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A delayed differentiation multi-product FPR model with scrap and a multi-delivery policy – II: Using two-machine production scheme

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SUMMARY

This paper concerns a delayed differentiation multi-product finite production rate (FPR) model with scrap and multi-delivery policy using a two-machine production scheme. Conventional FPR model considers a single product, single-stage production with all products fabricated being of perfect quality, and product demand satisfied by a continuous inventory issuing policy. However, in real vendor-buyer integrated systems, most vendors would adopt a multi-product production plan to maximize machine utilization. They often use a periodic or multi-shipment policy to distribute their finished products. When planning to produce a cluster of multiple products that share a common intermediate part, the vendor would often evaluate a two-stage production scheme. The first stage manufactures only the common parts for all products and the second stage separately manufactures the end products. The aim is to shorten the replenishment cycle time and reduce overall production-inventory related costs. This study considers a two-machine production scheme and the two-stage production process with the objective of determining the optimal production cycle time and number of deliveries. A numerical example with sensitivity analysis is provided to demonstrate practical use of the obtained results as well as to compare the proposed production scheme to that of using a single machine in the multi-product two-stage FPR model.

KEY WORDS: *finite production rate model, delayed product differentiation, two-stage production, two-machine production, multi-product system, common intermediate part, scrap, multi-delivery.*

1. INTRODUCTION

Delayed differentiation strategy is often considered when *common intermediate part* exists in multi-product manufacturing systems to shorten the production time of the finished products and/or to reduce the overall production-inventory related costs as presented in Refs. [1-14] and Part I of this paper. Gerchak et al. [1] developed a model for an arbitrary number of products with general joint demand distribution. Whereas utilizing commonality is beneficial, they stated that nothing general can be said about the resulting change in the components' stock levels. When the cost structure is of a particular simple form, though, some interesting general patterns do emerge. They discussed the case of using a service-level measure where rationing of common components might be required, and characterized the implied rationing rule. Davis and Sasser [2] discussed a rediscovery of 90's-style manufacturing. It is the idea of keeping the product generic as long as possible in the supply chain to balance inventory savings and customer service with product design, material, and manufacturing costs. The technique is commonly called postponement, because careful analysis generally has shown that postponing product differentiation beyond the factory leads to lower overall costs. Swaminathan and Tayur [3] indicated that, with the aim of lowering the cost while maintaining good customer service, some of the leading manufacturers in the computer industry are delaying product differentiation while managing broader product lines. However, they indicated that finding the optimal configurations and inventory levels of the vanilla boxes could be a challenging task. Accordingly, they constructed a model of two-stage integer program with recourse. By utilizing structural decomposition of the problem and derivative methods, they provided an effective solution procedure. They also compared the performance of the vanilla assembly process to make-to-stock and assemble-to-order processes and provided managerial insights on the conditions under which one might be better than the others. Thonemann and Brandeau [4] presented an approach to determine the optimal level of component commonality for end-product components that do not differentiate models from the customer's perspective. Their work was inspired by and applied to a wire-harness design problem faced by a major automobile manufacturer. They modelled the component design problem as a mathematical program that considers production, inventory holding, setup, and complexity costs (the cost in indirect functions caused by component variety). Two approaches were used to solve the problem: a branch-and-bound algorithm that can solve small and medium-size problems optimally, and a simulated annealing algorithm that can solve large-size problems heuristically. They applied both algorithms to the wire-harness design problem and showed that an optimal design achieves high cost savings by using significantly fewer variants than a no-commonality design but significantly more variants than a full-commonality design. Sensitivity analysis was conducted to identify extreme conditions under which the no- commonality and full-commonality designs perform well, and they identified the key cost drivers for their application. Gupta and Benjaafar [5] developed models to compute the costs and benefits of delaying differentiation in series production systems when the order lead times are load dependent. The models were then used to gain insights through analytical and numerical comparisons. Through examining patterns in a large number of numerical experiments they studied the effect of congestion in the make-to-stock (MTS) and make-to-order (MTO) stages is asymmetric with tighter capacity at the MTO stage having a greater detrimental effect on the desirability of delaying differentiation. They also concluded that if there is flexibility in choosing the point of differentiation, higher loading is observed to favour later differentiation, and if the sequence in which work is performed can be affected, then placing workstations that have a tighter capacity in the MTS stage lowers costs.

Heese and Swaminathan [6] analyzed a stylized model of a manufacturer who designs a product line consisting of two products for sale to two market segments with different valuations of quality. They assumed that the manufacturer determines the component quality levels, the amount of effort to reduce production costs, and whether to use common or different components for the two products. Explicitly considering potential interdependencies between cost-reduction effort and quality decisions, they characterized environments where the optimal product line involving component commonality features products of higher quality and yields higher revenues. Contrary to earlier research, they showed that it can be preferable to make those components common so that, relative to their production cost, they are attributed a higher importance by customers. Al-Salim and Choobineh [7] proposed two nonlinear binary optimization models for determining the optimal stage for differentiating each product. The first model maximizes the expected value of profit and the second model maximizes the value of options to postpone product differentiation. They used the results from financial basket options pricing theory to formulate and solve the second model. A taboo-constrained randomized search algorithm was used to obtain models solution, and parametric analysis of a numerical example is adopted to verify the models and gain insight into the product differentiation problem. Additional studies that are related to the delayed product differentiation issues may also be found in Refs. [8-14] and Part I of this paper. This study examines a delayed differentiation strategy in two-stage multi-product manufacturing systems using a two-machine production scheme. Details of problem statement and modelling are given in the following section.

2. THE PROPOSED PRODUCTION SCHEME

A delayed differentiation two-stage multi-product FPR model with common intermediate part, scrap, and a multi-delivery policy is explored in this study using a two-machine production scheme. Consider a family of L products to be fabricated and they share a common intermediate part. Under the proposed production scheme, a machine is used in the first stage to manufacture only the common parts for all products (see Figure 1(a)). Then, in the second stage, the end products are produced in turn by a separate machine using the common production cycle time policy (see Figure 1(b)). The purpose of utilizing such a two-machine production scheme for the multi-product manufacturing system is to shorten the production cycle time for fabricating the end products and lower overall production-inventory related cost.

In the first stage, the machine 1 produces all common intermediate parts at a rate of $P_{1,0.}$ Then, in the second stage, machine 2 fabricates the different L end products at a rate of $P_{1,i,}$ where $i = 1, 2, \ldots, L$, and it is under the common production cycle approach (see Figures 1(a) and 1(b)). It is assumed that during the production cycle in each stage, there is x_i portion of defective items produced randomly at a rate $d_{1,I}$; these random defective items are considered to be scrap, and they will be removed in the end of production cycle. The constant production rate $P_{1,i}$ is larger than the sum of demand rate λ_i and production rate of scrap items $d_{1,i}$. That is: $(P_{1,i} - d_{1,i} - \lambda_i) > 0$ for $i = 0, 1, 2, \ldots, L$; where $i = 0$ denote the *common intermediate part* and $d_{1,i} = P_{1,i}(x_i)$. In the proposed model, it is also assumed that the delivery of each end product in the second stage starts at the end of production when the whole production lot is quality assured for each product i. Fixed quantity n instalments of the finished batch are distributed to the customer at a fixed interval of time during the production downtime $t_{2,i}$ of each product (see Figure 1(b)). The cost related parameters used in the proposed model include the following: unit production cost C_i, production setup cost K_i, unit holding cost $h_{1,i}$, fixed delivery cost $K_{1,i}$

per shipment, delivery cost $C_{T,i}$ per item shipped to customers, and disposal cost per scrap item $C_{S,i}$. Other notations used in the modelling and analysis of this study can be referred to Appendix A.

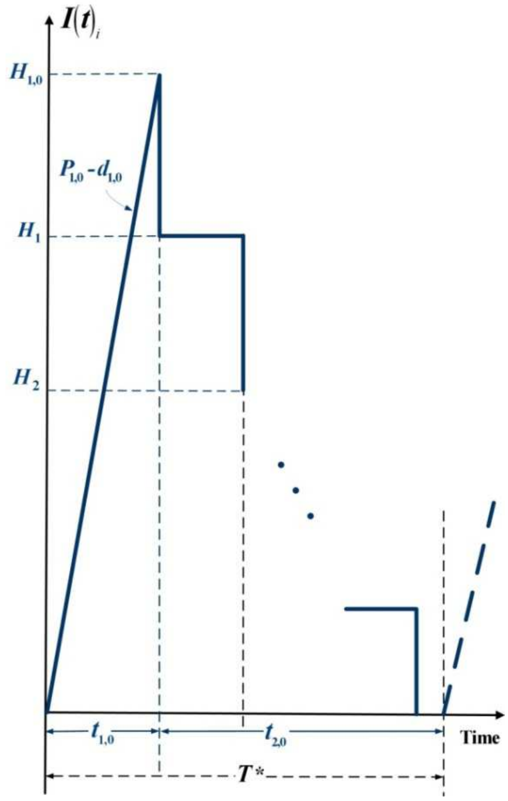

Fig. 1(a) *On-hand inventory level of perfect quality common intermediate parts in the stage 1 (machine 1) of the proposed multi-product FPR system with delayed product differentiation using a two-machine production scheme*

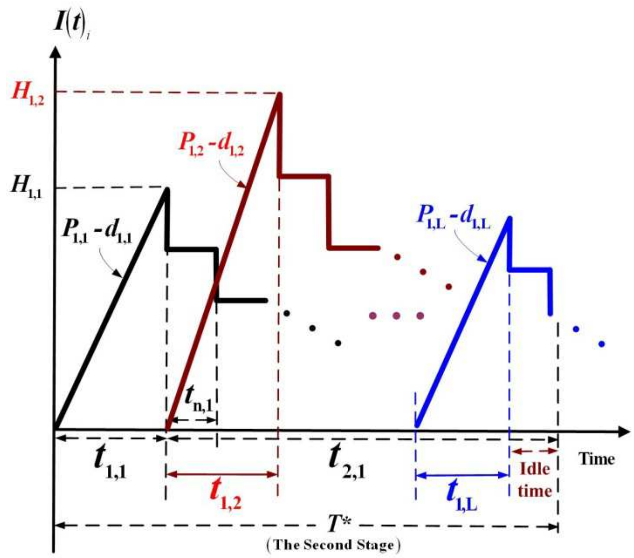

Fig. 1(b) *On-hand inventory level of perfect quality finished products in the stage 2 (machine 2) of the proposed multi-product FPR system with delayed product differentiation using a two-machine production scheme*

The on-hand inventory level of common intermediate parts waiting to be finished during the second stage is depicted in Figure 2. The on-hand inventory level of defective items produced during stages 1 and 2 of each production cycle is shown in Figures 3(a) and 3(b), respectively.

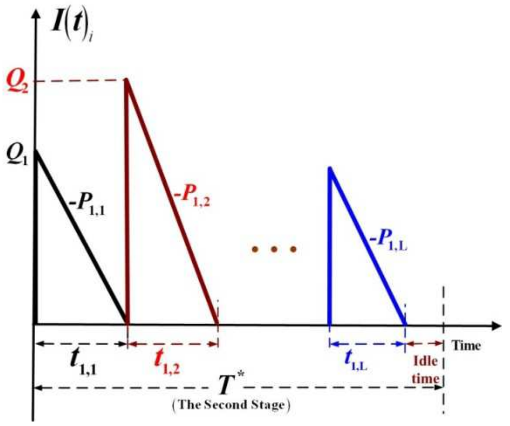

Fig. 2 *On-hand inventory level of common intermediate parts waiting to be finished during the second stage*

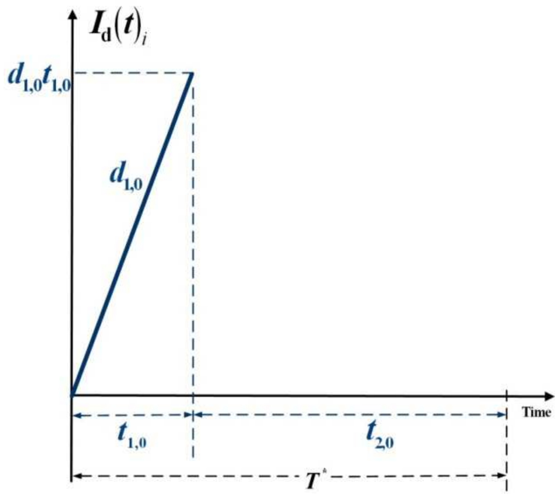

Fig. 3(a) *On-hand inventory level of defective common intermediate parts in the stage 1 (machine 1) of the proposed model during the production cycle*

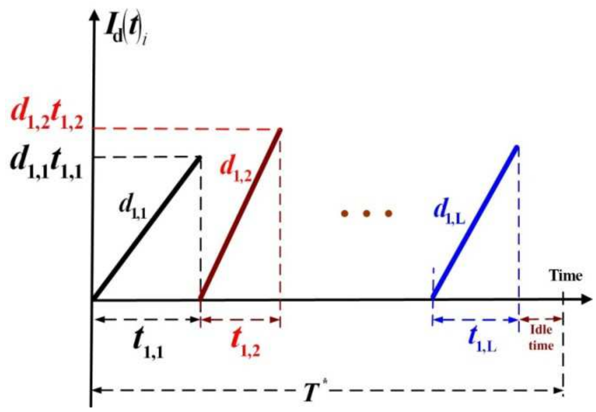

Fig. 3(b) *On-hand inventory level of defective end products in the stage 2 (machine 2) of the proposed model during the production cycle*

The proposed two-stage multi-product FPR model using a two-machine production scheme releases the workloads of fabricating common intermediate parts from the machine 2. Therefore, we expect to have a more efficient production of the end products during the second stage of production. The solution procedure starts with determining the optimal common production cycle time for the second stage. Then we apply the obtained cycle length to the stage of fabricating the common intermediate parts.

First, we focus on mathematical modelling of multi-product FPR model of machine 2. The following assumption must satisfy to ensure that there is *sufficient capacity* in machine 2 to fabricate these L products on a rotation cycle basis.

$$\sum_{i=1}^{L} t_{1,i} < T \quad or \quad \sum_{i=1}^{L} \frac{Q_i}{P_{1,i}} < T \tag{1}$$

or:

$$\sum_{i=1}^{L} \frac{\lambda_i}{\left[1-E[x_i]\right]P_{1,i}} < 1 \tag{2}$$

Then, by observing Figures 1(b), 2, and 3(b), we obtain the following equations directly:

$$T = t_{1,i} + t_{2,i} \quad for\ i = 0, 1, 2, ..., L \tag{3}$$

$$t_{1,i} = \frac{H_{1,i}}{P_{1,i} - d_{1,i}} \quad for\ i = 0, 1, 2, ..., L \tag{4}$$

$$H_{1,i} = \left(P_{1,i} - d_{1,i}\right)t_{1,i} \quad for\ i = 0, 1, 2, ..., L \tag{5}$$

$$t_{2,i} = nt_{n,i} \quad for\ i = 0, 1, 2, ..., L \tag{6}$$

$$t_{1,i} = \frac{Q_i}{P_{1,i}},\ for\ i = 0, 1, 2, ..., L \tag{7}$$

$$Q_i = \frac{\lambda_i T}{1-E[x_i]},\ for\ i = 0, 1, 2, ..., L \tag{8}$$

$$d_{1,i} \cdot t_{1,i} = t_{1,i} \cdot P_{1,i} \cdot x_i,\ for\ i = 0, 1, 2, ..., L \tag{9}$$

The on-hand inventory of end products at the customer's side during the cycle is depicted in Figure 4.

From Figures 1(b) and 4, one can observe the following:

$$D_i = \frac{H_{1,i}}{n},\ for\ i = 1...L \tag{10}$$

$$I_i = D_i - (\lambda_i)(t_{n,i}) \tag{11}$$

$$nI_i = \lambda_i\left(t_{1,i}\right) \tag{12}$$

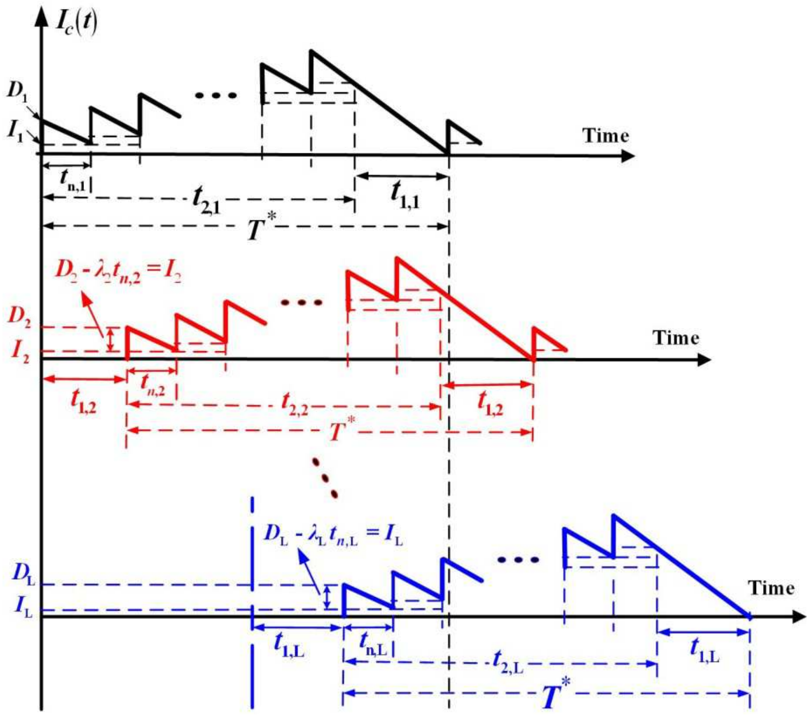

Fig. 4 *On-hand inventory of finished customized products at the customer's side during the production cycle*

Total production-inventory-delivery costs per cycle for stage 2 of the proposed model, $TC_2(T, n)$ includes the variable production costs, setup costs, disposal costs, holding costs for *multiple customized products*; costs of safety stocks; the fixed and variable delivery costs, and the holding costs for the stocks kept at customers' side [15]. Hence, $TC_2(T, n)$ can be expressed as follows:

$$TC_2(T, n) = \sum_{i=1}^{L} \left\{ \begin{array}{l} C_i Q_i + K_i + C_{S,i}\left[x_i Q_i\right] + nK_{1,i} + C_{T,i}\left[Q_i\left(1-x_i\right)\right] + h_{4,i}\left(x_i Q_i\right)T + \\ +h_{1,i}\left[\frac{Q_i}{2}\left(t_{1,i}\right) + \frac{H_{1,i}t_{1,i}}{2} + \left(\frac{n-1}{2n}\right)H_{1,i}t_{2,i} + \frac{d_{1,i}t_{1,i}}{2}\left(t_{1,i}\right)\right] + \\ +h_{3,i}\left[\frac{n\left(D_i - I_i\right)t_{n,i}}{2} + \frac{n(n+1)}{2}I_i t_{n,i} + \frac{nI_i\left(t_{1,i}\right)}{2}\right] \end{array} \right\} \quad (13)$$

In order to supply sufficient common intermediate parts in time to meet the demands of fabrication of end products in the second stage, machine 1 must start producing the common intermediate parts $t_{1,0}$ ahead of time (see Figures 1(a) and 2).

By observing Figures 1(a), 2, and 3(a), we obtain the following formulations directly:

$$H_{1,0} = \sum_{i=1}^{L} Q_i \quad (14)$$

$$Q_0 = \frac{\sum_{i=1}^{L} Q_i}{1 - E\left[x_0\right]} \quad (15)$$

$$T = t_{1,0} + t_{2,0} \quad (16)$$

$$\lambda_0 = \frac{\sum_{i=1}^{L} Q_i}{T} \quad , \; for \; i = 0, 1, 2, ..., L \tag{17}$$

$$Q_0 = \frac{\lambda_0 T}{1 - E[x_0]} \tag{18}$$

$$t_{1,0} = \frac{H_{1,0}}{P_{1,0} - d_{1,0}} \tag{19}$$

$$H_1 = H_{1,0} - Q_1 \tag{20}$$

$$H_i = H_{(i-1)} - Q_i \; , \; for \; i = 0, 1, 2, ..., L \tag{21}$$

$$H_L = H_{(L-1)} - Q_L = 0 \tag{22}$$

Total production-inventory-delivery costs per cycle in the stage 1 of the proposed model $TC_1(T, n)$ includes the variable production cost, setup cost, disposal cost, cost for safety stocks, and the holding cost [16]. Hence, $TC_1(T, n)$ can be expressed as follows:

$$TC_1(T, n) = C_0 Q_0 + K_0 + C_{S,0}[x_0 Q_0] + h_{4,0}(x_0 Q_0)T + + h_{1,0}\left[\frac{H_{1,0} t_{1,0}}{2} + \sum_{i=1}^{L}\left[H_i\left(t_{1,i}\right)\right] + \frac{d_{1,0} t_{1,0}}{2}\left(t_{1,0}\right)\right] \tag{23}$$

Again, the prerequisite assumption for the stage 1 of the proposed model is the following:

$$\frac{\lambda_0}{\left[1 - E[x_0]\right] P_{1,0}} < 1 \tag{24}$$

By substituting Eqs. (3) to (12) in Eq. (13) and taking the randomness of defective rate in production into account, and with further derivation the long-run average costs per unit time for stage 2 of the proposed model, $E[TCU_2(T, n)]$ can be derived as follows:

$$E\left[TCU_2(T, n)\right] = = \sum_{i=1}^{L}\left\{\begin{array}{l} \left[C_i \lambda_i E_{0i} + \dfrac{K_i}{T} + C_{s,i}\lambda_i E_{1i} + \dfrac{nK_{1,i}}{T} + C_{T,i}\lambda_i\right] + Th_{4,i}\lambda_i E_{1i} + \\ + \dfrac{h_{1,i} T \lambda_i^2}{2}\left\{\left(1 + \dfrac{1}{n}\right)\dfrac{E_{0i}^2}{P_{1,i}} + \left(1 - \dfrac{1}{n}\right)\left[\dfrac{1 - E[x_i]}{\lambda_i} E_{0i} + \dfrac{E_{0i} E_{1i}}{P_{1,i}}\right]\right\} + \\ + \dfrac{h_{3,i} T \lambda_i^2}{2}\left\{\dfrac{2E_{0i}}{P_{1,i}} - \dfrac{1}{\lambda_i} + \left(1 + \dfrac{1}{n}\right)\left[\dfrac{1 - E[x_i]}{\lambda_i} E_{0i} - \dfrac{E_{0i}^2}{P_{1,i}} + \dfrac{E_{0i} E_{1i}}{P_{1,i}}\right]\right\} \end{array}\right\} \tag{25}$$

where:

$$E_{0i} = \frac{1}{(1 - E[x_i])} \qquad for \;\; i = 1, 2, ..., L \; ;$$

$$E_{1i} = \frac{E[x_i]}{(1 - E[x_i])} \qquad for \;\; i = 1, 2, ..., L$$

Similarly, substituting Eqs. (14) to (22) in Eq. (23) and taking randomness of defective rate into account, and with further derivation, the long-run average costs per unit time for stage 1 of the proposed model $E[TCU_1(T, n)]$ can be derived as follows:

$$E\left[TCU_1(T,n)\right] = \\ = \left\{ \begin{array}{l} C_0\lambda_0 E_{00} + \dfrac{K_0}{T} + C_{S,0}\lambda_0 E_{10} + \dfrac{h_{1,0}\lambda_0^2 T}{2}\left(E_{00}\right)^2\left[\dfrac{1}{P_{1,0}}\right] + Th_{4,0}\lambda_0 E_{10} + \\ + T\cdot h_{1,0}\sum_{i=1}^{L}\left(\dfrac{\lambda_i E_{0i}}{P_{1,i}}\cdot\sum_{i=1}^{L}\left(\lambda_i E_{0i}\right)\right) + Th_{1,0}\sum_{i=1}^{L}\left(-\dfrac{\lambda_i E_{0i}}{P_{1,i}}\cdot\sum_{j=1}^{i}\left(\lambda_j E_{0j}\right)\right) \end{array} \right\} \tag{26}$$

where:

$$E_{00} = \frac{1}{\left(1-E[x_0]\right)};\; E_{10} = \frac{E[x_0]}{\left(1-E[x_0]\right)};$$

$$E_{0j} = \frac{1}{\left(1-E[x_j]\right)} \quad for\ j = 1,\ldots,i$$

The long-run average costs per unit time for the proposed model $E[TCU(T, n)]$ includes the expected system costs for stages 1 and 2. Hence, it is as follows:

$$E\left[TCU(T,n)\right] = E\left[TCU_1(T,n)\right] + E\left[TCU_2(T,n)\right] \tag{27}$$

3. DETERMINING THE OPTIMAL PRODUCTION-SHIPMENT POLICY

First, we apply the following Hessian matrix [17] to verify existence of minimum system cost function $E[TCU_2(T, n)]$.

$$\begin{bmatrix} T & n \end{bmatrix}\cdot\begin{bmatrix} \dfrac{\partial^2 E\left[TCU_2(T,n)\right]}{\partial T^2} & \dfrac{\partial^2 E\left[TCU_2(T,n)\right]}{\partial T\partial n} \\ \dfrac{\partial^2 E\left[TCU_2(T,n)\right]}{\partial T\partial n} & \dfrac{\partial^2 E\left[TCU_2(T,n)\right]}{\partial n^2} \end{bmatrix}\cdot\begin{bmatrix} T \\ n \end{bmatrix} > 0 \tag{28}$$

From Eq. (25), we have:

$$\frac{\partial E\left[TCU_2(T,n)\right]}{\partial T} = \sum_{i=1}^{L}\left\{ \begin{array}{l} \left[\dfrac{-K_i}{T^2} - \dfrac{nK_{1,i}}{T^2}\right] + h_{4,i}\lambda_i E_{1i} + \\ + \dfrac{h_{1,i}\lambda_i^2}{2}\left\{ + \left(1+\dfrac{1}{n}\right)\left[\dfrac{E_{0i}^2}{P_{1,i}}\right] + \left(1-\dfrac{1}{n}\right)\left[\dfrac{1-E[x_i]}{\lambda_i}E_{0i} + \dfrac{E_{0i}E_{1i}}{P_{1,i}}\right]\right\} + \\ + \dfrac{h_{3,i}\lambda_i^2}{2}\left\{\dfrac{2E_{0i}}{P_{1,i}} - \dfrac{1}{\lambda_i} + \left(1+\dfrac{1}{n}\right)\left[\dfrac{1-E[x_i]}{\lambda_i}E_{0i} - \dfrac{E_{0i}^2}{P_{1,i}} + \dfrac{E_{0i}E_{1i}}{P_{1,i}}\right]\right\} \end{array} \right\} \tag{29}$$

$$\frac{\partial E^2\left[TCU_2(T,n)\right]}{\partial T^2} = \sum_{i=1}^{L}\left\{\frac{2K_i}{T^3} + \frac{2nK_{1,i}}{T^3}\right\} \tag{30}$$

$$\frac{\partial E\left[TCU_2(T,n)\right]}{\partial n} = \sum_{i=1}^{L}\left\{\frac{K_{1,i}}{T} + \frac{T\lambda_i^2}{2n^2}\left\{\left(h_{1,i} - h_{3,i}\right)\left[\frac{1-E[x_i]}{\lambda_i}E_{0i} + \frac{E_{0i}E_{1i}}{P_{1,i}} - \frac{E_{0i}^2}{P_{1,i}}\right]\right\}\right\} \tag{31}$$

$$\frac{\partial E^2\left[TCU_2(T,n)\right]}{\partial n^2}=\sum_{i=1}^{L}\left\{\frac{T\lambda_i^{\,2}}{n^3}\left\{\left(h_{3,i}-h_{1,i}\right)\left[\frac{1-E[x_i]}{\lambda_i}E_{0i}+\frac{E_{0i}E_{1i}}{P_{1,i}}-\frac{E_{0i}^2}{P_{1,i}}\right]\right\}\right\} \tag{32}$$

$$\frac{\partial^2 E\left[TCU_2(T,n)\right]}{\partial T\partial n}=\sum_{i=1}^{L}\left\{-\frac{K_{1,i}}{T^2}+\frac{\lambda_i^{\,2}}{2n^2}\left\{\left(h_{1,i}-h_{3,i}\right)\left[\frac{1-E[x_i]}{\lambda_i}E_{0i}+\frac{E_{0i}E_{1i}}{P_{1,i}}-\frac{E_{0i}^2}{P_{1,i}}\right]\right\}\right\} \tag{33}$$

By substituting Eqs. (30), (32), and (33) in Eq. (28), we obtain:

$$\begin{bmatrix}T & n\end{bmatrix}\cdot\begin{bmatrix}\dfrac{\partial^2 E\left[TCU_2(T,n)\right]}{\partial T^2} & \dfrac{\partial^2 E\left[TCU_2(T,n)\right]}{\partial T\partial n}\\ \dfrac{\partial^2 E\left[TCU_2(T,n)\right]}{\partial T\partial n} & \dfrac{\partial^2 E\left[TCU_2(T,n)\right]}{\partial n^2}\end{bmatrix}\cdot\begin{bmatrix}T\\ n\end{bmatrix}=\sum_{i=1}^{L}\frac{2K_i}{T}>0 \tag{34}$$

Equation (34) results in positive value, because K_i and T are positive. Hence, $E[TCU_2(T, n)]$ is a strictly convex function for all T and n different from zero. Once the convexity of $E[TCU_2(T, n)]$ is proved, we know that there is a minimum $E[TCU_2(T, n)]$. In order to jointly determine the production-shipment policy for the stage 2 of the proposed multi-product FPR model, we can solve the linear system of equations obtained by setting partial derivatives defined by the Eqs. (29) and (31) equal to zero. With further derivations, the following replenishment cycle time and number of deliveries are obtained as follows:

$$T^*=\sqrt{\frac{\sum_{i=1}^{L}\left[K_i+nK_{1,i}\right]}{\sum_{i=1}^{L}\left\{\begin{array}{l}\dfrac{h_{1,i}\lambda_i^{\,2}}{2}\left\{\left(1+\dfrac{1}{n}\right)\left[\dfrac{1}{P_{1,i}}E_{0i}^2\right]+\left(1-\dfrac{1}{n}\right)\left[\dfrac{1-E[x_i]}{\lambda_i}E_{0i}+\dfrac{E_{0i}E_{1i}}{P_{1,i}}\right]\right\}+\\ +\dfrac{h_{3,i}\lambda_i^{\,2}}{2}\left\{\begin{array}{l}\dfrac{2E_{0i}}{P_{1,i}}-\dfrac{1}{\lambda_i}\\ +\left(1+\dfrac{1}{n}\right)\left[\dfrac{1-E[x_i]}{\lambda_i}E_{0i}-\dfrac{E_{0i}^2}{P_{1,i}}+\dfrac{E_{0i}E_{1i}}{P_{1,i}}\right]\end{array}\right\}+h_{4,i}\lambda_iE_{1i}\end{array}\right\}}} \tag{35}$$

and:

$$n^*=\sqrt{\frac{\sum_{i=1}^{L}\left[K_i\right]\cdot\sum_{i=1}^{L}\left\{\dfrac{\lambda_i^{\,2}}{2}\left(h_{3,i}-h_{1,i}\right)\left[\dfrac{1-E[x_i]}{\lambda_i}E_{0i}+\dfrac{E_{0i}E_{1i}}{P_{1,i}}-\dfrac{E_{0i}^2}{P_{1,i}}\right]\right\}}{\left(\sum_{i=1}^{L}\left(K_{1,i}\right)\right)\sum_{i=1}^{L}\left\{\begin{array}{l}\dfrac{h_{1,i}\lambda_i^{\,2}}{2}\left\{\dfrac{E_{0i}^2}{P_{1,i}}+\dfrac{1-E[x_i]}{\lambda_i}E_{0i}+\dfrac{E_{0i}E_{1i}}{P_{1,i}}\right\}+h_{4,i}\lambda_iE_{1i}+\\ +\dfrac{h_{3,i}\lambda_i^{\,2}}{2}\left\{\dfrac{2E_{0i}}{P_{1,i}}-\dfrac{1}{\lambda_i}+\dfrac{1-E[x_i]}{\lambda_i}E_{0i}-\dfrac{E_{0i}^2}{P_{1,i}}+\dfrac{E_{0i}E_{1i}}{P_{1,i}}\right\}\end{array}\right\}}} \tag{36}$$

In real production-shipment system, the number of deliveries n takes on integer values only. In order to determine the integer value of n^* that minimizes the cost function $E[TCU_2(T, n)]$, two adjacent integers to n (as derived from Eq.(36)) must be examined, respectively [16]. Let n^+ denote the smallest integer greater than or equal to n and n^- denote the largest integer less than or equal to n. We perform the substitution of n^+ and n^- respectively in Eq. (35), then apply the resulting (T, n^+) and (T, n^-) in Eq. (25), respectively, and finally select the one that gives the minimum value of $E[TCU_2(T, n)]$ as the optimal production-shipment policy (T^*, n^*).

4. NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS

In order to facilitate the comparison efforts, in this section we used the same numerical example as given by prior studies of a single-stage multi-product FPR model [15] and a two-stage single machine multi-product FPR model presented in Part I of the paper. The following system parameters are used in the proposed multi-product two-stage FPR model using two machines and $i = 1, 2, ..., L$:

$P_{1,0}$ = *120000* units, the production rate of the common intermediate part;

C_0 = *$40*, unit manufacturing cost for the common intermediate part;

K_0 = *$8500*, production setup cost for the common intermediate part;

$C_{S,0}$ = *$20*, unit scrap cost for the common intermediate part;

$h_{1,0}$ = *$5*, unit holding cost for the common intermediate part;

x_0 = [*0, 0.04*], the range of uniformly distributed defective rate in the production of the common intermediate part;

λ_i = annual product demands are *3000*, *3200*, *3400*, *3600*, and *3800* units respectively;

K_i = production set up costs are *$8500*, *$9000*, *$9500*, *$10000*, and *$10500* respectively;

C_i = unit manufacturing costs are *$40*, *$50*, *$60*, *$70*, and *$80* respectively;

h_i = unit holding costs are *$10*, *$15*, *$20*, *$25*, and *$30* respectively.

x_i = random defective rates follow the uniform distribution over the intervals of [*0, 0.01*], [*0, 0.06*], [*0, 0.11*], [*0, 0.16*], and [*0, 0.21*] respectively;

$C_{S,i}$ = unit scrap costs $C_{S,i}$ are *$10*, *$15*, *$20*, *$25*, and *$30* respectively;

K_{1i} = the fixed delivery costs per shipment are *$1800*, *$1900*, *$2000*, *$2100*, and *$2200*;

C_{Ti} = unit transportation costs are *$0.1*, *$0.2*, *$0.3*, *$0.4*, and *$0.5* respectively;

h_{3i} = unit holding costs in the customers' end are *$70*, *$75*, *$80*, *$85*, and *$90* respectively;

h_{4i} = unit holding costs for safety stocks in the producer's end is *$5*;

α = *0.5*, the completion rate of the common intermediate part, as compared to the finished end products;

$P_{1,i}$ = annual production rates for the end products in the second stage are *112258*, *116066*, *124068*, *128276*, and *600667* units (which based on a *50%* completion rate (i.e., $\alpha = 0.5$) of the common intermediate part, as compared to the finished end products.

In this numerical example, it is assumed that production rates and the production-inventory related system parameters in the second stage of the proposed model are in (linear) proportion to α. By applying Eqs. (36), (35) and (25), we obtain the optimal number of delivery $n^* = 3$, the optimal common production cycle time $T^* = 0.4437$ (years), and the long-run average costs per unit time for the proposed model $E[TCU(T, n)]$ = *$2,278,602*. Influence of number of deliveries n on $E[TCU(T, n)]$ is depicted in Figure 5.

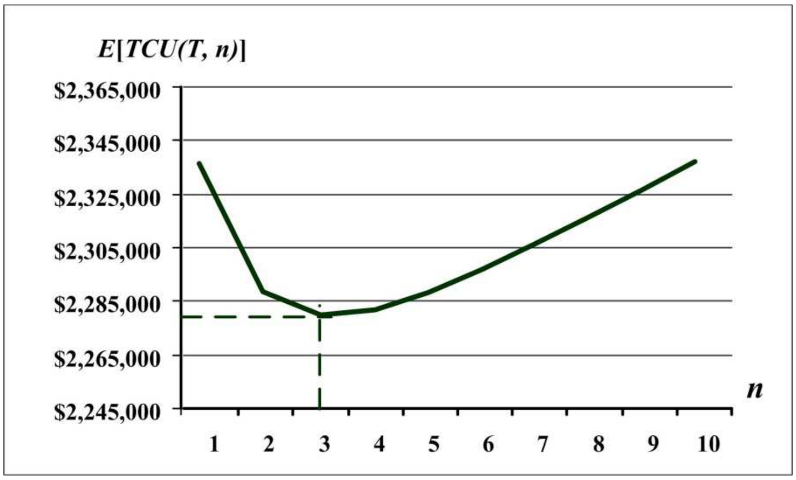

Fig. 5 *Influence of number of deliveries n on E[TCU(T, n)]*

The behaviour of the optimal production cycle time T^* with respect to the completion rate α of the common intermediate part is illustrated in Figure 6. It is noted as the completion rate α increases, T^* decreases significantly.

Figure 6 also shows the comparison of our research results with the results obtained using a two-stage single-machine production scheme presented in Part I of this paper and that of using a single-stage production scheme presented in [15]. It is noted that the proposed two-stage two-machine delayed differentiation multi-product FPR model has significantly shorter production cycle time than that obtained from the single-stage production scheme. One also notes that, as the completion rate α increases (specifically when it reaches *50%* and on), T^* decreases significantly as compared to that obtained using a two- stage single-machine production scheme.

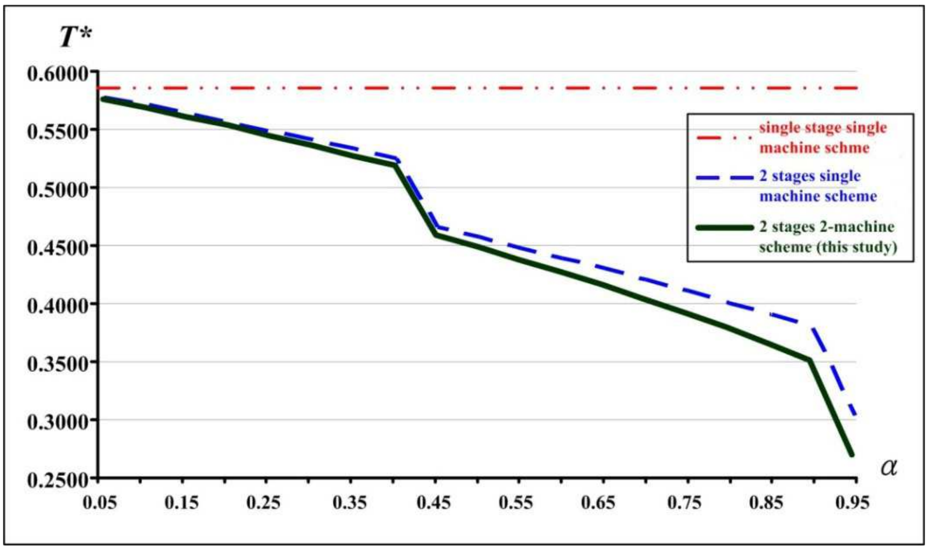

Fig. 6 *The behaviour of optimal production cycle time T* with respect to the completion rate (α) of the common intermediate part*

Variation of the completion rate α effects on the long-run average system cost $E[TCU(T, n)]$ is depicted in Figure 7. It is noted that, as the completion rate α increases, the expected system costs $E[TCU(T, n)]$ decreases significantly.

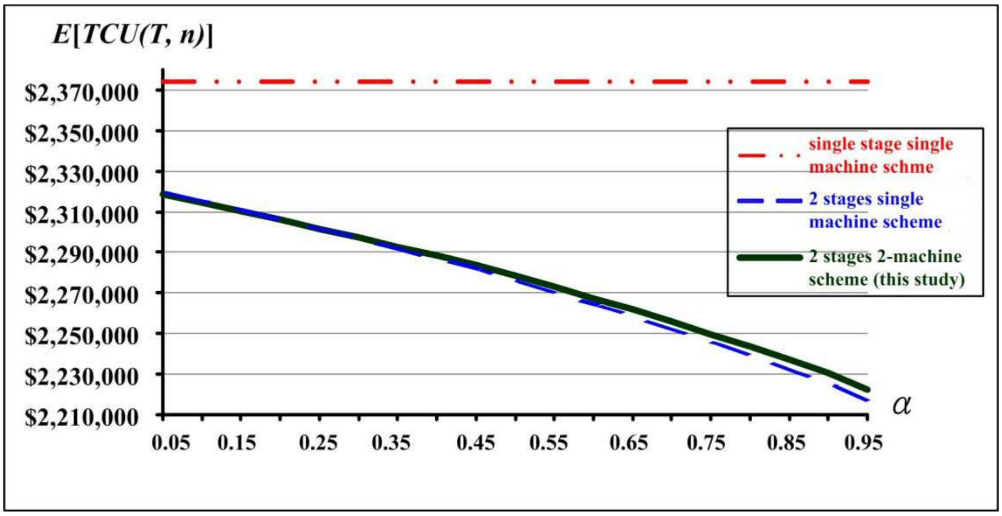

Fig. 7 *The behaviour of E[TCU(T, n)] with respect to the completion rate α of the common intermediate part*

Figure 7 also illustrates comparison of results obtained in this study to that obtained using a two-stage single-machine production scheme presented in Part I of this paper and that obtained using a single-stage production scheme [15]. It is noted that the proposed two-stage two-machine delayed differentiation multi-product FPR model has significantly lower system costs than that obtained from the single-stage multi-product FPR model [15]. When comparing the proposed model to the two stages single-machine scheme, excluding the cost of placement of an extra machine, although cost is slightly higher, as completion rate α increases, the overall production-inventory-delivery related costs $E[TCU(T, n)]$ shows no significant difference.

5. CONCLUSIONS

This paper concerns a delayed differentiation multi-product FPR model with scrap and a multi-delivery policy using a two-machine production scheme. It is inspired by evaluating all potential production plans when fabricating a cluster of multiple products that share a common intermediate part. The aim is to shorten production cycle time and reduce overall production- inventory related costs.

The use of a two-machine scheme is proposed to cope with the two-stage production processes. The first stage manufactures only the common parts for all products and the second stage separately manufactures the end products. This study assumes the production rates and production-inventory related parameters in the second stage of the model are in (linear) proportion to α, which is the completion rate of the common intermediate part as compared to the end products. With the help of mathematical modelling, we explored the proposed problem and were able to determine the closed-form optimal production cycle time and number of deliveries for the proposed two-stage two-machine multi-product FPR model. By using a numerical example with sensitivity analysis, we showed practical use of obtained result and also compared our result to that of using a two-stage single-machine production scheme (see Part I of this paper) and that of using a single-stage multi-product scheme [15].

The research results enable the management to better understand, plan, and control such a real multi-product FPR system with delayed differentiation, scrap, and multi-delivery policy.

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7. APPENDIX A

T = common production cycle time, one of the decision variables;

Q_i = production lot size for product i in a production cycle;

i = index for customized product number, where $i = 1, 2, \ldots L$, with $i = 0$ denotes the common intermediate product produced in the stage 1;

$t_{1,i}$ = production uptime for product i in a production cycle, where $i = 0, 1, 2, \ldots L$;

$t_{2,i}$ = delivery time for product i in a production cycle, where $i = 1, 2, \ldots L$;

$H_{1,i}$ = maximum level of finished product i in the end of production, except $i = 0$ stands for the maximum level of common intermediate parts;

H_i = inventory level of common intermediate parts during the production time of product i, where $i = 1, 2, \ldots L$;

n = number of fixed quantity instalments of the finished batch, to be delivered to customers in each cycle, other decision variables;

$t_{n,i}$ = a fixed interval of time between each instalment of end product i to be delivered to its buyer during production downtime $t_{2,i}$;

α = completion rate of common intermediate part as compared to the finished product;

$I(t)_i$ = on-hand inventory level of perfect quality product i at time t, where $i = 0, 1, 2, \ldots L$;

$I_d(t)_I$ = on-hand inventory level of defective product i at time t, with $i = 0$ stands for the inventory level of defective common intermediate parts;

$I_c(t)_I$ = on-hand inventory level of finished product i at time t, at the customer's side, where $i = 1, 2, \ldots L$;

$h_{3,i}$ = unit holding cost for stocks stored at the customer's side;

$h_{4,i}$ = unit holding cost for safety stocks stored at the producer's side;

D_i = number of finished items of product i distributed to the customer in each shipment;

I_i = the left-over number of finished items of product i in each $t_{n,i}$, at the customer's side;

$TC_1(T, n)$ = total production-inventory costs per cycle in the stage 1 of the proposed model;

$TC_2(T, n)$ = total production-inventory-delivery costs per cycle in the stage 2 of the proposed model;

$E[TCU_1(T, n)]$ = the long-run average costs per unit time for the stage 1 of the proposed model;

$E[TCU_2(T, n)]$ = the long-run average costs per unit time for the stage 2 of the proposed model;

$E[TCU(T, n)]$ = the long-run average costs per unit time for the proposed model.

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