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Determining cycle time for a multi-product FPR model with rework and an improved delivery policy by alternative approach

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SUMMARY

The present study determines the common cycle time for a multi-product finite production rate (FPR) model with rework and an improved delivery policy [1] by an alternative approach. Conventional method to the multi-product FPR problem employs the differential calculus to first prove convexity of the system cost function, then to derive the optimal common production cycle time that minimizes the long-run average system cost per unit time; whereas the proposed approach obtains the optimal cycle time without the need to reference the differential calculus. Such a simplified method may help those practitioners who have insufficient knowledge of calculus to effectively manage the real-life multi-product FPR problem.

KEY WORDS: *finite production rate model, algebraic approach, optimization, multi-product system, common production cycle time, rework, improved delivery policy.*

1. INTRODUCTION

In real-life manufacturing environments, production managers often plan to fabricate multiple products in sequence on a single machine under the common production cycle time policy in order to maximize machine utilization [2]. Dixon and Silver [3] determined lot-sizes for a group of products which are produced at a single work centre. They assumed that the requirements for each product are known period by period, until the end of some common time horizon. For each product, there is a fixed production setup cost, separate linear production and pertaining holding costs. All costs and production rates can vary from product to product. Their objective is to determine lot-sizes so that (1) costs are minimized; (2) no backlogging occurs; and (3) capacity is not exceeded. They proposed a simple heuristic to derive a feasible solution and, through examples, demonstrated that their heuristic can usually generate a very good solution with a relatively small amount of computational effort. Tamura [4] proposed an approximation procedure used for solving a production planning problem for a multistage production system which produces many different components and assembles them into finished products under capacity limitations. A generalized production planning model was built using mixed-integer programming. The solution procedure was approximated by a linear programming method. Different algorithms were developed in detail for a two-stage production problem. Numerical example was provided so as to examine the validity and efficiency of proposed algorithms. Denizel-Sivri and Selcuk Erenguc [5] considered a linear programming model for production planning in certain manufacturing environments where demand is not periodic. Their model was modelled after a T-period planning horizon and could be used for determining lot sizes and planned due dates for N products in a production order set. A polynomial time algorithm was developed for solving the linear program. The algorithm found an optimal solution to the linear program after solving most of T knapsack problems and had a worst case computational bound of O(NT). They also discussed the links between the model and the classical single machine static scheduling problem. Federgruen and Katalan [6] studied periodic base-stock policies for stochastic economic lot scheduling problems. Under periodic base-stock policies, items are produced according to a given periodic item-sequence. Their paper derived effective heuristics for the design of a periodic item-sequence minimizing system-wide costs. This sequence was constructed based on desirable production frequencies for the items, obtained as the solution of lower bound mathematical programs. An extensive numerical study gauges the quality of the proposed heuristics. Sambasivan and Schmidt [7] presented a heuristic procedure for solving multi-plateau, multi-item, capacitated lot sizing problems with inter-plant transfers. The solution procedure used the solution for the un-capacitated problem as a starting point. A smoothing routine has been employed to remove capacity violations. The smoothing routine consists of two modules. Extensive experimentation has been conducted comparing the heuristic solution procedure and LLNDO. The heuristic has been implemented on IBM 3090 mainframe using FORTRAN. Chiu et al. [8] derived the optimal common production cycle time for a multi-item finite production rate model with rework and multi-shipment policy. They focused on a multi-item production-delivery integrated system under a common cycle time policy, a rework process of all nonconforming items, and deliveries of *n* fixed quantity instalments of the finished lots upon completion of reworks, respectively. As a result, a closed-form optimal cycle time that minimizes the long-run average system cost is obtained. Studies of various different aspects of multi-item production planning and optimization issues can also be found in Refs. [9-11].

Multi-delivery policy for transporting finished goods is another commonly adopted inventory issuing policy in real life production-shipment systems. Schwarz et al. [12] examined the fill-rate of a one-warehouse *N*-identical retailer distribution system. An approximation model was adopted from a prior study to maximize system fill-rate subject to a constraint on system safety stock. As a result, properties of fill-rate policy were suggested to provide management when looking into system optimization. Sarker and Khan [13] studied a manufacturing system that procures raw materials from suppliers in a lot and processes them into finished products which are then delivered to outside buyers at fixed points in time. They formulated a general cost model considering both raw materials and finished products and developed a simple procedure accordingly to determine an optimal ordering policy for procurement of raw materials as well as the manufacturing batch size. Abdul-Jalbar et al. [14] examined a multi-echelon inventory system in which one vendor supplies an item to multiple buyers. It was assumed that the vendor produces the item at a finite rate and customer demand occurs for each buyer at a constant rate. Their goal is to determine the order quantities for the buyers and the production and shipment schedule for the vendor in order to minimize the average total cost per unit time. The problem was formulated in terms of integer-ratio policies, followed by the development of a heuristic procedure. Both solution procedures were illustrated with a numerical example. Performance of the heuristic for computing integer-ratio policies was demonstrated. With the purpose of cutting down producer's inventory holding cost in [8], Chiu et al. [1] incorporated an improved *n+1* shipment policy into model of [8]. Under the *n+1* policy, one extra delivery of finished items is made during producer's uptime to satisfy customer's product demands during the periods of producer's uptime and rework times. Then, *n* fixed quantity instalments of finished items are delivered to buyers at the end of rework when the rest of the production lot is quality assured. They derived the optimal common production cycle time that minimizes the long-run average system cost per unit time, and investigated effects of rework and the improved delivery policy on the optimal cycle time and system costs. Additional studies that addressed various aspects of periodic or multiple deliveries issues in vendor-buyer integrated systems can also be found in Refs. [15-20].

In a recent study, Grubbström and Erdem [21] presented an algebraic approach to the economic order quantity model with backlogging, and derived the optimal lot size without reference to derivatives, i.e., by neither applying the first- nor second-order differentiations. A few studies used similar method to solve various different aspects of production-inventory and/or vendor-buyer integrated problems [22-25]. This paper extends such an algebraic approach to re-examine the problem in Chiu et al. [1], and shows that the optimal common production cycle time in their model can be obtained with derivatives.

2. MODELLING AND ANALYSIS

Reconsidering the multi-item finite production rate model with rework and an improved delivery policy [1] as follows: a production plan for L products has been made on a single machine in turn under the common production cycle policy. In the production process, for each product i (where $i = 1, 2, \ldots, L$), an x_i portion of nonconforming items is randomly produced at a rate d_{1i}. All items produced are screened with an inspection cost that is included in unit production cost C_i. It is assumed that all nonconforming items can be repairable at a rate of P_{2i} when regular production ends in each cycle, with unit reworking cost C_{Ri}. In order to prevent shortages, it is assumed that constant production rate for product i, P_{1i} must satisfies $(P_{1i} - d_{1i} - \lambda_i) > 0$, where λ_i is the annual demand rate for product i, and $d_{1i,}$ can be expressed as $d_{1i} = x_i P_{1i}$. With the intention of reducing inventory holding cost, an improved *n+1* multi-shipment policy is considered. Under such a policy, an initial finished goods delivery is made to meet the customers' product demands during producer's uptime and reworking time. Once the rework process ends, n fixed quantity instalments of the finished products are distributed to customers at a fixed time interval t_n (Figure 1).

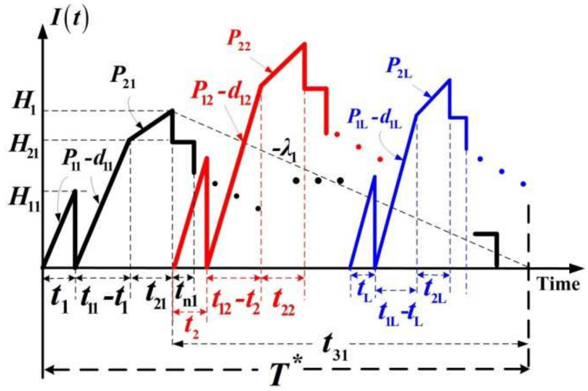

Fig. 1 *On-hand inventory of perfect quality items for product i in the proposed FPR model [1]*

Additional cost related variables used in the proposed cost analysis include: unit holding cost h_i, holding cost h_{1i} per reworked item, the production setup cost K_i, the fixed delivery cost K_{1i} for product i per delivery, and unit shipping cost C_{Ti} for each product i. Other notation used in the modelling and analysis of this paper is listed as follows:

Q_i = production lot size per cycle for product i,

n = number of fixed quantity instalments of the finished batch to be delivered to buyers in each cycle, it is assumed to be a constant for all products,

T = common production cycle length - the decision variable,

H_{1i} = level of on-hand inventory in units for meeting demand of product i during producer's uptime t_{1i} and reworking time t_{2i},

H_{2i} = maximum level of on-hand inventory for product i when the regular production ends,

H_i = maximum level of on-hand inventory in units for product i when the rework process ends,

t_{1i} = production uptime for product i,

t_{2i} = the reworking time for product i,

t_{3i} = the delivery time for product i,

t_i = time required for producing items to meet demand of product i during producer's uptime t_{1i} and reworking time t_{2i},

t_{ni} = fixed interval of time between each instalment of finished product i being delivered during t_{3i},

$I(t)$ = on-hand inventory of perfect quality items at time t,

$TC(Q_i)$ = total production-inventory-delivery costs per cycle for product i,

$E[TCU(Q_i)]$ = expected total production-inventory-delivery costs per unit time for L products in the proposed system,

$E[TCU(T)]$ = expected total production-inventory-delivery costs per unit time in the proposed system using common production cycle time T as decision variable.

For multiple product i, where $i = 1, 2, \dots, L$, the following formulas can be obtained directly from Figure 1:

$$t_i = \frac{H_{1i}}{P_{1i} - d_{1i}} = \frac{\lambda_i (t_{1i} + t_{2i})}{P_{1i} - d_{1i}} \tag{1}$$

$$t_{1i} = \frac{Q_i}{P_{1i}} = \frac{H_{1i} + H_{2i}}{P_{1i} - d_{1i}} \tag{2}$$

$$t_{2i} = \frac{H_i - H_{2i}}{P_{2i}} \tag{3}$$

$$t_{3i} = T - (t_{1i} + t_{2i}) = n t_{ni} \tag{4}$$

$$H_{1i} = \lambda_i (t_{1i} + t_{2i}) \tag{5}$$

$$H_{2i} = (P_{1i} - d_{1i})(t_{1i} - t_i) \tag{6}$$

$$H_i = H_{2i} + (P_{2i}) t_{2i} \tag{7}$$

$$\lambda = \sum_{i=1}^{L} \lambda_i \tag{8}$$

$$T = t_{1i} + t_{2i} + t_{3i} = \frac{Q_i}{\lambda_i} \tag{9}$$

$$d_{1i} \cdot t_{1i} = x_i \cdot Q_i \tag{10}$$

$$t_{2i} = \frac{x_i Q_i}{P_{2i}} \tag{11}$$

Variable holding costs for finished items of product *i* during t_{3i} are [17]:

$$h_i \left(\frac{n-1}{2n} \right) H_i \cdot t_{3i} \tag{12}$$

Fixed and variable transportation costs for product *i* in a production cycle are:

$$(n+1) K_{1i} + C_{Ti} Q_i \tag{13}$$

Total $TC(Q_i)$ for L products (as shown in Eq. (14)) consists of production setup cost, variable manufacturing and reworking costs, total transportation costs, holding cost in the periods of t_{1i} and t_{2i}, and holding cost for finished goods in t_{3i}.

$$\sum_{i=1}^{L} TC(Q_i) = \sum_{i=1}^{L} \left\{ K_i + C_i Q_i + C_{Ri}(x_i Q_i) + (n+1) K_{1i} + C_{Ti} Q_i + h_{1i} \left[\frac{d_{1i} t_{1i}}{2} \cdot (t_{2i}) \right] \right\}$$
$$+ \sum_{i=1}^{L} h_i \left[\frac{H_{1i}}{2}(t_i) + \frac{H_{2i}}{2}(t_{1i} - t_i) + \frac{d_{1i} t_{1i}}{2}(t_{1i}) + \frac{H_{2i} + H_i}{2}(t_{2i}) + \left(\frac{n-1}{2n} \right) H_i t_{3i} \right] \tag{14}$$

Since we assume that the defective rate x is random variable with a known probability density function, in order to cope with the randomness, the expected value of x is employed. Substituting all related system variables in Eq. (14) and with further derivations, $E[TCU(Q_i)]$ is obtained [1] as:

$$E\left[TCU(T)\right]=\sum_{i=1}^{L}\left\{C_i\lambda_i+\frac{K_i}{T}+C_{Ri}\lambda_i E\left[x_i\right]+C_{Ti}\lambda_i+\frac{(n+1)K_{1i}}{T}+\frac{h_{1i}T\lambda_i^{\,2}E\left(x_i\right)^2}{2P_{2i}}\right\}$$
$$+\sum_{i=1}^{L}\left\{\frac{h_iT\lambda_i^{\,2}}{2}\left\{\begin{aligned}&\lambda_i\left[\frac{1}{P_{1i}}+\frac{E\left[x_i\right]}{P_{2i}}\right]^2\left[\frac{2\lambda_i}{P_{1i}(1-E\left[x_i\right])}-1\right]-\frac{E\left[x_i\right]}{P_{2i}}\left[\frac{1}{P_{1i}}-\left[1-E\left[x_i\right]\right]\right]+\frac{1}{P_{1i}}\\&+\left[\frac{1}{\lambda_i}-\frac{2}{P_{1i}}-\frac{E\left[x_i\right]}{P_{2i}}\right]-\left(\frac{1}{n}\right)\left[\left[\frac{1}{\lambda_i}-\frac{2}{P_{1i}}-\frac{E\left[x_i\right]}{P_{2i}}\right]+\lambda_i\left[\frac{1}{P_{1i}}+\frac{E\left[x_i\right]}{P_{2i}}\right]^2\right]\end{aligned}\right\}\right\} \quad \textbf{(15)}$$

Suppose we let E_{ji} (where j = 1, 2, and 3) [1] denote the following:

$$E_{1i}=\left[\frac{1}{P_{1i}}+\frac{E\left[x_i\right]}{P_{2i}}\right];\ E_{2i}=\left[\frac{1}{\lambda_i}-\frac{2}{P_{1i}}-\frac{E\left[x_i\right]}{P_{2i}}\right];\ E_{3i}=\frac{2\lambda_i}{P_{1i}\left[1-E\left[x_i\right]\right]}$$

Equation (15) becomes:

$$E\left[TCU(T)\right]=\sum_{i=1}^{L}\left\{C_i\lambda_i+\frac{K_i}{T}+C_{Ri}\lambda_i E\left[x_i\right]+C_{Ti}\lambda_i+\frac{(n+1)K_{1i}}{T}+\frac{h_{1i}T\lambda_i^{\,2}E\left[x_i\right]^2}{2P_{2i}}\right\}$$
$$+\sum_{i=1}^{L}\left\{+\frac{h_iT\lambda_i^{\,2}}{2}\left\{\begin{aligned}&\lambda_iE_{1i}^2\cdot\left[E_{3i}-1\right]-\frac{E\left[x_i\right]}{P_{2i}}\cdot\left[\frac{1}{P_{1i}}-\left[1-E\left[x_i\right]\right]\right]+\frac{1}{P_{1i}}\\&+E_{2i}-\left(\frac{1}{n}\right)\left[E_{2i}+\lambda_iE_{1i}^2\right]\end{aligned}\right\}\right\} \quad \textbf{(16)}$$

3. OPTIMAL CYCLE TIME BY ALGEBRAIC APPROACH

Instead of using conventional differential calculus method, this study proposes an alternative algebraic approach. From Eq. (16) it can be seen that in the right-hand side (RHS) of Eq. (16) the decision variable T is in different forms as T^0, T^{-1}, and T^1. Let z_1, z_2, and z_3 denote the following:

$$z_1=\sum_{i=1}^{L}\left[C_i\lambda_i+C_{Ri}\lambda_iE\left(x_i\right)+C_{Ti}\lambda_i\right] \quad \textbf{(17)}$$

$$z_2=\sum_{i=1}^{L}\left[K_i+(n+1)K_{1i}\right] \quad \textbf{(18)}$$

$$z_3=\sum_{i=1}^{L}\frac{h_i\lambda_i^{\,2}}{2}\left\{\lambda_iE_{1i}^2\left[E_{3i}-1\right]-\frac{E\left[x_i\right]}{P_{2i}}\left[\frac{1}{P_{1i}}-\left[1-E\left[x_i\right]\right]\right]+\frac{1}{P_{1i}}+E_{2i}-\left(\frac{1}{n}\right)\left[E_{2i}+\lambda_iE_{1i}^2\right]\right\} \quad \textbf{(19)}$$

Eq. (16) can be rearranged as:

$$E\left[TCU(T)\right]=z_1+z_2T^{-1}+z_3T \quad \textbf{(20)}$$

One can further rearrange Eq. (20) as:

$$E\left[TCU(T)\right]=z_1+T^{-1}\left(z_2+z_3T^2\right) \quad \textbf{(21)}$$

or:

$$E\left[TCU(T)\right]=z_1+\left[\left(\sqrt{z_2}\right)^2+\left(\sqrt{z_3}T\right)^2\right]T^{-1}-2\sqrt{z_2}\left(\sqrt{z_3}T\right)T^{-1}+2\sqrt{z_2}\left(\sqrt{z_3}T\right)T^{-1} \quad \textbf{(22)}$$

or:

$$E\left[TCU(T)\right] = z_1 + \left[\sqrt{z_2} - \left(\sqrt{z_3}T\right)\right]^2 T^{-1} + 2\sqrt{z_2}\sqrt{z_3} \quad \textbf{(23)}$$

It can be seen that if the second term in the RHS of Eq. (23) equals zero, then $E[TCU(T)]$ is minimized. That is if:

$$\sqrt{z_2} = \sqrt{z_3}T \quad \textbf{(24)}$$

or:

$$T = \sqrt{\frac{z_2}{z_3}} \quad \textbf{(25)}$$

Substituting Eqs. (18) and (19) in Eq. (25) one obtains the optimal common production cycle time T^* as:

$$T^* = \sqrt{\frac{\sum_{i=1}^{L}\left[K_i + (n+1)K_{1i}\right]}{\sum_{i=1}^{L}\frac{h_i\lambda_i^2}{2}\left\{\lambda_i E_{1i}^2\left[E_{3i}-1\right] - \frac{E[x_i]}{P_{2i}}\left[\frac{1}{P_{1i}} - \left[1-E[x_i]\right]\right] + \frac{1}{P_{1i}} + E_{2i} - \left(\frac{1}{n}\right)\left[E_{2i} + \lambda_i E_{1i}^2\right]\right\}}} \quad \textbf{(26)}$$

It is noted that Eq. (26) is identical to that obtained by using conventional differential calculus method in Ref. [1]. Finally, by applying the optimal T^* (i.e., substituting Eq. (24) in Eq. (23)) the following simplified formula of $E[TCU(T)]$ can be obtained:

$$E\left[TCU(T)\right] = z_1 + 2\sqrt{z_2}\sqrt{z_3} \quad \textbf{(27)}$$

3.1 VERIFICATION WITH NUMERICAL EXAMPLE

In this section, in order to verify the aforementioned result we use the same numerical example as in Chiu et al. [1]. Consider that a production plan is to manufacture five products in turns on a single machine under the common production cycle policy. For each production i, where $i = 1, 2, \ldots, 5$, the production rates P_{1i} are *58000*, *59000*, *60000*, *61000*, and *62000*, and the annual demand λ_i for five different products is *3000*, *3200*, *3400*, *3600*, and *3800*, respectively. Defective rates x_i in production uptime for each product i follow a Uniform distribution over the intervals of [*0*, *0.05*], [*0*, *0.10*], [*0*, *0.15*], [*0*, *0.20*] and [*0*, *0.25*], respectively. All defective items are assumed to be repairable at the reworking rates P_{2i} of *1800*, *2000*, *2200*, *2400*, and *2600*, respectively. The additional unit cost for rework is *$50*, *$55*, *$60*, *$65*, and *$70*. Other parameters in the system include:

K_i = the production set up costs are *$3800*, *$3900*, *$4000*, *$4100*, and *$4200*, respectively.

K_{1i} = fixed costs per delivery are *$1800*, *$1900*, *$2000*, *$2100*, and *$2200*, respectively.

C_{Ti} = unit transportation costs are *$0.1*, *$0.2*, *$0.3*, *$0.4*, and *$0.5*, respectively.

n = number of shipments per cycle, in this study it is assumed to be 3 (i.e., $n+1 = 4$).

C_i = unit production costs are *$80*, *$90*, *$100*, *$110*, and *$120*, respectively.

h_i = unit holding costs are *$10*, *$15*, *$20*, *$25*, and *$30*, respectively.

h_{1i} = unit holding costs per reworked are $30, $35, $40, $45, and $50, respectively.

Applying Eq. (26), one obtains optimal common production cycle time $T^*=0.7238$ (years). Substituting Eqs. (17) to (18) in Eq. (27) one obtains the expected total production-inventory-delivery costs per unit time $E[TCU(T^*=0.7238)] = \$1,975,584$. Both the aforementioned results are identical to that obtained in Chiu et al. [1].

4. CONCLUDING REMARKS

A simplified algebraic approach is proposed in this study to re-examine the multi-product FPR problem with rework and an improved delivery policy [1]. As a result, it is demonstrated that the optimal common production cycle time can be derived without using the derivatives. Such an alternative approach may help those practitioners who have insufficient knowledge of calculus to manage more effectively the real-life multi-product FPR problems.

5. ACKNOWLEDGEMENTS

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