frequency, there is quite a significant difference in the results obtained by the author's and the finite element method. Values obtained by the author, should be regarded as accurate, since they are obtained by the exact solution of the original differential equation by the method of direct integration.

5 REFERENCES

[1] Babakov I. M. Teoriya kolebanii / I. M. Babakov. – M.: Nauka, 1976. – 592 p.

[2] Vasilenko M. V. Teoriya kolyvan' i stiykosti ruhu / M. V. Vasilenko, O. M. Alekseychuk. – K.: Vyscha shkola, 2004. – 525 p.

[3] Krylov A. N. Sobranie trudov. Tom 10: Vibratsii sudov. / A.N. Krylov. – M.: Izd-vo AN USSR, 1948. – 402 p.

[4] Timoshenko S. P. Kolebaniya v inzhenernom dele. M.: Nauka, 1967. – 444 p.
[5] Panovko Y. G. Vnutrennee trenie pri kolebaniyakh uprugikh sistem. / Y.G. Panovko. – M.: Fiznatgiz, 1960. – 193 p.
[6] Sorokin E. S. K teorii vnutrennego treniya pri kolebaniyakh uprugikh sistem. – M.: Gosstroyizdat, 1960. – 131 p.
[7] Olshanskiy V. P. Nestacionarnie kolebaniya mekhanicheskoy sistemi lineyno-peremennoy massy s kombinirovannim treniem / V.P. Olshanskiy, S.V. Olshanskiy // Visnyk Natsionalnogo tehnichnogo universitetu "Kharkivskiy politehnichniy institute". – 2013. – №4 (1041). – P. 324–333.
[8] Olshanskiy V. P. Nestacionarnie kolebaniya oscillatora peremennoy massi s uchetom viazkogo treniya / V. P. Olshanskiy, S. V. Olshanskiy // Vibratsii v tekhnitsi ta tekhologiyakh. – 2014. – Vyp. № 3 (75). – P. 18–27.
[9] Berendeev N. N. Issledovaniye vliyaniya vnutrennego treniya i sposoba vozbuzhdeniya na vynuzhdennie kolebaniya sistemy / Elektronnoe uchebno-metodicheskoye posobie. – Nizhniy Novgorod: Nizhegorodskiy universitet, 2012. – 50 p.
[10] Abu-Hilal M. Forced vibration of Euler–Bernoulli beams by means of dynamic Green functions. // Journal of Sound and Vibration. 267, (2003), pp. 191–207.
[11] Krutiy Yu. S. Forced harmonic oscillations of the Euler–Bernoulli beam with resistance forces: Odeskyi Politechichnyi Universytet. Pratsi. Machine building. process metallurgy. materials science / Yu. S. Krutiy // Pratsi OPU. 3(47)(2015), pp. 9–16.
[12] Daschenko A. F. ANSYS v zadachakh inzhenernoi mekhaniki / A. F. Daschenko, D. V. Lazareva, N. G. Surianinov / Izd. 2-e, pererab. i dop. Pod red. N. G. Surianinova. – Odesa. – Palmira, 2011. – 505 p.

Authors' contacts:

Yurii Krutii,
Odesa State Academy of Civil Engineering and Architecture
Didrikhsona str. 4, 65029 Odesa, Ukraine
yurii.krutii@gmail.com

Nikolay Suryaninov
Odesa State Academy of Civil Engineering and Architecture
Didrikhsona str. 4, 65029 Odesa, Ukraine
sng@ogasa.org.ua